United States
Department of
Agriculture

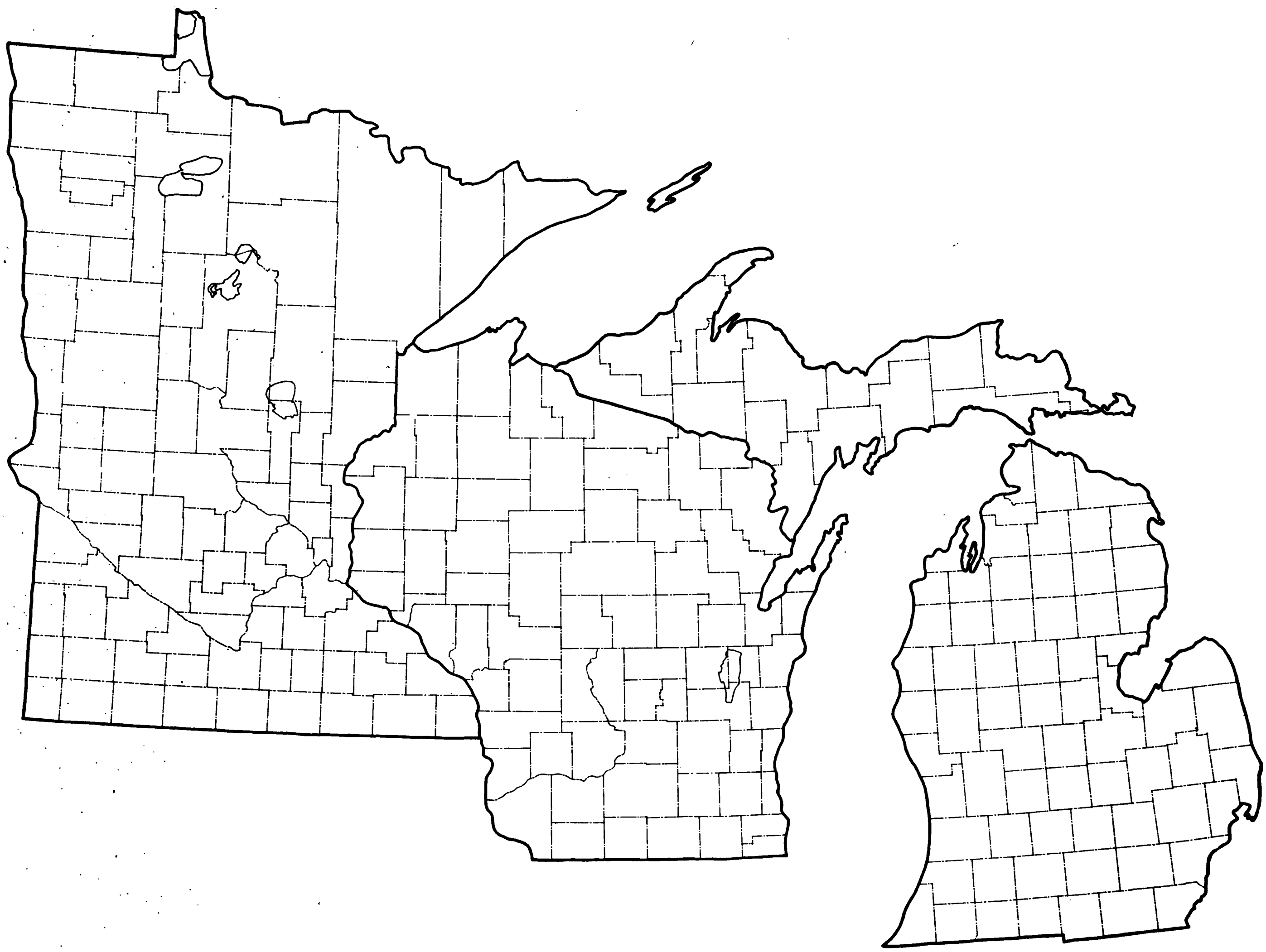
Forest
Service

North Central
Forest Experiment
Station

Research
Paper NC-250

Tree Volume and Biomass Equations for the Lake States

Jerold T. Hahn



CONTENTS

Hahn, Jerold T.

Tree volume and biomass equations for the Lake States. Res. Pap. NC-250. St. Paul, MN: U.S. Department of Agriculture, Forest Service, North Central Forest Experiment Station; 1984. 10 p.

Presents species specific equations and methods for computing tree height, cubic foot and board foot volume, and biomass for the Lake States (Michigan, Minnesota, and Wisconsin). Height equations compute either total or merchantable height to a variable top d.o.b. from d.b.h., site index, and basal area. Volumes and biomass are computed from d.b.h. and height.

KEY WORDS: Inventory, computer, height, cull, cubic foot, board foot, green tons.

North Central Forest Experiment Station
Forest Service—U. S. Department of Agriculture
1992 Folwell Avenue
St. Paul, Minnesota 55108

Manuscript approved for publication March 26, 1984
September 1984

TREE VOLUME AND BIOMASS EQUATIONS FOR THE LAKE STATES

Jerold T. Hahn, *Principal Mensurationist*

This paper presents volume and biomass equations for individual trees in the Lake States of Michigan, Wisconsin, and Minnesota. Equations and computation procedures are for:

gross cubic foot volume,
net cubic foot volume,
gross board foot volume (International ¼-inch rule),
net board foot volume (International ¼-inch rule), and
above ground biomass (green tons).

The Forest Inventory and Analysis (FIA) research work unit at the North Central Forest Experiment Station uses these equations to compute volume and biomass of sample trees to compile volume and growth estimates for State inventories. These equations are used in the current (1983) Wisconsin forest inventory. They are also included in the computerized data bases the FIA Unit has compiled for the Lake States (Hahn and Hansen in prep.). These equations could also be used by those needing similar volume-per-tree estimates.

