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A Generalized System of Models Forecasting Central States Tree Growth

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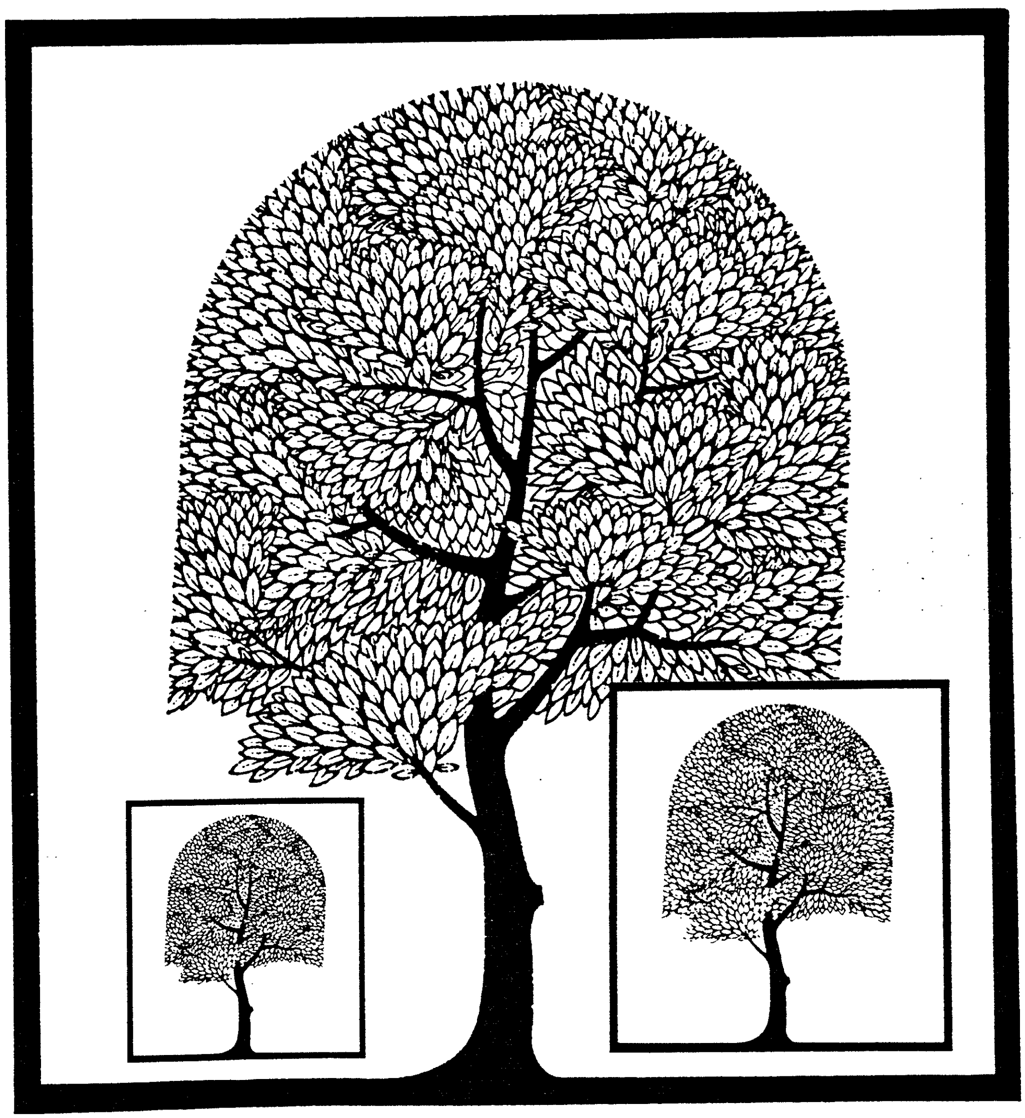

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Describes the development and testing of a system of individual-tree-based growth projection models applicable to species in Indiana, Missouri, and Ohio. Annual tree basal area growth is estimated as a function of tree size, crown ratio, stand density, and site index. Models are compatible with the STEMS and TWIGS projection system.

KEY WORDS: Yield, individual tree models, distance independent, STEMS, TWIGS.

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A GENERALIZED SYSTEM OF MODELS FORECASTING CENTRAL STATES TREE GROWTH

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Individual-tree-based growth projection systems such as STEMS (Belcher *et al.* 1982) and TWIGS (Belcher 1982) have demonstrated their value in solving a variety of inventory and management problems in the Lake States. The software for these projection systems has evolved over more than a decade, providing easy access for nonprogrammers, detailed options for simulating forest management, interface with inventory data bases, and detailed output summaries. These projection systems can be linked to other programs that evaluate the relationship of forest change to economic return, wildlife habitat, and pest activity.

