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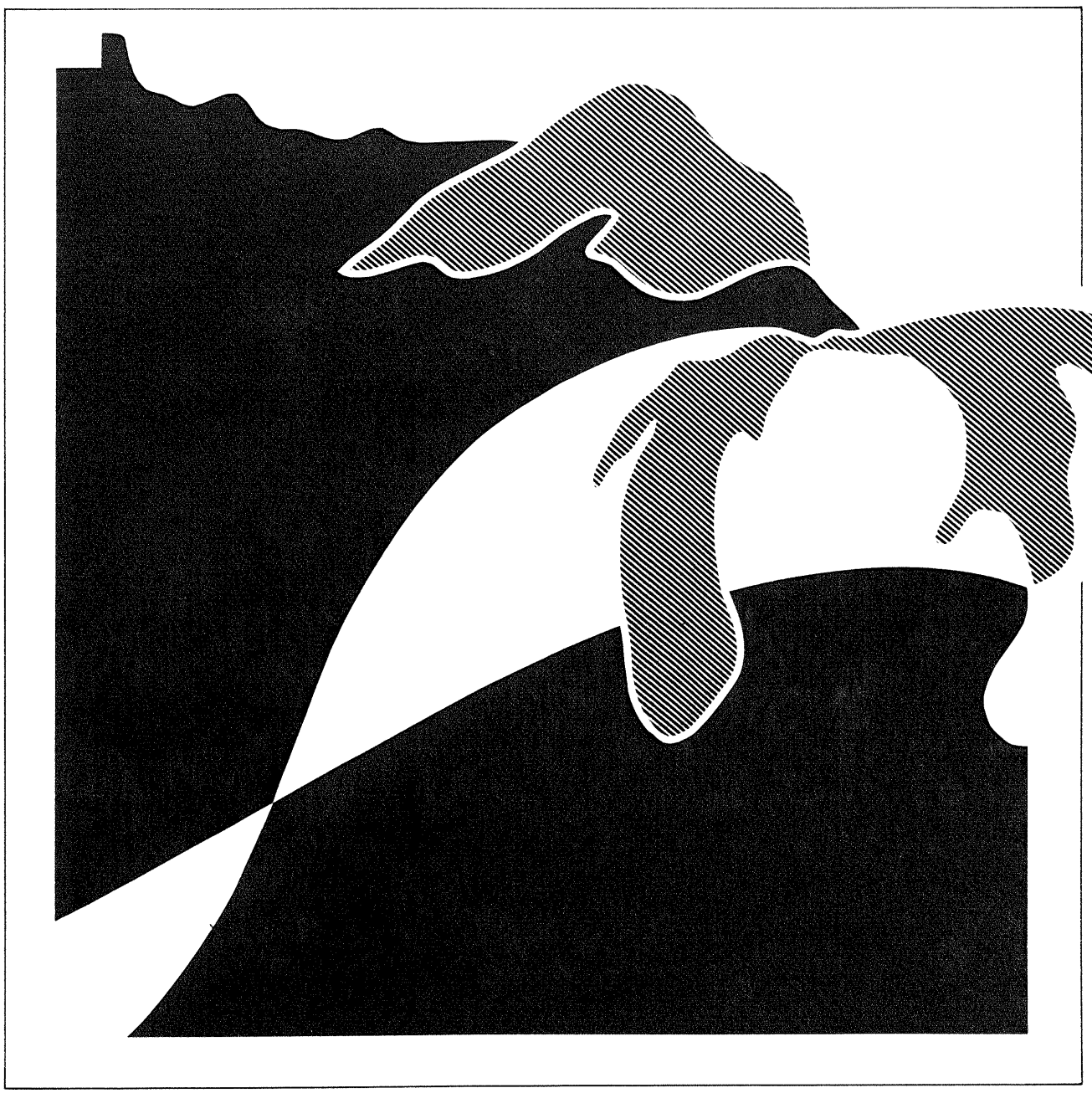

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LAKE STATES SITE INDEX CURVES FORMULATED

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Site index is the commonly used method of estimating site quality in the United States. For most species in the eastern United States, site index is the average total height of dominant and codominant trees at a total age of 50 years. Height and age values from suitable site trees are related to site index curves for each species, and then interpolations or extrapolations are made to estimate site index. However, such graphical procedures are tedious and subject to error. More rapid, convenient, and accurate estimates of site index are possible using site index formulae with computers or with programmable hand-held calculators.