Separate equations for major species and species groups of the region are presented as well as separate equations for growing-stock and cull trees (see Appendixes II and III for definitions and for species and species groups). Also included are the equations for merchantable and total tree height from Ek *et al.* (1981), for computing the volume of a 1 foot stump from Raile (1982), and for computing bark volume.

Data needed to compute volume and biomass are species, diameter at breast height (d.b.h.), and merchantable height. If height is not measured, merchantable height can be estimated from d.b.h., species site index, stand basal area, and diameter at the limit of merchantability (top diameter outside bark (top d.o.b.)).[1]

The volume equations assume a 4 inch top d.o.b. for cubic foot volume and a 7 (softwoods) or 9 (hardwoods) inch top d.o.b. for board foot volume. If species site index is not available, it can be computed using the equations and methods presented by Carmean *et al.* (in press) or stand site index may be used.

HOW THE EQUATIONS WERE DEVELOPED

Tree Volume Computation

Local net volume equations have been derived as a part of the forest inventories of Wisconsin (Hahn 1973), Minnesota (Raile 1981), and Michigan (Raile *et al.* 1982). The data files created in conjunction with the derivation of these equations serve as a data base for developing regional equations. This data base is comprised of field data collected on 101,642 trees measured during the most recent inventories of Wisconsin (1968), Minnesota (1977), and Michigan (1981). Data included are tree d.b.h., merchantable height, top d.o.b., species site index, species, tree class, and stand basal area (all trees 1 inch d.b.h. and larger). For sawtimber-size trees, measurements were taken at both the sawtimber and poletimber limits of merchantability. Field crews also estimated cull volume (both board foot and cubic foot) in each tree (see Appendix IV for the cull estimation procedure). The availability of regional height equations permitted the use of height estimates as an independent variable in the volume model (Ek *et al.* 1981). These height equations are included in this paper for the convenience of the reader.

Equations developed by Robert N. Stone and the author (see Appendix I) based on Gevorkiantz and Olsen's (1955) composite volume tables for the Lake States were used to estimate the cubic foot volume in

[1] *The poletimber merchantability limits include trees greater than or equal to 5.0 inches d.b.h. from a 1.0 foot stump to a 4.0 inch minimum top d.o.b. or to the point where the central stem breaks into limbs. Dimensions for sawtimber are d.b.h. greater than or equal to 9.0 inches to a 7.0 inch minimum top d.o.b. for softwoods and d.b.h. greater than or equal to 11.0 inches to a 9.0 inch minimum top d.o.b. for hardwoods or to the point where the central stem breaks into limbs (U.S. Department of Agriculture 1975).*

each tree and board foot volume in sawtimber-sized trees. These equations estimate tree volume inside bark from a 1 foot stump to the merchantable limit based on three measurements: d.b.h., merchantable height, and top d.o.b. This estimated volume was then corrected for species differences in bark thickness using equations developed for the region from felled tree measurements to arrive at adjusted gross volume. In addition, tree height to a 4 inch top d.o.b. for all trees and tree height to a 7 (softwoods) or 9 (hardwoods) inch top d.o.b. for sawtimber-size trees were computed as were tree heights to the observed top diameters if the diameters were greater than 4, 7, or 9 inches, respectively. The volume of the section between the observed top and the standard top was computed using Smailian's formula for short sections and added to both the above adjusted gross volume and the field estimated cull volume. The result of this process provided the dependent variable volume for each tree. The species specific volume equations developed are described in the next section.

Volume Estimation Equations

The common volume model—

$$V = b_0 + b_1 D^2 H \qquad (1)$$

where:

D = diameter breast height (d.b.h.) in inches,
H = merchantable height in feet,
V = gross volume in cubic feet,
and b_0 and b_1 are appropriate regression coefficients—was found to describe the data well and was chosen for this study.

Species specific equation coefficients were derived for cubic foot and board foot (International ¼-inch rule) volumes computed as described in the previous section for merchantable height to 4, 7, or 9 inch top d.o.b. as appropriate. Cull percentages (based on the individual tree estimates made in the field) were then developed for growing stock (desirable and acceptable trees), rough and short-log, and rotten tree classes (see Appendix II for definitions). Coefficients were computed for most of the species groups recognized by Forest Inventory and Analysis. Some grouping was necessary when data were insufficient for a viable regression analysis. Species for which no data were available were assigned coefficients on the basis of typical bole form (see Appendix III for species and species groups).

The FIA Unit divides inventoried States into geographical subunits based on similarity of forest types and economic/political boundaries—Michigan and Minnesota each have four units and Wisconsin has five (fig. 1). The influence of geographical location was investigated by regression analysis using dummy variables for unit effects. The F statistic showed no significant contribution to the gross volume equation by geographical unit.

The coefficients of model (1) were estimated by regression analysis using the Statistical Package for the Social Sciences package of statistical routines (tables 2 and 3).

HOW TO COMPUTE VOLUME

1. If an observed merchantable height is not available, compute merchantable height to the appropriate top d.o.b. (4 inches for cubic foot volume or biomass, 7 (softwood) or 9 (hardwood) inches for board foot volume) using the appropriate species group coefficients (b) from table 1 and the following equation from Ek *et al.* (1981).

$$\text{Height} = 4.5 + b_1 (1-e^{(-b_2 D)})^{b_3} \cdot S^{b_4} \cdot T^{b_5} \cdot B^{b_6} \qquad (2)$$

where:

Height = tree height (merchantable or total),
D = diameter breast height (d.b.h.),
e = base of natural logarithm,
S = site index (species specific if possible),
$T = (1.00001 - d/D)$, d = top d.o.b., a value of 0.0 gives total height, and
B = stand basal area.

