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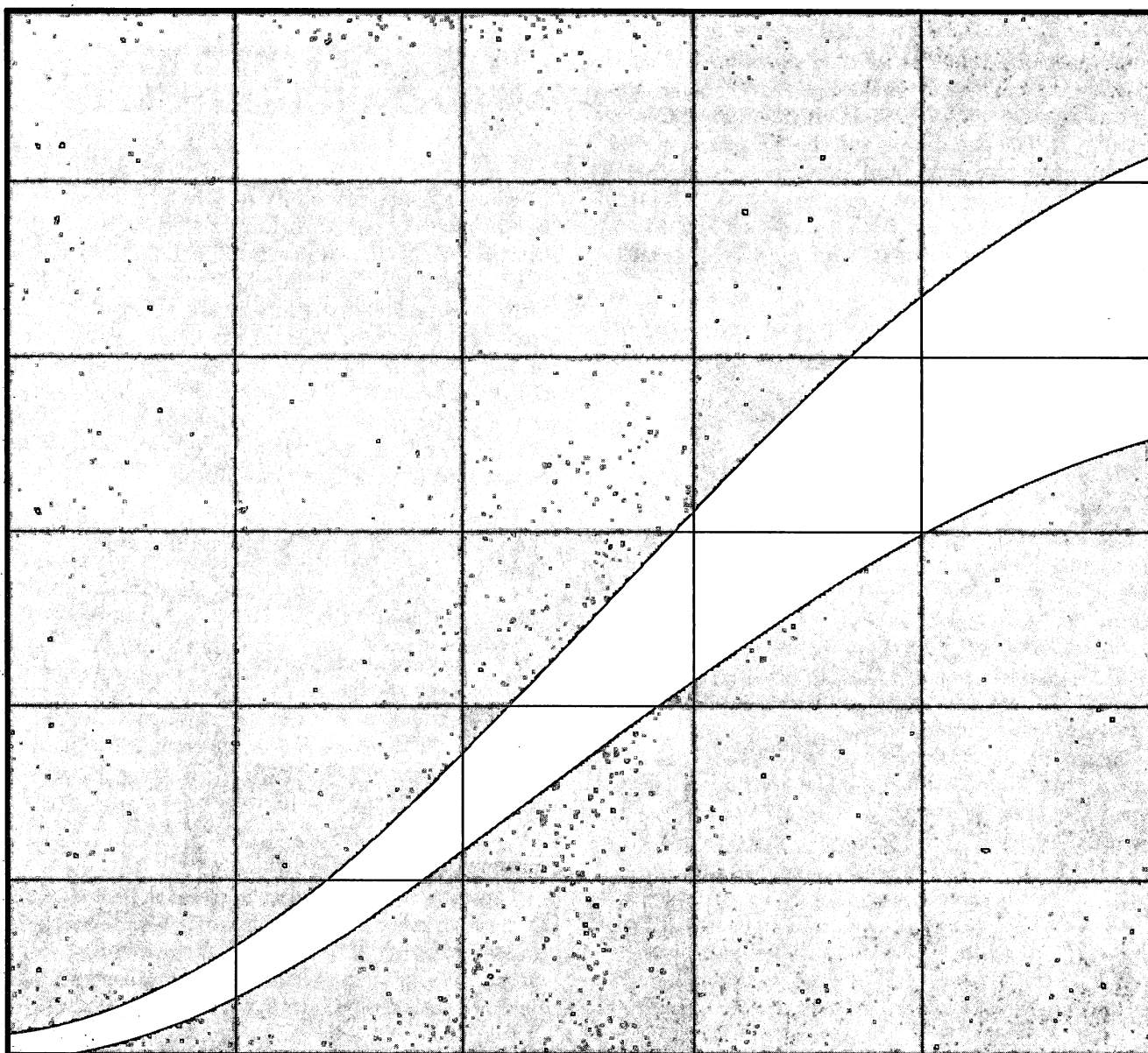
North Central
Forest Experiment
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General Technical
Report **NC-80**



A Net Volume Equation for Michigan's Upper & Lower Peninsulas

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A NET VOLUME EQUATION FOR MICHIGAN'S UPPER AND LOWER PENINSULAS

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A net merchantable tree volume equation and coefficients were developed for the Upper and Lower Peninsulas as part of the 1981 Michigan Forest Inventory (fig. 1). The equation is suitable for use on digital computers by foresters, land managers, researchers, and others in need of net volume-per-tree estimates over large forest areas. Both board-foot and cubic-foot volume can be estimated for most tree classes.

HOW THE EQUATION AND COEFFICIENTS WERE DEVELOPED

Equations developed by Robert N. Stone (see Appendix) based on Gevorkiantz and Olsen's (1955) composite volume tables for the Lake States were used to estimate the cubic-foot volume in each tree. Stone's equations compute gross volume in a tree from three measurements: diameter breast height (d.b.h.), merchantable height, and top diameter outside bark (t.d.o.b.). This estimated volume was then corrected for differences in bark thickness between species and the field estimate of cull volume was subtracted to arrive at net volume.

