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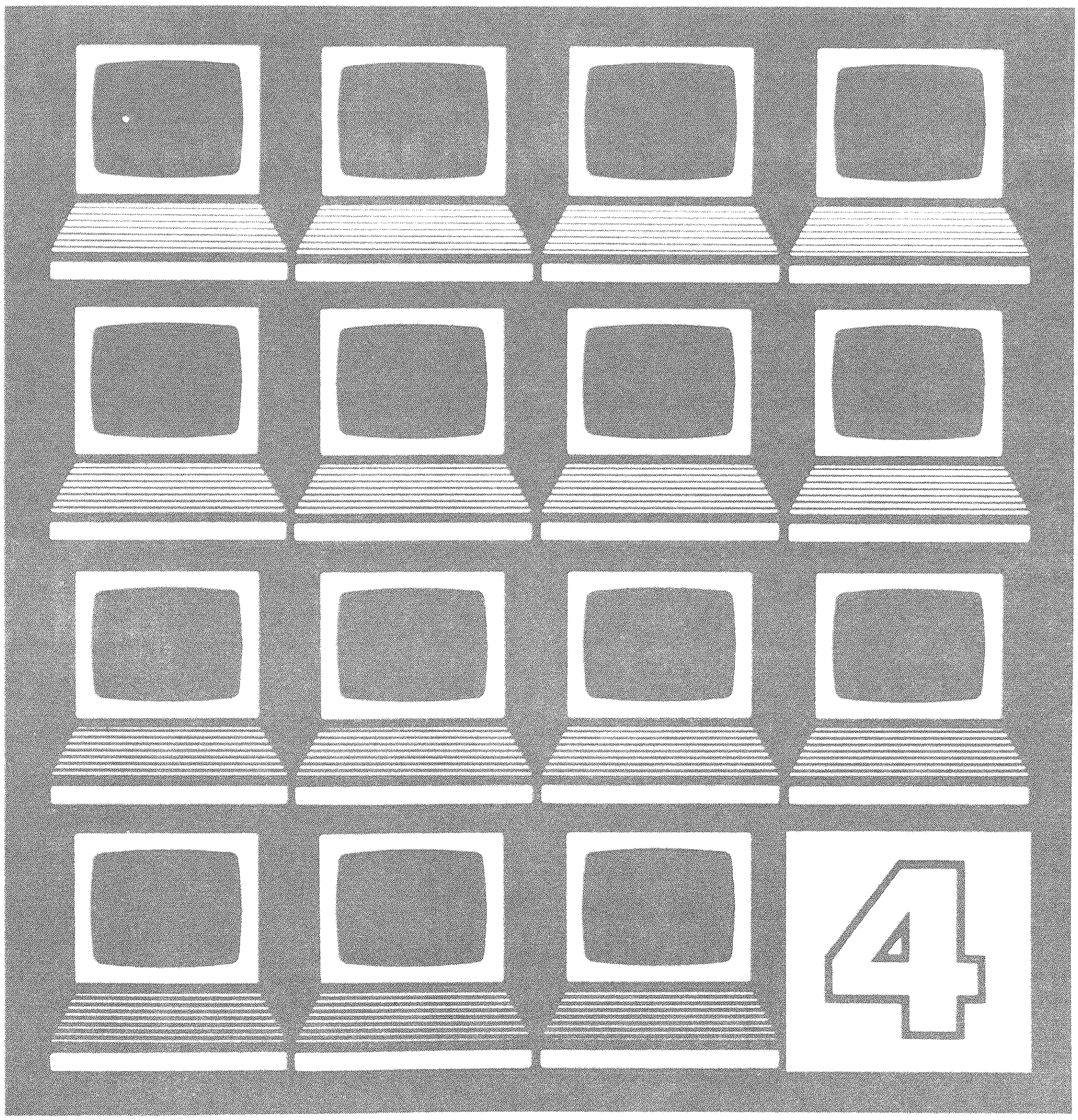
The Microcomputer Scientific Software Series 4

Testing Prediction Accuracy

Harold M. Rauscher



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THE MICROCOMPUTER SCIENTIFIC SOFTWARE SERIES 4: TESTING PREDICTION ACCURACY

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GENERAL INFORMATION

Identification

Name of Program:	ATEST
Name of Author:	H. Michael Rauscher
Location:	North Central Forest Experiment Station 1831 Highway 169 East Grand Rapids, MN 55744
Language:	MICROSOFT BASIC[1]
Operating System:	MSDOS
Computer:	IBM PC/XT[1]

This manual is designed as both a user's and programmer's guide to program ATEST. ATEST will be described in enough detail to allow a knowledgeable user to implement improvements and extensions. The complete BASIC language source code for this program is included in appendix A.

Objectives

In this paper I have two objectives:

(a) Provide a computer program for testing the accuracy of prediction models.

(b) Provide users with enough program details to allow implementation of extensions and refinements.

[1]*Mention of trade names does not constitute endorsement of the products by the USDA Forest Service.*

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APPLICATION

General

Models are constantly being used by forest managers and researchers for prediction. Most users of predictors fail to independently determine the predictive power (accuracy) of the models they are using for their specific applications. Mosteller and Tukey (1977) summarize the statistician's point of view:

"Users have often been disappointed by procedures, such as multiple regressions, that 'forecast' quite well for the data on which they were built. When tried on fresh data, the predictive power of these procedures fell dismally."

The forestry community, by and large, does not know what degree of uncertainty is reasonable for a given application nor have we established the maximum uncertainty levels that should lead us to reject a prediction model.

One of the reasons for this deplorable situation is the lack of discussion in the scientific literature on the philosophy and testing procedures for accuracy. The paper by Reynolds (1984) redresses this deficiency. A second reason is the lack of computer tools to make accuracy testing of prediction systems easily possible. Program ATEST has been developed to make available a simple computer software tool that can be used to determine the accuracy of predictions.

Operating Instructions

This section of the manual contains two examples to explain how program ATEST can be used to assess prediction uncertainty.

Example #1

The first example deals with determining the accuracy of cruise data. True and predicted volumes from a hypothetical timber cruise have been artificially generated:

Observation number	Mill scale	Cruiser
	Cords/ac	
1	35	33
2	18	19
3	43	44
4	15	16
5	28	27
6	32	30
7	19	18
8	24	26
9	33	30
10	38	33

Let us choose the mill scale as the observed values and the cruisers' data as the predicted values. The program advises the user to let Y represent the observed (standard) values and X represent the predicted (competitor) values. The user is then asked to enter the units of measure of these values and the number of <Y, X> pairs of data to be entered. The program queries for each data pair until they are all entered. The program next requests whether the user wants to review the data for accuracy. The data pairs are displayed 10 points at a time and the user can edit any or all points in each display segment.

The program then computes the differences between the observed and predicted values ($D_i = Y_i - X_i$, i = 1 to n). The program also computes the percentage errors between the observed and the predicted values (D_i (%) = 100 (D_i/Y_i), i = 1 to n). Both sets of errors are tested against the hypothesis that the differences (D_i) follow a normal distribution whose mean equals the mean of the sample data set and whose variance equals the variance of the sample data set. The probability plot correlation coefficient (Filliben 1975) is computed for both the raw errors and the errors in percent. Based on the sample size and the value of the correlation coefficient, the user is asked to refer to the table in appendix B or Filliben (1975, table 1) to decide if there is evidence to contradict the hypothesis of normality. In this example, the correlation coefficient for the raw data is 0.9699. Looking at appendix B where n = 10, we see that 0.9699 is well above the 0.05 critical level and lies between the 0.25 and 0.50 critical levels. We can conclude that we have no reason to reject the hypothesis of normality and can respond negatively to the question in the program. In this example the correlation coefficient for the percent errors is 0.9737 and likewise fails to produce evidence to reject the assumption of normality.

The user is then asked to enter the desired two-sided probability level, alpha, which is often 0.05 or 0.01. Finally, in order to compute the length of the tolerance interval, a value for K (equation 10 this paper) must be entered from appendix C or Eisenhart *et al*. (1947, table 2.1). For this example, with N = 10, gamma = 0.95, and P = 0.95 (alpha = 0.05), K = 3.379.

The program continues to process until the output is reported as:

RESULTS FOR ERRORS IN CDS/AC:
(THE TWO-SIDED PROBABILITY LEVEL (ALPHA) IS: 0.05)

BIAS	CONFIDENCE INTERVAL	PREDICTION INTERVAL	TOLERANCE INTERVAL	S	N
0.9	+/−1.56	+/−5.18	+/−7.37	2.18	10

RESULTS FOR ERRORS IN PERCENT:
(THE TWO-SIDED PROBABILITY LEVEL (ALPHA) IS: 0.05)

BIAS	CONFIDENCE INTERVAL	PREDICTION INTERVAL	TOLERANCE INTERVAL	S	N
2.0	+/−5.21	+/−17.29	+/−24.60	7.28	10

The average difference between the mill scale and the Cruiser's estimates ($Y_i - X_i$) is 0.9 CDS/AC. Notice that this indicates an *underestimation* of the true values. Is this bias significantly different from zero? The answer is no because the 95-percent confidence interval about the bias (−0.66 to +2.46) includes zero. Based on past performance, we may be interested in obtaining an interval to include the error in a future prediction. The *confidence interval* assesses uncertainty based on the sample at hand and almost certainly overestimates the performance of the predictor (Mosteller and Tukey 1977). The real question relates to the uncertainty associated with future predictions, so we must use either the prediction or the tolerance intervals. Using the *prediction interval*, we can be 95 percent confident that in a single future prediction the error will be between −4.28 and +6.08 CDS/AC (0.9 +/− 5.18). The estimate is likely to vary between a 4.28 CDS/AC overprediction to a 6.08 CDS/AC underprediction.

If we want to provide limits to contain future errors in the long run, then we must use a *tolerance interval*. In this example we are 95-percent confident that at least 95 percent of the population of errors fall between −6.47 and 8.27 CDS/AC (0.9 +/− 7.37). Thus the magnitude of the long-term errors should be no more than about 7.37 CDS/AC.

The conclusions reached concerning the accuracy of a system to predict future events apply *only* if the prediction system is used in the same way as it was used for assessing the accuracy in the first place. Making changes in the prediction system or applying it to a different population invalidates the assessed levels of accuracy (Reynolds 1984).

Errors can also be evaluated as percentage differences from the observed or standard values (D_i (%) = 100 (D_i/Y_i)). In this case the bias is + 2.0 percent, which is not significantly different from zero at the 95 percent probability level because the confidence interval contains zero (−3.21 to 7.21). We can be 95-percent confident that the error of a single future prediction will lie between −15.29 and 17.29 percent of the mill scale value (0.2 +/− 17.29). Finally, we can be 95 percent confident that 95 percent of the errors over the long term will fall between −22.6 and 26.6 percent of the mill scale values. The magnitude of the long-term errors should be no more than about 24.6 percent.

Example #2

The second example illustrates how to use ATEST to determine the accuracy of a basal area growth model. Buckman (1962) developed a basal area growth model for red pine based on 235 permanent sample plots in 14 stands. His model is:

$$BAG = 1.6889 + .041066 * BA - .00016303 * BA^2 - .076958 * A + .00022741 * A^2 + .06441 * SI \qquad (1)$$

where:

BAG = periodic net annual basal area growth in ft^2/acre

BA = basal area in ft^2/acre

A = age in years

SI = site index in feet at index age of 50 years

Since 1962 these same plots have been maintained and measured four additional times in 5-year increments. Every 5 years from 1959 to 1979, three replications of five density regimes were marked and cut to one of the desired nominal densities of 60, 80, 100, 120, and 140 ft^2 basal area per acre. From research records, I obtained the "true" basal area growth increments (Y) for the periods:

1. 1959-1964;
2. 1964-1969;
3. 1969-1974; and
4. 1974-1979.