Although the STEMS and TWIGS software has grown significantly in complexity and capability in recent years, the internal models projecting tree growth and mortality have changed little. At the heart of both these systems is a single growth processor, a system of species-specific, individual-tree-based growth and mortality models that forecast changes in the number and sizes of trees. All other system components are directly or indirectly linked to the ability to forecast tree growth and mortality.

The primary barrier to applying projection systems like STEMS and TWIGS in other regions is the lack of appropriate growth and mortality models calibrated for use with the species and conditions found in those regions. Other barriers such as replacing the computerized management algorithms (Brand 1981b), changing volume equations, and otherwise modifying the projection software are significant, but they can usually be solved without data analysis by consulting the existing literature and experienced individuals.

This paper describes an individual-tree-based growth projection model calibrated using an extensive data base from Indiana, Missouri, and Ohio. Although the model was specifically developed for use in Central States variants of STEMS and TWIGS, it is quite general and can be applied in a wide variety of simulation frameworks.

DATA BASE

Data used in this study were assembled from five different sources throughout the Midwest (table 1). All

Table 1.—*Data sources for model calibration and testing*[1]

Source	Location	Sample type	Years between measurement	Plots	Trees[2]
				-------*Number*-------	
Wayne-Hoosier National Forest	Indiana Ohio	10-point cluster plot 37.5 Basal Area Factor (BAF) Prism	10	160	3,387
Mark Twain National Forest	Missouri	10-point cluster plot 37.5 BAF Prism	7	224	4,947
Bottomland Hardwood Plots	Indiana	10-point cluster plot 37.5 BAF Prism	11	35	617
Missouri State Inventory	Missouri	Single Point Sample 5 BAF Prism	12	1,920	24,051
Pioneer Forest	Missouri	1/5 Acre Fixed Size Plot	10	417	11,617
Total				2,756	44,619

[1] I gratefully acknowledge the assistance and cooperation of the following organizations and individuals who provided these and other data essential to the conduct of this and related studies: USDA Forest Service, North Central Forest Experiment Station (Burton Essex, A. Jerry Ostrom, Robert Rogers, Ivan Sander, Richard Schlesinger, W. Brad Smith); Pioneer Forest, Salem, Missouri; Purdue University (John W. Moser, Jr., W. L. Mills, and W. Lloyd Fix); Tennessee Valley Authority (Robert Gregory and Robert Brooks); University of Illinois (George Gertner and Dieter Pelz).

[2] Includes cut, dead, ingrowth, and other trees not used in growth model analysis.

data had been collected for other purposes and were made available through the generous assistance of many cooperators (see table 1 footnote). The data were edited and combined into a data base of more than 2,700 sample plots.

Information recorded for each tree included species, initial diameter, diameter 5 to 15 years after the initial measurement, crown ratio, crown class, tree status, and per acre expansion factor. We recorded site index for each plot, and computed per acre estimates of basal area, number of trees, mean diameter, and other statistics from the combined tree records for each plot.

Before analysis, every fourth plot was systematically removed from the data base and reserved as a source of data for evaluating the models. The remaining calibration data, with dead and ingrowth trees deleted, were subdivided into 22 species groups for analysis (table 2). Where there were many observations for a single species, the species group included only that one species. Species with few observations were combined with other species from the same genus or with similar silvical characteristics in an effort to develop homogeneous groupings.

Groups 20, 21, and 22 are a repository for many species that occurred infrequently in the calibration data. Groupings with this high degree of species diversity are not desirable; they simply provide an expedient mechanism for dealing with the many minor species that occur in the Central States region.