Experience in working with volume-d.b.h. relations suggests the following equation for this type of data:

$$V = A \cdot SI^B (1 - e^{-C \cdot DBH})^D$$

where the dependent variable V is net volume of the tree, and the independent variables SI and DBH are tree site index and diameter at breast height, respectively. The value of e is the constant base of the natural logarithm (2.71828). A , B , C , and D are the equation coefficients. The upper asymptote, $A \cdot SI^B$, is a function of site index. A general form of this function can be seen graphically in figure 2.

The equation coefficients were developed using field data collected on 12,361 trees in the Upper Peninsula and 9,780 trees in the Lower. Data included d.b.h., merchantable height, t.d.o.b., site index, and tree class. For sawtimber-size trees, measurements were

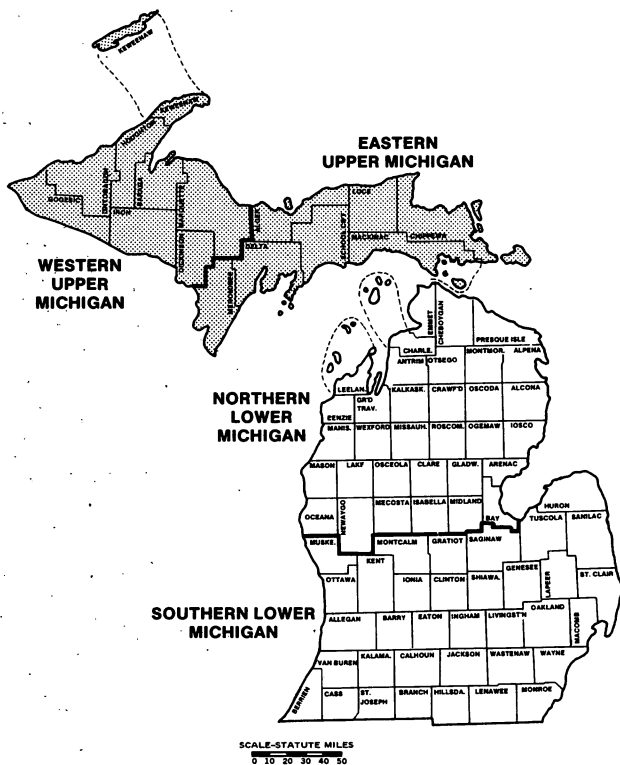


Figure 1.—Michigan's forest survey units.

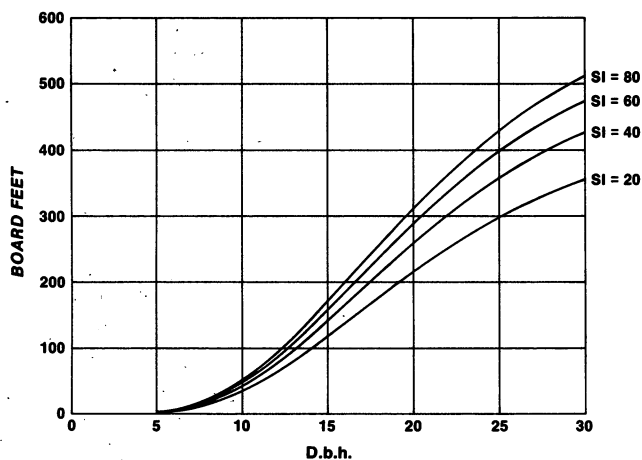


Figure 2.—Net board foot volume (International 1/4-inch rule) of desirable and acceptable basswood trees by d.b.h. and selected site index levels.

taken at both the sawtimber and poletimber limits of merchantability.¹ Field crews also estimated the volume of board foot and cubic foot cull in each tree (USDA Forest Service 1975).

Equation coefficients were developed for each of three tree class categories: "desirable and acceptable," "rough and short-log," and "rotten" (see Appendix for definitions). Equation coefficients were also developed for board-foot and cubic-foot volumes. The groupings of species for the equations were determined primarily by the number of observations and the similarity of bole form among species. Species for which no observations were available were assigned equations on the basis of bole form. (For species and species groups, see Appendix).

A computerized nonlinear regression program was used to fit the above equation to the tree data. The resulting coefficients and statistics for each regression are given in tables 1-7 for the Upper Peninsula and 8-14 for the Lower. Note that the standard error is the standard error of the regression.

¹The poletimber merchantability limits include trees 5.0 inches d.b.h. and over, from a 1-foot stump to a minimum of 4.0-inch t.d.o.b. or to the point where the central stem breaks into limbs. Dimensions for sawtimber are 9.0-inch d.b.h. to a minimum 7-inch t.d.o.b. for softwoods and 11.0-inch d.b.h. to a minimum 9.0-inch t.d.o.b. for hardwoods or to the point where the central stem breaks into limbs.

Because the average site index may be used when site index is unknown, the only other data needed to determine tree volumes using this model are species, d.b.h., and tree class. Net volume per tree is highly variable, so these equations should be applied to an adequate sample over a large forest area.

HOW TO USE THE EQUATION

1. Select the appropriate species group and coefficients from tables 1-14.
2. Substitute these coefficients in the above equation.
3. Solve the equation to obtain net volume.