Site index formulae have been published for 11 Lake States site index curves (Lundgren and Dolid 1970). New site index curves based on stem analyses are now available for 12 species in northern hardwood stands (Carmean 1978). Among these are aspen and paper birch site curves that supersede those formulated by Lundgren and Dolid. In addition, we have converted the site curves for balsam fir and white spruce from breast height age to total age (Carmean and Hahn 1981a). Because of these changes, we have summarized the formulations for 21 site index curves now recommended for use in the Lake States (table 1). The formulations we have computed use a single equation model and are convenient for computers or for field use with programmable hand calculators.

Many formulations are based on the anamorphic form of the Richards (1959) growth functions, (Lundren and Dolid 1970; Carmean 1971, 1972; Monserud and Ek 1976).

This function can easily be solved for site index but is suited only to anamorphic site index curves. But stem analyses for many tree species show that tree height growth patterns and site index curves are usually polymorphic rather than anamorphic (Stage 1963, Carmean 1970). Therefore, accurate site index estimations require height growth models that describe these polymorphic patterns of tree height growth. Ek (1971), Payandeh (1974a, 1974b), and Monserud and Ek (1976) have suggested a more general and descriptive form of the Richards function that expresses polymorphic height growth.

$$H = b_1 S^{b2} [1-e^{b3 A}]^{b4 S^{b5}} \qquad (1)$$

where H = height, S = site index, A = age, e = base of the natural logarithms, and the b_i are regression parameters.

We chose this model to estimate tree heights, given age and site index, for all the Lake States species in this paper. Parameters for using this model are given in table 2.

Model (1) cannot be easily solved for site index, but an analogue of it suggested by Payandeh (1974b) provides a form for computing site index directly. Curves computed with this model may not pass exactly through the index tree height specified at the index age of 50 years, but this problem can be solved by using a weighted regression with a weight of [50-2 | AGE-50 |]. This procedure ensures close agreement at 50 years but with a slightly poorer yet acceptable fit at the extremes of age and site index. The following general formula was used for estimating site index, given age and height, for the Lake States species. Parameters for using this model are given in table 3.

$$S = c_1 H^{c2} [1-e^{c3 A}]^{c4 H^{c5}} \qquad (2)$$

where S, H, A, and e are as above and c_i are regression parameters.

Table 1.—*Species and data sources used in developing height and site index functions.*

Species[1]	Age range	Site index range	Data source
	Years	*Feet*	
Balsam fir (*Abies balsamea* (L.) Mill.)	20-90	20-70	Carmean and Hahn (1981a)
Red maple (*Acer rubrum* L.)	20-85	45-75	Carmean (1978)
Sugar maple (*Acer saccharum* Marsh)	20-95	45-75	Carmean (1978)
Yellow birch (*Betula alleghaniensis* Britton)	20-95	40-75	Carmean (1978)
Paper birch (*Betula papyrifera* Marsh)	20-100	45-70	Carmean (1978)
American beech (*Fagus grandifolia* Ehrh.)	20-95	45-60	Carmean (1978)
White ash (*Fraxinus americana* L.)	20-80	45-85	Carmean (1978)
Black ash (*Fraxinus nigra* L.)	20-100	50-80	Carmean (1978)
Tamarack (*Larix laricina* (DuRoi) K. Koch)	20-120	20-60	Gevorkiantz (1957e)
White spruce (*Picea glauca* (Moench.) Voss	20-130	20-70	Carmean and Hahn (1981a)
Black spruce (*Picea mariana* (Mill.) B.S.P.	20-120	20-60	Gevorkiantz (1957b)
Jack pine (*Pinus banksiana* Lamb.)	20-80	30-70	Gevorkiantz (1956)
Red Pine (*Pinus resinosa* Ait.)	20-120	40-70	Gevorkiantz (1957d)
White pine (*Pinus strobus* L.)	20-120	40-80	Gevorkiantz (1957c)
Aspen (*Populus* spp.)	20-75	55-85	Carmean (1978)
Black cherry (*Prunus serotina* Ehrh.)	20-100	40-80	Carmean (1978)
White oak (*Quercus alba* L.)	1-100	40-80	Carmean and Hahn (1981b)
Northern red oak (*Quercus rubra* L.)	20-100	50-75	Carmean (1978)
Northern white-cedar (*Thuja occidentialis* L.)	20-100	20-60	Gevorkiantz (1957a)
American basswood (*Tilia americana* L.)	20-95	45-80	Carmean (1978)
American elm (*Ulmus americana* L.)	20-100	45-80	Carmean (1978)

[1]Scientific names after Little (1953).