For example, a 12 inch d.b.h. sugar maple to a 4 inch top d.o.b. in a stand with a site index of 65 feet and basal area 88 square feet will have a height of:

$$\text{Height} = 4.5 + 5.3416(1-e^{(-0.23044 \cdot 12.0)})^{1.1529} \cdot 65^{0.54194} \cdot (1.00001 - 4.0/12.0)^{0.83440} \cdot 88^{\,0.06372}$$
$$= 49.6 \text{ feet.}$$

2. Solve equation (1) to obtain gross volume and subtract the cull percentage to obtain net volume using the appropriate species coefficients and percentage from table 2 or 3 depending on type of volume desired. From model (1) the above tree has a gross cubic foot volume of:

$$\text{Gross volume} = 1.3746 + 0.002206(12.0)^2 \cdot 49.6$$
$$= 17.1 \text{ cubic feet}$$

and if it is a growing-stock tree, it has a net cubic foot volume of:

Net volume = 17.1 (1.0 – 0.063) = 16.0 cubic feet.

If board foot volume is desired, it is also determined from model (1) but use coefficients from table 3 and saw log merchantability limits as follows:

Height $= 4.5 + 5.3416\,(1-e^{(-0.23044 \cdot 12.0)})^{1.1529} \bullet 65^{0.54194} \bullet (1.00001 - 9.0/12.0)^{0.83440} \bullet 88^{0.06372}$
$= 24.4$ feet;
Gross volume $= 36.859 + 0.01534\,(12.00)^2 \bullet 24.4$
$= 90.8$ board feet; and
Net volume $= 90.8\,(1.0 - 0.156) = 76.6$ board feet.

HOW TO COMPUTE BIOMASS IN GREEN TONS

Biomass is computed using two systems—one for trees 5 inches d.b.h. and larger and the other for trees less than 5 inches d.b.h.

A. *Trees 5 inches d.b.h. or larger*

1. Compute gross cubic foot volume as above.
2. Compute volume inside bark of a 1 foot stump using appropriate stump volume species coefficient from column 1 of table 4 and the equation:

$$\text{Stump volume (ft}^3) = S \bullet D^2. \tag{3}$$

3. Compute Species Correction Factor (SCF) for bark differences using appropriate species coefficients from columns 2 and 3 of table 4 and the equation:

$$SCF = (b_0 + b_1 \bullet D)/100. \tag{4}$$

4. Compute bole bark weight using the equation:

$$\text{Bark (lbs)} = (\text{gross cubic volume} + \text{stump volume}) \bullet (1.1646\text{-SCF}) \bullet 37. \tag{5}$$

5. Compute merchantable bole green weight using appropriate species weight (WT) from column 5 of table 4 and the equation:

$$\text{Bole biomass (tons)} = (\text{bark} + (\text{gross cubic volume} + \text{stump volume}) \bullet WT)/2{,}000. \tag{6}$$

6. Compute weight of top (section above merchantable portion of tree) and limbs using the equation:

$$\text{top biomass (tons)} = 0.4545 \bullet (\text{bark} + \text{gross volume} \bullet WT)/2{,}000. \tag{7}$$

