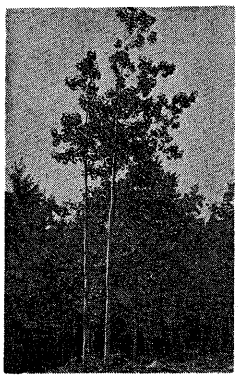
USDA FOREST SERVICE
GENERAL TECHNICAL REPORT NC- 20



$\ln Y = \ln a + b \ln X$

ESTIMATING ASPEN VOLUME AND WEIGHT FOR INDIVIDUAL TREES, DIAMETER CLASSES, OR ENTIRE STANDS

Bryce E. Schlaegel

NORTH CENTRAL FOREST EXPERIMENT STATION
FOREST SERVICE
U.S. DEPARTMENT OF AGRICULTURE

CONTENTS

At the time of this study Bryce E. Schlaegel was an Associate Mensurationist for the Station at its Northern Conifers Laboratory in Grand Rapids, Minnesota. He is now with the USDA Forest Service's Southern Forest Experiment Station in Stoneville, Mississippi.

North Central Forest Experiment Station
John H. Ohman, Director
Forest Service - U.S. Department of Agriculture
Folwell Avenue
St. Paul, Minnesota 55108

1975

ESTIMATING ASPEN VOLUME AND WEIGHT FOR INDIVIDUAL TREES, DIAMETER CLASSES, OR ENTIRE STANDS

Bryce E. Schlaegel

Increased aspen (*Populus tremuloides* Michx.) utilization and weight scaling have created a demand for methods to estimate volumes and weights of whole trees and stands. So far there has been no satisfactory method for predicting volumes or weights for whole trees on a stand basis.

This paper presents equations and tables to estimate gross aspen volume, green weight, and dry weight for individual trees, diameter classes, or entire stands. Estimates can be made for bole wood only, bole wood and bark, or for whole trees including limbs. Estimates for wood and bark to merchantable top diameters from 2 to 7 inches can also be made. The predicting equations are based on a sample of 491 trees from 47 stands in northern Minnesota.

PREDICTING ASPEN VOLUME AND WEIGHT

Volume, green weight, and dry weight for individual trees, diameter classes, or stands can be easily estimated from measurements of basal area (B) and total height (H). Appropriate tree estimates (tables 1 to 3) or stand estimates (tables 4 to 6) are made by substituting measurements of the combined variable (BH) into the following allometric equations, which have been corrected for the bias that occurs when converting from logarithmic units to arithmetic units (Baskerville 1972) (this is an approximate correction):

Volume (ft^3)[1]		$S_{y.x}$	
Ln (bole wood)	=0.99554(Ln BH)-0.85373	0.055	(1)
Ln (bole wood + bark)	=0.99788(Ln BH)-0.73529	0.053	(2)
Ln (bole wood + bark + branches)	=0.98717(Ln BH)-0.47559	0.134	(3)
Green weight (lbs)			
Ln (bole wood)	=0.99719(Ln BH)+3.0070	0.052	(4)
Ln (bole wood + bark)	=0.99718(Ln BH)+3.1879	0.045	(5)
Ln (bole wood + bark + branches)	=0.98800(Ln BH)+3.4329	0.096	(6)
Dry weight (lbs)			
Ln (bole wood)	=0.99461(Ln BH)+2.3465	0.054	(7)
Ln (bole wood + bark)	=0.99889(Ln BH)+2.5006	0.055	(8)
Ln (bole wood + bark + branches)	=0.98996(Ln BH)+2.7422	0.094	(9)

These equations can be used to estimate volume and weight for individual trees, individual diameter classes, or entire stands.

---

[1] *All volumes and weights are from a 6-inch stump to the tree tip. Ln is a natural logarithm.* $R^2 = 0.99$.

Table 1.--*Gross cubic-foot volume per tree for bole wood, bole wood + bark, and for complete trees (the table can be used for either individual trees, for individual diameter classes, or for the complete stand using the quadratic mean diameter)*

Tree diameter, diameter class, or quadratic mean stand diameter (inches)	Tree height, average height of diameter class, or average stand height (ft)								
	20	30	40	50	60	70	80	90	100
1	0.047[1]	0.070	--	--	--	--	--	--	--
	.053[2]	.079	--	--	--	--	--	--	--
	.070[3]	.104	--	--	--	--	--	--	--
2	.186	.279	.372	--	--	--	--	--	--
	.210	.314	.418	--	--	--	--	--	--
	.274	.409	.543	--	--	--	--	--	--
3	.418	.626	.834	1.04	--	--	--	--	--
	.471	.705	.940	1.17	--	--	--	--	--
	.610	.911	1.21	1.51	--	--	--	--	--
4	.741	1.11	1.48	1.85	2.21	--	--	--	--
	.836	1.25	1.67	2.09	2.50	--	--	--	--
	1.08	1.61	2.13	2.66	3.19	--	--	--	--
5	--	1.73	2.30	2.88	3.45	4.02	--	--	--
	--	1.96	2.61	3.25	3.90	4.55	--	--	--
	--	2.50	3.32	4.13	4.95	5.76	--	--	--
6	--	--	3.31	4.14	4.96	5.78	--	--	--
	--	--	3.75	4.68	5.62	6.55	--	--	--
	--	--	4.75	5.93	7.09	8.26	--	--	--
7	--	--	--	5.62	6.74	7.86	8.98	--	--
	--	--	--	6.37	7.64	8.91	10.2	--	--
	--	--	--	8.03	9.62	11.2	12.8	--	--
8	--	--	--	7.34	8.80	10.3	11.7	13.2	14.6
	--	--	--	8.32	9.98	11.6	13.3	14.9	16.6
	--	--	--	10.5	12.5	14.6	16.6	18.7	20.7
9	--	--	--	9.28	11.1	13.0	14.8	16.7	18.5
	--	--	--	10.5	12.6	14.7	16.8	18.9	21.0
	--	--	--	13.2	15.8	18.4	21.0	23.6	26.2
10	--	--	--	11.4	13.7	16.0	18.3	20.5	22.8
	--	--	--	13.0	15.6	18.2	20.7	23.3	25.9
	--	--	--	16.2	19.4	22.6	25.8	29.0	32.2
11	--	--	--	13.8	16.6	19.3	22.1	24.8	27.6
	--	--	--	15.7	18.8	22.0	25.1	28.2	31.4
	--	--	--	19.6	23.5	27.3	31.2	35.0	38.9
12	--	--	--	16.5	19.7	23.0	26.3	29.5	32.8
	--	--	--	18.7	22.4	26.1	29.9	33.6	37.3
	--	--	--	23.3	27.9	32.5	37.0	41.6	46.2
13	--	--	--	19.3	23.1	27.0	30.8	34.6	38.5
	--	--	--	21.9	26.3	30.7	35.0	39.4	43.8
	--	--	--	27.3	32.6	38.0	43.4	48.7	54.1
14	--	--	--	22.4	26.8	31.3	35.7	40.1	44.6
	--	--	--	25.4	30.5	35.5	40.6	45.7	50.7
	--	--	--	31.6	37.8	44.0	50.2	56.4	62.6
15	--	--	--	--	30.8	35.9	41.0	46.1	51.1
	--	--	--	--	35.0	40.8	46.6	52.4	58.2
	--	--	--	--	43.3	50.4	57.5	64.6	71.7
16	--	--	--	--	--	40.8	46.6	52.4	58.2
	--	--	--	--	--	46.4	53.0	59.6	66.2
	--	--	--	--	--	57.3	65.3	73.4	81.4
17	--	--	--	--	--	46.0	52.5	59.1	65.6
	--	--	--	--	--	52.4	59.8	67.3	74.8
	--	--	--	--	--	64.6	73.7	82.7	91.8

(Table 1 cont. on next page)

(Table 1 cont.)

