USDA FOREST SERVICE
RESEARCH PAPER NC-43
1970

the IMPACT of ESTIMATION ERRORS on EVALUATIONS of TIMBER PRODUCTION OPPORTUNITIES

Dennis L. Schweitzer

NORTH CENTRAL FOREST EXPERIMENT STATION
FOREST SERVICE
U.S. DEPARTMENT OF AGRICULTURE

The author, formerly Economist for the North Central Station, is now with the Pacific Northwest Forest & Range Experiment Station, Portland.

North Central Forest Experiment Station
D. B. King, Director
Forest Service — U.S. Department of Agriculture
Folwell, Avenue
St. Paul, Minnesota 55101

CONTENTS

The Impact of Estimation Errors On Evaluations of Timber Production Opportunities

Dennis L. Schweitzer

Much attention has been paid to efficiently allocating funds to timber production. Alternative investments are usually ranked by their present worths or internal rates of return or benefit-cost ratios; investment priorities are then established to follow these rankings (Marty *et al.* 1966, Marty and Newman 1969, Webster 1960). Unfortunately, if the costs or returns that define investment opportunities are incorrectly estimated, such rankings may be worthless. This paper is concerned with measuring the impact of estimation errors on calculated present worths. These errors may occur in specifying costs, returns, the length of time between investments and harvests, and the costs of using funds in particular investments.

The importance of errors made yesterday in forcasting today's costs and prices can be readily determined. It is only necessary to find the difference in value between what was done, based on erroneous information, and what would have been done, given correct information. Unfortunately, the same kind of absolute value cannot be placed (now) on the importance of errors concerning tomorrow's costs and prices.[1] Instead, a variety of more-or-less satisfactory approximation techniques must be relied upon if anything is to be said about how much effort should be spent in refining data or about the likelihood that resulting decisions will be less than the best possible.

Perhaps the most common approach is to make a series of "sensitivity analyses." Essentially, the aim is to determine how much the present worth, or other decision criterion, is altered if input data are

[1] *For a summary of probabilistic techniques that can be used to determine the odds that particular values will be realized, given less-than-perfect data, see Dennis L. Schweitzer. Evaluating forest investments under uncertainty: a synopsis of available techniques. (Unpublished report on file at Pacific Northwest Forest and Range Exp. Sta., Portland, Ore.)*

systematically varied. A general availability of investment analysis computer programs (Chappelle 1969, Forster 1968, Schweitzer *et al.* 1967, Row 1963) makes feasible repeated analyses of complex alternatives. "Canned" linear programming routines usually show the effects on the objective function of relaxing each constraint. Wikstrom and Alley (1967) suggest that a multiple regression analysis can lead to the same goal. In this paper, partial derivatives and a graphical analysis are used to illustrate the effects of various estimation errors on the present worths of individual investments in timber production.[2]

Measuring Impacts By Partial Derivatives

When present worth is used as a measure of the desirability of an investment, it is defined as:

$$\text{(1)} \qquad \text{Present worth} = x_0 + \frac{x_1}{(1+i)^1} + \ldots + \frac{x_j}{(1+i)^j} + \ldots + \frac{x_n}{(1+i)^n}$$

where x_j is the income or cost occurring in the j^{th} year, i is the (assumed constant) cost of using investment funds expressed as a decimal fraction, and n is the number of years in the investment period or the length of the rotation. We will consider timber production opportunities that are defined by establishment costs (EC), constant annual costs (AC), and returns (R) from clear cutting at the end of a rotation. The present worth of a single rotation can be calculated from:

$$\text{(2)} \qquad PW = \frac{R}{(1+i)^n} - AC\left[\frac{(1+i)^n - 1}{i\,(1+i)^n}\right] - EC.$$

Errors in Establishment Costs

The sensitivity of present worth to errors in estimating the values on the right side of this expression can be explored by varying each over some range while holding the others constant. A straightforward approach is to take partial derivatives of the expression for present worth with respect to each of these investment parameters and graph the results. Taking the partial derivative of present worth with respect to establishment cost yields:

$$\text{(3)} \qquad \frac{\delta PW}{\delta EC} = -1.$$

---

2 *This approach follows that of Canada who was concerned with applications of continuous discounting to engineering problems. (John Robert Canada. The effect of risk and uncertainty in economic analyses of investments in capital assets. Unpublished doctoral thesis on file at Georgia Inst. of Tech., Atlanta.) I acknowledge Allen L. Lundgren's major contribution in formulating the following partial derivatives.*

That is, if the establishment costs were underestimated by $1.00, present worth would be overestimated by the same amount.