7. Add bole weight to top weight for total biomass in green tons.

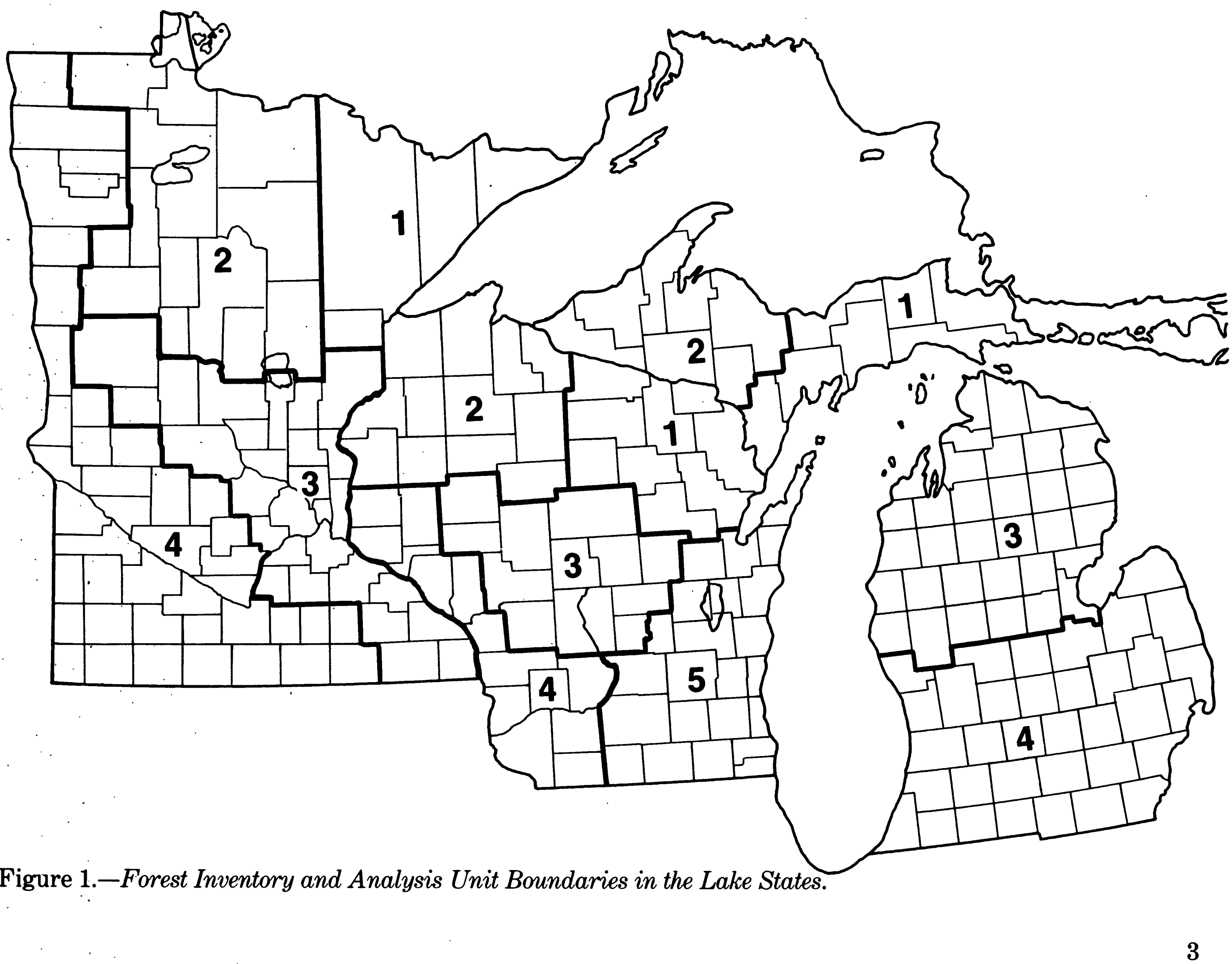


Figure 1.—*Forest Inventory and Analysis Unit Boundaries in the Lake States.*

Table 1.—*Coefficients and standard errors of height equations for model 2*[1]

Species group	Coefficient b1	b2	b3	b4	b5	b6	Standard error
Jack pine	16.9340	0.12972	1.0000	0.20854	0.77792	0.12902	7.2
Red pine	36.8510	.08298	1.0000	.00001	.63884	.18231	8.5
White pine	16.2810	.08621	1.0000	.16220	.86833	.23316	10.7
Ponderosa pine	36.8510	.08298	1.0000	.00001	.63884	.18231	(2)
White spruce	31.9570	.18511	1.7020	0	.68967	.16200	8.2
Black spruce	20.0380	.18981	1.2909	.17836	.57343	.10159	5.5
Balsam fir	14.3040	.19894	1.4195	.23349	.76878	.12399	5.9
Hemlock	5.3117	.10357	1.0000	.68454	.71410	0	7.2
Tamarack	13.6200	.24255	1.2885	.25831	.68128	.10771	5.6
Eastern redcedar	8.2079	.19672	1.3112	.33978	.76173	.11666	(3)
Northern white-cedar	8.2079	.19672	1.3112	.33978	.76173	.11666	5.4
Other softwoods	16.9340	.12972	1.0000	.20854	.77792	.12902	(4)
Select white oak	9.2078	.22208	1.0000	.31723	.82560	.13465	8.2
Select red oak	6.6844	.19049	1.0000	.43972	.82962	.10806	8.2
Other red oak	3.8011	.39213	2.9053	.55634	.84317	.09593	7.7
Select hickory	6.1034	.17368	1.0000	.44725	1.02370	.14610	7.3
Other hickory	6.1034	.17368	1.0000	.44725	1.02370	.14610	7.3
Basswood	6.3628	.27859	1.8677	.49589	.76169	.05841	7.7
Beech	7.1852	.28384	1.4417	.38884	.82157	.11411	(5)
Yellow birch	7.1852	.28384	1.4417	.38884	.82157	.11411	9.3
Hard maple	5.3416	.23044	1.1529	.54194	.83440	.06372	7.7
Soft maple	6.8600	.27725	1.4287	.40115	.85299	.12403	7.2
Elm	8.4580	.27527	1.9602	.34894	.89213	.12594	8.1
Black ash	11.2910	.25250	1.5466	.35711	.75060	.06859	7.2
White & green ash	8.1782	.27316	1.7250	.38694	.75822	.10847	8.3
Sycamore	6.3628	.27859	1.8677	.49589	.76169	.05841	(6)
Cottonwood	13.6250	.28668	1.6124	.30651	1.02920	.07460	8.1
Willow	13.6250	.28668	1.6124	.30651	1.02920	.07460	(7)
Hackberry	6.8600	.27725	1.4287	.40115	.85299	.12403	(8)
Balsam poplar	6.4301	.23545	1.3380	.47370	.73385	.08228	6.7
Bigtooth aspen	5.5346	.22637	1.0000	.46918	.72456	.11782	7.2
Quaking aspen	6.4301	.23545	1.3380	.47370	.73385	.08228	6.7
Paper birch	7.2773	.22721	1.0000	.41179	.76498	.11046	7.1
River birch	7.2773	.22721	1.0000	.41179	.76498	.11046	(9)
Sweetgum	5.3416	.23044	1.1529	.54194	.83440	.06372	(10)
Tupelo	5.3416	.23044	1.1529	.54194	.83440	.06372	(10)
Black cherry	5.3416	.23044	1.1529	.54194	.83440	.06372	(10)
Black walnut	6.3628	.27859	1.8677	.49589	.76169	.05841	(6)
Butternut	6.3628	.27859	1.8677	.49589	.76169	.05841	(6)
Yellow poplar	6.3628	.27859	1.8677	.49589	.76169	.05841	(6)
Other hardwoods	6.9572	.26564	1.0000	.48660	.76954	.01618	(11)
Noncommercial spp.	6.9572	.26564	1.0000	.48660	.76954	.01618	(11)

[1]From Ek *et al.*, 1981.
[2]No data available. Red pine equation used.
[3]No data available. Northern white-cedar used.
[4]No data available. Jack pine used.
[5]No data available. Yellow birch used.
[6]No data available. Basswood used.
[7]No data available. Cottonwood used.
[8]No data available. Soft maple used.
[9]No data available. Paper birch used.
[10]No data available. Hard maple used.
[11]No data available. Silver maple used.