I used model (1) to predict growth based on site index, age, and beginning basal area to generate the "competitor" (X) values. For the sake of brevity, I have presented the input values for only the first growth period (table 1).

The results show that the bias, (table 2), is significantly different from zero at the 95-percent probability level only for the final period, 1974-1979. Because the bias is negative, the predictor overestimates the observed values. The prediction model provides unbiased estimates for 15 years of basal

Table 1.—*True and competitor values of basal area growth for period 1: 1959–1964*

Site[1] index (feet)	1959[2] Starting basal area	1959–1964[3] True basal area growth	1959–1964 Predicted basal area growth
	ft^2/ac	-------- ft^2/ac/5 yrs	--------
47	60.0	8.1	6.6
50	59.0	7.3	7.6
48	59.8	6.5	6.9
48	79.2	9.2	9.7
49	80.1	8.6	9.1
48	79.8	8.2	8.7
47	99.5	11.8	9.5
48	100.5	8.8	9.8
51	100.4	11.5	10.8
49	120.0	10.8	10.5
48	119.9	10.7	10.2
49	119.6	8.6	10.5
51	140.3	11.9	10.9
48	141.0	10.9	9.9
51	140.0	8.6	10.9

[1]Base age 50.
[2]Stand was 95 years old in 1959.
[3]Average of three 1/5-acre plots.

Table 2.—*Results from program ATEST for example #2*

A. RESULTS FOR ERRORS IN FT^2/AC/5 YRS: (THE TWO-SIDED PROBABILITY LEVEL (ALPHA) IS: 0.05)

TIME PERIOD	BIAS	CONFIDENCE INTERVAL	PREDICTION INTERVAL	TOLERANCE INTERVAL	S	N
1959–1964	0.06	+/−0.68	+/−2.73	+/−3.63	1.23	15
1964–1969	−0.05	+/−0.88	+/−3.51	+/−4.70	1.59	15
1969–1974	0.17	+/−1.07	+/−4.28	+/−5.70	1.93	15
1974–1979	0.72	+/−0.65	+/−2.60	+/−3.46	1.17	15

B. RESULTS FOR ERRORS IN PERCENT: (THE TWO-SIDED PROBABILITY LEVEL (ALPHA) IS: 0.05)

TIME PERIOD	BIAS	CONFIDENCE INTERVAL	PREDICTION INTERVAL	TOLERANCE INTERVAL	S	N
1959–1964	−0.52	+/−7.29	+/−29.18	+/−38.90	13.17	15
1964–1969	−2.37	+/−11.21	+/−44.84	+/−59.79	20.24	15
1969–1974	−1.70	+/−14.23	+/−56.94	+/−75.92	25.70	15
1974–1979	−10.10	+/−17.97	+/−71.87	+/−95.83	32.44	15

area growth on the plots used originally for calibration. After year 15, estimates overpredict by about 0.72 ft^2/acre, which is about 10 percent of the observed basal area.

The magnitude of the long-term errors steadily increases with time except for the 1974-1979 period (see column marked TOLERANCE INTERVAL). This trend indicates that, in general, the precision of the predicted estimates decreases as the number of years from the original measurements increases. This is not surprising. We can be 95-percent confident that 95 percent of the errors fall within 3.6 – 5.7 ft^2/acre when using model (1) to predict 5-year basal area growth up to 20 years past the point where these plots were used to calibrate the model. Notice that most of the errors will fall between 39 percent and 96 percent of the observed values depending upon the 5-year interval selected.

PROCEDURES AND REQUIREMENTS

System Characteristics

Program ATEST is written in Microsoft BASICrunning under MSDOS on an IBM-PC microcomputer system. No machine-dependent statements have been used. The program takes approximately 7 kilobytes of random access memory (RAM) for storage. Program ATEST is written so that the required input data are generated by the user through the keyboard. This method increases universality. Users who desire data input from disk files must write a simple subroutine to enter it from the format in which the data is stored.

Program Availability

ATEST is distributed by FORS, the Forest Resources Systems Institute, a non-profit organization for forestry computer support, at a nominal price to cover disk, postage, and handling. For further information on ordering a machine-readable copy, write or call:

The Forest Resources Systems Institute
Courtview Towers, Suite 24
201 North Pine Street
Florence, Alabama 35630
(205) 767-0250

Statistical Background

The problem is to assess the accuracy of a predictor. The prediction system may be the best estimates of experts, it may be a regression equation, or it may be a complex simulation model. For this problem, the form of the prediction system is irrelevant as long as the predicted and observed values estimate the same thing. Given a prediction system, the only way to evaluate its accuracy safely is to test it against a set of data different from those used to develop it in the first place (Mosteller and Tukey 1977). In statistical terms, appraising prediction accuracy requires cross-validation. Simple cross-validation tests the predictor against data different from those used to fit the numerical coefficients of the model form. Double cross-validation tests the predictor against data different from those used to choose the model form as well as its coefficients. Setting aside a portion of the data to test the model against is simple cross-validation. Gathering a fresh and independent set of data to conduct the accuracy test is double cross-validation (Mosteller and Tukey 1977).

In the past, the error ($D_i = Y_i - X_i$, or $D_i (\%) = 100\ D_i/Y_i$ where $i = 1$ to n) between observed (Y_i) and predicted (X_i) values has been evaluated in a variety of ways. The errors have been presented in a histogram or displayed in tabular form as in Gevorkiantz and Olson (1955, page 15). Others have recognized that prediction errors are composed of two components. The first component, bias, is the average error between predicted and observed values. Precision, the second component, refers to the clustering of sample values about their own average. The following estimators have been used to predict the degree of bias in the errors:

$\overline{D}$ average of the set of Di (1)

$|\overline{D}|$ average of the absolute value of Di (2)

The following estimators have been used to predict the degree of bias or precision of the errors:

ΣD_i sum of the set of D_i, $i = 1$ to n (3)

ΣD_i^2 sum of the set of D_i^2, $i = 1$ to n (4)

$\Sigma|D_i|$ sum of the set of absolute values of D_i,

$$i = 1 \text{ to } n \tag{5}$$

$S(D)$ standard deviation of the set of D_i,

$$i = 1 \text{ to } n \tag{6}$$

$$S_D \text{ standard error of the mean, } \overline{D} \tag{7}$$

Freese (1960) took the prediction of accuracy a step further. He proposed a statistical procedure to determine whether a prediction system attains the level of accuracy required by its user. Unfortunately, Freese (1960) failed to explicitly state the basic underlying assumptions so that users were unaware when these were violated (Reynolds 1984). Freese adopted the hypothesis testing philosophy that is appropriate when the question is whether or not a prediction system meets a specified accuracy requirement. More generally, we are interested in determining the magnitude (bias) and the precision of the prediction errors, which is appropriately addressed using the statistical estimation philosophical approach (Reynolds 1984).

If the set of D_i's is normally distributed, then a confidence interval can be computed directly based on the student's t-statistic.

$$\overline{D} \pm S\left[t_{1-\alpha/2}(n-1)\right]/\sqrt{n} \tag{8}$$

where:

$$\overline{D} = \sum_{i=1}^{n} D_i/n$$

$$S = \text{Standard Deviation} = \sqrt{\frac{\sum_{i=1}^{n} (Di - \overline{D})^2}{n-1}}$$

$t_{1-\alpha/2}(n-1)$ = the $1-\alpha/2$ quartile of the t-distribution with $n-1$ degrees of freedom

n = number of observations

To compare accuracy of predictions on a relative basis (without reference to units), it is desirable to compute the errors in percent of the observed values. Unfortunately, if the errors in absolute terms are normally distributed, the percent errors may not be and vice versa (Reynolds 1984). When the normality assumption does not hold, (8) cannot be used without distorting of the alpha level of the t-test.

The jackknife procedure allows estimation of an approximate confidence interval without assuming normality. We proceed as follows. We check to ensure that the sample is at least unimodal. Because the mean is sensitive to non-normality, we choose the 10-percent trimmed mean (disregarding the upper and lower 10 percent of the observations) as a more robust location estimator (Mosteller and Tukey 1977). We resort to the jackknife procedure to calculate an estimate of the standard deviation of the errors (see Mosteller and Tukey 1977, pages 135-141, and Sokal and Rohlf 1981, pages 795-799). The jackknife pseudo-values have been shown to be "asymptotically distributed as a student's t with n-1 degrees of freedom as n approaches infinity" (Gray and Schucany 1972). Therefore, using the trimmed mean as an estimate of $\overline{D}$ and the jackknifed sample standard deviation as an estimate of S, we can compute the confidence interval as in (8).

The *confidence interval* is an indication of uncertainty based on the current sample and can be expected to overestimate the performance of the predictor (Mosteller and Tukey 1977). We are more often interested in the uncertainty associated with making future predictions.

A *prediction interval* is used to calculate the probability that k future values of D_i will fall within its range. Program ATEST assumes $K = 1$ and thus reports the range that a single future value of D_i should fall within given a specified probability level. Because the standard deviation (S) and the number of observations (n) are also reported, it is a simple matter for the user to compute the general case (where $K > 1$).

$$\overline{D} \pm \sqrt{1/n + 1/k}\ S\ t_{1-\alpha/2}(n-1) \tag{9}$$

where:

K = number of future estimates to be made.

If we want to provide limits to contain a specified proportion of the distribution D_i over the long term, we need to construct a *tolerance limit* (Reynolds 1984). A tolerance limit to contain at least a proportion P of the distribution of D_i is of the form:

$$\overline{D} \pm K(\gamma, n, P)\ S \tag{10}$$

where:

K = table value, appendix C (Eisenhart *et al.* 1947). The tolerance interval provides the range that over the long run will contain P percent of the distribution of D_i's with probability of γ.

To test for normality where location and scale are unspecified, we used the normal probability plot correlation coefficient (Filliben 1975). This test is conceptually and computationally simple and compares favorably with competing tests of the composite hypothesis of normality.

Warning! The prediction and tolerance intervals are very sensitive to non-normality. Nonparametric prediction intervals can be used, but these are not very satisfactory for small n (Reynolds, personal communication). The use of the trimmed mean and the jackknifed variance should provide a more robust confidence and prediction interval. Alternately, some type of data transformation could be used to achieve approximate normality (Mosteller and Tukey 1977).

PROGRAM CODE DOCUMENTATION

The initialization section of the program (lines 100-250) requests that the user enter the units of measure, the number of [Y,X] pairs, and the [Y,X] pairs themselves (fig. 1). The differences are computed both in raw units and in percent (line 240) and set into arrays DA(•) and DR(•), respectively. The user is given the chance to review and correct the data entry (lines 20000-20220). These two arrays are then sorted into ascending order (lines 10000-10230) by an in-place Shell-Metzner sorting algorithm.

Each array, in turn, is placed into the temporary array TMP(•) (lines 290-390) and tested for the composite hypothesis of normality (lines 11000-11360). The program reports the computed correlation coefficient in both raw and percent units and asks the user to decide whether there is enough evidence to contradict the assumption of normality (lines 400-530).

Based on whether the differences are normally distributed or not, results are computed by code (lines 12000-12160) or code (lines 13000-13300), respectively. The t-table value is computed (lines 14000-15240) based on an algorithm reported by Shammas (1984).