The meaningfulness of such a derivative can be illustrated by considering how an opportunity to produce jack pine with clear cutting after 40 years might be evaluated. The investment costs and incomes have been estimated to be $10.00 per acre for establishment costs, $.40 per acre per year for annual costs, and $200.00 per acre income from harvesting at age 40. Further, a 6 percent interest rate must be charged against using funds in this investment. The anticipated present worth would be:

$$(4) \qquad PW = \frac{\$200.00}{(1.06)^{40}} - \$.40 \left[\frac{(1.06)^{40} - 1}{(.06)\ (1.06)^{40}} \right] - \$10 = \$3.42 \text{ per acre.}$$

That is, discounted returns are expected to exceed discounted costs by $3.42 per acre. The above derivative suggests that if establishment costs were underestimated by, say, $4.00, the investment would be a loss.

Errors in Harvest Returns

Another error might arise in estimating harvest returns. The partial derivative of present worth with respect to harvest returns is:

$$(5) \qquad \frac{\delta PW}{\delta R} = \left[\frac{1}{(1 + i)^n} \right].$$

When this expression is graphed, the impact of assuming returns to be, say, $180.00 rather than $200.00 can be quickly determined (fig. 1). For example, assuming the year of harvesting to be 40 and using 6 percent interest, present worth will change about $.10 for each $1.00 change in harvest returns. Therefore, returns reduced by $20.00 would lower present worth for this investment by about $2.00 to less than $1.50 per acre.

Errors in Annual Costs

Similarly, figure 2 can be prepared after determining that the partial derivative with respect to annual costs is:

$$(6) \qquad \frac{\delta PW}{\delta AC} = - \left[\frac{(1 + i)^n - 1}{i\ (1 + i)^n} \right].$$

Underestimating annual costs by, say, $.25 a year leads to an overestimate of present worth of about $3.75—enough difference to change an apparently profitable venture to one that will lose money.

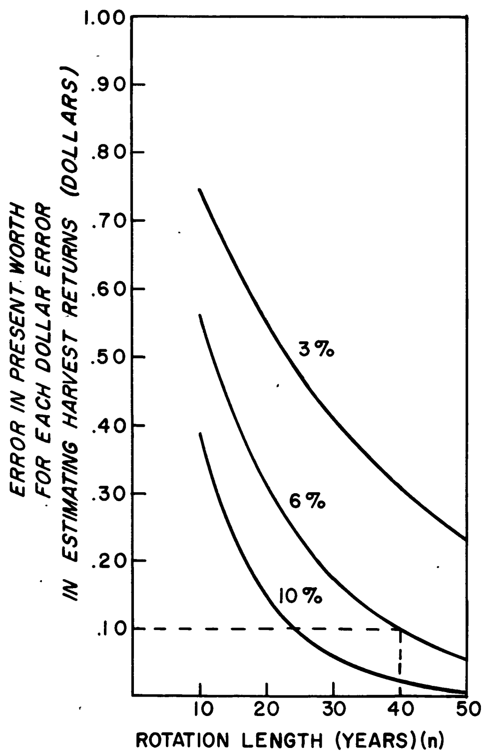

Figure 1.—Impact on present worth of erroneously estimating harvest returns.

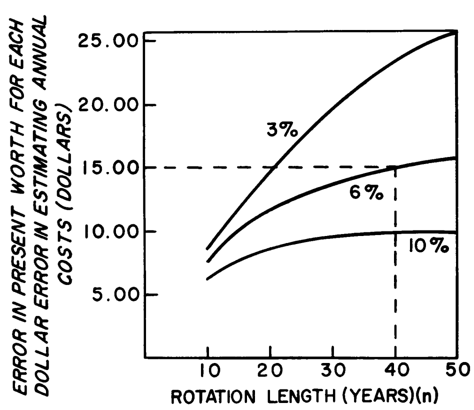

Figure 2.—Impact on present worth of erroneously estimating annual costs.