Tree diameter, diameter class, or quadratic mean stand diameter (inches)	Tree height, average height of diameter class, or average stand height (ft)								
	20	30	40	50	60	70	80	90	100
18	--	--	--	--	--	51.6	58.9	66.2	73.5
	--	--	--	--	--	58.7	67.1	75.4	83.8
	--	--	--	--	--	72.3	82.5	92.6	103
19	--	--	--	--	--	57.4	65.6	73.7	81.9
	--	--	--	--	--	65.4	74.7	84.0	93.3
	--	--	--	--	--	80.4	91.7	103	114
20	--	--	--	--	--	63.6	72.6	81.7	90.7
	--	--	--	--	--	72.4	82.8	93.1	103
	--	--	--	--	--	89.0	102	114	127

[1] Bole wood volume only, from 6-inch stump to tip of tree.
[2] Bole volume of wood + bark from 6-inch stump to tip of tree.
[3] Complete tree volume, including branches, from 6-inch stump to tip of tree.

Table 2.--*Gross green weight for bole wood, bole wood + bark, and for complete trees (the table can be used for either individual trees, for individual diameter classes, or for the complete stand using the quadratic mean diameter)*

(In lb/tree)

Tree diameter, diameter class, or quadratic mean stand diameter (inches)	Tree height, average height of diameter class, or average stand height (ft)								
	20	30	40	50	60	70	80	90	100
1	2[1]	3	--	--	--	--	--	--	--
	3[2]	4	--	--	--	--	--	--	--
	3[3]	5	--	--	--	--	--	--	--
2	9	13	18	--	--	--	--	--	--
	11	16	21	--	--	--	--	--	--
	14	20	27	--	--	--	--	--	--
3	20	30	40	50	--	--	--	--	--
	24	36	47	59	--	--	--	--	--
	30	45	60	75	--	--	--	--	--
4	35	53	70	88	105	--	--	--	--
	42	63	84	105	126	--	--	--	--
	54	80	106	133	159	--	--	--	--
5	--	82	110	137	164	192	--	--	--
	--	99	132	164	197	230	--	--	--
	--	125	165	206	247	288	--	--	--
6	--	--	158	197	237	276	--	--	--
	--	--	189	236	284	331	--	--	--
	--	--	237	296	354	412	--	--	--
7	--	--	--	268	322	375	429	--	--
	--	--	--	322	386	450	514	--	--
	--	--	--	401	480	559	638	--	--
8	--	--	--	350	420	490	560	629	699
	--	--	--	420	503	587	670	754	838
	--	--	--	522	625	728	831	933	1,040
9	--	--	--	443	531	619	708	796	884
	--	--	--	531	637	742	848	954	1,060
	--	--	--	659	789	919	1,050	1,180	1,310

(Table 2 cont. on next page)

(Table 2 cont.)

Tree diameter, diameter class, or quadratic mean stand diameter (inches)	Tree height, average height of diameter class, or average stand height (ft)								
	20	30	40	50	60	70	80	90	100
10	--	--	--	546	655	764	873	982	1,090
	--	--	--	655	785	916	1,050	1,180	1,310
	--	--	--	812	972	1,130	1,290	1,450	1,610
11	--	--	--	661	793	924	1,060	1,190	1,320
	--	--	--	792	950	1,110	1,270	1,420	1,580
	--	--	--	980	1,170	1,370	1,560	1,750	1,940
12	--	--	--	786	943	1,100	1,260	1,410	1,570
	--	--	--	942	1,130	1,320	1,510	1,690	1,880
	--	--	--	1,160	1,390	1,620	1,850	2,080	2,310
13	--	--	--	922	1,110	1,290	1,470	1,660	1,840
	--	--	--	1,110	1,330	1,550	1,770	1,990	2,210
	--	--	--	1,360	1,630	1,900	2,170	2,440	2,700
14	--	--	--	1,070	1,280	1,500	1,710	1,920	2,130
	--	--	--	1,280	1,540	1,790	2,050	2,300	2,560
	--	--	--	1,580	1,890	2,200	2,510	2,820	3,130
15	--	--	--	--	1,470	1,720	1,960	2,200	2,450
	--	--	--	--	1,760	2,060	2,350	2,640	2,930
	--	--	--	--	2,170	2,520	2,880	3,230	3,590
16	--	--	--	--	--	1,950	2,230	2,510	2,790
	--	--	--	--	--	2,340	2,670	3,000	3,340
	--	--	--	--	--	2,860	3,270	3,670	4,070
17	--	--	--	--	--	2,200	2,520	2,830	3,140
	--	--	--	--	--	2,640	3,010	3,390	3,770
	--	--	--	--	--	3,230	3,680	4,140	4,590
18	--	--	--	--	--	2,470	2,820	3,170	3,520
	--	--	--	--	--	2,960	3,380	3,800	4,220
	--	--	--	--	--	3,620	4,130	4,630	5,140
19	--	--	--	--	--	2,750	3,140	3,530	3,920
	--	--	--	--	--	3,290	3,760	4,230	4,700
	--	--	--	--	--	4,020	4,590	5,160	5,720
20	--	--	--	--	--	3,050	3,480	3,910	4,350
	--	--	--	--	--	3,650	4,170	4,690	5,210
	--	--	--	--	--	4,450	5,080	5,710	6,330

[1] Bole wood weight only, from 6-inch stump to tip of tree.
[2] Bole weight of wood + bark from 6-inch stump to tip of tree.
[3] Complete tree weight, including branches, from 6-inch stump to tip of tree.

Table 3.--*Gross oven-dry weight for bole wood, bole wood + bark, and for complete trees (the table can be used for either individual trees, for individual diameter classes, or for the complete stand using the quadratic mean diameter)*

(In lb/tree)

Tree diameter, diameter class, or quadratic mean stand diameter (inches)	Tree height, average height of diameter class, or average stand height (ft)								
	20	30	40	50	60	70	80	90	100
1	[1] 1/2	2	--	--	--	--	--	--	--
	[2] 1/3	2	--	--	--	--	--	--	--
	[3] 2	3	--	--	--	--	--	--	--

(table 3 cont. on next page)

(Table 3 cont.)