Finally, results are displayed (lines 680-870) and the user is asked if another problem is to be solved. If not, the program ends.

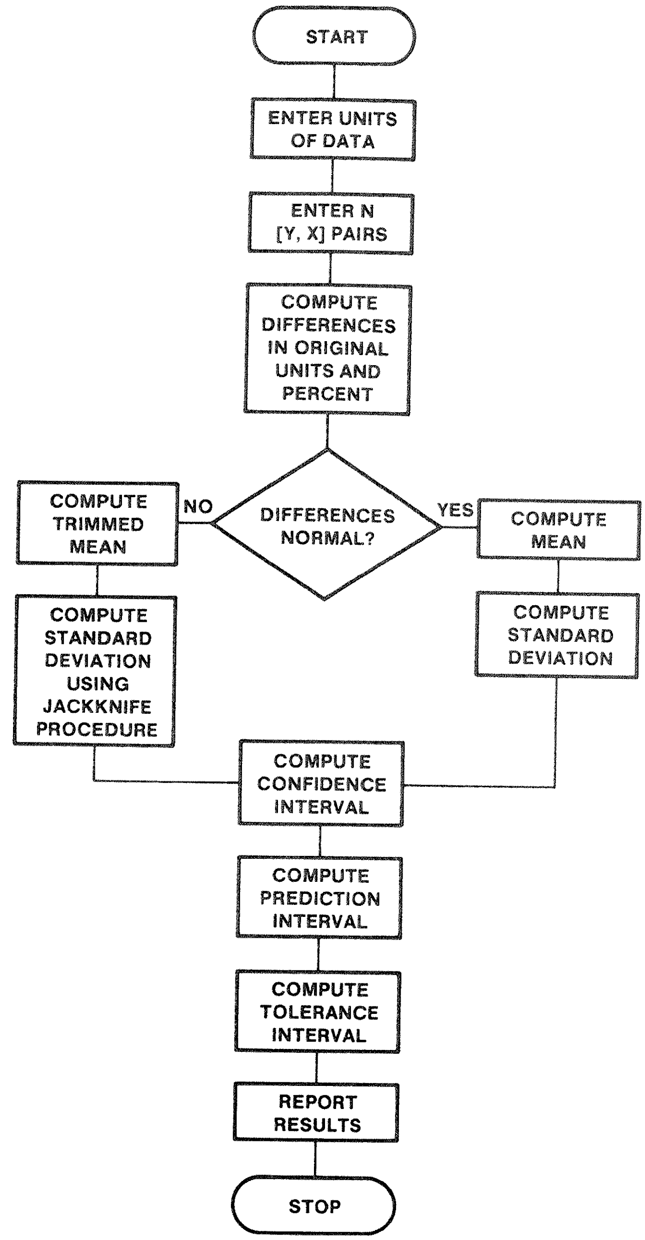

Figure 1.—*Flowchart of the logical blocks of program ATEST.*

APPENDIX A. COMPUTER SOURCE CODE FOR PROGRAM ATEST

```
100 REM  ***************************************************************
110 REM  **   PROGRAM TO TEST ACCURACY
120 REM  **   VERSION :  2/12/85
130 REM  ***************************************************************
140 CLS
142 CLEAR
145 PRINT"PROGRAM TO COMPUTE THE ACCURACY OF PREDICTIONS."
146 PRINT:PRINT
150 PRINT"LET Y REPRESENT THE STANDARD AND X REPRESENT THE COMPETITOR."
160 PRINT
170 PRINT"ENTER UNITS OF MEASURE OF THE DATA --> ";: INPUT UNITS$
180 PRINT:PRINT
190 PRINT"HOW MANY Y,X PAIRS WILL YOU ENTER --> ";: INPUT ROWS
200 DIM DA(ROWS),DR(ROWS),TMP(ROWS),MI(ROWS),Y(ROWS),X(ROWS)
210 FOR I=1 TO ROWS
220 PRINT
230 PRINT"PAIR NUMBER = ";I;"    ENTER [Y,X] --> ";: INPUT Y(I),X(I)
240 DA(I)=Y(I)-X(I): DR(I)=100*(DA(I)/Y(I))
250 NEXT I
255 GOSUB 20000:' CHECK DATA FOR CORRECT ENTRY
260 REM   SORT THE ERROR ARRAYS INTO ASCENDING ORDER
270 ITAG$="DA": GOSUB 10000
280 ITAG$="DR": GOSUB 10000
290 REM   FILL IN THE TEMPORARY ARRAY AND TEST FOR NORMALITY IN EACH CASE
300 FOR I=1 TO ROWS
310 TMP(I)=DA(I)
320 NEXT I
330 GOSUB 11000
340 DARHO=RHO
350 FOR I=1 TO ROWS
360 TMP(I)=DR(I)
370 NEXT I
380 GOSUB 11000
390 DRRHO=RHO
400 REM   DISPLAY RESULTS TO USER AND ASK FOR SIGNIFICANCE DETERMINATION
410 CLS
420 PRINT"THE NORMALITY TEST HAS BEEN COMPLETED FOR THE ABSOLUTE AND"
430 PRINT"     AND RELATIVE VALUES OF THE ERRORS."
440 PRINT: PRINT
450 PRINT" NUMBER OF OBSERVATIONS = ";ROWS
460 PRINT
470 PRINT"CORRELATION COEFFICIENT FOR THE RAW ERRORS = ";DARHO
480 PRINT
490 PRINT"CORRELATION COEFFICIENT FOR THE PERCENTAGE ERRORS = ";DRRHO
500 PRINT: PRINT
505 PRINT"BASED ON THE CRITICAL LEVELS IN APPENDIX B (FILLIBEN 1975) "
510 PRINT"IS THERE EVIDENCE TO CONTRADICT THE NORMALITY ASSUMPTION :
520 PRINT"           FOR THE RAW        ERRORS [Y/N]? --> ";: INPUT ANS1$
530 PRINT"         FOR THE PERCENTAGE ERRORS [Y/N]? --> ";: INPUT ANS2$
540 REM  BASED ON USER INPUT SELECT SEPARATE PATHS TO CALCULATE BIAS AND
550 REM  PRECISION OF ERRORS
552 DF=ROWS-1: REM SET DEGREES OF FREEDOM FOR T-TEST AS N-1
554 P=.025
556 PRINT:PRINT
557 PRINT"ENTER THE TWO-SIDED PROBABILITY LEVEL (DEFAULT=.05) --> ";: INPUT ANS$
558 IF VAL(ANS$)>0 THEN P=VAL(ANS$)/2
560 REM  COMPUTE RESULTS FOR THE ABSOLUTE ERRORS
570 FOR I=1 TO ROWS
580 TMP(I)=DA(I)
590 NEXT I
```

```
595 PRINT:PRINT"COMPUTE RESULTS FOR THE ABSOLUTE ERRORS": PRINT
600 IF ANS1$="Y" OR ANS1$="y" THEN GOSUB 13000 ELSE IF ANS1$="N" OR ANS1$="n" THEN
610 XBAR1=XBAR: CI1=CI: S1=S: PI1=PI
620 REM COMPUTE RESULTS FOR THE RELATIVE ERRORS
630 FOR I=1 TO ROWS
640 TMP(I)=DR(I)
650 NEXT I
655 PRINT:PRINT"COMPUTE RESULTS FOR THE RELATIVE ERRORS": PRINT
660 IF ANS2$="Y" OR ANS2$="y" THEN GOSUB 13000 ELSE IF ANS2$="N" OR ANS2$="n" THEN
670 XBAR2=XBAR: CI2=CI: S2=S: PI2=PI
680 REM DISPLAY THE RESULTS OF THE ANALYSIS
682 CLS
683 PRINT"THE NUMBER OF OBSERVATIONS [N] = ";ROWS: PRINT
684 PRINT"PLEASE ENTER THE VALUE FOR K FROM APPENDIX C (EISENHART, HASTAY"
686 PRINT",AND WALLIS 1947) SO THAT THE PROBABILITY IS GAMMA THAT AT LEAST"
688 PRINT"A PROPORTION P OF THE DISTRIBUTION OF FUTURE VALUES WILL BE"
689 PRINT"INCLUDED IN THE CALCULATED TOLERANCE INTERVALE --> ";: INPUT K
690 CLS
692 PRINT"RESULTS FOR ERRORS IN:";TAB(30) UNITS$; TAB(50) "PERCENT"
693 PRINT
694 PRINT"TWO SIDED PROB. LEVEL:";TAB(30) P*2; TAB(50) P*2
696 PRINT" BIAS ";TAB(30) XBAR1;TAB(50) XBAR2
698 PRINT"CONFIDENCE INTERVAL [+/-] :";TAB(30) CI1; TAB(50) CI2
700 PRINT"PREDICTION INTERVAL [+/-] :";TAB(30) PI1; TAB(50) PI2
705 PRINT"TOLERANCE  INTERVAL [+/-] :";TAB(30) S1*K;TAB(50) S2*K
710 PRINT"STANDARD DEVIATION :";TAB(30) S1;TAB(50) S2
715 PRINT"NUMBER OF OBSERVATIONS = ";ROWS
790 PRINT:PRINT:PRINT
800 IF DF> 5 THEN GOTO 820
810 PRINT"WARNING!!  THE PROCEDURE TO CALCULATE THE T-TABLE VALUE LOOSES"
812 PRINT"           ACCURACY WHEN THE DEGREES OF FREEDOM FALL BELOW 6."
820 PRINT:PRINT
830 PRINT"WHEN READY TO CONTINUE PRESS [RETURN] --> ";: INPUT ANS$
840 CLS
850 PRINT"ANOTHER SET OF DATA TO ANALYZE [Y/N]? --> ";: INPUT ANS$
860 IF ANS$="Y" OR ANS$="y" THEN GOTO 140
870 END
10000 REM  ***********************************************************
10010 REM  **  SHELL-METZNER INPLACE, IN RAM SORT
10020 REM  **  INPUT   ROWS= NUMBER OF PLACES IN ARRAY
10030 REM  **          ITAG$ CONTAINS ARRAY POINTER DA OR DR
10040 REM  ***********************************************************
10050 M=ROWS: C1=0: S1=0
10060 M=INT(M/2)
10070 IF M=0 THEN GOTO 10230
10080 J=1: K=ROWS-M
10090 I=J
10100 L=I+M
10110 C1=C1+1
10120 IF ITAG$="DR" THEN GOTO 10150
10130 IF DA(I)<DA(L) THEN GOTO 10200
10140 T=DA(I): DA(I)=DA(L): DA(L)=T
10150 IF DR(I)<DR(L) THEN GOTO 10200
10160 T=DR(I): DR(I)=DR(L): DR(L)=T
10170 S1=S1+1: I=I-M
10180 IF I<1 THEN GOTO 10200
10190 GOTO 10100
10200 J=J+1
10210 IF J>K THEN GOTO 10060
10220 GOTO 10090
```

```
10230 RETURN
11000 REM  ************************************************************
11010 REM  **  TEST FOR HYPOTHESIS OF NORMALITY
11020 REM  **        LOCATION AND SCALE UNSPECIFIED
11030 REM  **  REFERENCE:  FILLIBEN,J.J. 1975. THE PROBABILITY PLOT
11040 REM  **              CORRELATION COEFFICIENT TEST FOR NORMALITY.
11050 REM  **              TECHNOMETRICS  17: 111-117
11060 REM  **  VERSION:  1/10/84
11070 REM  **  WRITTEN BY:  H.M. RAUSCHER, NCFES, GRAND RAPIDS, MN