Table 2.—*Parameters for a height growth equation expressing height as a function of site index and age for published site index curves*[1]

Species	Parameters					R^2	Standard error of mean (unweighted)	Maximum difference[2]
	b_1	b_2	b_3	b_4	b_5			
Balsam fir	2.0901	0.9296	-0.0280	2.8280	-0.1403	.99	0.54	1.7
Red maple	2.9435	.9132	-.0141	1.6580	-1.1095	.99	0.49	2.0
Sugar maple	6.1308	.6904	-.0195	10.1563	-.5330	.99	1.29	5.3
Yellow birch	6.0522	.6768	-.0217	15.4232	-.6354	.99	1.29	5.0
Paper birch	2.4321	.9207	-.0168	1.5297	-.1042	.99	1.06	4.2
American beech	29.7300	.3631	-.0127	16.7616	-.6804	.99	1.62	1.3
White ash	4.1492	.7531	-.0269	14.5384	-.5811	.99	1.37	5.1
Black ash	4.2286	.7857	-.0178	4.6219	-.3591	.99	.70	2.4
Tamarack	1.5470	1.0000	-.0225	1.1129	-.0000	.99	.52	1.4
White spruce	10.8738	.5529	-.0343	34.6880	-.6139	.99	2.33	7.2
Black spruce	1.7620	1.0000	-.0201	1.2307	.0000	.99	.52	1.9
Jack pine	1.6330	1.0000	-.0223	1.2419	.0000	.99	.50	1.1
Red pine	1.8900	1.0000	-.0198	1.3892	.0000	.99	.64	1.4
White pine	1.9660	1.0000	-.0240	1.8942	.0000	.99	.66	1.7
Aspen	11.4804	.5039	-.0291	105.9678	-1.0590	.99	1.31	4.9
Black cherry	5.0844	.6974	-.0250	20.7996	-.7114	.99	1.62	5.5
White oak	4.5437	.8170	-.0128	2.1089	-.1770	.98	4.25	([3])
Northern red oak	6.1785	.6619	-.0241	25.0185	-.7400	.99	1.31	4.9
Northern white-cedar	1.9730	1.0000	-.0154	1.0875	.0000	.99	.66	1.7
American basswood	4.7633	.7576	-.0194	6.5110	-.4156	.99	.70	2.7
American elm	6.4362	.6827	-.0194	10.9767	-.5477	.99	1.12	2.7

[1]$\text{Height} = b_1 S^{b2} [1-e^{b3 A}]^{b4 S^{b5}}$

S = Site index, A = Age, e = base of the natural logarithms.

[2]Maximum difference between site index estimated by the equation and the site index listed in published curves.

[3]Value not calculated because model was fitted to raw data rather than to site index curves.

Table 3.—*Parameters for a site index equation expressing site index as a function of height and age for published site index curves*[1]

Species	Parameters					R^2	Standard error of mean (unweighted)	Maximum difference[2]
	c_1	c_2	c_3	c_4	c_5			
Balsam fir	0.2198	1.1644	-0.0110	-2.0364	-0.1775	.0.99	1.10	3.1
Red maple	.3263	1.0634	-.0106	-1.2573	-.0646	.99	1.99	2.2
Sugar maple	.1984	1.2089	-.0110	-2.4917	-.2542	.98	1.90	6.7
Yellow birch	.1817	1.2430	-.0110	-3.0184	-.3180	.99	1.25	4.5
Paper birch	.5119	1.0229	-.0167	-1.0284	-.0049	.99	1.07	4.3
American beech	.2376	1.1312	-.0109	-1.8550	-.1430	.99	1.99	6.5
White ash	.1728	1.2560	-.0110	-3.3605	-.3452	.99	1.99	9.5
Black ash	.2388	1.1583	-.0102	-1.8455	-.1833	.99	0.51	3.4
Tamarack	.6464	1.0000	-.0225	-1.1129	.0000	.99	0.52	1.4
White spruce	.0833	1.3965	-.0196	-8.0895	-.3659	.98	3.22	9.8
Black spruce	.5675	1.0000	-.0201	-1.2307	.0000	.99	0.72	1.9
Jack pine	.6124	1.0000	-.2233	-1.2419	.0000	.99	0.50	1.1
Red pine	.5291	1.0000	-.0198	-1.3892	.0000	.99	0.64	1.4
White pine	.5086	1.0000	-.0240	-1.8942	.0000	.99	0.66	1.7
Aspen	.1271	1.3330	-.0100	-4.8354	-.4568	.95	2.75	11.6
Black cherry	.1738	1.2707	-.0110	-3.5467	-.3823	.98	2.24	7.2
White oak	.4455	0.9895	-.0107	-0.9656	-.0004	.79	5.47	[3]
Northern red oak	.1692	1.2648	-.0110	-3.4334	-.3557	.97	2.09	7.8
Northern white-cedar	.5068	1.0000	-.0154	-1.0895	.0000	.99	0.66	1.7
American basswood	.1921	1.2010	-.0100	-2.3009	-.2331	.99	1.90	4.5
American elm	.1898	1.2186	-.0110	-2.6865	-.2717	.99	2.05	6.6