For example, to compute the board foot volume of a 15-inch acceptable basswood tree with a site index of 60 in the Lower Peninsula, we would first obtain the coefficients for basswood from table 11. Substituting into the general net volume equation:

$$V = A \cdot SI^B (1 - e^{-C \cdot DBH})^D$$

$$V = (2076.0) (60)^{(.17993)} (1 - e^{-0.02575(15.0)})^{2.921}$$

$$V = 156 \text{ board feet}$$

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Table 1.--Net cubic foot volume regression coefficients for desirable and acceptable trees, Upper Peninsula, Michigan, 1981

Species group	Coefficient			Site index			D.b.h.			Observations Number	Standard error Cubic feet
	A	B	C	D	Range	Average	Range	Average			
Jack pine	4.325 x 10 ¹	.004942	0.06338	3.398	25-82	63	5-18	9	163	2.0	
Red pine	2.083 x 10 ²	.04348		3.281	43-80	62	5-23	12	104	4.1	
White pine	7.116 x 10 ⁶	.24602	.00052	2.780	30-82	56	5-32	16	287	15.7	
White spruce	7.942 x 10 ²	.08698	.02469	3.034	20-82	53	5-26	11	333	4.6	
Black spruce	2.398 x 10 ⁷	.14839	.00036	2.694	21-92	46	5-21	7	374	1.3	
Balsam fir	5.907 x 10 ⁶	.21264	.00056	2.716	19-92	54	5-28	8	898	1.8	
Hemlock	7.702 x 10 ⁶	.30587	.00027	2.564	43-95	71	5-34	14	516	7.3	
Tamarack	4.351 x 10 ¹	0	.16016	5.623	27-63	48	5-16	8	68	1.7	
N. white-cedar	8.297 x 10 ⁵	.05522	.00067	2.339	10-58	30	5-25	9	953	2.7	
Red oak	2.260 x 10 ²	.06018	.04078	3.068	43-87	63	5-27	12	161	3.6	
Basswood	2.208 x 10 ²	.12196	.03593	2.944	48-91	67	5-25	11	298	3.8	
Beech	1.156 x 10 ²	.23099	.03388	2.617	46-97	67	5-22	12	95	4.7	
Yellow birch	4.118 x 10 ⁶	.29140	.00021	2.301	30-83	61	5-36	13	522	6.2	
Hard maple	5.004 x 10 ²	.00742	.02825	2.758	20-85	63	5-28	11	2,272	4.7	
Soft maple	8.410 x 10 ¹	.13855	.05764	3.184	12-90	61	5-26	10	1,318	3.3	
Elm	1.398 x 10 ²	.43871	.01914	2.539	42-87	65	5-27	11	211	3.3	
Ash	2.692 x 10 ⁴	.14046	.00313	2.428	33-90	59	5-25	9	226	2.8	
Cottonwood- balsam poplar	7.257 x 10 ⁶	1.01628	.00018	2.837	22-96	67	5-33	12	165	7.2	
Aspen	1.577 x 10 ²	.21361	.03154	2.717	32-96	71	5-25	10	994	3.2	
Paper birch	7.256 x 10 ³	.10993	.00363	2.093	19-86	59	5-25	10	520	2.4	
Other hardwoods	1.444 x 10 ¹	.33869	.11817	4.489	43-87	62	5-19	9	83	2.0	

Table 2.--Net cubic foot volume regression coefficients for rough and short-log trees, Upper Peninsula, Michigan, 1981

Species group	Coefficient				Site index			D.b.h.		Observations Number	Standard error Cubic feet
	A	B	C	D	Range		Average				
Softwoods	1.218 x 10 ⁷	0.31779	0.00017	2.472	10-88	45	5-30	10	236	4.3	
Beech	2.133 x 10 ²	0	.04384	3.161	53-92	68	6-24	14	19	10.8	
Yellow birch	5.424 x 10 ¹	.71702	.01995	3.021	30-83	60	5-33	14	84	11.6	
Hard maple	2.914 x 10 ⁶	0	.00031	2.207	39-85	62	5-28	11	212	4.2	
Soft maple	4.162 x 10 ¹	.20572	.05977	3.073	31-84	60	5-25	11	121	4.4	
Aspen	1.492 x 10 ⁵	.45481	.00034	2.079	45-96	69	5-20	11	61	3.6	
Paper birch	2.697 x 10 ⁴	0	.00161	1.959	33-76	56	5-21	10	26	2.9	
Other hardwoods	7.614 x 10 ³	.44924	.10296	3.961	45-78	61	6-14	9	36	1.6	
Noncommercial species	4.543 x 10 ⁵	.51085	.00035	2.308	45-90	66	5-15	8	58	1.9	

Table 3.--Net cubic foot volume regression coefficients for rotten trees, Upper Peninsula, Michigan, 1981