Errors in Length of Rotation

Considering the effect of poorly estimating the time of final harvest is slightly more complex. The partial derivative is :[3]

$$\frac{\delta PW}{\delta n} = -\left[R\left(\frac{\ln\ (1 + i)}{(1 + i)^n}\right) + AC\left(\frac{\ln\ (1 + i)}{i\ (1 + i)^n}\right) \right]. \tag{7}$$

Because a change in the rotation length, *n*, will affect the discounted values of both annual costs and harvest returns, both figures 3A and 3B must be used to determine the full effect. Receiving the income, *R*, after 39 years rather than after 40, will increase present worth about $.006 for each dollar of *R* because that sum will be discounted for 1 less year (fig. 3A). Such a change is seen to further increase present worth by

---

3 *Recall that all variables in the present worth equation have been assumed correct except the length of the rotation. If, because* n *is incorrect, the clear-cut yield and return,* R, *is also wrong, a more complex expression is required:*

$$\frac{\delta PW}{\delta n} = -\left[R\left(\frac{\ln\ (1 + i)}{(1 + i)^n}\right) + AC\left(\frac{\ln\ (1 + i)}{i\ (1 + i)^n}\right) \right] + \frac{\delta R}{\delta n}\left[\frac{1}{(1 + i)^n}\right]. \tag{8}$$

If the new term is assumed to be roughly equivalent to the value of the mean annual increment, in the jack pine example $\delta R / \delta n$ *would be about* $200 ÷ 40 *years or* $5.00. *Discounting at* 6 *percent per year, then, would result in additional change of about* $.50 *for each year change in rotation length.*

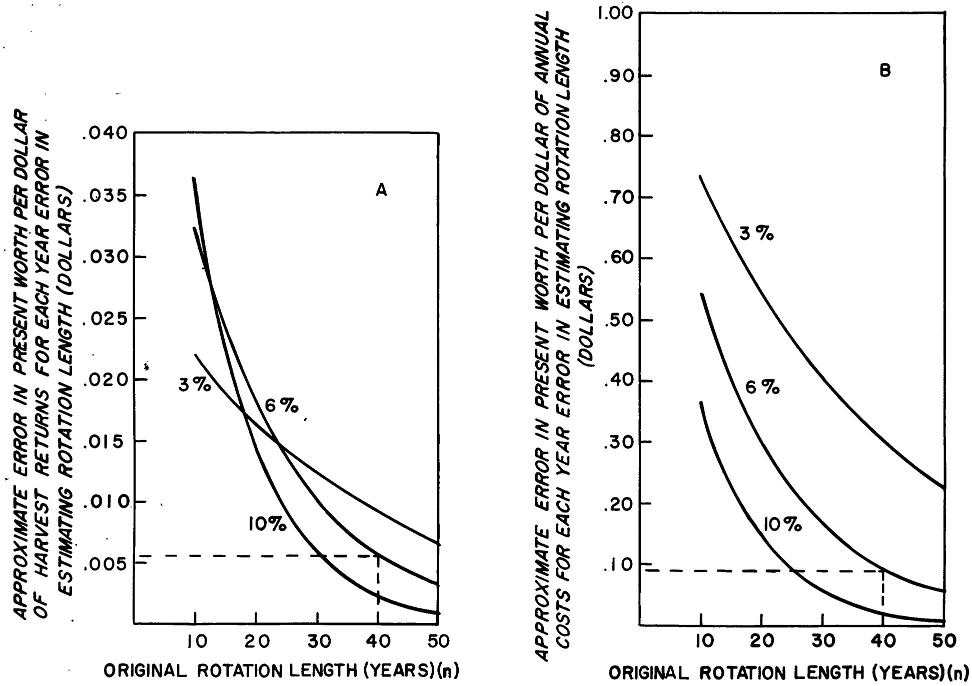

Figure 3.—(A) Approximate impact on present worth per dollar of harvest returns for each year error in estimating rotation length. (B) Approximate impact on present worth per dollar of annual costs for each year error in estimating rotation length.

about $.10 for each dollar in annual costs because those costs will be borne for 1 less year (fig. 3B). In terms of the jack pine example, the total effect would be to increase the discounted value of net profits by ($.006) x ($200) + ($.10) x ($.40) = $1.24 per acre.