Table 2.—*Species group definition and composition*

Species group	Common name	Percent of group	Observations	DBH		Stand basal area	
				Min	Max	Min.	Max
			Number	*Inches*		*Ft²/ac*	
1	Eastern redcedar	100	187	1	18	15	120
2	Shortleaf pine	97	1,604	1	21	8	131
	Virginia pine	3					
3	Boxelder	26	349	1	59	15	195
	Red maple	44					
	Silver maple	30					
4	Sugar maple	100	330	1	31	23	154
5	Bitternut hickory	4	2,495	1	33	8	170
	Pignut hickory	21					
	Shellbark hickory	4					
	Shagbark hickory	37					
	Black hickory	17					
	Mockernut hickory	17					
6	White ash	71	430	1	32	5	160
	Black ash	4					
	Green ash	25					
7	Butternut	3	264	2	27	8	170
	Black walnut	97					
8	Black tupelo	38	231	1	28	15	135
	Swamp tupelo	62					
9	White oak	100	3,766	1	43	10	153
10	Scarlet oak	100	1,100	1	24	8	128
11	Blackjack oak	100	566	1	24	5	115
12	Chinkapin oak	100	286	1	34	14	130
13	Southern red oak	3	1,040	1	39	5	154
	Northern red oak	97					
14	Post oak	100	2,226	1	30	5	140
15	Black oak	100	3,562	1	37	1	140
16	Bur oak	22	191	2	36	20	130
	Chestnut oak	78					
17	Swamp white oak	27	232	1	41	5	140
	Cherrybark oak	3					
	Shingle oak	31					
	Overcup oak	3					
	Swamp chestnut oak	6					
	Pin oak	21					
	Willow oak	5					
	Shumard oak	4					
18	Hackberry	18	594	1	39	10	160
	Winged elm	7					
	American elm	45					
	Slippery elm	25					
	Rock elm	5					
19	Yellow poplar	100	128	1	36	37	113
20	Ohio buckeye	4	461	1	37	5	17
	Persimmon	13					
	American beech	14					
	Honey locust	10					
	Bigtooth aspen	5					
	Black cherry	11					
	Black locust	7					
	Sassafras	26					
	American basswood	10					
21	Baldcypress	5	336	1	63	5	195
	River birch	8					
	Sweetgum	9					
	Sycamore	56					
	Eastern cottonwood	12					
	Black willow	10					
22	Eastern redbud	2	511	1	17	15	170
	Flowering dogwood	40					
	Kentucky coffee tree	2					
	Osage orange	5					
	Red mulberry	7					
	Ironwood	2					
	Misc. noncommercial	42					

(Table 2 continued)

(Table 2 continued)

MODEL FORM

The objective of developing a STEMS-compatible tree growth model for Central States application imposes a number of constraints necessary to make the model compatible with the continuous forest inventory data available for model calibration and compatible with forest inventory data bases used as the starting point for future projections. Specifically:

1. the model must be distance-independent (i.e., must not rely on direction and distance from one tree to another),
2. the dependent variable must be diameter growth or some direct transformation of diameter growth (e.g., basal area growth or log of diameter growth),
3. the independent (predictor) variables must be those that are commonly available from forest inventory plots (e.g., species, diameter, crown ratio, crown class, site index, percent stocking, basal area), and
4. the models must be applicable to stands with mixed species and mixed size or age structure.

In a prior study, Shifley (1982) compared the performance of three alternative models on a portion of this data set. These models were (1) the Lake States STEMS diameter increment model (Belcher *et al.* 1982), (2) the Prognosis diameter increment model (Wykoff *et al.* 1982), and (3) a new model combining several features of both. Each of the models predicted periodic change in diameter (or some transformation thereof) as a function of initial diameter, crown ratio, site index, relative tree size, basal area per acre, basal area of larger trees, and/or crown competition factor. The ability of these three models to estimate change in tree diameter and basal area growth was essentially the same over the range of conditions tested.

Each of the three preliminary models (Shifley 1982) used a different dependent variable; the first predicted change in tree diameter, the second predicted the log of the change in squared tree diameter, and the third predicted change in tree basal area. Based on comparisons of prediction accuracy, there was little statistical evidence favoring any one of these dependent variables over the other two. West (1980) came to the same conclusion when he compared the accuracy of models predicting tree diameter growth with models predicting tree basal area growth.

Hilt (1983) points out that the relationship of tree growth to tree size is graphically much more distinct when tree basal area growth is used as the dependent variable than when tree diameter growth is used. Moreover, volume estimates are usually functions of tree basal area; so it is intuitively logical to forecast tree basal area growth directly, minimizing the error sum of squares for tree basal area growth. Consequently, annual tree basal area growth was used as the dependent variable in this system of equations.

Ideally, models should only be applied under conditions that are representative of those encompassed in the calibration data. But when alternative methods are lacking, models are often used to simulate tree growth for conditions that were poorly represented in the calibration data.

Therefore, we wanted to find a growth model that was carefully constrained to give biologically reasonable estimates over a wide range of conditions. Of particular concern were growth rate estimates for very large and very small trees growing at extremes of stand density.

Like the growth model used in Lake States STEMS, this model has two components: (1) a growth potential function estimating maximum expected growth for trees of a specific species, size, and crown ratio on a specific site, and (2) a modifier function to reduce potential growth based on the amount of competition from other trees. There are some practical advantages to splitting growth estimation into potential and modifier components. First, it breaks a complex modeling problem into two more tractable parts. Second, the growth potential function identifies an upper bound to tree growth and total size. A carefully selected potential function ensures that all projected growth rates, even those falling outside of the range of the calibration data, are biologically reasonable. Potential growth is estimated from trees that have experienced little competition. The modifier function, developed from growth observations over a wide range of stand densities, causes information about competition to bear upon the growth estimate.

Growth Potential Function

Potential growth estimates for this system of models were selected as a percentage of the fastest growing trees by diameter class. Generally the fastest growing 5 percent of trees were included. For groups with few observations, the fastest growing 20 percent were included to better describe the shape of the function.