Table 2.—*Cubic foot volume regression coefficients, standard errors, and coefficients of determination for model 1 and percent cull for three tree classes*

Species group	Trees	Coefficients b_0	b_1	Standard error	R^2	Percent cull by tree class: Growing stock	Rough	Rotten
	Number			*Feet*^{3}				
Jack pine	1,671	1.2446	0.002165	1.70	0.93	2.6	17.8	63.1
Red pine	1,005	2.0822	.002046	4.09	.93	1.3	11.2	74.3
White pine	1,952	0	.002364	13.21	.96	3.2	16.6	64.6
Ponderosa pine	[1]—	2.0822	.002046	—	—	1.3	11.2	74.3
White spruce	1,030	.3365	.002599	4.57	.93	1.6	11.7	56.0
Black spruce	1,338	.2631	.002706	1.70	.90	1.4	12.1	57.7
Balsam fir	2,752	.2514	.002679	1.82	.93	2.1	11.1	63.4
Hemlock	1,368	0	.001856	9.75	.89	5.4	24.9	71.9
Tamarack	1,591	1.4109	.002227	1.65	.89	2.7	16.8	58.4
Eastern redcedar	[2]43	.5905	.002168	1.47	.94	3.4	22.0	52.2
Northern white-cedar	3,124	1.0406	.002408	3.42	.87	3.8	17.2	61.2
Other softwoods	[2]151	.5905	.002168	1.47	.94	3.4	22.0	52.2
Select white oak	2,081	.7316	.001951	6.31	.92	5.5	14.0	65.6
Select red oak	3,401	1.6378	.002032	5.91	.92	5.3	14.4	63.6
Other red oak	2,463	.7554	.002174	14.36	.57	6.1	10.7	66.5
Select hickory	[3]264	2.4364	.001881	3.59	.91	4.7	15.2	65.7
Other hickory	[3]224	2.4364	.001881	3.59	.91	4.7	15.2	65.7
Basswood	1,763	.9239	.002206	5.16	.95	4.4	14.0	63.3
Beech	627	2.2793	.002395	8.15	.90	8.1	21.8	55.3
Yellow birch	1,566	0	.002480	14.39	.88	8.9	19.7	64.0
Hard maple	3,780	1.3746	.002206	6.72	.90	6.3	17.7	61.7
Soft maple	3,173	1.7283	.002189	9.80	.87	5.8	16.8	62.3
Elm	[4]3,692	0	.002325	11.42	.91	6.3	13.2	64.0
Black ash	1,256	2.5990	.001792	3.23	.93	5.3	20.2	67.2
White & green ash	1,516	1.5280	.002021	6.84	.91	4.6	17.8	62.2
Sycamore	[5]21	0	.002485	28.13	.90	5.5	19.7	70.6
Cottonwood	[5]173	0	.002485	28.13	.90	5.5	19.7	70.6
Willow	[5]161	0	.002485	28.13	.90	5.5	19.7	70.6
Hackberry	[4]31	0	.002325	11.42	.91	6.3	13.2	64.0
Balsam poplar	2,631	0	.002335	6.15	.87	4.5	19.0	65.2
Bigtooth aspen	3,398	.9461	.002247	13.37	.42	4.4	20.7	59.7
Quaking aspen	2,419	2.0756	.001913	4.08	.81	5.2	21.8	61.8
Paper birch	3,760	2.3037	.001810	3.22	.79	4.5	14.3	64.9
River birch	[5]43	0	.002485	28.13	.90	5.5	19.7	70.6
Sweetgum	[6]—	0	.002485	—	—	5.5	19.7	70.6
Tupelo	[6]—	0	.002485	—	—	5.5	19.7	70.6
Black cherry	[7]821	2.6341	.001887	5.86	.88	6.8	15.5	61.2
Black walnut	[7]85	2.6341	.001887	5.86	.88	6.8	15.5	61.2
Butternut	[7]160	2.6341	.001887	5.86	.88	6.8	15.5	61.2
Yellow poplar	[7]35	2.6341	.001887	5.86	.88	6.8	15.5	61.2
Other hardwoods	290	1.4824	.001796	4.82	.86	8.7	21.0	64.2
Noncommercial spp.	393	.8670	.001940	2.30	.86	8.7	16.8	64.2

[1]No data available, red pine equation used.
[2]Eastern redcedar and other softwoods data combined.
[3]Select and other hickory data combined.
[4]Elm and hackberry data combined.
[5]Sycamore, cottonwood, willow and river birch data combined.
[6]No data available, river birch equation used.
[7]Black cherry, black walnut, butternut and yellow poplar data combined.

Table 3.—*Board foot volume regression coefficients, standard errors, and coefficients of determination for model 1 and percent cull for two tree classes*