[1]Site = $c_1 H^{c2} [1-e^{c3 A}]^{C4 H^{c5}}$
H = Total Height, A = Age, e = base of the natural logarithms.
[2]Maximum difference between site index estimated by the equation and the site index listed in published curves.
[3]Value not calculated because curves were fitted to raw data rather than to curved data.

The recommended site index curves used as data sources for 21 Lake States tree species are listed in table 1; the range of ages and the range of site indices for each of these species are also listed. No new site curve information is available for jack pine, red pine, white pine, black spruce, tamarack, and northern white-cedar. Therefore, our formulations for these species are based on the original Gevorkiantz site index curves and the parameters computed for these species by Lundgren and Dolid (1970), adding the values of 1.0 and 0.0 for parameters b_2 and b_5 respectively. The white oak curves are based on individual tree stem analyses gathered by Carmean (1971, 1972) in the Central States. We consider that these curves also apply to white oak in the Lake States. These original oak site index curves have been revised (Carmean and Hahn 1981b).

For the northern hardwood species (Carmean 1978) and for balsam fir and white spruce (Carmean and Hahn 1981a), data points for fitting the models were read directly from published site index curves over the ranges of ages and site indices indicated in table 1. Models (1) and (2) were then fitted to these readings using the weighted non-linear least squares procedures and the weights as described above. Table 2 lists the parameters and fit statistics for model (1) and table 3 lists the parameters and fit statistics for model (2). Each table lists the maximum difference between site index estimated by the equation and site index listed in the published site curves. These differences usually occur for the oldest trees on the best sites.

When these equations are used with a programmable calculator to estimate site index from age and total height, the parameters (table 3) and a simple program for solving the equation are entered and then stored. Estimates of site index can then be obtained by manually entering the species code, total height, and total age.

The standard precautions in using site index curves also apply in using these site index equations: site trees should be at least 20 years old and should be free growing, uninjured, dominant and codominant trees. Such trees commonly occur in well stocked, even-aged stands that have not been disturbed by past cutting. We recommend that increment cores for tree age be taken at breast height. Breast height age and total height should then be compared to the site index curves to estimate site class. D.b.h. age can be converted to total age by using the number

Table 4.—*Years to reach breast height by site index class*

	All classes	Site index class							
		20	30	40	50	60	70	80	90
		Years							
Balsam fir	—	15	15	13	11	10	9	8	8
Red maple	4	—	—	—	—	—	—	—	—
Sugar maple	4	—	—	—	—	—	—	—	—
Yellow birch	4	—	—	—	—	—	—	—	—
Paper birch	4	—	—	—	—	—	—	—	—
American beech	4	—	—	—	—	—	—	—	—
White ash	4	—	—	—	—	—	—	—	—
Black ash	4	—	—	—	—	—	—	—	—
Tamarack	—	12	10	7	5	5	5	5	5
White spruce	—	15	15	13	11	10	9	8	8
Black spruce	—	15	13	11	10	9	9	9	9
Jack pine	—	—	9	8	7	6	5	4	4
Red pine	—	—	10	8	6	5	4	4	4
White pine	—	—	12	12	12	10	8	6	5
Aspen	4	—	—	—	—	—	—	—	—
Black cherry	4	—	—	—	—	—	—	—	—
White oak	4	—	—	—	—	—	—	—	—
Northern red oak	4	—	—	—	—	—	—	—	—
Northern white-cedar	—	20	15	15	10	10	10	10	10
American basswood	4	—	—	—	—	—	—	—	—
American elm	4	—	—	—	—	—	—	—	—

of years specified by site class. Table 4 summarizes these values that are given in the original publications and/or that are used by the forest survey field crews. The resulting total age and total height values are then used to graphically estimate site index from the curves or to estimate site index with the formulae.

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Lake States site index curves formulated. Gen. Tech. Rep. NC-88. St. Paul, MN: U.S. Department of Agriculture, Forest Service, North Central Forest Experiment Station; 1982. 5 p.

Functions and parameters are given for computing site index for 21 Lake States tree species. Functions and parameters also are given for computing total height of dominant and codominant trees.

KEY WORDS: Equations, functions, computerized, tree height.

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