Tree diameter, diameter class, or quadratic mean stand diameter (inches)	Tree height, average height of diameter class, or average stand height (ft)								
	20	30	40	50	60	70	80	90	100
2	5	7	9	--	--	--	--	--	--
	5	8	11	--	--	--	--	--	--
	7	10	14	--	--	--	--	--	--
3	10	15	20	26	--	--	--	--	--
	12	18	24	30	--	--	--	--	--
	15	23	30	38	--	--	--	--	--
4	18	27	36	45	54	--	--	--	--
	21	32	42	53	64	--	--	--	--
	27	40	54	67	80	--	--	--	--
5	--	42	56	71	85	99	--	--	--
	--	50	66	83	99	116	--	--	--
	--	63	83	104	124	145	--	--	--
6	--	--	81	101	121	142	--	--	--
	--	--	96	119	143	167	--	--	--
	--	--	119	149	178	208	--	--	--
7	--	--	--	138	165	192	220	--	--
	--	--	--	162	195	227	260	--	--
	--	--	--	202	242	282	322	--	--
8	--	--	--	180	215	251	287	322	358
	--	--	--	212	254	297	339	381	424
	--	--	--	263	315	367	419	471	523
9	--	--	--	227	272	317	362	407	452
	--	--	--	268	322	376	429	483	536
	--	--	--	332	398	464	529	595	660
10	--	--	--	280	336	391	447	502	558
	--	--	--	331	397	464	530	596	662
	--	--	--	409	490	571	652	733	813
11	--	--	--	338	406	473	540	607	674
	--	--	--	401	481	561	641	721	801
	--	--	--	494	592	690	787	885	982
12	--	--	--	402	482	562	642	722	802
	--	--	--	477	572	667	762	858	953
	--	--	--	587	704	820	935	1,050	1,170
13	--	--	--	472	565	659	753	846	940
	--	--	--	559	671	783	895	1,010	1,120
	--	--	--	688	824	960	1,100	1,230	1,370
14	--	--	--	547	655	764	872	981	1,090
	--	--	--	649	778	908	1,040	1,170	1,300
	--	--	--	797	955	1,110	1,270	1,430	1,580
15	--	--	--	--	752	876	1,000	1,130	1,250
	--	--	--	--	893	1,040	1,190	1,340	1,490
	--	--	--	--	1,090	1,270	1,460	1,640	1,810
16	--	--	--	--	--	996	1,140	1,280	1,420
	--	--	--	--	--	1,190	1,350	1,520	1,690
	--	--	--	--	--	1,450	1,650	1,860	2,060
17	--	--	--	--	--	1,120	1,280	1,440	1,600
	--	--	--	--	--	1,340	1,530	1,720	1,910
	--	--	--	--	--	1,630	1,860	2,090	2,330
18	--	--	--	--	--	1,260	1,440	1,620	1,800
	--	--	--	--	--	1,500	1,710	1,930	2,140
	--	--	--	--	--	1,830	2,090	2,350	2,600
19	--	--	--	--	--	1,400	1,600	1,800	2,000
	--	--	--	--	--	1,670	1,910	2,150	2,390
	--	--	--	--	--	2,040	2,320	2,610	2,900
20	--	--	--	--	--	1,550	1,770	1,990	2,210
	--	--	--	--	--	1,850	2,120	2,380	2,640
	--	--	--	--	--	2,250	2,570	2,890	3,210

[1] [2] Bole wood weight only, from 6-inch stump to tip of tree.
[3] Bole weight of wood + bark from 6-inch stump to tip of tree.
Complete tree weight, including branches, from 6-inch stump to tip of tree.

Table 4.--*Gross volume for bole wood, bole wood + bark, and for complete trees (the table can be used to estimate volumes of either individual diameter classes or the total stand)*
(In ft^3/acre)

Diameter class: or stand basal: area per acre : (ft^2) :	Average height of diameter class, or average stand height (ft)								
	20	30	40	50	60	70	80	90	100
10	83[1]	125	166	207	248	289	331	372	413
	95[2]	142	189	237	284	331	378	425	472
	116[3]	173	230	287	344	400	456	513	569
20	166	248	331	413	495	577	659	741	823
	189	284	378	472	567	661	755	849	943
	230	344	456	569	681	793	905	1,020	1,130
30	248	372	495	618	741	864	987	1,110	1,230
	284	425	567	708	849	990	1,130	1,270	1,410
	344	513	681	849	1,020	1,180	1,350	1,520	1,680
40	331	495	659	823	987	1,150	1,310	1,480	1,640
	378	567	755	943	1,130	1,320	1,510	1,700	1,880
	456	681	905	1,130	1,350	1,570	1,790	2,010	2,240
50	413	618	823	1,030	1,230	1,440	1,640	1,850	2,050
	472	708	943	1,180	1,410	1,650	1,880	2,120	2,350
	569	849	1,130	1,410	1,680	1,960	2,240	2,510	2,790
60	495	741	987	1,230	1,480	1,720	1,970	2,210	2,460
	567	849	1,130	1,410	1,700	1,980	2,260	2,540	2,820
	681	1,020	1,350	1,680	2,010	2,350	2,680	3,010	3,340
70	577	864	1,150	1,440	1,720	2,010	2,290	2,580	2,870
	661	990	1,320	1,650	1,980	2,310	2,640	2,960	3,290
	793	1,180	1,570	1,960	2,350	2,730	3,120	3,500	3,880
80	659	987	1,310	1,640	1,970	2,290	2,620	2,950	3,270
	755	1,130	1,510	1,880	2,260	2,640	3,010	3,390	3,760
	905	1,350	1,790	2,240	2,680	3,120	3,550	3,990	4,430
90	741	1,110	1,480	1,850	2,210	2,580	2,950	3,310	3,680
	849	1,270	1,700	2,120	2,540	2,960	3,390	3,810	4,230
	1,020	1,520	2,010	2,510	3,010	3,500	3,990	4,490	4,980
100	--	1,230	1,640	2,050	2,460	2,870	3,270	3,680	4,090
	--	1,410	1,880	2,350	2,820	3,290	3,760	4,230	4,700
	--	1,680	2,240	2,790	3,340	3,880	4,430	4,980	5,520
110	--	1,360	1,800	2,250	2,700	3,150	3,600	4,050	4,490
	--	1,550	2,070	2,590	3,110	3,620	4,140	4,650	5,170
	--	1,850	2,460	3,060	3,660	4,270	4,870	5,470	6,070
120	--	--	1,970	2,460	2,950	3,440	3,920	4,410	4,900
	--	--	2,260	2,820	3,390	3,950	4,510	5,080	5,640
	--	--	2,680	3,340	3,990	4,650	5,300	5,960	6,610
130	--	--	2,130	2,660	3,190	3,720	4,250	4,780	5,310
	--	--	2,450	3,060	3,670	4,280	4,890	5,500	6,110
	--	--	2,900	3,610	4,320	5,030	5,740	6,450	7,180
140	--	--	2,290	2,870	3,440	4,010	4,570	5,140	5,710
	--	--	2,640	3,290	3,950	4,610	5,260	5,920	6,580
	--	--	3,120	3,880	4,650	5,410	6,180	6,940	7,700
150	--	--	--	3,070	3,680	4,290	4,900	5,510	6,120
	--	--	--	3,530	4,230	4,940	5,640	6,340	7,050
	--	--	--	4,160	4,980	5,790	6,610	7,430	8,240
160	--	--	--	3,270	3,920	4,570	5,230	5,880	6,530
	--	--	--	3,760	4,510	5,260	6,010	6,760	7,510
	--	--	--	4,430	5,300	6,180	7,050	7,920	8,780
170	--	--	--	3,480	4,170	4,860	5,550	6,240	6,930
	--	--	--	4,000	4,790	5,590	6,390	7,190	7,980
	--	--	--	4,700	5,630	6,560	7,480	8,400	9,320
180	--	--	--	3,680	4,410	5,140	5,880	6,610	7,340
	--	--	--	4,230	5,080	5,920	6,760	7,610	8,450
	--	--	--	4,980	5,960	6,940	7,920	8,890	9,870

(Table 4 cont. on next page)

(Table 4 cont.)