Species group	Coefficient				Site index		D.b.h.		Observations Number	Standard error Cubic feet
	A	B	C	D	Range	Average	Range	Average		
Softwoods	1.835 x 10 ⁶	0.15210	0.00133	3.271	10-98	38	5-40	12	248	7.8
Beech	1.731 x 10 ⁴	1.29100	.00022	2.149	53-75	61	6-22	14	39	5.9
Yellow birch	5.915 x 10 ¹	0	.09740	5.555	40-83	60	5-37	17	124	13.8
Hard maple	1.058 x 10 ⁶	.73128	.00016	2.367	43-85	62	5-30	13	125	6.6
Soft maple	3.962 x 10 ⁴	1.42808	.00012	2.180	36-82	60	5-27	12	104	6.2
Aspen	9.412 x 10 ⁵	.13671	.00062	2.484	35-96	68	5-24	12	143	4.8
Paper birch	1.433 x 10 ¹	0	.28871	10.861	19-86	57	5-26	12	31	4.0
Other hardwoods	1.000 x 10 ³	.10882	.02847	4.891	14-94	60	5-33	11	53	6.2

Table 4.--Net board foot¹ volume regression coefficients for desirable and acceptable trees, Upper Peninsula, Michigan, 1981

Species group	Coefficient				Site index		D.b.h.		Observations Number	Standard error Cubic feet
	A	B	C	D	Range	Average	Range	Average		
Jack pine	1.686 x 10 ²	0.39511	0.05124	3.196	25-72	66	9-18	12	75	19.2
Red pine	2.969 x 10 ³	.25519	.01790	2.761	44-77	61	9-23	14	77	32.2
White pine	3.628 x 10 ⁷	.14236	.00083	2.973	30-82	56	9-35	17	260	98.1
White spruce	3.525 x 10 ⁴	0	.01109	2.866	20-82	54	9-26	13	215	36.0
Black spruce	2.894 x 10 ⁷	.06307	.00140	3.165	21-92	49	9-21	11	45	18.7
Balsam fir	2.798 x 10 ⁴	.33410	.00969	3.261	25-92	55	9-28	11	221	21.3
Hemlock	2.587 x 10 ⁶	.05437	.00139	2.636	43-95	72	9-34	15	442	52.6
Tamarack	6.298 x 10 ¹	.22695	.32850	26.891	27-59	52	9-16	11	17	12.1
N. white-cedar	5.756 x 10 ⁶	0	.00079	2.463	10-58	32	9-25	13	375	31.2
Red oak	1.007 x 10 ³	0	.06887	4.382	43-87	64	11-26	15	95	38.4
Basswood	2.014 x 10 ²	.26549	.11156	6.420	48-85	66	11-25	15	114	47.4
Beech	3.153 x 10 ²	.17511	.08716	4.435	46-97	67	11-22	14	53	47.9
Yellow birch	4.005 x 10 ²	.31603	.04587	3.418	36-83	60	11-36	16	303	63.3
Hard maple	7.117 x 10 ²	0	.09952	5.866	39-85	63	11-28	15	905	56.4
Soft maple	1.203 x 10 ²	.27194	.17342	11.676	38-84	61	11-26	14	431	90.9
Elm	1.211 x 10 ⁶	.57652	.00013	1.834	42-87	64	11-27	15	95	58.0
Ash	1.093 x 10 ⁷	.02244	.00082	2.564	33-90	58	11-25	14	64	37.6
Cottonwood-										
balsam poplar	1.594 x 10 ⁷	.75718	.00029	2.740	41-96	71	11-33	16	88	51.9
Aspen	3.933 x 10 ²	.19532	.06404	3.681	42-96	73	11-25	14	381	38.4
Paper birch	9.104 x 10 ⁶	0	.00017	1.882	23-86	58	11-25	14	144	28.8
Other hardwoods	8.070 x 10 ³	.63809	.00040	1.346	48-87	65	11-19	13	20	19.3

1International 1/4-inch rule.

Table 5.--Net board foot¹ volume regression coefficients for rough and short-log trees, Upper Peninsula, Michigan, 1981

Species group	Coefficient				Site index		D.b.h.		Observations Number	Standard error Cubic feet
	A	B	C	D	Range	Average	Range	Average		
N. white-cedar	1.910 x 10 ³	0	.00138	1.014	14-45	30	9-20	12	37	15.4
Softwoods ²	1.012 x 10 ⁷	0	0.00016	2.009	35-87	65	9-29	13	41	17.8
Yellow birch	4.006 x 10 ⁴	0	.00204	1.757	46-80	62	11-33	18	37	155.8
Hard maple	9.304 x 10 ¹	0.02006	.23417	16.842	39-85	64	11-21	15	63	28.9
Soft maple	3.015 x 10 ²	0	.06176	3.223	31-74	57	11-26	15	42	31.8
Aspen	1.357 x 10 ⁴	.77549	.00012	1.419	49-96	71	11-20	14	26	31.8
Other hardwoods	2.236 x 10 ²	.43694	.03429	3.519	33-92	65	11-24	15	40	31.2

¹International 1/4-inch rule.²Softwoods other than white-cedar.