Errors in Discount Rate

Finally, the partial derivative of present worth with respect to the discount rate can be written as:[4]

(9) $$\frac{\delta PW}{\delta i} = -\left[R\left(\frac{n}{(1 + i)^{n+1}}\right) - AC\left(\frac{1}{i^2} - \frac{1 + i + ni}{i^2\ (1 + i)^{n+1}}\right)\right].$$

The amounts of future costs and returns of a particular investment are assumed independent of *i*, the cost of using investment funds. If the cost of investment funds should increase from 6 to 7 percent, present worth would be decreased because discounted harvest returns would be less but

4 *This expression gives the average of the impact of decreasing the interest rate* 1 *percent and the somewhat smaller impact of increasing the rate a like amount. See section entitled "Impact Analysis by Computer."*

increased because discounted annual costs would be less (figs. 4A and 4B). For the jack pine example, present worth would decline to the point where this investment could only be undertaken at a loss.

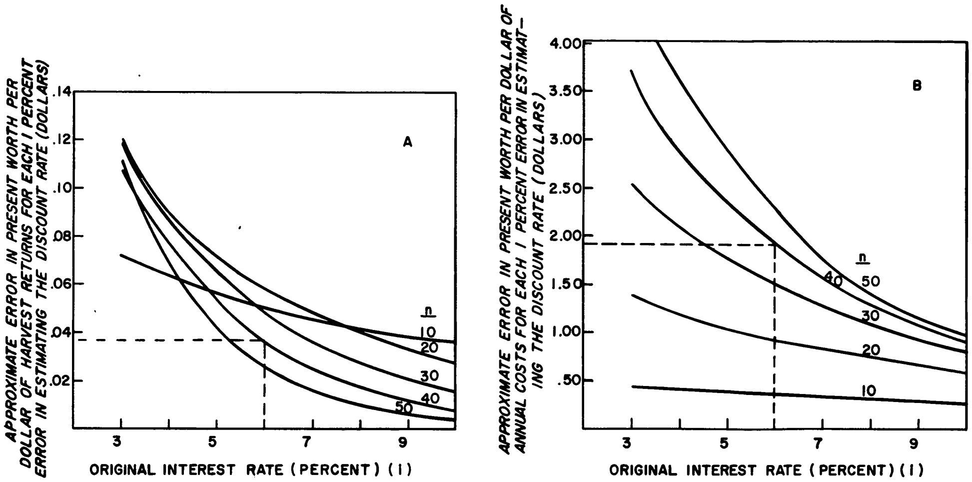

Figure 4.—(A) Approximate impact on present worth per dollar of harvest returns for each 1 percent error in estimating the discount rate. (B) Approximate impact on present worth per dollar of annual costs for each 1 percent error in estimating the discount rate.

Relative Importance of These Errors

We might summarize the findings of this jack pine example by noting that present worth will be altered by $1.00 if any of the following estimating errors are made:

	Error in absolute terms	*Error in relative terms (Percent)*
Establishment costs	$ 1.00	10.0
Annual costs	$ .07	17.5
Harvest returns	$10.00	5.0
Rotation length	1 year	2.5
Discount rate	.2 percent	3.0

This summary suggests that choosing an appropriate discount rate is important. If that rate is set too high, perhaps in an attempt to allow for uncertainties (Flora 1964), a desirable investment opportunity may be lost. On the other hand, using an artificially low rate (2 or 3 percent) will lead to investments that cannot be justified using a present worth criterion. The critical role of the discount rate suggests that its determination deserves expert attention: a rule-of-thumb approach is clearly inappropriate.

A measure of the relative importance of correctly estimating costs and returns can be derived by dividing the partial derivative of present worth with respect to a cost by that with respect to harvest returns. When these ratios for annual costs are plotted, for example, we see that at 6 percent over 40 years a dollar error in estimating annual costs will have about 150 times the effect of a dollar error in estimating harvest returns (fig. 5). Being off $.07 a year is equivalent to about a $10.00 error in estimating harvest returns, as illustrated by the jack pine problem. In general, mistakes in forcasting harvest returns are relatively important for short rotations while annual costs are the dominating factor over longer periods. All errors, except those concerning establishment costs (which are not discounted), are most critical when interest rates are low.