Diameter class or stand basal area per acre (ft^2)	Average height of diameter class, or average stand height (ft)								
	20	30	40	50	60	70	80	90	100
190	--	--	--	3,880	4,660	5,430	6,200	6,970	7,740
	--	--	--	4,470	5,360	6,250	7,140	8,030	8,920
	--	--	--	5,250	6,290	7,320	8,350	9,380	10,410
200	--	--	--	4,090	4,900	5,710	6,530	7,340	8,150
	--	--	--	4,700	5,640	6,580	7,510	8,450	9,390
	--	--	--	5,520	6,610	7,700	8,780	9,870	10,950
210	--	--	--	4,290	5,140	6,000	6,850	7,700	8,550
	--	--	--	4,940	5,920	6,900	7,890	8,870	9,860
	--	--	--	5,790	6,940	8,080	9,220	10,350	11,490
220	--	--	--	4,490	5,390	6,280	7,170	8,070	8,960
	--	--	--	5,170	6,200	7,230	8,260	9,290	10,320
	--	--	--	6,070	7,260	8,460	9,650	10,840	12,030
230	--	--	--	4,700	5,630	6,570	7,500	8,430	9,360
	--	--	--	5,400	6,480	7,560	8,640	9,720	10,790
	--	--	--	6,340	7,590	8,840	10,080	11,330	12,570
240	--	--	--	4,900	5,880	6,850	7,820	8,800	9,770
	--	--	--	5,640	6,760	7,890	9,010	10,140	11,260
	--	--	--	6,610	7,920	9,220	10,510	11,810	13,110
250	--	--	--	5,100	6,120	7,130	8,150	9,160	10,180
	--	--	--	5,870	7,050	8,220	9,390	10,560	11,730
	--	--	--	6,880	8,240	9,600	10,950	12,300	13,640

[1] Bole wood volume only, from 6-inch stump to tip of tree.
[2] Bole volume of wood + bark from 6-inch stump to tip of tree.
[3] Complete tree volume, including branches, from 6-inch stump to tip of tree.

Table 5.--*Gross green weight for bole wood, bole wood + bark, and for complete trees (the table can be used to estimate weights of either individual diameter classes or the total stand)*
(In thousand lb/acre)

Diameter class or stand basal area per acre (ft^2)	Average height of diameter class, or average stand height (ft)								
	20	30	40	50	60	70	80	90	100
10	3.99[1]	5.97	7.96	9.94	11.9	13.9	15.9	17.9	19.8
	4.78[2]	7.16	9.53	11.9	14.3	16.7	19.0	21.4	23.8
	5.81[3]	8.68	11.5	14.4	17.2	20.0	22.9	25.7	28.5
20	7.96	11.9	15.9	19.8	23.8	27.7	31.7	35.6	39.6
	9.53	14.3	19.0	23.8	28.5	33.2	38.0	42.7	47.4
	11.5	17.2	22.9	28.5	34.1	39.7	45.3	50.9	56.5
30	11.9	17.9	23.8	29.7	35.6	41.6	47.5	53.4	59.3
	14.3	21.4	28.5	35.6	42.7	49.8	56.9	64.0	71.1
	17.2	25.7	34.1	42.5	50.9	59.3	67.7	76.0	84.4
40	15.9	23.8	31.7	39.6	47.5	55.4	63.3	71.2	79.0
	19.0	28.5	38.0	47.4	56.9	66.4	75.8	85.3	94.7
	22.9	34.1	45.3	56.5	67.7	78.8	89.9	101	112
50	19.8	29.7	39.6	49.5	59.3	69.2	79.0	88.9	98.7
	23.8	35.6	47.4	59.3	71.1	82.9	94.7	107	118
	28.5	42.5	56.5	70.5	84.4	98.3	112	126	140
60	23.8	35.6	47.5	59.3	71.2	83.0	94.8	107	118
	28.5	42.7	56.9	71.1	85.3	99.4	114	128	142
	34.1	50.9	67.7	84.4	101	118	134	151	167
70	27.7	41.6	55.4	69.2	83.0	96.8	111	124	138
	33.2	49.8	66.4	82.9	99.4	116	132	149	165
	39.7	59.3	78.8	98.3	118	137	156	176	195

(Table 5 cont. on next page)

(Table 5 cont.)

Diameter class: or stand basal: area per acre (ft^2)	Average height of diameter class, or average stand height (ft)								
	20	30	40	50	60	70	80	90	100
80	31.7	47.5	63.3	79.0	94.8	111	126	142	158
	38.0	56.9	75.8	94.7	114	132	151	170	189
	45.3	67.7	89.9	112	134	156	178	200	222
90	35.6	53.4	71.2	88.9	107	124	142	160	177
	42.7	64.0	85.3	107	128	149	170	191	213
	50.9	76.0	101	126	151	176	200	225	250
100	--	59.3	79.0	98.7	118	138	158	177	197
	--	71.1	94.7	118	142	165	189	213	236
	--	84.4	112	140	167	195	222	250	277
110	--	65.2	86.9	109	130	152	174	195	217
	--	78.2	104	130	156	182	208	234	260
	--	92.7	123	154	184	214	244	275	305
120	--	--	94.8	118	142	166	189	213	236
	--	--	114	142	170	198	227	255	283
	--	--	134	167	200	233	266	299	332
130	--	--	103	128	154	179	205	231	256
	--	--	123	154	184	215	246	276	307
	--	--	145	181	217	253	288	324	359
140	--	--	111	138	166	193	221	248	276
	--	--	132	165	198	231	264	297	330
	--	--	156	195	233	272	310	348	387
150	--	--	--	148	177	207	236	266	295
	--	--	--	177	213	248	283	319	354
	--	--	--	209	250	291	332	373	414
160	--	--	--	158	189	221	252	284	315
	--	--	--	189	227	264	302	340	377
	--	--	--	222	266	310	354	398	441
170	--	--	--	168	201	234	268	301	335
	--	--	--	201	241	281	321	361	401
	--	--	--	236	283	329	376	422	468
180	--	--	--	177	213	248	284	319	354
	--	--	--	213	255	297	340	382	424
	--	--	--	250	299	348	398	447	496
190	--	--	--	187	225	262	299	337	374
	--	--	--	224	269	314	359	403	448
	--	--	--	264	316	367	419	471	523
200	--	--	--	197	236	276	315	354	393
	--	--	--	236	283	330	377	424	471
	--	--	--	277	332	387	441	496	550
210	--	--	--	207	248	289	331	372	413
	--	--	--	248	297	347	396	446	495
	--	--	--	291	348	406	463	520	577
220	--	--	--	217	260	303	346	390	433
	--	--	--	260	311	363	415	467	518
	--	--	--	305	365	425	485	544	604
230	--	--	--	227	272	317	362	407	452
	--	--	--	271	326	380	434	488	542
	--	--	--	318	381	444	506	569	631
240	--	--	--	236	284	331	378	425	472
	--	--	--	283	340	396	453	509	565
	--	--	--	332	398	463	528	593	658
250	--	--	--	246	295	344	393	442	491
	--	--	--	295	354	413	471	530	589
	--	--	--	346	414	482	550	618	686