11080 REM  ************************************************************
11090 REM  THE ORDERED ARRAY, SMALLEST TO LARGEST, MUST BE IN TMP(ROWS)
11100 REM  THE WORKING ARRAY MI(ROWS) MUST HAVE BEEN DIMENSIONED ALREADY
11102 REM  INITIALIZATION
11105 C0=2.515517: C1=.802853: C2=.010328
11106 D1=1.432788: D2=.189269: D3=.001308
11110 REM
11120 REM  FILL IN THE MI(.) ARRAY
11130 MI(ROWS)=.5^(1/ROWS)
11140 MI(1)=1-MI(ROWS)
11150 FOR I=2 TO ROWS-1
11160 MI(I)=(I-.3175)/(ROWS+.365)
11170 NEXT I
11175 J=0
11180 FOR I= 1 TO ROWS
11184 IF MI(I)<=.5 THEN J=J+1
11185 IF MI(I)>.5 THEN MI(I)=1-MI(I)
11190 TN=SQR(LOG(1/(MI(I)*MI(I))))
11200 MI(I)=TN-(C0+C1*TN+C2*TN*TN)/(1+D1*TN+D2*TN*TN+D3*TN*TN*TN)
11207 IF ABS(MI(I))<.00001 THEN MI(I)=0
11210 NEXT I
11212 FOR I=1 TO J
11214 MI(I)=-MI(I)
11216 NEXT I
11220 REM  COMPUTE THE AVERAGE VALUE FOR TMP(.)
11230 XBAR=0:SUM1=0
11240 FOR I= 1 TO ROWS
11250 SUM1=SUM1+TMP(I)
11260 NEXT I
11262 XBAR=SUM1/ROWS
11270 REM COMPUTE THE CORRELATION COEFFICIENT
11280 SUM1=0: SUM2=0: SUM3=0
11290 FOR I=1 TO ROWS
11300 SUM1=SUM1+TMP(I)*MI(I)
11310 SUM2=SUM2+MI(I)*MI(I)
11320 SUM3=SUM3+(TMP(I)-XBAR)^2
11330 NEXT I
11340 RHO=SUM1/SQR(SUM2*SUM3)
11350 REM  RETURN THE VALUE OF RHO AND ROWS TO CALLING PROGRAM
11360 RETURN
12000 REM  ****************************************************
12010 REM  **  ROUTINE TO COMPUTE BIAS AND PRECISION WHEN NORMALITY ASSUMED
12020 REM  ****************************************************
12030 SUM1=0: SUM2=0
12040 FOR I=1 TO ROWS
12050 SUM1=SUM1+TMP(I)
12060 SUM2=SUM2+TMP(I)*TMP(I)
12070 NEXT I
12080 XBAR=SUM1/ROWS
12090 S=SUM2-(SUM1*SUM1)/ROWS:S=SQR(S/(ROWS-1))
12100 REM  COMPUTE THE T-TABLE VALUE , TT
```

```
12110 GOSUB 15000
12120 REM  COMPUTE CONFIDENCE INTERVAL
12130 CI=S*TT/SQR(ROWS)
12140 REM COMPUTE THE PREDICTION INTERVAL
12150 PI=S*TT*SQR(1+1/ROWS)
12160 RETURN
13000 REM  **********************************************************
13010 REM  ** ROUTINE TO COMPUTE ACCURACY WHEN ERROR NOT NORMAL
13020 REM  **********************************************************
13030 REM COMPUTE THE 10% TRIMMED MEAN
13040 J=INT(ROWS*.1+.5)
13050 IF J=0 THEN J=1: REM AT LEAST TRIM THE EXTREMES
13060 SUM1=0:N=0
13070 FOR I=J+1 TO ROWS-J
13080 SUM1=SUM1+TMP(I): N=N+1
13090 NEXT I
13100 XBAR=SUM1/N
13102 REM COMPUTE THE FULL SAMPLE S, FSS
13104 SUM1=0:SUM2=0
13106 FOR I=1 TO ROWS: SUM1=SUM1+TMP(I):SUM2=SUM2+TMP(I)*TMP(I): NEXT I
13108 FSS=SUM2-(SUM1*SUM1)/ROWS: FSS=SQR(FSS/(ROWS-1))
13110 REM COMPUTE THE JACKKNIFED STANDARD DEVIATION
13120 SUM3=0
13130 FOR J=1 TO ROWS
13140 SUM1=0:SUM2=0
13150 FOR I=1 TO ROWS
13160 IF I=J THEN GOTO 13190
13170 SUM1=SUM1+TMP(I)
13180 SUM2=SUM2+TMP(I)*TMP(I)
13190 NEXT I
13200 S=SUM2-(SUM1*SUM1)/(ROWS-1): S=SQR(S/(ROWS-2))
13210 SUM3=SUM3+(ROWS*FSS-(ROWS-1)*S)
13220 NEXT J
13230 S=SUM3/ROWS
13240 REM COMPUTE THE T-TABLE VALUE, TT
13250 GOSUB 15000
13260 REM COMPUTE CONFIDENCE INTERVAL
13270 CI=S*TT/SQR(ROWS)
13280 REM COMPUTE PREDICTION INTERVAL
13290 PI=S*TT*SQR(1+1/ROWS)
13300 RETURN
14000 REM  ***********************************************************
14010 PRINT:PRINT
14020 PRINT"AN INPUT ERROR HAS BEEN DETECTED.  PLEASE ENTER ONLY [Y],[y],[N],"
14030 PRINT"OR [n] WHEN ASKED TO ENTER [Y/N] ! "
14040 PRINT"PLEASE PRESS [RETURN] TO TRY AGAIN";: INPUT ANS1$
14050 GOTO 410
15000 REM  *************************************************************
15010 REM  ** ROUTINE TO COMPUTE THE T-TABLE VALUE GIVEN DEGREES OF
15020 REM  **     FREEDOM (DF) AND THE TWO SIDED ALPHA LEVEL
15030 REM  **REFERENCE:  SHAMMAS,NAMIR C. 1984. STATISTICAL TABLE
15040 REM  **              SUBROUTINES.  ACCESS 3:19-22
15050 REM  ** VERSION:  1/10/85
15060 REM  *************************************************************
15070 REM  WARNING:  ROUTINE ACCURATE TO 3 DIGITS, LOOSING ACCURACY AT
15080 REM            LOW VALUES OF THE DEGREES OF FREEDOM
15130 IF P>.5 THEN U0=SQR(LOG(1/(1-P)^2)) ELSE U0=SQR(LOG(1/P/P))
15140 U1=((.010328*U0)+.802853)*U0+2.515517
15150 U2=(((.001308*U0)+.189269)*U0+1.432788)*U0+1
15160 XP=U0-U1/U2
```

```
15170 IF P > .5 THEN XP=-XP
15180 U0=XP^2
15190 U1=XP/4*(U0+1)
15200 U2=(5*U0^2+16*U0+3)*XP/96
15210 U3=(3*U0^3+19*U0^2+17*U0-15)*XP/384
15220 U4=(79*U0^4+776*U0^3+1482*U0^2-1920*U0-945)*XP/92160!
15230 TT=XP+U1/DF+U2/DF^2+U3/DF^3+U4/DF^4
15240 RETURN
20000 '*******************************************************************
20010 '*   ROUTINE TO CHECK AND EDIT DATA IN CASE OF ENTRY ERROR
20020 '*******************************************************************
20030 CLS:INPUT"DO YOU WANT TO REVIEW YOUR DATA (Y/N)  --> ";ANS$
20040 IF ANS$="N" OR ANS$="n" THEN GOTO 20210
20050 PRINT:PRINT
20060 I=0
20070 FOR J=1 TO 10
20075 I=I+1: IF I>ROWS THEN GOTO 20100
20080 PRINT"PAIR NO.:  ";I;" STANDARD (Y):  ";Y(I);"    COMPETITOR (X):  ";X(I)
20090 NEXT J
20100 PRINT:PRINT
20110 INPUT "CORRECT (Y/N)  --> ";ANS$
20120 IF ANS$="Y" OR ANS$="y" THEN GOTO 20180
20130 ' ALLOW EDITING OF DATA
20140 PRINT"ENTER A TRIPLET: [PAIR NO., STANDARD(Y), COMPETITOR(X)]"
20150 PRINT"          TO FINISH:  ENTER [0,0,0]"
20160 PRINT: INPUT II,Y(II),X(II)
20162 IF II<=0 THEN GOTO 20175
20165 DA(II)=Y(II)-X(II):DR(II)=100*(DA(II)/Y(II))
20170 IF II>0 THEN GOTO 20160
20175 I=INT((I-1)/10)*10:' DISPLAY AGAIN THE LAST PAGE
20178 GOTO 20070
20180 IF I>ROWS THEN GOTO 20210
20185 PRINT:INPUT"REVIEW THE NEXT 10 PAIRS (Y/N) --> ";ANS$
20186 PRINT:PRINT
20190 IF ANS$="N" OR ANS$="n" THEN GOTO 20210
20200 GOTO 20070
20210 RETURN
20220 END
```

APPENDIX B. PERCENT POINTS OF THE NORMAL PROBABILITY CORRELATION COEFFICIENT (Filliben 1975, Table 1)