Table 6.--Net cubic foot saw-log volume regression coefficients for desirable and acceptable trees, Upper Peninsula, Michigan, 1981

Species group	Coefficient				Site index		D.b.h.		Observations Number	Standard error Cubic feet
	A	B	C	D	Range	Average	Range	Average		
Jack pine	8.947 x 10 ⁰	0.53396	0.08549	4.216	25-72	66	9-18	12	75	2.7
Red pine	1.727 x 10 ²	.13296	.04191	3.166	44-77	61	9-23	14	77	4.7
White pine	2.797 x 10 ⁷	.15545	.00034	2.776	30-82	56	9-35	18	260	14.1
White spruce	7.649 x 10 ²	.02644	.02773	3.168	20-82	54	9-26	13	215	5.6
Black spruce	1.321 x 10 ⁷	.02868	.00090	3.047	21-92	49	9-20	11	45	3.1
Balsam fir	3.496 x 10 ²	.39522	.02444	3.503	25-92	55	9-28	11	221	3.2
Hemlock	1.374 x 10 ⁴	.09324	.00506	2.643	43-95	72	9-34	15	442	7.2
Tamarack	1.185 x 10 ¹	.22306	.29331	20.224	27-59	52	9-16	11	17	2.0
N. white-cedar	3.293 x 10 ²	.04549	.03184	3.132	10-58	32	9-24	13	375	4.3
Red oak	9.210 x 10 ¹	0	.11298	6.721	43-87	64	11-26	15	95	4.5
Basswood	3.254 x 10 ¹	.23508	.13247	8.230	48-85	66	11-25	15	114	6.3
Beech	5.942 x 10 ¹	.20844	.06226	3.507	46-97	67	11-22	14	53	7.1
Yellow birch	2.327 x 10 ²	.18111	.02860	2.971	36-83	60	11-36	16	303	8.3
Hard maple	1.157 x 10 ²	0	.09714	5.752	39-85	63	11-28	15	905	7.6
Soft maple	3.425 x 10 ¹	.18168	.14025	8.552	38-84	61	11-26	14	431	9.5
Elm	3.158 x 10 ⁵	.63848	.00010	1.886	42-87	64	11-27	15	95	9.4
Ash	2.361 x 10 ⁶	.08194	.00049	2.400	33-90	58	11-25	14	64	5.7
Cottonwood-										
balsam poplar	9.227 x 10 ⁶	.62755	.00017	2.606	41-96	71	11-33	16	88	7.1
Aspen	3.552 x 10 ¹	.25513	.08955	4.727	42-96	73	11-25	14	381	5.3
Other hardwoods	2.327 x 10 ²	.18111	.02860	2.971	36-83	60	11-36	16	303	8.3

Table 7.--Net cubic foot saw-log volume regression coefficients for rough and short-log trees, Upper Peninsula, Michigan, 1981

Species group	Coefficient			Site index		D.b.h.		Observations Number	Standard error Cubic feet
	A	B	C	Range	Average	Range	Average		
Softwoods	3.985 x 10 ⁴	0.07070	0.00175	14-45	30	9-20	12	37	1.9
Yellow birch	8.576 x 10 ⁵	.00850	.00048	46-80	62	11-33	18	37	16.8
Hard maple	2.639 x 10 ¹	.17348	.15635	39-85	64	11-21	15	63	6.3
Soft maple	5.080 x 10 ¹	0	.13449	31-74	57	11-26	15	42	5.0
Aspen	2.334 x 10 ⁰	1.20985	.01793	49-96	71	11-20	14	26	4.0
Other hardwoods	1.985 x 10 ⁵	0	.00182	33-92	65	11-24	15	40	4.2

Table 8.--Net cubic foot volume regression coefficients for desirable and acceptable trees, Lower Peninsula, Michigan, 1981

Species group	Coefficient			Site index		D.b.h.		Observations Number	Standard error Cubic feet
	A	B	C	Range	Average	Range	Average		
Jack pine	4.325 x 10 ¹	0.24942	0.06338	40-82	58	5-17	8	206	1.7
Red pine	1.143 x 10 ²	0	.07601	41-82	66	5-22	10	292	2.3
White pine	7.519 x 10 ⁶	.15234	.00054	34-86	57	5-31	15	195	16.3
White spruce	3.559 x 10 ²	0	.04443	25-68	48	5-19	10	111	4.6
Black spruce-tamarack	1.296 x 10 ⁷	.19981	.00043	22-87	45	5-14	7	160	1.3
Balsam fir	2.455 x 10 ⁷	0	.00059	25-77	54	5-18	7	343	1.3
Hemlock	6.908 x 10 ⁶	.57395	.00016	48-94	69	5-27	13	218	6.7
N. white-cedar	3.396 x 10 ⁵	0	.00100	15-58	32	5-23	8	975	1.9
Other softwoods	1.143 x 10 ²	0	.07601	56-75	64	5-12	7	34	1.0
White oak	5.598 x 10 ⁵	0	.00069	30-84	64	5-42	13	312	6.3
Red oak	2.835 x 10 ²	.04236	.04104	40-99	65	5-38	12	739	5.0
Hickory	6.638 x 10 ¹	.05243	.10201	50-83	66	5-29	11	50	2.2
Basswood	2.458 x 10 ⁵	.34244	.00072	46-95	70	5-25	12	307	4.5
Beech	6.180 x 10 ²	.12032	.01730	46-97	70	5-25	12	225	5.6
Yellow birch	4.095 x 10 ²	.22434	.01586	39-80	61	6-26	13	105	5.2
Hard maple	2.411 x 10 ²	.20816	.02332	34-98	68	5-31	10	1,136	3.9
Soft maple	3.455 x 10 ¹	.50730	.04165	27-95	66	5-50	11	891	5.0
Elm	2.574 x 10 ²	.05352	.03621	41-92	68	5-26	10	121	2.1
Ash	3.267 x 10 ²	.17118	.02287	22-97	68	5-38	10	366	2.6
Cottonwood	4.447 x 10 ³	.28134	.00686	63-97	78	7-36	16	52	9.5
Balsam poplar	8.655 x 10 ²	.01915	.02039	37-94	66	5-22	11	144	3.6
Aspen	3.140 x 10 ¹	.40269	.05728	39-99	72	5-27	10	1,039	2.9
Paper birch	5.045 x 10 ¹	.10011	.08744	19-90	62	5-18	9	363	2.0
Black cherry	1.155 x 10 ¹	.48724	.08182	39-93	71	5-33	11	105	4.5
Other hardwoods	9.051 x 10 ⁵	.05103	.00072	48-96	79	5-41	15	63	15.6