[1] Bole wood weight only, from 6-inch stump to tip of tree.
[2] Bole weight of wood + bark from 6-inch stump to tip of tree.
[3] Complete tree weight, including branches, from 6-inch stump to tip of tree.

Table 6.--*Gross oven-dry weight for bole wood, bole wood + bark, and for complete trees (the table can be used to estimate weights of either individual diameter classes or the total stand)*
(In thousand lb/acre)

Diameter class: or stand basal area per acre (ft^2)	Average height of diameter class, or average stand height (ft)								
	20	30	40	50	60	70	80	90	100
10	2.03[1]	3.04	4.05	5.05	6.06	7.06	8.06	9.07	10.1
	2.42[2]	3.63	4.84	6.05	7.26	8.47	9.68	10.9	12.1
	2.94[3]	4.40	5.85	7.29	8.73	10.2	11.6	13.0	14.5
20	4.05	6.06	8.06	10.1	12.1	14.1	16.1	18.1	20.1
	4.84	7.26	9.68	12.1	14.5	16.9	19.3	21.8	24.2
	5.85	8.73	11.6	14.5	17.3	20.2	23.1	25.9	28.8
30	6.06	9.07	12.1	15.1	18.1	21.1	24.0	27.0	30.0
	7.26	10.9	14.5	18.1	21.8	25.4	29.0	32.6	36.2
	8.73	13.0	17.3	21.6	25.9	30.2	34.5	38.7	43.0
40	8.06	12.1	16.1	20.1	24.0	28.0	32.0	36.0	40.0
	9.68	14.5	19.3	24.2	29.0	33.8	38.7	43.5	48.3
	11.6	17.3	23.1	28.8	34.5	40.1	45.8	51.5	57.1
50	10.1	15.1	20.1	25.0	30.0	35.0	40.0	44.9	49.9
	12.1	18.1	24.2	30.2	36.2	42.3	48.3	54.3	60.4
	14.5	21.6	28.8	35.9	43.0	50.1	57.1	64.2	71.2
60	12.1	18.1	24.0	30.0	36.0	42.0	47.9	53.9	59.8
	14.5	21.8	29.0	36.2	43.5	50.7	58.0	65.2	72.4
	17.3	25.9	34.5	43.0	51.5	60.0	68.4	76.9	85.3
70	14.1	21.1	28.0	35.0	42.0	48.9	55.9	62.8	69.7
	16.9	25.4	33.8	42.3	50.7	59.2	67.6	76.1	84.5
	20.2	30.2	40.1	50.1	60.0	69.8	79.7	89.6	99.4
80	16.1	24.0	32.0	40.0	47.9	55.9	63.8	71.7	79.6
	19.3	29.0	38.7	48.3	58.0	67.6	77.3	86.9	96.6
	23.1	34.5	45.8	57.1	68.4	79.7	91.0	102	113
90	18.1	27.0	36.0	44.9	53.9	62.8	71.7	80.6	89.5
	21.8	32.6	43.5	54.3	65.2	76.1	86.9	97.8	109
	25.9	38.7	51.5	64.2	76.9	89.6	102	115	127
100	--	30.0	40.0	49.9	59.8	69.7	79.6	89.5	99.4
	--	36.2	48.3	60.4	72.4	84.5	96.6	109	121
	--	43.0	57.1	71.2	85.3	99.4	113	127	142
110	--	33.0	43.9	54.9	65.8	76.7	87.6	98.4	109
	--	39.9	53.1	66.4	79.7	92.9	106	119	133
	--	47.2	62.8	78.3	93.8	109	125	140	156
120	--	--	47.9	59.8	71.7	83.6	95.5	107	119
	--	--	58.0	72.4	86.9	101	116	130	145
	--	--	68.4	85.3	102	119	136	153	169
130	--	--	51.9	64.8	77.7	90.5	103	116	129
	--	--	62.8	78.5	94.1	110	125	141	157
	--	--	74.1	92.4	111	129	147	165	183
140	--	--	55.9	69.7	83.6	97.5	111	125	139
	--	--	67.6	84.5	101	118	135	152	169
	--	--	79.7	99.4	119	139	158	178	197
150	--	--	--	74.7	89.5	104	119	134	149
	--	--	--	90.5	109	127	145	163	181
	--	--	--	106	127	149	169	190	211
160	--	--	--	79.6	95.5	111	127	143	159
	--	--	--	96.6	116	135	154	174	193
	--	--	--	113	136	158	181	203	225
170	--	--	--	84.6	101	118	135	152	169
	--	--	--	103	123	144	164	185	205
	--	--	--	120	144	168	192	216	239
180	--	--	--	89.5	107	125	143	161	178
	--	--	--	109	130	152	174	195	217
	--	--	--	127	153	178	203	228	253

(Table 6 cont. on next page)

(Table 6 cont.)

Diameter class or stand basal area per acre (ft^2)	Average height of diameter class, or average stand height (ft)								
	20	30	40	50	60	70	80	90	100
190	--	--	--	94.5	113	132	151	170	188
	--	--	--	115	138	160	183	206	229
	--	--	--	134	161	188	214	241	267
200	--	--	--	99.4	119	139	159	178	198
	--	--	--	121	145	169	193	217	241
	--	--	--	142	169	197	225	253	281
210	--	--	--	104	125	146	167	187	208
	--	--	--	127	152	177	203	228	253
	--	--	--	149	178	207	236	266	295
220	--	--	--	109	131	153	174	196	218
	--	--	--	133	159	186	212	239	265
	--	--	--	156	186	217	248	278	309
230	--	--	--	114	137	160	182	205	228
	--	--	--	139	166	194	222	250	277
	--	--	--	162	195	227	259	291	323
240	--	--	--	119	143	167	190	214	238
	--	--	--	145	174	203	231	260	289
	--	--	--	169	203	236	270	303	337
250	--	--	--	124	149	173	198	223	247
	--	--	--	151	181	211	241	271	301
	--	--	--	176	211	246	281	316	351

[1] Bole wood weight only, from 6-inch stump to tip of tree.
[2] Bole weight of wood + bark from 6-inch stump to tip of tree.
[3] Complete tree weight, including branches, from 6-inch stump to tip of tree.