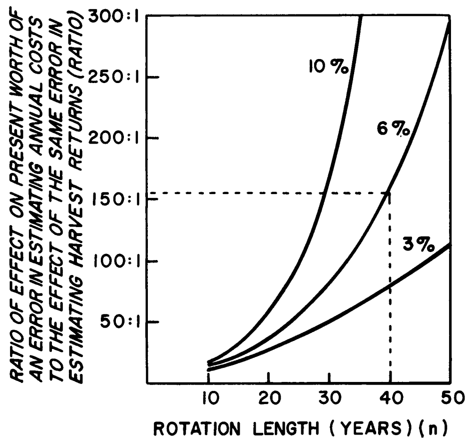

Figure 5.—The relative impacts on present worth of equal errors in estimating annual costs and harvest returns.

Impact Analysis By Computer

A computer routine titled IMPACT (see the Appendix for programing details) enables us to verify that these general conclusions remain valid for complex investments in which costs and receipts occur at irregular intervals. As an illustration consider a proposed investment in Douglas-fir management defined by the following anticipated costs and returns:

Year	*Cost or return per acre*	
Annually	–$ 1.50	Administration
0	– 50.00	Planting
5	– 25.00	Brush control
25	100.00	Thinning
40	550.00	Thinning
55	600.00	Thinning
70	650.00	Thinning
85	2,000.00	Clear cut

The computer printout quantifies the effects of errors in estimating each of these costs and returns (fig. 6).

SUB-PROGRAM IMPACT

INVESTMENT PERIOD = 85 YEARS
DISCOUNT RATE = 6.00 PERCENT

CHANGE IN DISCOUNTED VALUE DUE TO ONE PERCENT CHANGE IN DISCOUNT RATE APPLIED TO FUTURE PAYMENT......
CHANGE IN DISCOUNTED VALUE DUE TO ONE YEAR CHANGE IN TIMING OF FUTURE PAYMENT......
CHANGE IN DISCOUNTED VALUE DUE TO ONE DOLLAR CHANGE IN FUTURE PAYMENT

TYPE OF PAYMENT	(LAST) YEAR	FUTURE PAYMENT	DISCOUNTED VALUE	PERCENT OF COSTS	DISCOUNTED INCOMES	D (DISC. VALUE) / D (PAYMENT)	D (DISC. VALUE) / D (YEAR)	D (DISC. VALUE) / D (RATE)
ANNUAL	85	-1.50	-24.82	26.5	0.0	16.54895	.01029	3.99564
SINGLE	0	-50.00	-50.00	53.5	0.0	1.00000	2.91345	0.00000
SINGLE	5	-25.00	-18.68	20.0	0.0	.74726	1.08855	.88120
SINGLE	25	100.00	23.30	0.0	18.5	.23300	1.35766	5.49525
SINGLE	40	550.00	53.47	0.0	42.4	.09722	3.11577	20.17819
SINGLE	55	600.00	24.34	0.0	19.3	.04057	1.41829	12.62948
SINGLE	70	650.00	11.00	0.0	8.7	.01693	.64112	7.26599
SINGLE	85	2000.00	14.13	0.0	11.2	.00706	.82313	11.32777

32.74 = PRESENT WORTH

INFINITE ROTATIONS ADJUSTMENT FACTOR = 1.00711

Figure 6.—Analyzing the importance of estimation errors in the Douglas-fir problem by the IMPACT computer routine.

The size of each income and cost (a minus sign), whether it is a one-time or annual payment, and the year when it occurs (for annuities, the last year it occurs) are printed in the first three columns. The present values of the payments, which are calculated by using the discount rate and investment period printed at the top of the page, are listed and summed in the fourth column.

The percentage contributions of each of the individual discounted incomes and costs to total discounted incomes and costs are given in the next two columns. In the current example, planting costs (age 0) make up over half of total discounted costs, while the major item in discounted incomes is the thinning at age 40. This single thinning accounts for 42 percent of discounted incomes, compared to only 11 percent contributed by a much larger volume harvested at age 85. Further study shows that the first three thinnings account for 80 percent of all discounted incomes. Thus, in this preliminary analysis, one can quickly identify the important items for further study.