Table 9.--Net cubic foot volume regression coefficients for rough and short-log trees, Lower Peninsula, Michigan, 1981

Species group	Coefficient				Site index			D.b.h.		Observations Number	Standard error Cubic feet
	A	B	C	D	Range	Average	Range	Average			
N. white-cedar	2.407 x 10 ⁷	0	0.00016	2.352	17-58	30	5-20	9	107	2.3	
Softwoods ¹	1.218 x 10 ⁷	.29455	.00006	2.165	33-84	62	5-20	8	97	2.5	
Hardwoods	1.749 x 10 ³	.44290	.00403	2.238	31-99	66	5-43	12	436	7.9	
Noncommercial species	5.725 x 10 ²	.55858	.00457	2.150	35-99	69	5-15	8	61	2.3	

¹Softwoods other than white-cedar.

Table 10.--Net cubic foot volume regression coefficients for rotten trees, Lower Peninsula, Michigan, 1981

Species group	Coefficient				Site index		D.b.h.		Observations Number	Standard error Cubic feet
	A	B	C	D	Range	Average	Range	Average		
N. white-cedar	1.835 x 10 ⁶	0.10408	0.00008	1.870	18-58	32	5-30	10	2.8	
Softwoods ¹	1.218 x 10 ⁷	.29455	.00006	2.165	34-98	61	5-40	14	36.6	
Hardwoods	6.170 x 10 ¹	.35651	.02913	2.816	19-99	65	5-47	13	8.4	

¹Softwoods other than white-cedar.

Table 11.--Net board foot¹ volume regression coefficients for desirable and acceptable trees, Lower Peninsula, Michigan, 1981

Species group	Coefficient				Site index		D.b.h.		Observations	Standard error
	A	B	C	D	Range		Average			
					Number	Cubic feet				
Jack pine	2.507 x 10 ²	0	0.14611	6.309	40-75	59	9-17	11	57	13.2
Red pine	3.449 x 10 ²	0	.14728	7.536	41-82	62	9-22	13	132	22.9
White pine	7.435 x 10 ⁶	.18431	.00133	2.960	34-86	57	10-31	17	158	116.1
White spruce	1.232 x 10 ⁴	0	.01818	3.025	26-68	47	9-19	13	63	34.4
Black spruce-tamarack	1.137 x 10 ⁶	.46220	.00342	3.471	34-87	49	9-14	11	27	10.0
Balsam fir	1.212 x 10 ³	0	.08642	6.011	25-69	53	9-18	11	48	18.7
Hemlock	7.805 x 10 ²	.37007	.03753	3.990	48-94	68	9-27	15	158	55.8
N. white-cedar	5.756 x 10 ⁶	0	.00075	2.437	17-58	33	9-23	12	271	26.4
White oak	1.901 x 10 ³	.13500	.02718	2.949	45-84	67	11-42	18	166	59.4
Red oak	1.110 x 10 ³	0	.07531	5.081	40-99	67	11-38	16	372	49.9
Hickory	3.183 x 10 ²	0	.21096	14.474	58-73	65	11-20	15	25	24.9
Basswood	2.076 x 10 ³	.17993	.02575	2.921	46-95	71	11-29	15	134	42.5
Beech	4.626 x 10 ²	0	.14504	8.794	46-97	69	11-25	15	115	53.7
Yellow birch	7.302 x 10 ²	0	.08150	4.721	47-80	62	11-26	15	67	46.1
Hard maple	6.809 x 10 ²	.07082	.06900	4.079	46-98	69	11-31	15	405	50.6
Soft maple	2.571 x 10 ²	.43970	.03910	2.950	31-95	69	11-50	16	348	105.3
EJm	1.513 x 10 ²	.37200	.07717	4.435	41-92	67	11-26	14	36	29.2
Ash	1.073 x 10 ³	.05396	.05372	3.748	42-97	74	11-38	15	106	34.1
Cottonwood	9.202 x 10 ⁴	0	.00581	2.572	63-97	78	12-36	18	42	75.1
Balsam poplar	1.970 x 10 ⁶	.17284	.00017	1.708	45-94	67	11-22	15	64	81.0
Aspen	1.997 x 10 ²	.21261	.12888	7.269	43-99	74	11-27	14	373	35.5
Paper birch	9.296 x 10 ⁶	0	.00024	1.995	23-81	64	11-18	13	71	27.3
Black cherry	2.557 x 10 ¹	.46049	.44366	148.962	39-93	76	11-33	15	36	44.1
Other hardwoods	3.651 x 10 ⁶	.01140	.00106	2.475	48-96	80	11-41	18	42	120.4