Potential tree growth was modeled as a function of tree size, crown ratio, and site index. The majority of trees in the data base ranged from 1 to 30 inches in diameter. Most of the growth potential function models tested performed well over this range of tree sizes. However, several models tested did not give reasonable growth potential estimates for trees larger than those included in the data base. For example, some models allowed trees to achieve an unreasonably large total size. In other cases, tree growth ceased at diameters that were unrealistically small compared to record sizes reported in the National Register of Big Trees (American Forestry Association 1982).

Considering the intended application of these models and the expectation that they will occasionally be applied under conditions not encompassed by the calibration data base, the model was structured to allow the maximum tree size to be constrained within biologically reasonable limits. This goal was accomplished by using a variation of the Chapman-Richards growth function that allows maximum tree size to be expressed explicitly (Shifley and Brand 1984).

The general form of the model selected to estimate potential annual tree basal area growth is as follows:

$$POT = [b_1 TBA^{b_2} - b_3\, TBA]\,[b_4 + b_5 SI + b_6\, CR] \quad (1)$$

where:

POT = potential annual tree basal area growth (ft^2/yr)
TBA = current tree basal area (ft^2)
SI = site index of dominant species on plot (feet at age 50)
CR = crown ratio class (0-10% = 1, 11-20% = 2,...71-80% = 8, 81-100% = 9)
b_i = species specific regression coefficients.

The first bracketed term of equation 1 is simply the growth form of the Chapman-Richards function (Pienaar and Turnbull 1973). Maximum size (i.e., the point at which tree growth equals zero) occurs when tree basal area reaches

$$A = (b_1/b_3)^{1/(1-b_2)} \quad (2)$$

where A stands for maximum or asymptotic tree size.

Regression coefficients for equation 1 were fit in two steps. Initially only coefficients b_1, b_2, and b_3 of the first bracketed term were regressed against potential basal area growth with the entire second bracketed term set equal to one. We obtained estimates of b_1, b_2, and b_3 for each species group using nonlinear least-squares regression (table 3). The regression was weighted by the inverse of diameter squared to correct for a nonhomogeneous error variance.

Maximum size for each species group was computed (equation 2) from the fitted regression coefficients and compared to the maximum size recorded for the same species in the National Register of Big Trees (American Forestry Association 1982). If the fitted maximum size was larger than the record size reported in the National Register of Big Trees or less than 75 percent of the record size, the first term in equation 1 was modified so that a reasonable maximum size could be forced (Shifley and Brand 1984). Solving equation 2 for b_3 gives

$$b_3 = b_1/A^{(1-b_2)} \quad (3)$$

Substituting equation 3 into equation 1 gives

$$POT = [b_1 TBA^{b_2} - b_1/A^{(1-b_2)}\, TBA]\,[b_4 + b_5 SI + b_6\, CR] \quad (4)$$

Setting A equal to an estimate of maximum tree basal area and refitting b_1 and b_2 (with the second bracketed term still set equal to one) forces tree growth to approach zero as tree basal area approaches A. Using equation 4, we forced the growth potential function to assume a specified (presumably reasonable) maximum tree size. This additional constraint had little effect on the fit of the equation to the data for smaller trees. Biologically reasonable values of A for each species group were usually two to three times larger than the largest trees in the calibration data for each species group.

For 6 of the 22 species groups, the maximum tree size obtained by fitting b_1, b_2, and b_3 (equation 1) to the potential growth data fell between 75 and 100 percent of the record size reported in the National Register of Big Trees. For the 16 groups where the fitted values of b_1, b_2, and b_3 resulted in a maximum size outside of this range, we used equation 4, setting A equal to 80 percent of the tree basal area reported in the National Register of Big Trees.

The second bracketed term of the potential function (equation 1 or 4) is a component to adjust potential tree basal area growth for the effects of tree crown ratio and site index. Coefficients b_4, b_5, and b_6 were fit in a second step using ordinary unweighted least squares with b_1 through b_3 fixed and b_5 and b_6 constrained to be greater than or equal to zero.

The site index-crown ratio term of equations 1 and 4 increases or decreases the annual growth potential. The maximum tree size for each species group remains constant. For a site index of 25 feet and a crown ratio code of 1 (0-10 percent), the average value of the site index-crown ratio term for the 22 species groups is 0.7 (minimum 0.3, maximum 0.9). When site index is 99 feet and crown ratio code is 9 (81-100 percent), the average value of the site index-crown ratio term for the 22 species groups is 1.3 (minimum 1.1, maximum 1.7).