In the seventh column are listed the first three types of partial derivatives we have considered: those taken with respect to individual annual ($\delta PW/\delta AC$) and one-time or single ($\delta PW/\delta EC$ and $\delta PW/\delta R$) payments. For example, the present value of a \$2.50 annual payment (for 85 years at 6 percent) would be \$16.55 greater than the listed present value of \$24.82, which is for a \$1.50 annuity. An increase in the receipts expected at age 25 from \$100 to \$120 would increase present worth by (\$20 x .233 =) \$4.66. These partials (which have been tabulated by

Lundgren,[5] Marty and Neebe (1966) and others as discounting "multipliers") are multiplied by the corresponding future payments to get the column of discounted values.

The approximate effect of varying the investment period for each payment is quantified in the next-to-last column. If the $1.50 annuity were paid for 84 rather than 85 years, its present value would be about 1 cent less. That these entries are approximations is apparent when considering again the receipts expected in the 25th year. Receiving the $100 24 years from now would increase the present value by $1.39; waiting for 26 years would decrease it by $1.31. The column entry averages these two changes. The sum of this column of entries, when the directions of individual income and cost effects are considered, is comparable to $\delta PW / \delta n$, the total effect of delaying all investment elements 1 year (e.g., as when there is a 1-year regeneration lag).

The magnitude of error introduced by these approximations can be illustrated by summing the individual impacts, suggested by the partials, of delaying all payments 1 year in the Douglas-fir example. We assume annual administrative costs must be extended 1 year.

Year of payment		*Change in present worth*
Example	*+1*	*(Dollars)*
Annual		–0.01029
0	1	+2.91345
5	6	+1.08855
25	26	–1.35766
40	41	–3.11577
55	56	–1.41829
70	71	– .64112
85	86	– .82313
Total negative:		7.36626
Total positive:		4.00200

The net change in present worth estimated by this method is the algebraic sum of negative and positive changes, –$7.36 + $4.00 = –$3.36. The true net change, calculated by subtracting the present worth of the example ($32.74) from the present worth of the same investment with all payments delayed 1 year ($29.47), is –$3.27. Thus, in this example, the error in estimating the change in net worth from partials rather than calculating it directly is only $.09 (about 3 percent of the change in net worth).

[5] *Allen L. Lundgren. Compound-discount interest rate multipliers for evaluating forest investments. (Unpublished tables on file at N. Cent. Forest Exp. Sta., St. Paul.)*

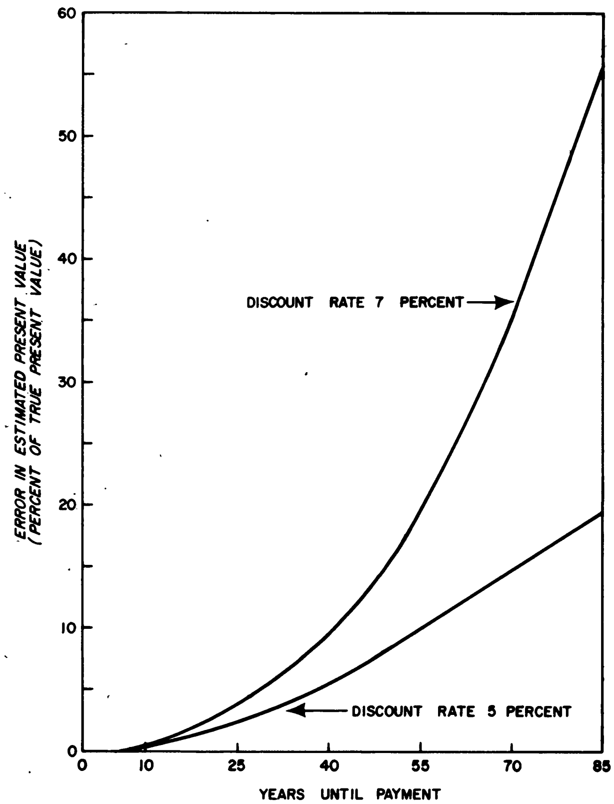

Figure 7.—The errors in present values introduced by using partials to estimate impacts of changing the discount rate from 6 percent.