Individual tree volume or weight estimates are obtained from the equations when B is the individual tree basal area and H is the total tree height. Bole wood volume estimates agree closely with Table 3 of the composite volume tables (Gevorkiantz and Olsen 1955).

A further extension of the individual tree estimate is to estimate volume or weight by diameter classes to derive stocking tables. These estimates can be readily calculated from the equations by using diameter class basal area for B and the average total height of the trees in that diameter class for H.

Finally, volume or weight estimates can be made on a total stand basis. In this case B is the total stand basal area and H is the average weighted total height of all the trees in the stand (the weighting factor for each tree is its squared diameter). This average height estimate is the same as the height of the tree of mean basal area.

A simple way to find diameter class heights and average stand height is to measure total tree heights on a sample of trees while cruising the stands. Make sure that all diameter classes are represented, usually 15 to 25 trees. Fit a height/diameter curve to these data by regression. Any suitable model can be used, but one of the more convenient is the second degree polynomial of the form:

$$\text{total tree height} = b_0 + b_1(\text{dbh}) + b_2(\text{dbh})^2. \quad (10)$$

Diameter-class heights can now be found from the height/diameter equation using diameter class midpoint as the estimator. Average total stand height can be found using the quadratic mean stand diameter in the height /diameter equation. Or, it can be found from the sample tree height/diameter data:

$$\text{average stand height} = \text{sum}(D^2H)/\text{sum } D^2.$$

Because the volume estimates from eq. 1 agree so closely with the composite tables, which are applicable for a number of Lake States species, dry weight equations presented here for aspen bole wood could probably be extended to these other species, after correcting for specific gravity differences between species, as described in a later section. Bark volumes, green

weights, and total weights for other species could not be extended in this manner due to differences between species in wood/bark ratios, moisture content, and crowns.

PREDICTING MERCHANTABLE STAND VOLUME AND WEIGHT

Although whole-tree harvesting of aspen is becoming increasingly important, there is still a need to estimate volumes and weights to various top diameters. Tree and stand volumes and weights can be estimated to top diameters of 2 to 7 inches by using the ratio-estimation technique (Schlaegel 1971, 1974). The percent of weight and volume to various top diameters can be estimated from the following asymptotic equations:

Percent[2] *of bole to*

2-inch top[3] $= 0.994 - (1.468)(0.383)^{D}$ (11)
3-inch top $= 0.984 - (4.469)(0.389)^{D}$ (12)
4-inch top $= 0.981 - (2.826)(0.556)^{D}$ (13)
5-inch top $= 0.967 - (5.248)(0.597)^{D}$ (14)
6-inch top $= 0.947 - (9.662)(0.598)^{D}$ (15)
7-inch top $= 0.971 - (5.752)(0.697)^{D}$ (16)

where D = diameter of tree of mean basal area.

From these equations (or table 7) the merchantable bole as a percent of the total bole can be used with the estimate of total bole volume or weight (tables 1 to 6) to estimate volumes and weights to specified top diameters for individual trees, for all the trees in a diameter class, or for the total stand. NOTE: When using the ratio equations to estimate merchantable volume or weight, only the bole volumes and weights of trees with a d.b.h. larger than the indicated top diameter can be used.

CORRECTING FOR FORM AND SPECIFIC GRAVITY DIFFERENCES BETWEEN STANDS

Additional accuracy can be obtained by adjusting for form and specific gravity differences when estimating bole volume and weight. The average form factor for these Minnesota aspen estimates is 0.413 and the average wood specific gravity is 0.391.

---

[2] *Expressed as a decimal*

[3] *Top diameters were measured outside bark.*

Table 7.--*Merchantable bole based on top diameter outside bark and quadratic mean stand diameter*[1]
(In decimal percent)

Quadratic mean stand diameter (inches)	Top diameter outside bark (inches)					
	2	3	4	5	6	7
2	0.78	0.31	--	--	--	--
3	.91	.72	0.50	--	--	--
4	.96	.88	.71	0.30	--	--
5	.98	.94	.83	.57	0.21	--
6	.99	.97	.90	.73	.51	0.31
7	.99	.98	.93	.83	.68	.51
8	.99	.98	.96	.88	.79	.65
9	.99	.98	.97	.92	.85	.75
10	.99	.98	.97	.94	.89	.82
11	.99	.98	.98	.95	.91	.86
12	.99	.98	.98	.96	.93	.90
13	.99	.98	.98	.96	.93	.92
14	.99	.98	.98	.96	.94	.93
15	.99	.98	.98	.96	.94	.95

[1] Model form: decimal percent = $a + b(r)^{D}$. Where a = the asymptote of percent, b = the change in percent as D goes from 0 to infinity, r = the factor by which the deviation of percent is reduced from its asymptotic value by each unit step of D, D = quadratic mean stand diameter (Stevens 1951).

Form factor can be estimated several ways. The best way is to cut a few sample trees in the stand, calculate tree volumes from stem measurements, and calculate form factors as the ratio of tree volume to the volume of a cylinder, with diameter equal to d.b.h. and a length equal to total height. Tree form factors differ little from tree to tree in the same stand except for intermediate and suppressed trees. These differences in small trees can be minimized when averaging the tree form factors if each tree is weighted by its diameter squared and a weighted average calculated.

If destructive sampling is not feasible, average stand form factor can be estimated from the stand age and average stand form quotient (FQ), i.e., diameter at ½-tree height over d.b.h.:

$$\text{average stand form factor} = -0.073 + 0.737\,(FQ) - 0.000545\,(age). \quad (17)$$

This equation accounts for 84 percent of regression variation and has a standard error of 0.009.

After average stand form factor has been estimated, adjust stand bole wood volume and weight estimates by the percent difference that exists between the actual stand form and the average stand form given above. For instance, if you know that the stand in question has an average form factor of 0.445 instead of 0.413, you would adjust the weight or volume estimate up 7.7 percent.

Adjust stand ovendry wood weight for differences in specific gravities between stands by relating average increment core specific gravity to average stand specific gravity.[4] Take enough large diameter (½-inch) increment cores to give a reliable and accurate estimate (Freese 1967), then compute an arithmetic average of the core specific gravities.

$$\text{Avg. stand wood specific gravity} = 0.031 + 0.963\ (\text{avg. core SG}). \qquad (18)$$

$$R^2 = 0.86 \qquad Sy.x = 0.008$$

Ovendry wood weight estimates from equation 7 can then be adjusted by the percent difference between 0.391 and the stand specific gravity estimate.

LITERATURE CITED

Baskerville, G. L. 1972. Use of logarithmic regression in the estimation of plant biomass. Can. J. For. Res. 2:49-53.

Freese, Frank. 1964. Linear regression methods for forest research. USDA For. Serv. Res. Pap. FPL-17, 136 p. For. Prod. Lab., Madison, Wis.

Freese, Frank. 1967. Elementary statistical methods for foresters. U.S. Dep. Agric., Agric. Handb. 317, 87 p., illus.