Species group	Trees	Coefficients b_0	b_1	Standard error	R^2	Percent cull by tree class: Growing stock	Short log
	Number			*Board feet*			
Jack pine	1,184	19.149	0.01307	14.84	0.85	4.6	46.2
Red pine	1,231	24.848	.01298	28.38	.90	2.9	38.2
White pine	1,511	0	.01625	89.30	.90	5.6	46.6
Ponderosa pine	[1]—	24.848	.01298	—	—	2.9	38.2
White spruce	608	6.810	.01611	34.73	.90	3.1	20.6
Black spruce	342	0	.01735	20.11	.95	2.9	47.3
Balsam fir	1,231	4.658	.01694	17.81	.85	4.7	50.1
Hemlock	989	0	.01054	68.62	.94	8.1	59.4
Tamarack	311	17.592	.01427	14.49	.79	5.0	35.7
Eastern redcedar	[2]32	17.167	.01404	15.88	.81	5.2	31.3
Northern white-cedar	2,117	12.532	.01560	17.81	.90	7.4	45.8
Other softwoods	[2]32	17.167	.01404	15.88	.81	5.2	31.3
Select white oak	1,669	46.038	.01173	38.58	.93	16.9	60.6
Select red oak	3,224	41.410	.01326	39.99	.91	15.6	65.4
Other red oak	782	34.677	.01370	41.93	.86	16.1	62.2
Select hickory	[3]91	57.449	.01122	28.33	.88	11.3	58.3
Other hickory	[3]53	57.449	.01122	28.33	.88	11.3	58.3
Basswood	1,310	36.821	.01435	41.61	.90	15.2	66.5
Beech	296	56.500	.01465	53.84	.86	14.6	62.2
Yellow birch	631	14.575	.01766	77.04	.84	15.9	63.6
Hard maple	2,563	36.859	.01534	55.67	.87	15.6	64.4
Soft maple	1,788	63.992	.01215	61.83	.89	15.9	59.7
Elm	[4]1,639	28.875	.01466	67.68	.89	17.8	57.0
Black ash	457	70.167	.01015	30.62	.92	15.0	65.0
White & green ash	536	65.124	.01124	71.73	.81	15.8	66.6
Sycamore	[5]12	34.832	.01458	189.95	.83	13.6	47.3
Cottonwood	[5]140	34.832	.01458	189.95	.83	13.6	47.3
Willow	[5]89	34.832	.01458	189.95	.83	13.6	47.3
Hackberry	[4]12	28.875	.01466	67.68	.89	17.8	57.0
Balsam poplar	795	17.978	.01578	32.70	.93	16.9	62.8
Bigtooth aspen	932	31.842	.01483	18.23	.94	15.1	75.5
Quaking aspen	2,080	29.329	.01480	16.98	.93	16.4	68.9
Paper birch	981	37.619	.01404	19.45	.87	18.2	61.2
River birch	[5]10	34.832	.01458	189.95	.83	13.6	47.3
Sweetgum	[6]—	34.832	.01458	—	—	13.6	47.3
Tupelo	[6]—	34.832	.01458	—	—	13.6	47.3
Black cherry	[7]173	67.801	.01109	57.97	.76	16.0	62.8
Black walnut	[7]51	67.801	.01109	57.97	.76	16.0	62.8
Butternut	[7]34	67.801	.01109	57.97	.76	16.0	62.8
Yellow poplar	[7]30	67.801	.01109	57.97	.76	16.0	62.8
Other hardwoods	45	36.341	.01339	25.07	.88	19.4	55.3
Noncommercial spp.	[8]—	36.341	.01339	—	—	19.4	55.3

[1]No data available, red pine equation used.
[2]Eastern redcedar and other softwoods data combined.
[3]Select and other hickory data combined.
[4]Elm and hackberry data combined.
[5]Sycamore, cottonwood, willow, and river birch data combined.
[6]No data available, river birch equation used.
[7]Black cherry, black walnut, butternut, and yellow poplar data combined.
[8]No data available, other hardwoods equation used.

Table 4.—*Coefficients for stump volume, bark corrections, and biomass*

Species group	Stump coef.	Bark correction coef.		Biomass[1] lbs/ft³
		b_0	b_1	
Jack pine	0.007017	91.227	0.325	50
Red pine	.007176	90.365	.325	42
White pine	.008269	88.582	.325	36
Ponderosa pine	.007176	90.365	.325	42
White spruce	.010699	99.669	.325	35
Black spruce	.008877	100.100	.325	32
Balsam fir	.009967	97.520	.325	45
Hemlock	.008579	88.582	.325	50
Tamarack	.008877	102.560	.325	47
Eastern redcedar	.008877	100.080	.325	28
Northern white-cedar	.011946	100.080	.325	28
Other softwoods	.008877	91.227	.325	43
Select white oak	.009727	87.433	.325	62
Select red oak	.008908	90.051	.325	62
Other red oak	.008980	89.543	.325	63
Select hickory	.008980	85.094	.597	63
Other hickory	.008980	85.094	.597	63
Basswood	.009639	88.054	.325	43
Beech	.010202	94.853	.325	50
Yellow birch	.009968	92.943	.325	57
Hard maple	.008894	93.164	.325	56
Soft maple	.008476	94.853	.325	48
Elm	.010422	91.894	.325	54
Black ash	.011016	91.834	.325	49
White & green ash	.008728	91.834	.325	49
Sycamore	.008980	89.579	.325	50
Cottonwood	.011145	91.894	.325	54
Willow	.011145	91.894	.325	54
Hackberry	.010422	91.894	.325	54
Balsam poplar	.006594	89.579	.325	40
Bigtooth aspen	.006594	89.579	.325	43
Quaking aspen	.007369	92.838	.325	43
Paper birch	.008380	97.285	.325	50
River birch	.008380	97.285	.325	50
Sweetgum	.008980	89.579	.325	50
Tupelo	.008980	89.579	.325	50
Black cherry	.008980	88.688	.325	52
Black walnut	.008980	88.688	.325	52
Butternut	.008980	88.688	.325	52
Yellow poplar	.008980	88.688	.325	52
Other hardwoods	.008980	89.579	.325	50
Noncommercial spp.	.008980	89.579	.325	50

[1]Green wt/ft³ from Markwardt 1930.

B. *Trees less than 5 inches d.b.h.*

For trees less than 5 inches d.b.h. the following green weight equation can be used.[2]

Total biomass (tons) = $(4.8900625 \cdot D^{2.4323866}) \cdot 0.8/2{,}000$

where biomass is in green tons and D is d.b.h. (inches).