Finally, the right-most entries express the sensitivity of the individual discounted values to a 1-percent change in the discount rate. The sum of these entries is analogous to the last graphed derivative, $\delta PW / \delta i$. Again, because the impact on present worth is greater for decreasing the discount rate than for increasing it a like amount, these partials are approximations. In the present case, though, the estimates are so poor that they might well be unacceptable. If, in the Douglas-fir problem, we were to rely on partials to determine present worth in substituting discount rates of 5 percent and 7 percent for the assumed 6 percent, the errors would be large. Present worth would be estimated as \$84.77 in the former case, but it would actually be \$102.53; if increasing the rate to 7 percent, –\$19.29 would be estimated when –\$7.45 is correct. The partials will always overestimate the impacts of increasing the rate and underestimate the impacts of decreasing the rate (fig. 7).

Obviously, the IMPACT routine is not adequate to tell everything about the sensitivity of present worth to the parameters defining an investment. It does, however, provide a fast and accurate first suggestion of which data items are likely to be most critical.

Sensitivity Of The Internal Rate Of Return

The rate earned by an investment when discounted costs and returns are equal is called the internal rate of return. For the earlier jack pine problem, i would be that rate if:

$$\text{(10)} \qquad \frac{R}{(1 + i)^n} = AC\left[\frac{(1 + i)^n - 1}{i\ (1 + i)^n}\right] + EC.$$

Rankings on the basis of this measure provide a means of choosing between investments. Again, though, the possibility of forecasting errors must be considered in evaluating the reliability of such rankings.

Starting from the earlier assumptions of $10.00 for establishment costs and $.40 a year in annual costs, internal rates of return can be calculated for several possible jack pine rotation lengths (table 1). The internal-rate-of-return criterion of desirability suggests that a 30-year rotation is most desirable, given our assumptions. However, if estimated prices are too high or yields too low, the ranking might be radically changed (fig. 8). Although a 30-year rotation would be preferred over the range of errors considered here, the shorter rotation varies all the way from being least desirable to second-most desirable. If errors were to vary by years because pulpwood prices change unexpectedly, any desirability ranking of the rotation lengths would be possible.

Table 1.—*Illustrative rates of return for alternative rotation lengths in growing jack pine*

Rotation	Estimated yield	Estimated price per cord	Estimated harvest returns	Internal rate of return
Years	Cords	Dollars	Dollars	Percent
20	9	6.00	50	6.4
30	20	6.50	130	7.5
40	28	7.00	200	6.6
50	34	7.50	250	5.6

In the case of the present worth criterion, errors in estimating final harvests were found to be most critical for short rotations. This is also true for the internal rate of return (fig. 8). The impact of an error is less for the longer rotations (the slopes of the curves decrease).

In considering how conservative estimates of returns should be to compensate for possible underestimates of costs, it is helpful to multiply both sides of the definitive equation by $(1 + i)^n$. This leads to:

$$\text{(11)} \qquad R = AC\left[\frac{(1 + i)^n - 1}{i}\right] + EC(1 + i)^n.$$

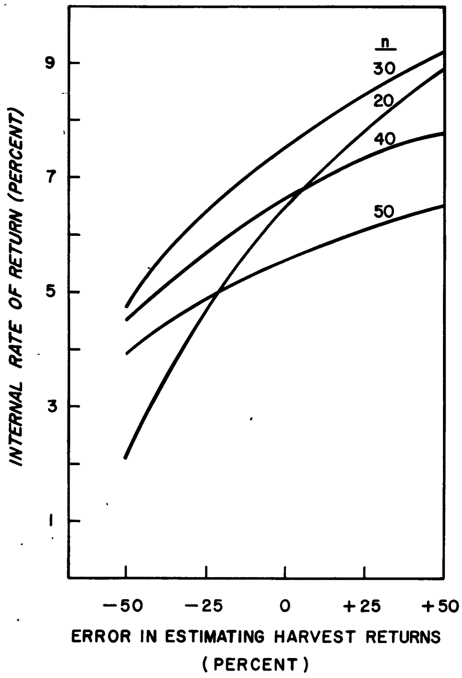

Figure 8.—The sensitivity of internal rate of return to errors in estimating harvest returns for the jack pine problem.