Gevorkiantz, S. R., and L. P. Olsen. 1955. Composite volume tables for timber and their application in the Lake States. USDA Tech. Bull. 1104, 51 p., illus.

Heinrichs, J. Frank, and L. E. Lassen. 1970. Improved technique for determining the volume of irregularly shaped wood blocks. For. Prod. J. 20(4):p. 24.

Schlaegel, Bryce E. 1971. Growth and yield of quaking aspen in north-central Minnesota. USDA For. Serv. Res. Pap. NC-58, 11 p. North Cent. For. Exp. Stn., St. Paul, Minn.

Schlaegel, Bryce E. 1974. Estimating the weight yield of Minnesota quaking aspen (*Populus tremuloides* Michx.). In IUFRO Biomass Studies. p. 387-399. College Life Sci. & Agric., Univ. Maine.

Stevens, W. L. 1951. Asymptotic regression. Biometrics 7(3):247-267.

[4] *Bryce E. Schlaegel. Estimating aspen specific gravity from increment cores. (In preparation.)*

APPENDIX I.--PROCEDURES

In an aspen weight-yield study begun in 1970, a total of 491 trees in 47 stands were sampled. Stand characteristics measured were:

	Mean	*Range*
Age (yr)	42	11-99
Site index, height at age 50 years (ft)	70	47-85
Basal area (ft^2/acre)	126	59-215
Average stand diameter (in. at 4.5 ft)	6.4	2-13
Average total stand height (ft)	62	28-86
Average stand wood (sp. gr.)	0.39	0.34-0.45
Average stand bark (sp. gr.)	0.55	0.47-0.66
Average stand wood moisture content (percent)	82	60-105
Average stand bark moisture content (percent)	80	62-120

A wide range of growing conditions is represented. Soils ranged from very dry coarse sands to clay loams to peat. Climate in the study area is continental with mild summers and cold winters. Precipitation averages 25 inches per year, two-thirds falling from May through September. Average temperatures range from 6°F in January to 67°F in July. About 75 percent of the commercial aspen range in Minnesota is represented by the 47 sample stands.

Sample locations were chosen in stands where aspen composed at least 75 percent of the overstory. At each sample location a plot was measured for stand characteristics. Plot size ranged from 1/100 to 1/10 acre, depending on the number of trees in the stand; the higher the density, the smaller the plot. Sample trees were selected using a basal area factor (BAF) prism to allow cutting 9 to 13 trees. Height to crown and total height were measured on each felled sample tree; then the tree was cut into 4-foot bolts and weighed. A 1-inch-thick disc was removed from each 4-foot section and sealed in a polyethylene bag for laboratory determination of moisture content and specific gravity. All limbs were weighed from a subsample of four trees on each plot.

In the laboratory, both wood and bark moisture content and specific gravity were determined as follows:

1. Wood and bark were separated with a knife.
2. Each component was weighed green.
3. Both wood and bark were soaked in water for an hour to ensure complete swelling.
4. Volumes were obtained by immersion (Heinrichs and Lassen 1970).
5. Wood and bark were ovendried at 103°C for at least 24 hours or until weight loss was completed.

$$\text{Wood or bark moisture content} = \frac{\text{green weight - ovendry weight}}{\text{ovendry weight}} .$$

$$\text{Specific gravity} = \frac{\text{ovendry wood (bark) weight (gm)}}{\text{green wood (bark) volume (cc)}} .$$

Because the amount of wood and bark decreases up the tree and the ratio of wood to bark also decreases up the tree, a simple arithmetic mean for tree moisture content and specific gravity cannot be used. Each disc wood and bark moisture content and specific gravity is weighted by its respective diameter squared. Thus, average tree moisture content = sum (disc diameter squared x disc moisture content)/sum disc diameters squared.

Bolt volumes were calculated from Smalian's formula. Total tree volumes and green weights were found by summing the values for each 4-foot bolt. Ovendry wood and bark weights were estimated for each bolt from average disc specific gravity and the appropriate bolt component volume.

Sample tree volumes and weights were related to BH by ratio regression estimates (Freese 1964). These ratios were used to calculate unit area component volumes and weights to various stem top diameters. Allometric regression equations were then used to estimate total stand volume and weight from various stand characteristics.

APPENDIX II.--EXAMPLE OF ESTIMATING VOLUME, GREEN WEIGHT, AND DRY WEIGHT

This example uses average stand table and height data from three 40-year-old stands, site index 75, used in this study. Table 8 gives the stand table and height data that were obtained from field measurements. It also gives results from intermediate steps in calculating some specific yield information.

Table 8.--*Example showing averages from three stands in this study, each 40 years old with a site index of 75*

Instruction step		2,3	4	7	8b	8b	8b	9	9
Equation used		10			1	5	9	1,14	5,12
D class	Trees	Height	Basal area	BxH	Bole wood volume	Green wt. wood + bark	Dry wood, bark, branches	Bole wood to 5 inch top	Green bole wood + bark to 3 inch top
	No.	*ft.*	*ft^2*		*ft^3*	*lbs*	*lbs*	*ft^3*	*lbs*
3	7	38	0.3	11	5	265	167	--	--
4	73	48	6.4	307	127	7,322	4,499	--	7,141
5	60	56	8.2	459	190	10,934	6,699	--	10,663
6	93	61	18.3	1,116	461	26,519	16,143	365	25,862
7	93	64	24.9	1,594	657	37,839	22,975	521	36,901
8	87	66	30.4	2,006	826	47,589	28,847	655	46,409
9	40	66	17.7	1,168	482	27,751	16,888	382	27,063
10	--	--	--	--	--	--	--	--	--
11	7	67	4.6	308	128	7,345	4,513	101	7,163
Total	460		110.8	6,969	2,876	165,564	100,731	2,024	161,202
		Sum of classes 6 inches +			2,554				
		Sum of classes 4 inches +				165,299			

Step

1. Establish a stand table for the area to be estimated.

2. Measure a representative sample of trees for diameter and height and fit a height/diameter curve by either freehand or by regression methods.

3. Find the average height for the midpoint of each diameter class from the curve or equation.

4. Compute the basal area of each diameter class.

5. Find the diameter of the tree of average basal area for use in equations 11 to 16 from:

$$\bar{D} = \sqrt{\frac{\text{Total basal area}}{\text{Total number of trees}} \Big/ 0.005454} = \sqrt{\frac{110.8}{460} \Big/ 0.005454} =$$

$$\sqrt{\frac{0.241}{0.005454}} = \sqrt{44.16} = 6.6 \text{ inches}$$

6a. Find the height of the tree of mean stand basal area from the ht./d.b.h. curve, in this case 63 feet, or

b. Calculate weighted average stand height from sample tree ht./d.b.h. data (weighted average height = sum $((\text{d.b.h.})^2$ x total height)/sum $(\text{d.b.h.})^2$), as in the following:

D	H	D^2	D^2H
4	50	16	800
5	58	25	1,425
6	63	36	2,232
7	66	49	3,185
8	68	64	4,288
		190	11,930

$\bar{D}$ = 11,930/190 = 63 ft.