¹International 1/4-inch rule.Table 12.--Net board foot¹ volume regression coefficients for rough and short-log trees, Lower Peninsula, Michigan, 1981

Species group	Coefficient				Site index		D.b.h.		Observations	Standard error
	A	B	C	D	Range	Average	Range	Average		
					Number	Cubic feet				
N. white-cedar	6.854 x 10 ²	0.21868	0.00182	1.025	19-48	30	9-20	12	39	12.2
Softwoods ²	6.854 x 10 ²	.21868	.00182	1.025	35-84	59	9-20	12	25	22.6
Hardwoods	6.622 x 10 ⁴	0	.00244	2.132	31-99	68	11-40	17	132	66.0

¹International 1/4-inch rule.²Softwoods other than white-cedar.

Table 13.--Net cubic foot saw-log volume regression coefficients for desirable and acceptable trees, Lower Peninsula, Michigan, 1981

Species group	Coefficient				Site index		D.b.h.		Observations Number	Standard error Cubic feet
	A	B	C	D	Range		Average			
					Range	Average	Range	Average		
Jack pine	7.341 x 10 ¹	0	0.11096	5.616	40-75	59	9-17	11	57	2.5
Red pine	6.309 x 10 ¹	0	.12942	6.202	41-82	62	9-22	13	132	3.1
White pine	1.619 x 10 ⁵	.14896	.00231	2.786	34-86	57	10-31	17	158	16.2
White spruce	2.260 x 10 ²	0	.06044	4.051	26-68	47	9-19	13	63	5.6
Black spruce-tamarack	2.006 x 10 ⁴	.48355	.00726	3.611	34-87	49	9-14	11	27	2.1
Balsam fir	1.883 x 10 ²	0	.09892	6.826	25-69	53	9-18	11	48	2.6
Hemlock	7.393 x 10 ¹	.43473	.03939	3.775	48-94	68	9-27	15	158	7.3
N. white-cedar	4.333 x 10 ³	0	.00821	2.534	17-58	33	9-23	12	271	3.4
White oak	5.830 x 10 ²	0	.02283	2.673	45-84	67	11-42	18	166	8.2
Red oak	1.700 x 10 ²	0	.07359	4.831	40-99	67	11-38	16	372	7.4
Hickory	4.403 x 10 ¹	0	.27354	28.212	58-73	65	11-20	15	25	4.1
Basswood	8.881 x 10 ⁴	.30682	.00108	2.287	46-95	71	11-29	15	134	6.3
Beech	9.970 x 10 ¹	0	.09524	5.123	46-97	69	11-25	15	115	7.8
Yellow birch	1.252 x 10 ²	0	.07606	4.522	47-80	62	11-26	15	67	6.2
Hard maple	1.042 x 10 ²	.07535	.06888	4.024	46-98	69	11-31	15	405	6.8
Soft maple	5.000 x 10 ¹	.41519	.03792	2.982	31-95	69	11-50	16	348	11.3
Elm	5.794 x 10 ⁵	.25782	.00031	2.111	41-92	67	11-26	14	36	4.4
Ash	1.925 x 10 ²	.18900	.03068	2.935	42-97	74	11-38	15	106	4.9
Cottonwood	1.841 x 10 ⁷	0	.00021	2.347	63-97	78	12-36	18	42	10.1
Balsam poplar	4.189 x 10 ⁶	.07181	.00013	1.982	45-94	67	11-22	15	64	13.1
Aspen	1.882 x 10 ¹	.32999	.13786	8.007	43-99	74	11-27	14	373	4.8
Paper birch	3.256 x 10 ¹	0	.21722	12.303	23-81	64	11-18	13	71	4.0
Black cherry	2.950 x 10 ⁰	.64209	.17603	9.090	39-93	76	11-33	15	36	6.7
Other hardwoods	7.655 x 10 ⁵	.06225	.00065	2.308	48-96	80	11-41	18	42	16.2

Table 14.--Net cubic foot saw-log volume regression coefficients for rough and short-log trees, Lower Peninsula, Michigan, 1981

Species group	Coefficient				Site index		D.b.h.		Observations Number	Standard error Cubic feet
	A	B	C	D	Range	Average	Range	Average		
N. white-cedar	1.597 x 10 ⁷	0	0.00028	2.563	19-48	30	9-20	39	2.0	
Softwoods ¹	1.597 x 10 ⁷	0	.00028	2.563	35-84	59	9-20	25	5.2	
Hardwoods	8.525 x 10 ¹	.34516	.02714	3.028	31-99	68	11-40	132	10.2	

¹Softwoods other than white-cedar.