Modifier Function

The second part of the growth function is the competition modifier, a multiplicative term that reduces potential growth in response to competition. The modifier term describes the relationship:

modifier = (actual growth)/(estimated potential growth)

as a function of tree and stand variables. Preliminary analyses showed that the strongest linear relation-

Table 3.—*Potential function coefficients and fit statistics*

Species group	No. of obs.	First term: b_1	b_2	b_3	r^2	$\sqrt{MSE}$[1]	Second term: b_4	b_5	b_6	r^2	$\sqrt{MSE}$[1]	Max dbh (inches)
1	44	0.39224814E − 01	0.51499115E + 00	0.14858530E − 01	0.63	0.016	0.39741560E + 00	0.71802997E − 03	0.74913965E − 01	0.04	0.243	37
2	91	.39963074E − 01	.14593018E + 00	.97097430E − 02	.15	.016	.82518771E + 00	.47569978E − 03	.45452244E − 01	.03	.056	31
3	82	.16952048E + 00	.72999623E + 00	.76329602E − 01	.85	.031	.49370041E + 00	.57751026E − 02	.49691855E − 02	.05	.139	59
4	77	.48570057E − 01	.61582056E + 00	.14586389E − 01	.82	.012	.85146939E + 00	.11077119E − 02	.11212651E − 01	0	.136	65
5	135	.52744503E − 01	.55319832E + 00	.18400205E − 01	.71	.016	.42131961E + 00	.53479814E − 02	.43100701E − 01	.05	.161	44
6	35	.65654684E − 01	.50860453E + 00	.14956433E − 01	.69	.022	.72314405E + 00	.30529956E − 03	.58527400E − 01	.02	.143	61
7	25	.64466063E − 01	.43471091E + 00	.10537837E − 01	.64	.026	.80160555E − 01	.74173819E − 02	.80556935E − 01	.08	.178	67
8	24	.35025816E − 01	.49430180E + 00	.14747469E − 01	.47	.015	.49661284E + 00	.77438829E − 02	0	.06	.269	32
9	199	.60901323E − 01	.55887542E + 00	.13610611E − 01	.86	.013	.73497649E + 00	.34681864E − 02	.84744098E − 02	.02	.068	74
10	231	.87972567E − 01	.65940355E + 00	.41193333E − 01	.91	.009	.72584313E + 00	.13168097E − 02	.34728069E − 01	.04	.038	41
11	123	.38250878E − 01	.54171348E + 00	.13671775E − 01	.75	.010	.85592983E + 00	.28003458E − 02	0	.01	.124	42
12	28	.36276511E − 01	.60464863E + 00	.10893541E − 01	.87	.008	.20650912E + 00	.68989163E − 02	.73312664E − 01	.12	.096	62
13	221	.63318474E − 01	.59455266E + 00	.15945405E − 01	.88	.012	.69043253E + 00	.38525880E − 02	.11731807E − 01	.04	.062	74
14	454	.45092032E − 01	.62527056E + 00	.21865447E − 01	.77	.009	.67031645E + 00	.17213157E − 03	.58600031E − 01	.04	.146	36
15	192	.71763840E − 01	.53296542E + 00	.18148446E − 01	.89	.011	.79907058E + 00	.20395398E − 02	.13021288E − 01	.02	.041	59
16	25	.86549455E − 01	.69260309E + 00	.36875250E − 01	.72	.021	.62666089E − 02	.12731064E − 01	.26572205E − 01	.16	.178	54
17	27	.81021551E − 01	.41667582E + 00	.13232815E − 01	.71	.025	.93047877E − 01	.32288682E − 02	.11176141E + 00	.33	.093	64
18	45	.93662312E − 01	.62629256E + 00	.34261393E − 01	.83	.018	.68950711E + 00	.43781142E − 02	.42879169E − 02	.03	.111	52
19	38	.77536771E − 01	.59211834E + 00	.16180938E − 01	.84	.023	.61431290E + 00	0	.87708460E − 01	.12	.142	92
20	38	.93056063E − 01	.64110355E + 00	.30538159E − 01	.85	.023	.62208533E + 00	0	.85091328E − 01	.12	.137	64
21	30	.13392107E − 00	.54183673E + 00	.30730406E − 01	.82	.035	.75132310E + 00	.31738981E − 02	0	.01	.123	67
22	36	.71576415E − 01	.64291031E + 00	.29429822E − 01	.66	.017	.29425402E + 00	.12965451E − 02	.13964282E + 00	.22	.162	47

[1]Equal to $[\Sigma(y_i - \hat{y}_i)^2/n]^{1/2}$

ships were between modifier values and basal area per acre, sum of diameters per acre, and basal area per acre of all trees as large as or larger than the subject tree. We avoided indirect estimates of competition such as percent stocking or crown competition factor (Krajicek *et al.* 1961) because the intended uses of the equations included stands with species mixtures and size structures for which applicable stocking charts and equations to estimate crown diameter from tree diameter do not exist.