7. Multiply the basal area in each D class by the height of that class to obtain BH.

8. Find the estimated volumes or weights from equations 1 to 9 by any of the following:

a. Individual trees--using tree basal area x height

b. Diameter class--using D class basal area x D class height.

ex.: table 8--6-inch diameter class

Ln (bole wood volume)
=0.99554(Ln BH) - 0.85373 Equation 1
=0.99554(Ln 1116) - 0.85373
=0.99554(7.02) - 0.85373
=6.13
Bole wood volume
=antilog (6.13)
=461 ft^3

or

Ln (Green bole wood + bark)
=0.99718(Ln BH) + 3.1879 Equation 5
=0.99718(Ln 1116) + 3.1879
=0.99718(7.02) + 3.1879
=10.19
Green bole wood + bark
=antilog (10.19)
=26,519 lbs

c. Total stand--using stand basal area x height of mean tree:

Stand basal area (B)
=110.8 ft^2
Mean tree height (H)
=63 ft

BH
=6,980.4
Bole wood volume (total stand)
=antilog (0.99554 (Ln 6,980.4) - 0.85373)
=2,857 ft^3

d. Total stand--by finding volume of mean tree and multiplying by the number of trees in the stand.

Diameter of mean tree
=6.6 inches
Basal area of mean tree
=0.005454 $(6.6)^2$ = 0.24 ft^2
Height of mean tree
=63 ft
BH
=14.97
Volume of mean tree
=6.3 ft^3 Equation 1
Numbers of trees in stand
=460
Stand volume
=6.3 x 460 = 2,897 ft^3

9. Find bole volume and weight information to the desired top diameter for:

a. Each diameter class:
By using mean basal area to find the appropriate ratio from equations 11 to 16 and multiplying that ratio times the total bole volume or weight of that D class.
In this example:
Find ratio of volume to 5-inch top using equation 14.

Percent of bole to 5-inch top
=0.967 - (5.248) $(0.597)^{\overline{D}}$ Equation 14

$\overline{D}$ =average stand diameter = 6.6 inches
Percent to 5-inch top
=0.967 - (5.248) antilog ((6.6) (Ln 0.597))
=0.967 - (5.248) antilog ((6.6) (-0.52))
=0.967 - (5.248) antilog (-3.40)
=0.967 - (5.248) (0.033)
=0.967 - 0.174
=0.793
Bole wood volume to 5-inch top in 6-inch class=0.793 x 461 = 365 ft^3
Bole wood volume to 5-inch top in 7-inch class=0.793 x 657 = 521 ft^3

b. For total stand:
By using the mean basal area to find the appropriate ratio from equations 11 to 16 and multiplying this ratio times the stand volume or weight of only those trees that are larger than the top diameter you are using--i.e., if you are finding stand volume to a 5-inch top, find the total stand volume for boles of all trees 6 inches and larger. For our example apply the percent found in (a) above to the total bole volume of all trees in 6-inch class and larger. Total stand bole wood volume to 5-inch top = 0.793 x 2,554 = 2,024.

10. Various stand estimates can be found by using equations 1 to 9 and 11 to 16 as in the examples presented.

APPENDIX III.--NATURAL LOGARITHMS

Finding Natural Logarithms

Large logarithms and antilogs can be easily handled by hand if you have a natural logarithm table available for numbers from 0.1 to 10. First, divide the number by 10, 100, etc., to reduce it to a number we can find in the table. Then, find the natural log of the reduced number. Finally, add to that logarithm the natural logarithm of 10, 100, etc. The following tabulation will help the user:

ln 10 = ln 10^1 = 1 ln 10 = 2.303
ln 100 = ln 10^2 = 2 ln 10 = 4.605
ln 1000 = ln 10^3 = 3 ln 10 = 6.908
ln 10,000 = ln 10^4 = 4 ln 10 = 9.210
ln 100,000 = ln 10^5 = 5 ln 10 = 11.513.

As an example, suppose we wish to find the natural logarithm of 6,980.4--as in 8c, Appendix II. Because the largest number for which we can find the logarithm in our table is 10, we have to reduce the 6,980.4 by some factor, 10, 100, etc., to a number that we can handle. Logically, we divide by 1,000 obtaining 6.9804 that has a natural log from the table of 1.943. Now, to obtain the natural logarithm of 6,980.4 we must add to 1.943 the logarithm of 1,000. Because 1,000 = 10^3, the natural log of 1,000 = 3 ln 10 = 6.908. So the natural log of 6,980.4 = ln 6.9804 + 3 ln 10 = 1.943 + 6.908 = 8.851.

Finding Natural Antilogarithms

Finding the antilogarithm of a number that is larger than those in our table is somewhat more complicated. With our natural log table for numbers to 10, the largest number we can find the antilog of is 2.303

(which is ln 10). Again, we must reduce the large number, in this case now a logarithm, to a number less than 2.303, that is available in the table.

As an example, find the natural antilogarithm of 10.19 (from 8b, Appendix II). We have to divide 10.19 by some number (X) so that $10.19 \div X \leq 2.303$. If we divide 10.19 by 5, we get 2.038, that has a natural antilog, from the table, of 7.675. If we now multiply 7.675 times itself X - 1 times we will obtain the natural antilogarithm of 10.19, which is 26,631. Or, 7.675 X 7.675 X 7.675 X 7.675 X 7.675 = 26,631.

We could also have divided by any other number larger than 5, say 6. Then we would have $10.19 \div 6 = 1.698$, the antilog of which is 5.463. Multiply 5.463 times itself 5 times (6-1) to obtain the antilog of 10.19 = 26,582. Notice this does not agree with the example above due to rounding errors. The smaller value of X will mean less paperwork and more accurate estimates.

☆U.S. GOVERNMENT PRINTING OFFICE: 1975—668-748/97

Schlaegel, Bryce E.
1975. Estimating aspen volume and weight for individual trees, diameter classes, or entire stands. USDA For. Serv. Gen. Tech. Rep. NC-20, 16 p. North Cent. For. Exp. Stn., St. Paul, Minn.

Presents allometric volume and weight equations for Minnesota quaking aspen. Volume, green weight, and dry weight estimates can be made for wood, bark, and limbs on the basis of individual trees, diameter classes, or entire stands.

OXFORD: 524.2:176.1(*Populus tremuloides*). KEY WORDS: specific gravity, weight-yield, merchantable ratio estimates, top diameter, form.

Schlaegel, Bryce E.
1975. Estimating aspen volume and weight for individual trees, diameter classes, or entire stands. USDA For. Serv. Gen. Tech. Rep. NC-20, 16 p. North Cent. For. Exp. Stn., St. Paul, Minn.

Presents allometric volume and weight equations for Minnesota quaking aspen. Volume, green weight, and dry weight estimates can be made for wood, bark, and limbs on the basis of individual trees, diameter classes, or entire stands.

OXFORD: 524.2:176.1(*Populus tremuloides*). KEY WORDS: specific gravity, weight-yield, merchantable ratio estimates, top diameter, form.


AMERICAN REVOLUTION BICENTENNIAL
1776-1976
™