Continuing the above example:

Stump volume = $0.008894 \cdot 12^2 = 1.2807$ cubic feet

SCF = $(93.164 + 0.325 \cdot 12)/100 = 0.97064$

Bark = $(17.1 + 1.2807) \cdot (1.1646 - 0.97064) \cdot 37 = 131.91$ lbs

Bole weight = $(131.91 + (7.1 + 1.2807) \cdot 56)/2{,}000 = 581$ tons

Top weight = $0.4545 \cdot (131.91 + 17.1 \cdot 56)/2{,}000 = 248$ tons

Total biomass = $581 + 248 = 829$ tons.

Cubic foot volume in the saw log portion of the tree can be computed using the saw log merchantable height and coefficients from table 2 or

Gross volume = $1.3746 + 0.002206 \cdot 12^2 \cdot 24.4 = 9.1$ cubic feet

Net volume = $9.1\,(1 - 0.063) = 8.5$ cubic feet

The example tree is computed to have a merchantable height of 49.6 feet, gross volume of 17.1 cubic feet, net volume of 16 cubic feet, and above ground biomass of 0.829 green tons. The saw log portion of the tree contains gross volume of 90.8 board feet (9.1 cubic feet) and a net volume of 76.6 board feet (8.5 cubic feet).

The above data can be used to obtain a breakdown by tree component for the example tree (table 5).

Table 5.—*Volume and biomass by tree component*

Tree component	Volume		Biomass	
	Ft³	Percent	Green tons	Percent
Bole (gross)				
Saw log portion	9.1	29	0.254	31
Upper stem	8.0	25	0.225	27
Top and limbs	7.8	25	[1]0.218	26
Bark	5.2	17	0.096	12
Stump	1.3	4	0.036	4
Total	31.4	100	0.829	100

[1]The biomass in bole and tops and limbs does not include bark.

[2]*Equation developed by Gerhard K. Raile (personal communication 1982) using data from Young* et al. *(1976).*

LITERATURE CITED

Carmean, W. H.; Hahn, Jerold T.; Raile, Gerhard K. Site index comparisons for forest species in the Lake States. St. Paul, MN: U.S. Department of Agriculture, Forest Service, North Central Forest Experiment Station; [in prep].

Doman, A. P.; Ennis, Robert; Weigel, Dale. North Central Forest Experiment Station Renewable Resources Evaluation Field Manual. St. Paul, MN: U.S. Department of Agriculture, Forest Service, North Central Forest Experiment Station; 1981. 118 p.

Ek, Allan R.; Birdsall, Earl T.; Spears, R. J. Total and merchantable tree height equation for Lake States tree species. Staff Pap. 27. St. Paul, MN: University of Minnesota, College of Forestry and Department of Forest Resources; 1981. 26 p.

Gevorkiantz, S. R.; Olsen, L. P. Composit volume tables for timber and their application in the Lake States. Gen. Tech. Bull. 1104. St. Paul, MN: U.S. Department of Agriculture, Forest Service, Lake States Forest Experiment Station; 1955. 55 p.

Hahn, Jerold T. Local net timber volume equations for Wisconsin. Res. Note NC-149. St. Paul, MN: U.S. Department of Agriculture, Forest Service, North Central Forest Experiment Station; 1973. 4 p.

Hahn, Jerold T.; Hansen, Mark H. Forest Inventory and Analysis Data Bases for the North Central Region - Description and Users Manual; [in prep.]

Markwardt, L. J. Comparative strength properties of woods grown in the United States. Tech. Bull, 158. Washington, DC: U.S. Department of Agriculture; 1930. 38 p.

Raile, Gerhard K. A net volume equation for northeastern Minnesota. Gen. Tech. Rep. NC-86. St. Paul, MN: U.S. Department of Agriculture, Forest Service, North Central Forest Experiment Station; 1981. 8 p.

Raile, Gerhard K. Estimating stump volume. Res. Pap. NC-224. St. Paul, MN: U.S. Department of Agriculture, Forest Service, North Central Forest Experiment Station; 1982. 4 p.

Raile, Gerhard K.; Smith, W. Brad; Weist, Carol A. A net volume equation for Michigan's upper and lower peninsulas. Gen. Tech. Rep. NC-80. St. Paul, MN: U.S. Department of Agriculture, Forest Service, North Central Forest Experiment Station; 1982. 12 p.

U.S. Department of Agriculture, Forest Service. Forest Service Handbook. Supplement FSH-4809.11-Forest Survey Handbook. Washington, DC: U.S. Department of Agriculture, Forest Service; 1975. 127 p.

Young, H. E.; Hoar, L. E.; Tryon, T. C. A forest biomass inventory of some public land in Maine. In:

Oslo Biomass Studies. Life Sciences and Agricultural Experiment Station. Orno, ME: University of Maine; 1976: 285-302.

APPENDIX I
Tree Volume Equations

Stone's cubic foot volume equation uses diameter at breast height (D), merchantable height (H), and diameter outside bark at merchantable height (T) to estimate cubic foot volume inside bark (CV) from a 1 foot stump to the merchantable top as:

$$CV = (3.0086 \times 10^{-3} + S1 + S2 + S3 + S4 + S5 + S6 + S7 + S8) \bullet 79.0$$

where

$S1 = (2.0355 \times 10^{-3})D$
$S2 = (-3.0018 \times 10^{-3})T$
$S3 = (6.2381 \times 10^{-5})D^2$
$S4 = (2.5705 \times 10^{-5})D^2 \bullet H$
$S5 = (-7.0090 \times 10^{-6})H^2$
$S6 = (3.6708 \times 10^{-5})H \bullet T^2$
$S7 = (8.1400 \times 10^{-10})D2 \bullet H^3$
$S8 = (-1.90000 \times 10^{-9})D^2 \bullet H^2 \bullet T$

The board foot volume equation developed by Hahn using Stone's original data is:

$$BV = 17.7488 + 7.3846 \bullet CV - 2.3523 \bullet D - 0.89945 \bullet H + 2.0726 \bullet T$$

where

BV = board foot volume (International ¼-inch rule).