We evaluated several modifier functions. Good theoretical and empirical results were obtained with the following model:

$$MOD = b_9 \{1 - \exp[-(b_7/BAL + b_8 D^2)(1 - BA/BAMAX)^{1/2}]\} \quad (5)$$

where:

MOD = proportion of potential growth actually achieved
BAL = basal area (ft^2/acre) of all trees with d.b.h. equal to or larger than the subject tree
D = tree diameter at breast height (inches)
BA = basal area (ft^2/acre)
BAMAX = maximum basal area per acre (always 200 ft^2/acre in this application)
b_i = species specific regression coefficients.

The general form of this function is similar to, although simpler than, the modifier function used in Lake States STEMS (Belcher *et al.* 1982, Holdaway 1984). The value of the modifier function is always less than or equal to b_9. Tree growth declines as the basal area of larger trees increases. Large diameter trees receive a larger proportion of potential growth than do small diameter trees. The function imposes a maximum basal area per acre, BAMAX, at which growth is zero, regardless of the potential growth. For all species in this study, BAMAX was set at 200. Consequently, if projected plot basal area reaches 200 ft^2/acre tree growth stops until the basal area per acre is reduced through mortality. The BAMAX term is a constraint designed to maintain a biologically reasonable maximum stand density, even under conditions outside the range of calibration data. The maximum basal area observed for any plot in the calibration data was 195 ft^2/acre. Fully stocked normal yield tables for oak-hickory stands, the most common forest type in this data base, reach maximum basal areas of about 150 ft^2/acre at age 100 (Schnur 1937). Based on these observations, 200 ft^2/acre was judged to be a reasonable (and perhaps generous) upper limit to stand basal area, and appropriate for the intended application.

We classified data for the modifier values into classes of 10 square feet for basal area (BA), 10 square feet for basal area of larger trees (BAL), and 2-inches for diameter (D). The mean modifier value for each class was the dependent variable, and the mean values of BA, BAL, and D were the independent variables. Regression coefficients for each of the 22 species groups were determined using nonlinear regression with each case weighted by the number of observations included in the class (table 4).

Table 4.—*Modifier function coefficients and fit statistics*

Species group	Observations	b_7	b_8	b_9	r^2	$\sqrt{MSE}$ [1]
	Number					
1	92	0.78283139E + 02	0.15577296E − 01	0.63526767E + 00	0.14	0.323
2	303	.16663427E + 02	.81767360E − 02	.70244761E + 00	.54	.289
3	186	.48525191E + 02	.10556878E + 00	.49498135E + 00	.10	.402
4	175	.97036785E + 02	.25603789E − 01	.60090233E + 00	.11	.371
5	402	.46404881E + 02	.12405189E − 01	.45019000E + 00	.33	.315
6	215	.41560896E + 02	.49641886E − 01	.42627347E + 00	.09	.305
7	180	.10945997E + 03	.31777816E − 01	.34065485E + 00	.02	.325
8	131	.44063464E + 02	.15989066E − 01	.52659283E + 00	.22	.330
9	550	.75413915E + 02	.67906582E − 02	.49141103E + 00	.37	.278
10	309	.71642264E + 02	.26097951E − 02	.74849531E + 00	.32	.313
11	211	.44031639E + 02	.90519736E − 02	.70245101E + 00	.23	.344
12	160	.12343504E + 03	.46581135E − 01	.52519352E + 00	.01	.344
13	352	.11541287E + 03	.75255275E − 02	.62906138E + 00	.09	.309
14	445	.80004772E + 02	.80789753E − 02	.59464374E + 00	.21	.396
15	518	.84947371E + 02	.29337395E − 02	.51575499E + 00	0	.276
16	130	.66338433E + 02	.20981329E − 02	.58700478E + 00	.15	.325
17	165	.36377003E + 02	.86953273E − 02	.62320871E + 00	.18	.316
18	271	.46112062E + 02	.86852193E − 02	.46062667E + 00	.14	.342
19	78	0	.31005625E + 00	.64505472E + 00	.01	.436
20	236	.68776678E + 02	.14202922E − 01	.46704069E + 00	.12	.311
21	192	.53176655E + 02	.71978128E − 02	.40176103E + 00	.06	.296
22	119	.14134518E + 03	.13699517E − 01	.35210465E + 00	.06	.367

[1] Equal to $[\Sigma(y_i - \hat{y}_i)^2/n]^{1/2}$

EVALUATION RESULTS

Statistics describing the fit of regression equations 1 or 4, and 5 to the data are included in tables 3 and 4. These figures tell how the components fit separately, but they shed little light on the ability of the combined model (potential times modifier) to estimate tree basal area growth.

To provide greater insight into the accuracy of tree growth projections, we computed the predicted tree basal area growth and the diameter growth for each tree in the calibration data base and compared them with the actual growth recorded for those trees (table 5). The remeasurement interval ranged from 7 to 12 years. To facilitate comparisons, growth estimates were linearly adjusted to a 10-year base.