APPENDIX

VOLUME EQUATIONS

Stone's cubic foot volume equation uses the following dependent variables: diameter at breast height (D), merchantable height (H), and diameter outside bark at merchantable height (T) to estimate cubic foot volume (CV).

$$CV = (3.0086 \times 10^{-3} + S1 + S2 + S3 + S4 + S5 + S6 + S7 + S8) \cdot 79.0$$

where

$$S1 = (2.0355 \times 10^{-3})D$$

$$S2 = (-3.0018 \times 10^{-3})T$$

$$S3 = (6.2381 \times 10^{-5})D^2$$

$$S4 = (2.5705 \times 10^{-5})D^2H$$

$$S5 = (-7.0090 \times 10^{-6})H^2$$

$$S6 = (3.6708 \times 10^{-5})H \cdot T^2$$

$$S7 = (8.1400 \times 10^{-10})D^2 \cdot H^3$$

$$S8 = (-1.90000 \times 10^{-9})D^2 \cdot H^2 \cdot T$$

Stone's board foot volume equation as modified by Hahn (1975, 1976) is as follows:

$$BV = 17.7488 + 7.3846 \cdot CV - 2.3523 \cdot D - 0.89945 \cdot H + 2.0726 \cdot T$$

where

BV = board foot volume (International 1/4-inch rule).

SPECIES GROUPS AND SPECIES FOR MICHIGAN TREES

Species group	Scientific names of included species
SOFTWOODS	
Jack pine	<i>Pinus banksiana</i>
Red pine	<i>Pinus resinosa</i>
White pine	<i>Pinus strobus</i>
White spruce	<i>Picea glauca</i>
Black spruce	<i>Picea mariana</i>
Balsam fir	<i>Abies balsamea</i> var. <i>balsamea</i>
Hemlock	<i>Tsuga canadensis</i>
Tamarack	<i>Larix laricina</i>
Northern white-cedar	<i>Thuja occidentalis</i>
Other softwoods	<i>Pinus sylvestris</i> <i>Pinus nigra</i> <i>Juniperus virginia</i> <i>Larix decidua</i> <i>Picea abies</i> <i>Pseudotsuga menziesii</i>
HARDWOODS	
White oak	<i>Quercus alba</i> <i>Quercus bicolor</i> <i>Quercus macrocarpa</i> <i>Quercus muehlenbergii</i>

Red oak	<i>Quercus rubra</i> <i>Quercus velutina</i> <i>Quercus palustris</i> <i>Quercus ellipsoidalis</i> <i>Carya cordiformis</i> <i>Carya ovata</i> <i>Carya glabra</i> <i>Tilia americana</i> <i>Fagus grandifolia</i> <i>Betula alleghaniensis</i> <i>Acer nigrum</i> <i>Acer saccharum</i> <i>Acer rubrum</i> var. <i>rubrum</i> <i>Acer saccharum</i> <i>Ulmus americana</i> <i>Ulmus rubra</i> <i>Ulmus thomasii</i> <i>Fraxinus americana</i> <i>Fraxinus nigra</i> <i>Fraxinus pennsylvanica</i> <i>Populus deltoides</i> <i>Populus balsamifera</i> <i>Betula papyrifera</i> var. <i>papyrifera</i> <i>Populus grandidentata</i> <i>Populus tremuloides</i> <i>Prunus serotina</i> <i>Acer negundo</i> <i>Betula nigra</i> <i>Celtis occidentalis</i> <i>Juglans cinerea</i> <i>Juglans nigra</i> <i>Salix nigra</i> <i>Liriodendron tulipifera</i>
Hickory	
Basswood	
Beech	
Yellow birch	
Hard maple	
Soft maple	
Elm	
Ash	
Cottonwood	
Balsam poplar	
Paper birch	
Aspen	
Black cherry	
Other hardwoods	
Noncommercial species	

METRIC EQUIVALENTS

1,000 board feet (International 1/4-inch log rule)
= 3.48 cubic meters.

Breast height = 1.37 meters above the ground.

1 cubic foot = 0.0283 cubic meter.

1 foot = 30.48 centimeters or 0.3048 meter.

1 inch = 25.4 millimeters or 2.54 centimeters or 0.0254 meter.

DEFINITION OF TERMS (USDA Forest Service 1975)

Desirable trees.—Live trees that have no serious defects that limit present or prospective use and that have relatively high vigor and have no visible signs of pathogens that may result in death or serious deterioration before rotation age. They would be favored by forest managers in silvicultural operations.

Acceptable trees.—Live trees having no serious defects that limit present or prospective use but that have pathogens or damage that may affect quality.

Rough trees.—Live trees that do not contain at least one merchantable 12-foot log, now or prospectively, because of roughness, poor form, or non-commercial species.

Rotten trees.—"Rough" trees in which more than 50 percent of the cull volume is rotten.

Short-log trees.—Live sawtimber-size trees that contain at least one merchantable 8- to 11-foot saw log but not a 12-foot saw log, now or prospectively.

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A net volume equation for Michigan's Upper and Lower Peninsulas. Gen. Tech. Rep. NC-80. St. Paul, MN: U.S. Department of Agriculture, Forest Service, North Central Forest Experiment Station; 1982. 12 p.

Describes a volume equation for Michigan's Upper and Lower Peninsulas developed as part of the 1981 Michigan Forest Inventory. Equation coefficients are presented by species groupings for both cubic-foot and board-foot volumes for three tree categories.

KEY WORDS: Inventory, computer, growing stock, cull, cubic foot, board foot.

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