n	Probability Level .000	.005	.01	.025	.05	.10	.25	.50	.75	.90	.95	.975	.99	.995
3	.866	.867	.869	.872	.879	.891	.924	.966	.991	.999	1.000	1.000	1.000	1.000
4	.784	.813	.822	.845	.868	.894	.931	.958	.979	.992	.996	.998	.999	1.000
5	.726	.803	.822	.855	.879	.902	.935	.960	.977	.988	.992	.995	.997	.998
6	.683	.818	.835	.868	.890	.911	.940	.962	.977	.986	.990	.993	.996	.997
7	.648	.828	.847	.876	.899	.916	.944	.965	.978	.986	.990	.992	.995	.996
8	.619	.841	.859	.886	.905	.924	.948	.967	.979	.986	.990	.992	.995	.996
9	.595	.851	.868	.893	.912	.929	.951	.968	.980	.987	.990	.992	.994	.995
10	.574	.860	.876	.900	.917	.934	.954	.970	.981	.987	.990	.992	.994	.995
11	.556	.868	.883	.906	.922	.938	.957	.972	.982	.988	.990	.992	.994	.995
12	.539	.875	.889	.912	.926	.941	.959	.973	.982	.988	.990	.992	.994	.995
13	.525	.882	.895	.917	.931	.944	.962	.975	.983	.988	.991	.993	.994	.995
14	.512	.888	.901	.921	.934	.947	.964	.976	.984	.989	.991	.993	.994	.995
15	.500	.894	.907	.925	.937	.950	.965	.977	.984	.989	.991	.993	.994	.995
16	.489	.899	.912	.928	.940	.952	.967	.978	.985	.989	.991	.993	.994	.995
17	.478	.903	.916	.931	.942	.954	.968	.979	.986	.990	.992	.993	.994	.995
18	.469	.907	.919	.934	.945	.956	.969	.979	.986	.990	.992	.993	.995	.995
19	.460	.909	.923	.937	.947	.958	.971	.980	.987	.990	.992	.993	.995	.995
20	.452	.912	.925	.939	.950	.960	.972	.981	.987	.991	.992	.994	.995	.995
21	.445	.914	.928	.942	.952	.961	.973	.981	.987	.991	.993	.994	.995	.996
22	.437	.918	.930	.944	.954	.962	.974	.982	.988	.991	.993	.994	.995	.996
23	.431	.922	.933	.947	.955	.964	.975	.983	.988	.991	.993	.994	.995	.996
24	.424	.926	.936	.949	.957	.965	.975	.983	.988	.992	.993	.994	.995	.996
25	.418	.928	.937	.950	.958	.966	.976	.984	.989	.992	.993	.994	.995	.996
26	.412	.930	.939	.952	.959	.967	.977	.984	.989	.992	.993	.994	.995	.996
27	.407	.932	.941	.953	.960	.968	.977	.984	.989	.992	.994	.995	.995	.996
28	.402	.934	.943	.955	.962	.969	.978	.985	.990	.992	.994	.995	.995	.996
29	.397	.937	.945	.956	.962	.969	.979	.985	.990	.992	.994	.995	.995	.996
30	.392	.938	.947	.957	.964	.970	.979	.986	.990	.993	.994	.995	.996	.996
31	.388	.939	.948	.958	.965	.971	.980	.986	.990	.993	.994	.995	.996	.996
32	.383	.939	.949	.959	.966	.972	.980	.986	.990	.993	.994	.995	.996	.996
33	.379	.940	.950	.960	.967	.973	.981	.987	.991	.993	.994	.995	.996	.996
34	.375	.941	.951	.960	.967	.973	.981	.987	.991	.993	.994	.995	.996	.996
35	.371	.943	.952	.961	.968	.974	.982	.987	.991	.993	.995	.995	.996	.997
36	.367	.945	.953	.962	.968	.974	.982	.987	.991	.994	.995	.996	.996	.997
37	.364	.947	.955	.962	.969	.975	.982	.988	.991	.994	.995	.996	.996	.997
38	.360	.948	.956	.964	.970	.975	.983	.988	.992	.994	.995	.996	.996	.997
39	.357	.949	.957	.965	.971	.976	.983	.988	.992	.994	.995	.996	.996	.997
40	.354	.949	.958	.966	.972	.977	.983	.988	.992	.994	.995	.996	.996	.997
41	.351	.950	.958	.967	.972	.977	.984	.989	.992	.994	.995	.996	.996	.997
42	.348	.951	.959	.967	.973	.978	.984	.989	.992	.994	.995	.996	.997	.997
43	.345	.953	.959	.967	.973	.978	.984	.989	.992	.994	.995	.996	.997	.997
44	.342	.954	.960	.968	.973	.978	.984	.989	.992	.994	.995	.996	.997	.997
45	.339	.955	.961	.969	.974	.978	.985	.989	.993	.994	.995	.996	.997	.997
46	.336	.956	.962	.969	.974	.979	.985	.990	.993	.995	.995	.996	.997	.997
47	.334	.956	.963	.970	.974	.979	.985	.990	.993	.995	.995	.996	.997	.997
48	.331	.957	.963	.970	.975	.980	.985	.990	.993	.995	.996	.996	.997	.997
49	.329	.957	.964	.971	.975	.980	.986	.990	.993	.995	.996	.996	.997	.997
50	.326	.959	.965	.972	.977	.981	.986	.990	.993	.995	.996	.996	.997	.997
55	.315	.962	.967	.974	.978	.982	.987	.991	.994	.995	.996	.997	.997	.997
60	.305	.965	.970	.976	.980	.983	.988	.991	.994	.995	.996	.997	.997	.998
65	.296	.967	.972	.977	.981	.984	.989	.992	.994	.996	.996	.997	.997	.998
70	.288	.969	.974	.978	.982	.985	.989	.993	.995	.996	.997	.997	.998	.998
75	.281	.971	.975	.979	.983	.986	.990	.993	.995	.996	.997	.997	.998	.998
80	.274	.973	.976	.980	.984	.987	.991	.993	.995	.996	.997	.997	.998	.998
85	.268	.974	.977	.981	.985	.987	.991	.994	.995	.997	.997	.997	.998	.998
90	.263	.976	.978	.982	.985	.988	.991	.994	.996	.997	.997	.998	.998	.998
95	.257	.977	.979	.983	.986	.989	.992	.994	.996	.997	.997	.998	.998	.998
100	.252	.979	.981	.984	.987	.989	.992	.994	.996	.997	.998	.998	.998	.998

APPENDIX C. TABULATION OF TOLLERANCE FACTORS FOR THE NORMAL DISTRIBUTION
(Eisenhart *et al*. 1947, Table 2.1)

TOLERANCE FACTORS FOR NORMAL DISTRIBUTIONS

Factors K such that the probability is γ that at least a proportion P of the distribution will be included between $\bar{X} \pm Ks$, where $\bar{X}$ and s are estimates of the mean and the standard deviation computed from a sample of N (Eisenhart et al. 1947).

	γ = 0.75					γ = 0.90					γ = 0.95					γ = 0.99				
N/P	0.75	0.90	0.95	0.99	0.999	0.75	0.90	0.95	0.99	0.999	0.75	0.90	0.95	0.99	0.999	0.75	0.90	0.95	0.99	0.999
2	4.498	6.301	7.414	9.531	11.920	11.407	15.978	18.800	24.167	30.227	22.858	32.019	37.674	48.430	60.573	114.363	160.193	188.491	242.300	303.054
3	2.501	3.538	4.187	5.431	6.844	4.132	5.847	6.919	8.974	11.309	5.922	8.380	9.916	12.861	16.208	13.378	18.930	22.401	29.055	36.616
4	2.035	2.892	3.431	4.471	5.657	2.932	4.166	4.943	6.440	8.149	3.779	5.369	6.370	8.299	10.502	6.614	9.398	11.150	14.527	18.383
5	1.825	2.599	3.088	4.033	5.117	2.454	3.494	4.152	5.423	6.879	3.002	4.275	5.079	6.634	8.415	4.643	6.612	7.855	10.260	13.015
6	1.704	2.429	2.889	3.779	4.802	2.196	3.131	3.723	4.870	6.188	2.604	3.712	4.414	5.775	7.337	3.743	5.337	6.345	8.301	10.548
7	1.624	2.318	2.757	3.611	4.593	2.034	2.902	3.452	4.521	5.750	2.361	3.369	4.007	5.248	6.676	3.283	4.613	5.488	7.187	9.142
8	1.568	2.238	2.663	3.491	4.444	1.921	2.743	3.264	4.278	5.446	2.197	3.136	3.732	4.891	6.226	2.905	4.147	4.936	6.468	8.234
9	1.525	2.178	2.593	3.400	4.330	1.839	2.626	3.125	4.098	5.220	2.078	2.967	3.532	4.631	5.899	2.677	3.822	4.550	5.966	7.600
10	1.492	2.131	2.537	3.328	4.241	1.775	2.535	3.018	3.959	5.046	1.987	2.839	3.379	4.433	5.649	2.508	3.582	4.265	5.594	7.129
11	1.465	2.093	2.493	3.271	4.169	1.724	2.463	2.933	3.849	4.906	1.916	2.737	3.259	4.277	5.452	2.378	3.397	4.045	5.308	6.766
12	1.443	2.062	2.456	3.223	4.110	1.683	2.404	2.863	3.758	4.792	1.858	2.655	3.162	4.150	5.291	2.274	3.250	3.870	5.079	6.477
13	1.425	2.036	2.424	3.183	4.059	1.648	2.355	2.805	3.682	4.697	1.810	2.587	3.081	4.044	5.158	2.190	3.130	3.727	4.893	6.240
14	1.409	2.013	2.398	3.148	4.016	1.619	2.314	2.756	3.618	4.615	1.770	2.529	3.012	3.955	5.045	2.120	3.029	3.608	4.737	6.043
15	1.395	1.994	2.375	3.118	3.979	1.594	2.278	2.713	3.562	4.545	1.735	2.480	2.954	3.878	4.949	2.060	2.945	3.507	4.605	5.876
16	1.383	1.977	2.355	3.092	3.946	1.572	2.246	2.676	3.514	4.484	1.705	2.437	2.903	3.812	4.865	2.009	2.872	3.421	4.492	5.732
17	1.372	1.962	2.337	3.069	3.917	1.552	2.219	2.643	3.471	4.430	1.679	2.400	2.858	3.754	4.791	1.965	2.808	3.345	4.393	5.607
18	1.363	1.948	2.321	3.048	3.891	1.535	2.194	2.614	3.433	4.382	1.655	2.366	2.819	3.702	4.725	1.926	2.753	3.279	4.307	5.497
19	1.355	1.936	2.307	3.030	3.867	1.520	2.172	2.588	3.399	4.339	1.635	2.337	2.784	3.656	4.667	1.891	2.703	3.221	4.230	5.399
20	1.347	1.925	2.294	3.013	3.846	1.506	2.152	2.564	3.368	4.300	1.616	2.310	2.752	3.615	4.614	1.860	2.659	3.168	4.161	5.312
21	1.340	1.915	2.282	2.998	3.827	1.493	2.135	2.543	3.340	4.264	1.599	2.286	2.723	3.577	4.567	1.833	2.620	3.121	4.100	5.234
22	1.334	1.906	2.271	2.984	3.809	1.482	2.118	2.524	3.315	4.232	1.584	2.264	2.697	3.543	4.523	1.808	2.584	3.078	4.044	5.163
23	1.328	1.898	2.261	2.971	3.793	1.471	2.103	2.506	3.292	4.203	1.570	2.244	2.673	3.512	4.484	1.785	2.551	3.040	3.993	5.098
24	1.322	1.891	2.252	2.959	3.778	1.462	2.089	2.489	3.270	4.176	1.557	2.225	2.651	3.483	4.447	1.764	2.522	3.004	3.947	5.039
25	1.317	1.883	2.244	2.948	3.764	1.453	2.077	2.474	3.251	4.151	1.545	2.208	2.631	3.457	4.413	1.745	2.494	2.972	3.904	4.985
26	1.313	1.877	2.236	2.938	3.751	1.444	2.065	2.460	3.232	4.127	1.534	2.193	2.612	3.432	4.382	1.727	2.469	2.941	3.865	4.935
27	1.309	1.871	2.229	2.929	3.740	1.437	2.054	2.447	3.215	4.106	1.523	2.178	2.595	3.409	4.353	1.711	2.446	2.914	3.828	4.888
28	1.305	1.865	2.222	2.920	3.728	1.430	2.044	2.435	3.199	4.085	1.514	2.164	2.579	3.388	4.326	1.695	2.424	2.888	3.794	4.845
29	1.301	1.860	2.216	2.911	3.718	1.423	2.034	2.424	3.184	4.066	1.505	2.152	2.564	3.368	4.301	1.681	2.404	2.864	3.763	4.805
30	1.297	1.855	2.210	2.904	3.708	1.417	2.025	2.413	3.170	4.049	1.497	2.140	2.549	3.350	4.278	1.668	2.385	2.841	3.733	4.768
31	1.294	1.850	2.204	2.896	3.699	1.411	2.017	2.403	3.157	4.032	1.489	2.129	2.536	3.332	4.256	1.656	2.367	2.820	3.706	4.732