The fit index in table 5 is analogous to the r^2 statistic in linear regression. It measures the ability of the model to explain the variation in the observed growth. Perfect predictions result in a fit index of 1. Assigning the mean growth estimate to every observation results in a fit index of 0.

Note that the fit index is always higher for basal area growth than for diameter growth. This will always be the case, due to the mathematical relationship between these two variables. The range of diameter growth values is small relative to the mean value; diameter growth values have a range of about two times the mean diameter growth. For the same trees, basal area growth has a range of about four times the mean basal area growth. Consequently, the fit index (i.e., the relative improvement over assigning every tree to the mean growth rate) will always be greater in the basal area growth scale than in the diameter growth scale.

The same evaluation statistics were computed for the validation data, the one-fourth of the data withheld from the calibration base (table 6). For species groups 8, 11, 17, and 19, the fit index for diameter growth was negative, indicating that change in diameter growth could have been better estimated simply by using the mean diameter growth from the validation data for that species group, had it been known. The basal area growth fit index for each of these species groups was positive.

Table 5.—*Comparison of observed and predicted 10-year growth for calibration data base*[1]

		Basal area				D.b.h.			
Species group [2]	Observations	Mean 10-year growth	Mean 10-year prediction error [3]	10-year root mean square error	Fit [4] index	Mean 10-year growth	Mean 10-year prediction error [3]	10-year root mean square error	Fit [4] index
	Number	*Ft²/tree*				*In/tree*			
1	187	0.063	0.000	0.044	0.38	0.92	0.086	0.52	0.22
2	1,604	.123	.000	.071	.31	1.30	.156	.755	.08
3	349	.342	−.033	.315	.37	2.14	.280	1.572	.15
4	330	.120	.001	.093	.43	1.03	.105	.657	.14
5	2,495	.072	−.002	.067	.38	.76	.061	.508	.13
6	430	.143	−.008	.127	.36	1.21	.138	.872	.02
7	264	.124	−.016	.126	.19	1.06	.125	.923	.07
8	231	.061	−.002	.052	.38	.57	.043	.446	.09
9	3,766	.143	.000	.086	.50	1.29	.103	.586	.17
10	1,100	.189	.002	.082	.55	1.74	.205	.677	.16
11	566	.094	.000	.053	.47	.92	.074	.487	.06
12	286	.101	−.012	.154	.19	.80	.023	.575	.02
13	1,040	.230	−.001	.120	.51	1.66	.156	.729	.09
14	2,226	.084	.000	.056	.37	.78	.051	.454	.15
15	3,562	.178	.000	.091	.50	1.49	.160	.656	.07
16	191	.200	−.003	.145	.42	.85	.089	.766	.17
17	232	.293	−.004	.186	.47	1.93	.340	1.037	.13
18	594	.134	−.007	.156	.68	1.07	.150	.919	.07
19	128	.366	.004	.217	.42	2.17	.214	1.200	.19
20	461	.132	.010	.123	.44	1.18	.198	.855	.12
21	336	.399	−.030	.324	.31	2.05	.175	1.497	.03
22	511	.030	.001	.057	.41	.59	.135	.573	−.02

[1] The remeasurement interval for trees ranged from 7 to 12 years. Values were linearly adjusted to a 10-year base.
[2] See table 1 for species group definitions.
[3] Predicted minus observed growth. Positive values signify overprediction.
[4] Computed as 1 − (error sum of squares/total sum of squares) = $1 - \Sigma(Y_i - \hat{Y}_i)^2/\Sigma(Y_i - \bar{Y})^2$
Analogous to the r^2 statistic in regression, the fit index measures the ability of the model to describe growth relative to the alternative of assigning the mean growth rate to each observation.

Table 6.—*Comparison of observed and predicted 10-year growth for validation data base*[1]