APPENDIX II
Metric Equivalents

1,000 board feet (International ¼-inch log rule) = 3.48 cubic meters.

Breast height = 1.37 meters above the ground.

1 cubic foot = 0.0283 cubic meter.

1 foot = 0.3048 meter.

1 inch = 2.54 centimeters.

Definition of Terms

Desirable trees.—Live trees having no serious defects that limit present or prospective use. Trees have relatively high vigor and have no visible signs of pathogens that may kill or seriously deteriorate them before rotation age. They would be favored by forest managers in silvicultural operations.

Acceptable trees.—Live trees having no serious defects that limit present or prospective use but that have pathogens or damage that may affect quality.

Growing-stock trees.—Live trees of commercial species qualifying as desirable and acceptable trees. (Note: excludes rough, rotten, and short-log trees.)

Rough trees.—Live trees that do not contain at least one merchantable 12-foot log, now or prospectively, because of roughness, poor form, or noncommercial species.

Rotten trees.—"Rough" trees in which more than 50 percent of the cull volume is rotten.

Short-log trees.—Sawtimber-size trees of commercial species that contain at least one merchantable 8- to 11-foot saw log but not a 12-foot saw log.

APPENDIX III
Species Groups and Species for Lake States Trees

Species group	Scientific names of included species
SOFTWOODS	
Jack pine	*Pinus banksiana*
Red pine	*Pinus resinosa*
White pine	*Pinus strobus*
White spruce	*Picea glauca*
Black spruce	*Picea mariana*
Balsam fir	*Abies balsamea* var. *balsamea*
Hemlock	*Tsuga canadensis*
Tamarack	*Larix laricina*
Northern white-cedar	*Thuja occidentalis*
Other softwoods	*Pinus sylvestris* *Pinus nigra* *Juniperus virginiana* *Larix decidua* *Picea abies* *Pseudotsuga menziesii*
HARDWOODS	
White oak	*Quercus alba* *Quercus bicolor* *Quercus macrocarpa* *Quercus muehlenbergii*
Red oak	*Quercus rubra* *Quercus velutina* *Quercus palustris* *Quercus ellipsoidalis*
Hickory	*Carya cordiformis* *Carya ovata* *Carya glabra*

Basswood	*Tilia americana*
Beech	*Fagus grandifolia*
Yellow birch	*Betula alleghaniensis*
Hard maple	*Acer nigrum*
	Acer saccharum
Soft maple	*Acer rubrum* var. *rubrum*
	Acer saccharum
Elm	*Ulmus americana*
	Ulmus rubra
	Ulmus thomasii
Ash	*Fraxinus americana*
	Fraxinus nigra
	Fraxinus pennsylvanica
Cottonwood	*Populus deltoides*
Balsam poplar	*Populus balsamifera*
Paper birch	*Betula papyrifera* var. *papyrifera*
Aspen	*Populus grandidentata*
	Populus tremuloides
Black cherry	*Prunus serotina*
Other hardwoods	*Acer negundo*
	Betula nigra
	Celtis occidentalis
	Juglans cinerea
	Juglans nigra
	Salix nigra
	Liriodendron tulipifera
Noncommercial species	*Acer pensylvanicum*
	Acer spicatum
	Adanthus altessimor
	Corpinius carolinian
	Ostrya virginiana
	Circus canadensis
	Crataegus spp.
	Prunus pensylvanica
	Prunus virginiana
	Sorbus americana

APPENDIX IV
Cubic-Foot Cull[3]

For growing-stock, live and dead trees, cubic-foot cull is the cubic-foot volume of decayed or missing wood in merchantable sections and the total cubic-foot volume in sections that do not meet pulpwood specifications up to the growing stock top. A section meets pulpwood specifications unless:

1. It is less than 4 feet long.
2. It has a diameter less than 4 inches outside bark.
3. It is a fork.
4. A limb has a knot collar greater than the stem diameter at that point, or several limbs more than 2 inches d.o.b. within a 1-foot span have an aggregate knot collar diameter greater than the stem d.o.b. of the section.
5. A 4-foot section of bole is so crooked that a line drawn between the center of the ends falls outside the bark at any point.
6. A rotten section is unusable for industrial products. Regional standards are a 4-foot section less than 50 percent sound.

For short-log, rough, and rotten class live trees, or dead trees that were cull trees at time of death, record only the volume of decayed or missing wood up to the bole length top.

Compute cubic foot cull by determining the length of the section affected and the midpoint d.o.b. Then look up the volume of the section in a "Cubic Foot Volumes of Short Logs" table and record to the last 1/10 cubic foot.

Board Foot Cull[3]

Board-foot cull is the volume within the saw log portion of trees that cannot be recovered for use as lumber because of rot, sweep, crook, forks, or other defects.

Cull volumes include:

1. The entire volumes of tree sections that do not meet minimum log grade requirements.
2. The entire volume in any foot or longer section of a tree that is less than 50 percent sound.
3. The cull volume *only*, in any 1-foot or longer section of a tree that is greater than 50 percent sound.
4. Computed volumes for sweep and crook.
5. Forks or stoppers (any limb or group of 2 inch limbs within a 1-foot span the sum of whose diameters exceeds the d.o.b. at that point).

Board foot measure is computed from a squared-off section within the circular form of a log. This is the only portion that contains lumber. Therefore, ignore shallow defects expected to be cut in slabbing for lumber and rounding for veneer.

Determine the board foot cull volume in logs and/or cull sections by estimating the length and d.o.b. at midpoint and looking up the board foot volume in the "Board-foot Volume of Short Logs" table. In determining cull due to sweep and crook, minimize the defect by logical log-making aimed at obtaining maximum high grade material. Then use sweep and crook tables to determine the cull volume.

---
[3]*Doman, A. P.* et al. *1981.*