	γ = 0.75					γ = 0.90					γ = 0.95					γ = 0.99				
N/P	0.75	0.90	0.95	0.99	0.999	0.75	0.90	0.95	0.99	0.999	0.75	0.90	0.95	0.99	0.999	0.75	0.90	0.95	0.99	0.999
32	1.291	1.846	2.199	2.890	3.690	1.405	2.009	2.393	3.145	4.016	1.481	2.118	2.524	3.316	4.235	1.644	2.351	2.801	3.680	4.699
33	1.288	1.842	2.194	2.883	3.682	1.400	2.001	2.385	3.133	4.001	1.475	2.108	2.512	3.300	4.215	1.633	2.335	2.782	3.655	4.668
34	1.285	1.838	2.189	2.877	3.674	1.395	1.994	2.376	3.122	3.987	1.468	2.099	2.501	3.286	4.197	1.623	2.320	2.764	3.632	4.639
35	1.283	1.834	2.185	2.871	3.667	1.390	1.988	2.368	3.112	3.974	1.462	2.090	2.490	3.272	4.179	1.613	2.306	2.748	3.611	4.611
36	1.280	1.830	2.181	2.866	3.660	1.386	1.981	2.361	3.102	3.961	1.455	2.081	2.479	3.258	4.161	1.604	2.293	2.732	3.590	4.585
37	1.278	1.827	2.177	2.860	3.653	1.381	1.975	2.353	3.092	3.949	1.450	2.073	2.470	3.246	4.146	1.595	2.281	2.717	3.571	4.560
38	1.275	1.824	2.173	2.855	3.647	1.377	1.969	2.346	3.083	3.938	1.446	2.068	2.464	3.237	4.134	1.587	2.269	2.703	3.552	4.537
39	1.273	1.821	2.169	2.850	3.641	1.374	1.964	2.340	3.075	3.927	1.441	2.060	2.455	3.226	4.120	1.579	2.257	2.690	3.534	4.514
40	1.271	1.818	2.166	2.846	3.635	1.370	1.959	2.334	3.066	3.917	1.435	2.052	2.445	3.213	4.104	1.571	2.247	2.677	3.518	4.493
41	1.269	1.815	2.162	2.841	3.629	1.366	1.954	2.328	3.059	3.907	1.430	2.045	2.437	3.202	4.090	1.564	2.236	2.665	3.502	4.472
42	1.267	1.812	2.159	2.837	3.624	1.363	1.949	2.322	3.051	3.897	1.426	2.039	2.429	3.192	4.077	1.557	2.227	2.653	3.486	4.453
43	1.266	1.810	2.156	2.833	3.619	1.360	1.944	2.316	3.044	3.888	1.422	2.033	2.422	3.183	4.065	1.551	2.217	2.642	3.472	4.434
44	1.264	1.807	2.153	2.829	3.614	1.357	1.940	2.311	3.037	3.879	1.418	2.027	2.415	3.173	4.053	1.545	2.208	2.631	3.458	4.416
45	1.262	1.805	2.150	2.826	3.609	1.354	1.935	2.306	3.030	3.871	1.414	2.021	2.408	3.165	4.042	1.539	2.200	2.621	3.444	4.399
46	1.261	1.802	2.148	2.822	3.605	1.351	1.931	2.301	3.024	3.863	1.410	2.016	2.402	3.156	4.031	1.533	2.192	2.611	3.431	4.383
47	1.259	1.800	2.145	2.819	3.600	1.348	1.927	2.297	3.018	3.855	1.406	2.011	2.396	3.148	4.021	1.527	2.184	2.602	3.419	4.367
48	1.258	1.798	2.143	2.815	3.596	1.345	1.924	2.292	3.012	3.847	1.403	2.006	2.390	3.140	4.011	1.522	2.176	2.593	3.407	4.352
49	1.256	1.796	2.140	2.812	3.592	1.343	1.920	2.288	3.006	3.840	1.399	2.001	2.384	3.133	4.002	1.517	2.169	2.584	3.396	4.337
50	1.255	1.794	2.138	2.809	3.588	1.340	1.916	2.284	3.001	3.833	1.396	1.996	2.379	3.126	3.993	1.512	2.162	2.576	3.385	4.323
51	1.253	1.792	2.135	2.806	3.584	1.338	1.913	2.279	2.995	3.826	1.393	1.992	2.373	3.119	3.984	1.507	2.155	2.568	3.374	4.310
52	1.252	1.790	2.133	2.803	3.581	1.336	1.910	2.276	2.990	3.820	1.390	1.988	2.368	3.112	3.975	1.503	2.148	2.560	3.364	4.297
53	1.251	1.789	2.131	2.801	3.577	1.334	1.907	2.272	2.985	3.813	1.387	1.984	2.363	3.106	3.967	1.498	2.142	2.552	3.354	4.284
54	1.250	1.787	2.129	2.798	3.574	1.331	1.904	2.268	2.981	3.807	1.384	1.980	2.359	3.100	3.959	1.494	2.136	2.545	3.344	4.272
55	1.249	1.785	2.127	2.795	3.571	1.329	1.901	2.265	2.976	3.801	1.382	1.976	2.354	3.094	3.951	1.490	2.130	2.538	3.335	4.260
56	1.247	1.784	2.125	2.793	3.567	1.327	1.898	2.261	2.972	3.796	1.379	1.972	2.350	3.088	3.944	1.486	2.124	2.531	3.326	4.249
57	1.246	1.782	2.123	2.790	3.564	1.325	1.895	2.258	2.967	3.790	1.377	1.968	2.345	3.082	3.937	1.482	2.119	2.524	3.318	4.238
58	1.245	1.781	2.122	2.788	3.561	1.323	1.892	2.255	2.963	3.785	1.374	1.965	2.341	3.076	3.930	1.478	2.113	2.518	3.309	4.227
59	1.244	1.779	2.120	2.786	3.558	1.322	1.890	2.252	2.959	3.779	1.372	1.961	2.337	3.071	3.923	1.474	2.108	2.512	3.301	4.216
60	1.243	1.778	2.118	2.784	3.556	1.320	1.887	2.248	2.955	3.774	1.369	1.958	2.333	3.066	3.916	1.471	2.103	2.506	3.293	4.206
61	1.242	1.776	2.117	2.781	3.553	1.318	1.885	2.245	2.951	3.769	1.367	1.955	2.329	3.061	3.909	1.467	2.098	2.500	3.285	4.196
62	1.241	1.775	2.115	2.779	3.550	1.316	1.882	2.243	2.947	3.765	1.365	1.951	2.325	3.056	3.903	1.464	2.093	2.494	3.278	4.187
63	1.240	1.774	2.113	2.777	3.548	1.315	1.880	2.240	2.944	3.760	1.363	1.948	2.322	3.051	3.897	1.461	2.089	2.489	3.271	4.178
64	1.240	1.772	2.112	2.775	3.545	1.313	1.878	2.237	2.940	3.755	1.361	1.945	2.318	3.046	3.891	1.458	2.084	2.483	3.264	4.169
65	1.239	1.771	2.110	2.773	3.543	1.312	1.875	2.235	2.937	3.751	1.359	1.943	2.315	3.042	3.886	1.455	2.080	2.478	3.257	4.160
66	1.238	1.770	2.109	2.771	3.540	1.310	1.873	2.232	2.933	3.747	1.357	1.940	2.311	3.037	3.880	1.452	2.076	2.473	3.250	4.152

	$\gamma = 0.75$					$\gamma = 0.90$					$\gamma = 0.95$					$\gamma = 0.99$				
N/P	0.75	0.90	0.95	0.99	0.999	0.75	0.90	0.95	0.99	0.999	0.75	0.90	0.95	0.99	0.999	0.75	0.90	0.95	0.99	0.999
67	1.237	1.769	2.108	2.770	3.538	1.309	1.871	2.229	2.930	3.742	1.355	1.937	2.308	3.033	3.874	1.449	2.071	2.468	3.244	4.143
68	1.236	1.768	2.106	2.768	3.536	1.307	1.869	2.227	2.927	3.738	1.358	1.934	2.305	3.029	3.869	1.446	2.067	2.463	3.237	4.135
69	1.235	1.766	2.105	2.766	3.533	1.306	1.867	2.225	2.923	3.734	1.351	1.932	2.302	3.025	3.864	1.443	2.063	2.459	3.231	4.127
70	1.235	1.765	2.104	2.764	3.531	1.304	1.865	2.222	2.920	3.730	1.349	1.929	2.299	3.021	3.859	1.440	2.060	2.454	3.225	4.120
71	1.234	1.764	2.102	2.763	3.529	1.303	1.863	2.220	2.917	3.727	1.347	1.927	2.296	3.017	3.854	1.438	2.056	2.450	3.219	4.112
72	1.233	1.763	2.101	2.761	3.527	1.302	1.861	2.218	2.915	3.723	1.346	1.924	2.293	3.013	3.849	1.435	2.052	2.445	3.214	4.105
73	1.233	1.762	2.100	2.760	3.525	1.300	1.859	2.216	2.912	3.719	1.344	1.922	2.290	3.009	3.844	1.433	2.049	2.441	3.208	4.098
74	1.232	1.761	2.099	2.758	3.523	1.299	1.858	2.214	2.909	3.716	1.343	1.920	2.287	3.006	3.840	1.430	2.045	2.437	3.203	4.091
75	1.231	1.760	2.098	2.757	3.521	1.298	1.856	2.211	2.906	3.712	1.341	1.917	2.285	3.002	3.835	1.428	2.042	2.433	3.197	4.084
76	1.230	1.759	2.096	2.755	3.519	1.297	1.854	2.209	2.904	3.709	1.339	1.915	2.282	2.999	3.831	1.426	2.039	2.429	3.192	4.078
77	1.230	1.758	2.095	2.754	3.517	1.296	1.853	2.207	2.901	3.706	1.338	1.913	2.279	2.996	3.826	1.423	2.035	2.425	3.187	4.071
78	1.229	1.758	2.094	2.752	3.516	1.295	1.851	2.206	2.898	3.702	1.336	1.911	2.277	2.992	3.822	1.421	2.032	2.421	3.182	4.065
79	1.229	1.757	2.093	2.751	3.514	1.293	1.849	2.204	2.896	3.699	1.335	1.909	2.274	2.989	3.818	1.419	2.029	2.418	3.177	4.059
80	1.228	1.756	2.092	2.749	3.512	1.292	1.848	2.202	2.894	3.696	1.334	1.907	2.272	2.986	3.814	1.417	2.026	2.414	3.173	4.053
81	1.227	1.755	2.091	2.748	3.510	1.291	1.846	2.200	2.891	3.693	1.332	1.905	2.270	2.983	3.810	1.415	2.023	2.411	3.168	4.047
82	1.227	1.754	2.090	2.747	3.509	1.290	1.845	2.198	2.889	3.690	1.331	1.903	2.267	2.980	3.806	1.413	2.020	2.407	3.163	4.041
83	1.226	1.753	2.089	2.746	3.507	1.289	1.843	2.196	2.887	3.687	1.329	1.901	2.265	2.977	3.803	1.411	2.017	2.404	3.159	4.035
84	1.226	1.753	2.088	2.744	3.506	1.288	1.842	2.195	2.884	3.684	1.328	1.899	2.263	2.974	3.799	1.409	2.014	2.400	3.155	4.030
85	1.225	1.752	2.087	2.743	3.504	1.287	1.841	2.193	2.882	3.682	1.327	1.897	2.261	2.971	3.795	1.407	2.012	2.397	3.150	4.024
86	1.225	1.751	2.086	2.742	3.503	1.286	1.839	2.191	2.880	3.679	1.326	1.896	2.259	2.968	3.792	1.405	2.009	2.394	3.146	4.019
87	1.224	1.750	2.086	2.741	3.501	1.285	1.838	2.190	2.878	3.676	1.324	1.894	2.257	2.966	3.788	1.403	2.007	2.391	3.142	4.014
88	1.224	1.749	2.085	2.740	3.500	1.284	1.837	2.188	2.876	3.674	1.323	1.892	2.255	2.963	3.785	1.402	2.004	2.388	3.138	4.000
89	1.223	1.749	2.084	2.738	3.498	1.284	1.835	2.187	2.874	3.671	1.322	1.890	2.253	2.960	3.781	1.400	2.001	2.385	3.134	4.004
90	1.223	1.748	2.083	2.737	3.497	1.283	1.834	2.185	2.872	3.669	1.321	1.889	2.251	2.958	3.778	1.398	1.999	2.382	3.130	3.999
91	1.222	1.747	2.082	2.736	3.495	1.282	1.833	2.184	2.870	3.666	1.320	1.887	2.249	2.955	3.775	1.396	1.997	2.379	3.127	3.994
92	1.222	1.747	2.081	2.735	3.494	1.281	1.832	2.182	2.868	3.664	1.319	1.886	2.247	2.953	3.772	1.395	1.994	2.376	3.123	3.989
93	1.221	1.746	2.081	2.734	3.493	1.280	1.830	2.181	2.866	3.661	1.318	1.884	2.245	2.950	3.769	1.393	1.992	2.373	3.119	3.985
94	1.221	1.745	2.080	2.733	3.491	1.279	1.829	2.180	2.864	3.659	1.317	1.882	2.243	2.948	3.766	1.392	1.990	2.371	3.116	3.980
95	1.220	1.745	2.079	2.732	3.490	1.278	1.828	2.178	2.863	3.657	1.315	1.881	2.241	2.945	3.763	1.390	1.987	2.368	3.112	3.976
96	1.220	1.744	2.078	2.731	3.489	1.278	1.287	2.177	2.861	3.654	1.314	1.880	2.240	2.943	3.760	1.388	1.985	2.366	3.109	3.971
97	1.219	1.743	2.077	2.730	3.488	1.277	1.826	2.175	2.859	3.652	1.313	1.878	2.238	2.941	3.757	1.387	1.983	2.363	3.105	3.967
98	1.219	1.743	2.077	2.729	3.486	1.276	1.825	2.174	2.857	3.650	1.312	1.877	2.236	2.939	3.754	1.385	1.981	2.360	3.102	3.963
99	1.219	1.742	2.076	2.728	3.485	1.275	1.824	2.173	2.856	3.648	1.311	1.875	2.234	2.936	3.751	1.384	1.979	2.358	3.099	3.958
100	1.218	1.742	2.075	2.727	3.484	1.275	1.822	2.172	2.854	3.646	1.311	1.874	2.233	2.934	3.748	1.383	1.977	2.355	3.096	3.954
101	1.218	1.741	2.075	2.726	3.483	1.274	1.821	2.170	2.852	3.644	1.310	1.872	2.231	2.932	3.746	1.381	1.975	2.353	3.092	3.950