		Basal area				D.b.h.			
Species group[2]	Observations	Mean 10-year growth	Mean 10-year prediction error[3]	10-year root mean square error	Fit[4] index	Mean 10-year growth	Mean 10-year prediction error[3]	10-year root mean square error	Fit[4] index
	Number	*Ft²/tree*				*In/tree*			
1	61	0.054	0.000	0.034	0.39	0.86	0.061	0.481	0.16
2	614	.129	–.003	.073	.30	1.34	.143	.797	.09
3	58	.255	–.029	.287	.48	1.76	.153	1.591	.24
4	117	.137	–.025	.097	.57	1.13	–.004	.663	.22
5	881	.073	.001	.053	.46	.75	.097	.479	.12
6	149	.148	–.015	.141	.34	1.24	.072	.925	.02
7	118	.106	.009	.084	.14	1.03	.202	.824	.07
8	72	.091	–.021	.080	.07	.66	–.046	.533	–.21
9	1,147	.148	.002	.082	.44	1.21	.107	.587	.19
10	367	.191	–.005	.086	.55	1.77	.184	.736	.06
11	170	.093	.000	.078	.26	.90	.082	.585	–.07
12	114	.074	–.004	.069	.33	.77	.012	.476	.12
13	257	.243	–.005	.137	.53	1.65	.017	.762	.04
14	808	.088	–.004	.066	.29	.82	.014	.533	.10
15	1,078	.188	–.008	.180	.49	1.55	.119	.680	.10
16	81	.194	–.005	.067	.67	1.35	.080	.488	.25
17	108	.292	.001	.167	.57	1.85	.265	.951	.17
18	174	.087	.025	.090	.34	.87	.337	.840	–.13
19	168	.206	.052	.158	.41	2.08	.680	1.217	–.26
20	119	.124	.007	.106	.54	1.12	.203	.873	.05
21	51	.325	.002	.266	.08	2.06	.066	1.335	.03
22	170	.026	.003	.042	.27	.57	.158	.579	–.06

[1] The remeasurement interval for trees ranged from 7 to 12 years. Values were linearly adjusted to a 10-year base.
[2] See table 1 for species group definitions.
[3] Predicted minus observed growth. Positive values signify overprediction.
[4] Computed as 1 – (error sum of squares/total sum of squares) = $1 - \Sigma(Y_i - \hat{Y}_i)^2/\Sigma(Y_i - \bar{Y})^2$
Analogous to the r^2 statistic in regression, the fit index measures the ability of the model to describe growth relative to the alternative of assigning the mean growth rate to each observation.

For most species, at least half of the variation in tree basal area growth remains unexplained. Thus, an opportunity exists to improve upon these predictions. However, based on trials with several different model forms, it does not appear that simply rearranging the terms in this model without also changing the set of independent variables will offer a major improvement in prediction capability for this data set. Many factors influence the degree to which projections for an individual tree correspond to the observed changes for that tree. The effects of weather, microclimate, macroclimate, irregular spacing, soils, pollutants, genetics, and other factors are unaccounted for in the current model and are undoubtedly responsible for a significant portion of unexplained variation in tree growth.

The number of observations per group from table 1 provides a rough guide to the relative strength of the fitted models. Models based on the most observations will generally provide satisfactory results over a wider range of conditions than will models developed from comparatively few observations.

APPLICATION

Annual tree basal area growth is estimated by substituting the appropriate species group coefficients from tables 3 and 4 into equations 1 and 5, solving each equation, and multiplying the results:

Annual tree basal area growth = {equation (1)} {equation (5)}

This tree growth projection model has been incorporated into Central States variants of the STEMS (Miner and Walters 1984) and TWIGS (Belcher 1982) projection systems. The models can also be incorporated into user-written software to forecast changes in forest inventory. Brand (1981a) has shown that software for implementing individual-tree-based projection models need not be complex to be functional.

When coupled with mortality information for Central States species (for example, USDA Forest Service unpublished, Smith and Shifley 1984, or Shifley and Smith 1982), the growth model provides a powerful technique for exploring a variety of forest management problems. The tree growth projection models are operable with the tree detail data collected in most contemporary forest inventories.

Because the model can be applied using typical forest inventory data, it is useful in updating past inventories and projecting future inventory levels (e.g., Smith 1984). The uniform treatment of a comprehensive set of species groups makes the model especially useful for applications involving stands with mixed species.

The same model can also be used for site specific management planning. Beginning with stand inventory information, stand development can be forecast under a variety of cutting strategies. Brand *et al.* (1986) and Buchman (1985) give examples using these models in the TWIGS program to evaluate stand management options. The capacity of the models to forecast growth of trees in stands with mixed species and multiple age classes makes them adaptable to the current conditions of most Central States hardwood stands.

Although the trend is toward comprehensive computer software to implement models of this kind, the models can be used with equal ease on a programmable hand-held calculator to estimate such things as number of years until a tree reaches merchantable size.

During application, users must remain aware that these models were developed from a regional data base. They predict the average change expected from a large number of trees on many different inventory plots. These models have been designed to outline the general trends and patterns of tree growth in the Central States; they do not provide all the fine details. The models should not be expected to give accurate predictions of growth response to new management techniques, or of growth in stands with rare or unnatural species mixtures. Whenever possible, adjustment techniques such as those described by Smith (1983), Stage (1981), and Gertner (1984) should be used to tailor the projections to more closely match specific local conditions.

Many factors must be considered in deciding to use or not to use a particular growth estimation model (Forrester 1968, Buchman and Shifley 1983). Ultimately, each user must evaluate the utility of these models against the alternative tree and forest growth projection methodologies that are available to meet his or her specific objectives.

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