	γ = 0.75					γ = 0.90					γ = 0.95					γ = 0.99				
N/P	0.75	0.90	0.95	0.99	0.999	0.75	0.90	0.95	0.99	0.999	0.75	0.90	0.95	0.99	0.999	0.75	0.90	0.95	0.99	0.999
102	1.217	1.741	2.074	2.726	3.482	1.273	1.820	2.169	2.851	3.642	1.309	1.871	2.230	2.930	3.743	1.380	1.973	2.351	3.089	3.946
104	1.217	1.739	2.073	2.724	3.480	1.272	1.818	2.167	2.848	3.638	1.307	1.869	2.227	2.926	3.738	1.377	1.969	2.346	3.083	3.939
106	1.216	1.738	2.071	2.722	3.477	1.270	1.816	2.164	2.845	3.634	1.305	1.866	2.224	2.922	3.733	1.374	1.965	2.342	3.077	3.931
108	1.215	1.737	2.070	2.721	3.475	1.269	1.815	2.162	2.842	3.630	1.303	1.864	2.221	2.918	3.728	1.372	1.962	2.337	3.072	3.924
110	1.214	1.736	2.069	2.719	3.473	1.268	1.813	2.160	2.839	3.626	1.302	1.861	2.218	2.915	3.723	1.369	1.958	2.333	3.066	3.917
112	1.214	1.735	2.068	2.717	3.471	1.267	1.811	2.158	2.836	3.623	1.300	1.859	2.215	2.911	3.719	1.367	1.955	2.329	3.061	3.910
114	1.213	1.734	2.067	2.716	3.469	1.265	1.809	2.156	2.833	3.619	1.299	1.857	2.212	2.908	3.714	1.365	1.951	2.325	3.056	3.904
116	1.212	1.733	2.065	2.714	3.468	1.264	1.808	2.154	2.831	3.616	1.297	1.855	2.210	2.904	3.710	1.363	1.948	2.321	3.051	3.897
118	1.212	1.733	2.064	2.713	3.466	1.263	1.806	2.152	2.828	3.613	1.296	1.852	2.207	2.901	3.706	1.360	1.945	2.318	3.046	3.891
120	1.211	1.732	2.063	2.712	3.464	1.262	1.804	2.150	2.826	3.610	1.294	1.850	2.205	2.898	3.702	1.358	1.942	2.314	3.041	3.885
122	1.210	1.731	2.062	2.710	3.462	1.261	1.803	2.148	2.823	3.607	1.293	1.848	2.203	2.895	3.698	1.356	1.939	2.311	3.037	3.879
124	1.210	1.730	2.061	2.709	3.461	1.260	1.801	2.147	2.821	3.604	1.291	1.847	2.200	2.892	3.694	1.354	1.936	2.307	3.032	3.873
126	1.209	1.729	2.060	2.708	3.459	1.259	1.800	2.145	2.819	3.601	1.290	1.845	2.198	2.889	3.690	1.352	1.934	2.304	3.028	3.868
128	1.209	1.728	2.060	2.707	3.458	1.258	1.799	2.143	2.816	3.598	1.289	1.843	2.196	2.886	3.686	1.350	1.931	2.301	3.024	3.862
130	1.208	1.728	2.059	2.705	3.456	1.257	1.797	2.141	2.814	3.595	1.288	1.841	2.194	2.883	3.683	1.349	1.928	2.298	3.019	3.857
132	1.208	1.727	2.058	2.704	3.455	1.256	1.796	2.140	2.812	3.592	1.286	1.839	2.192	2.880	3.679	1.347	1.926	2.295	3.015	3.852
134	1.207	1.726	2.057	2.703	3.453	1.255	1.795	2.138	2.810	3.590	1.285	1.838	2.190	2.878	3.676	1.345	1.923	2.292	3.012	3.847
136	1.207	1.725	2.056	2.702	3.452	1.254	1.793	2.137	2.808	3.587	1.284	1.836	2.188	2.875	3.673	1.343	1.921	2.289	3.008	3.842
138	1.206	1.725	2.055	2.701	3.450	1.253	1.792	2.135	2.806	3.585	1.283	1.834	2.186	2.873	3.669	1.342	1.918	2.286	3.004	3.837
140	1.206	1.724	2.054	2.700	3.449	1.252	1.791	2.134	2.804	3.582	1.282	1.833	2.184	2.870	3.666	1.340	1.916	2.283	3.000	3.833
142	1.205	1.723	2.054	2.699	3.448	1.252	1.790	2.132	2.802	3.580	1.281	1.831	2.182	2.868	3.663	1.338	1.914	2.280	2.997	3.828
144	1.205	1.723	2.053	2.698	3.446	1.251	1.788	2.131	2.801	3.578	1.280	1.830	2.180	2.865	3.660	1.337	1.912	2.278	2.993	3.824
146	1.204	1.722	2.052	2.697	3.445	1.250	1.787	2.130	2.799	3.575	1.279	1.828	2.178	2.863	3.657	1.335	1.909	2.275	2.990	3.820
148	1.204	1.722	2.051	2.696	3.444	1.249	1.786	2.128	2.797	3.573	1.278	1.827	2.177	2.861	3.654	1.334	1.907	2.273	2.987	3.815
150	1.204	1.721	2.051	2.695	3.443	1.248	1.785	2.127	2.795	3.571	1.277	1.825	2.175	2.859	3.652	1.332	1.905	2.270	2.983	3.811
152	1.203	1.720	2.050	2.694	3.441	1.248	1.784	2.126	2.794	3.569	1.276	1.824	2.173	2.856	3.649	1.331	1.903	2.268	2.980	3.807
154	1.203	1.720	2.049	2.693	3.440	1.247	1.783	2.125	2.792	3.567	1.275	1.823	2.172	2.854	3.646	1.330	1.901	2.265	2.977	3.803
156	1.202	1.719	2.049	2.692	3.439	1.246	1.782	2.123	2.791	3.565	1.274	1.821	2.170	2.852	3.644	1.328	1.899	2.263	2.974	3.799
158	1.202	1.719	2.048	2.691	3.438	1.246	1.781	2.122	2.789	3.563	1.273	1.820	2.169	2.850	3.641	1.327	1.897	2.261	2.971	3.796
160	1.202	1.718	2.047	2.691	3.437	1.245	1.780	2.121	2.787	3.561	1.272	1.819	2.167	2.848	3.638	1.326	1.896	2.259	2.968	3.792
162	1.201	1.718	2.047	2.690	3.436	1.244	1.779	2.120	2.786	3.559	1.271	1.818	2.166	2.846	3.636	1.324	1.894	2.256	2.965	3.788
164	1.201	1.717	2.046	2.689	3.435	1.244	1.778	2.119	2.785	3.557	1.270	1.816	2.164	2.844	3.634	1.323	1.892	2.254	2.963	3.785
166	1.201	1.717	2.045	2.688	3.434	1.243	1.777	2.118	2.783	3.555	1.269	1.815	2.163	2.843	3.631	1.322	1.890	2.252	2.960	3.781
168	1.200	1.716	2.045	2.687	3.433	1.242	1.776	2.117	2.782	3.553	1.269	1.814	2.162	2.841	3.629	1.321	1.888	2.250	2.957	3.778
170	1.200	1.716	2.044	2.687	3.432	1.242	1.775	2.116	2.780	3.552	1.268	1.813	2.160	2.839	3.627	1.320	1.887	2.248	2.955	3.774

	γ = 0.75					γ = 0.90					γ = 0.95					γ = 0.99				
N/P	0.75	0.90	0.95	0.99	0.999	0.75	0.90	0.95	0.99	0.999	0.75	0.90	0.95	0.99	0.999	0.75	0.90	0.95	0.99	0.999
172	1.199	1.715	2.044	2.686	3.431	1.241	1.775	2.115	2.779	3.550	1.267	1.812	2.159	2.837	3.624	1.318	1.885	2.246	2.952	3.771
174	1.199	1.715	2.043	2.685	3.430	1.240	1.774	2.114	2.778	3.548	1.266	1.811	2.158	2.835	3.622	1.317	1.884	2.244	2.950	3.768
176	1.199	1.714	2.043	2.684	3.429	1.240	1.773	2.113	2.776	3.547	1.266	1.810	2.156	2.834	3.620	1.316	1.882	2.243	2.947	3.765
178	1.199	1.714	2.042	2.684	3.428	1.239	1.772	2.112	2.775	3.545	1.265	1.809	2.155	2.832	3.618	1.315	1.880	2.241	2.945	3.762
180	1.198	1.713	2.042	2.683	3.427	1.239	1.771	2.111	2.774	3.543	1.264	1.808	2.154	2.831	3.616	1.314	1.879	2.239	2.942	3.759
185	1.197	1.712	2.040	2.681	3.425	1.237	1.769	2.108	2.771	3.539	1.262	1.805	2.151	2.827	3.611	1.311	1.875	2.234	2.937	3.751
190	1.197	1.711	2.039	2.680	3.423	1.236	1.767	2.106	2.768	3.536	1.261	1.803	2.148	2.823	3.606	1.309	1.872	2.230	2.931	3.744
195	1.196	1.710	2.038	2.678	3.421	1.235	1.766	2.104	2.765	3.532	1.259	1.800	2.145	2.819	3.601	1.307	1.868	2.226	2.926	3.738
200	1.195	1.709	2.037	2.677	3.419	1.234	1.764	2.102	2.762	3.529	1.258	1.798	2.143	2.816	3.597	1.304	1.865	2.222	2.921	3.731
205	1.195	1.708	2.036	2.675	3.418	1.233	1.762	2.100	2.760	3.526	1.256	1.796	2.140	2.812	3.593	1.302	1.862	2.219	2.916	3.725
210	1.194	1.708	2.035	2.674	3.416	1.231	1.761	2.098	2.757	3.522	1.255	1.794	2.138	2.809	3.589	1.300	1.859	2.215	2.911	3.719
215	1.194	1.707	2.034	2.673	3.414	1.230	1.759	2.096	2.755	3.519	1.253	1.792	2.135	2.806	3.585	1.298	1.856	2.212	2.907	3.713
220	1.193	1.706	2.033	2.671	3.413	1.229	1.758	2.095	2.753	3.516	1.252	1.790	2.133	2.803	3.581	1.296	1.854	2.209	2.903	3.708
225	1.192	1.705	2.032	2.670	3.411	1.228	1.756	2.093	2.750	3.514	1.251	1.788	2.131	2.800	3.577	1.294	1.851	2.205	2.898	3.703
230	1.192	1.704	2.031	2.669	3.409	1.227	1.755	2.091	2.748	3.511	1.249	1.787	2.129	2.798	3.574	1.293	1.848	2.202	2.894	3.697
235	1.191	1.704	2.030	2.668	3.408	1.226	1.754	2.090	2.746	3.508	1.248	1.785	2.127	2.795	3.571	1.291	1.846	2.199	2.890	3.692
240	1.191	1.703	2.029	2.667	3.407	1.226	1.752	2.088	2.744	3.506	1.247	1.783	2.125	2.792	3.567	1.289	1.843	2.197	2.887	3.688
245	1.190	1.702	2.028	2.666	3.405	1.225	1.751	2.087	2.742	3.503	1.246	1.782	2.123	2.790	3.564	1.288	1.841	2.194	2.883	3.683
250	1.190	1.702	2.028	2.665	2.404	1.224	1.750	2.085	2.740	3.501	1.245	1.780	2.121	2.788	3.561	1.286	1.839	2.191	2.880	3.678
255	1.190	1.701	2.027	2.664	3.403	1.223	1.749	2.084	2.739	3.499	1.244	1.779	2.119	2.785	3.558	1.284	1.837	2.189	2.876	3.674
260	1.189	1.700	2.026	2.663	3.401	1.222	1.748	2.083	2.737	3.496	1.243	1.777	2.118	2.783	3.555	1.283	1.835	2.186	2.873	3.670
265	1.189	1.700	2.025	2.662	3.400	1.222	1.747	2.081	2.735	3.494	1.242	1.776	2.116	2.781	3.552	1.282	1.833	2.184	2.870	3.666
270	1.188	1.699	2.025	2.661	3.399	1.221	1.746	2.080	2.734	3.492	1.241	1.774	2.114	2.779	3.550	1.280	1.831	2.181	2.867	3.662
275	1.188	1.699	2.024	2.660	3.398	1.220	1.745	2.079	2.732	3.490	1.240	1.773	2.113	2.777	3.547	1.279	1.829	2.179	2.864	3.658
280	1.188	1.698	2.023	2.659	3.397	1.219	1.744	2.078	2.730	3.488	1.239	1.772	2.111	2.775	3.544	1.278	1.827	2.177	2.861	3.655
285	1.187	1.698	2.023	2.658	3.396	1.219	1.743	2.076	2.729	3.486	1.238	1.771	2.110	2.773	3.542	1.276	1.825	2.175	2.858	3.651
290	1.187	1.697	2.022	2.658	3.395	1.218	1.742	2.075	2.727	3.484	1.237	1.769	2.108	2.771	3.540	1.275	1.823	2.173	2.855	3.647
295	1.186	1.697	2.022	2.657	3.394	1.217	1.741	2.074	2.726	3.482	1.237	1.768	2.107	2.769	3.537	1.274	1.822	2.170	2.853	3.644
300	1.186	1.696	2.021	2.656	3.393	1.217	1.740	2.073	2.725	3.481	1.236	1.767	2.106	2.767	3.535	1.273	1.820	2.169	2.850	3.641
310	1.185	1.695	2.020	2.655	3.391	1.216	1.738	2.071	2.722	3.477	1.234	1.765	2.103	2.764	3.531	1.271	1.817	2.165	2.845	3.634
320	1.185	1.694	2.019	2.653	3.389	1.215	1.737	2.069	2.719	3.474	1.233	1.763	2.100	2.760	3.526	1.268	1.814	2.161	2.840	3.628
330	1.184	1.693	2.018	2.652	3.388	1.213	1.735	2.067	2.717	3.471	1.231	1.761	2.098	2.757	3.522	1.266	1.811	2.158	2.836	3.623
340	1.184	1.693	2.017	2.651	3.386	1.212	1.734	2.066	2.715	3.468	1.230	1.759	2.096	2.754	3.519	1.265	1.808	2.154	2.831	3.617
350	1.183	1.692	2.016	2.649	3.384	1.211	1.732	2.064	2.713	3.465	1.229	1.757	2.094	2.752	3.515	1.263	1.805	2.151	2.827	3.612
360	1.183	1.691	2.015	2.648	3.383	1.210	1.731	2.062	2.710	3.463	1.228	1.755	2.092	2.749	3.512	1.261	1.803	2.148	2.823	3.607

	γ = 0.75					γ = 0.90					γ = 0.95					γ = 0.99				
N/P	0.75	0.90	0.95	0.99	0.999	0.75	0.90	0.95	0.99	0.999	0.75	0.90	0.95	0.99	0.999	0.75	0.90	0.95	0.99	0.999
370	1.182	1.690	2.014	2.647	3.382	1.210	1.730	2.061	2.708	3.460	1.227	1.754	2.090	2.746	3.508	1.259	1.801	2.146	2.820	3.602
380	1.182	1.690	2.013	2.646	3.380	1.209	1.728	2.059	2.707	3.458	1.225	1.752	2.088	2.744	3.505	1.258	1.798	2.143	2.816	3.598
390	1.181	1.689	2.013	2.645	3.379	1.208	1.727	2.058	2.705	3.455	1.224	1.751	2.086	2.742	3.502	1.256	1.796	2.140	2.813	3.593
400	1.181	1.688	2.012	2.644	3.378	1.207	1.726	2.057	2.703	3.453	1.223	1.749	2.084	2.739	3.499	1.255	1.794	2.138	2.809	3.589
425	1.180	1.687	2.010	2.642	3.375	1.205	1.723	2.054	2.699	3.448	1.221	1.746	2.080	2.734	3.492	1.251	1.789	2.132	2.802	3.579
450	1.179	1.686	2.009	2.640	3.372	1.204	1.721	2.051	2.695	3.443	1.219	1.743	2.077	2.729	3.486	1.248	1.785	2.127	2.795	3.570
475	1.178	1.685	2.007	2.638	3.370	1.202	1.719	2.048	2.692	3.438	1.217	1.740	2.073	2.725	3.481	1.245	1.781	2.122	2.788	3.562
500	1.177	1.683	2.006	2.636	3.368	1.201	1.717	2.046	2.689	3.434	1.215	1.737	2.070	2.721	3.475	1.243	1.777	2.117	2.783	3.555
525	1.177	1.682	2.005	2.635	3.366	1.199	1.715	2.043	2.686	3.431	1.213	1.735	2.067	2.717	3.471	1.240	1.773	2.113	2.772	3.548
550	1.176	1.681	2.004	2.633	3.364	1.198	1.713	2.041	2.683	3.427	1.212	1.733	2.065	2.713	3.466	1.238	1.770	2.109	2.772	3.541
575	1.175	1.681	2.003	2.632	3.362	1.197	1.712	2.039	2.680	3.424	1.210	1.731	2.062	2.710	3.462	1.236	1.767	2.106	2.767	3.535
600	1.175	1.680	2.002	2.631	3.360	1.196	1.710	2.038	2.678	3.421	1.209	1.729	2.060	2.707	3.458	1.234	1.764	2.102	2.763	3.530
625	1.174	1.679	2.001	2.629	3.359	1.195	1.709	2.036	2.676	3.418	1.208	1.727	2.058	2.704	3.455	1.232	1.762	2.099	2.759	3.525
650	1.174	1.678	2.000	2.628	3.357	1.194	1.707	2.034	2.674	3.416	1.207	1.725	2.056	2.702	3.451	1.230	1.759	2.096	2.755	3.520
675	1.173	1.678	1.999	2.627	3.356	1.193	1.706	2.033	2.672	3.413	1.205	1.724	2.054	2.699	3.448	1.229	1.757	2.094	2.752	3.515
700	1.173	1.677	1.998	2.626	3.355	1.192	1.705	2.032	2.670	3.411	1.204	1.722	2.052	2.697	3.445	1.227	1.755	2.091	2.748	3.511
725	1.172	1.676	1.998	2.625	3.354	1.192	1.704	2.030	2.668	3.408	1.203	1.721	2.050	2.694	3.442	1.226	1.753	2.089	2.745	3.507
750	1.172	1.676	1.997	2.624	3.352	1.191	1.703	2.029	2.667	3.406	1.202	1.719	2.049	2.692	3.439	1.225	1.751	2.086	2.742	3.503
800	1.171	1.675	1.996	2.623	3.350	1.189	1.701	2.027	2.663	3.402	1.201	1.717	2.046	2.688	3.434	1.222	1.747	2.082	2.736	3.495
850	1.171	1.674	1.994	2.621	3.348	1.188	1.699	2.025	2.661	3.399	1.199	1.714	2.043	2.685	3.430	1.220	1.744	2.078	2.731	3.489
900	1.170	1.673	1.993	2.620	3.347	1.187	1.697	2.023	2.658	3.396	1.198	1.712	2.040	2.682	3.426	1.218	1.741	2.075	2.726	3.483
950	1.169	1.672	1.992	2.619	3.345	1.186	1.696	2.021	2.656	3.393	1.196	1.710	2.038	2.679	3.422	1.216	1.738	2.071	2.722	3.477
1000	1.169	1.671	1.992	2.617	3.344	1.185	1.695	2.019	2.654	3.390	1.195	1.709	2.036	2.676	3.418	1.214	1.736	2.068	2.718	3.472
	1.150	1.645	1.960	2.576	3.291	1.150	1.645	1.960	2.576	3.291	1.150	1.645	1.960	2.576	3.291	1.150	1.645	1.960	2.576	3.291

Rauscher, H. Michael.

The microcomputer scientific software series 4: testing prediction accuracy. Gen. Tech. Rep. NC-107. St. Paul, MN: U.S. Department of Agriculture, Forest Service, North Central Forest Experiment Station; 1986. 19 p.

A computer program, ATEST, is described in this combination user's guide/programmer's manual. ATEST provides users with an efficient and convenient tool to test the accuracy of predictors. As input ATEST requires observed-predicted data pairs. The output reports the two components of accuracy, bias and precision.

KEY WORDS: Statistics, model validation, microcomputers, jackknife statistic, confidence intervals.

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20th Anniversary