By again taking partial derivatives, we can determine the trade-offs of returns for costs that will just maintain the equality and that will leave the internal rate of return unchanged. For the current example where $i = 7.5$ percent in the optimal 30-year rotation:

(12)
$$\frac{\delta R}{\delta AC} = \frac{(1 + i)^n - 1}{i} = 103.40$$
$$\frac{\delta R}{\delta EC} = (1 + i)^n = 8.75.$$

That is, each $1.00 underestimate of annual costs or establishment costs must be balanced, respectively, by underestimates of $103.40 and $8.75 of harvest returns if the internal rate of return is to be maintained. Substituting these values back into the preceding equation permits the calculation of the required returns for any levels of costs as:

(13)
$$R = 103.40\ (AC) + 8.75\ (EC).$$

A Final Word

This paper has explored how an arithmetic technique, the taking of partial derivatives, can be used to evaluate the "goodness" of data. In particular, the question of how much a calculated present worth will vary, given unit changes in a set of defining costs and returns, has been examined. We know what the impact will be if there is an estimating error — the companion question, which is not considered here, is what is the chance that there will be an estimating error. Given answers to both of these questions, a rational plan of data collection and analysis can be designed.

A basic premise is that data are collected to answer questions that guide actions. Ultimately, a decisionmaker must use his judgment to select a few of the many possible timber management opportunities. A sensitivity analysis such as this one can help to insure it will be an informed judgment.

Literature Cited

Chappelle, Daniel E. 1969. A computer program for evaluating forestry opportunities under three investment criteria. USDA Forest Serv. Res. Pap. PNW-78, 64 p., illus. Pacific Northwest Forest & Range Exp. Sta., Portland Ore.

Flora, Donald F. 1964. Uncertainty in forest investment decisions. J. Forest. 62: 376-380.

Forster, Robert B. 1968. A computer technique for the evaluation of investment alternatives. Forest Econ. Res. Inst., Ottawa, Info. Rep. E-X-1, 19 p.

Marty, Robert, and Neebe, David J. 1966. Compound interest tables for long-term planning in forestry. U.S. Dep. Agr., Agr. Handb. 311, 103 p.

Marty, Robert, and Newman, Walker. 1969. Opportunities for timber management intensification on the National Forests. J. Forest. 67: 482-485.

Marty, Robert, Rindt, Charles, and Fedkiw, John. 1966. A guide for evaluating reforestation and stand improvement projects in timber management planning on the National Forests. U.S. Dep. Agr., Agr. Handb. 304, 24 p.

Row, Clark. 1963. Determining forest investment rates of return by electronic computer. USDA Forest Serv. Res. Pap. SO-6, 13 p., illus. Southern Forest Exp. Sta., New Orleans, Louisiana.

Schweitzer, Dennis L. 1968. A computer program to evaluate timber production investments under uncertainty. USDA Forest Serv. Res. Note NC-65, 3 p., illus. N. Cent. Forest Exp. Sta., St. Paul, Minn.

Schweitzer, Dennis L., Lundgren, Allen L., and Wambach, Robert F. 1967. A computer program for evaluating long-term forestry investments. USDA Forest Serv. Res. Pap. NC-10, 34 p., illus. N. Cent. Forest Exp. Sta., St. Paul, Minn.

Webster, Henry H. 1960. Timber management opportunities in Pennsylvania. USDA Forest Serv. Res. Pap. NE-137, 37 p., illus. Northeast. Forest Exp. Sta., Upper Darby, Penn.

Wikstrom, J. H., and Alley, J. R. 1967. Cost control in timber growing on the National Forests of the northern region. USDA Forest Serv. Res. Pap. INT-42, 37 p., illus. Intermount. Forest and Range Exp. Sta., Ogden, Utah.

Appendix

The IMPACT Computer Routine[6]

The IMPACT routine was written in FORTRAN II in the form of a "subroutine." It was designed to be used with some standard investment analysis computer program. All of the following values, for each of as many as 100 annuities and/or single (nonrecurring) payments which define an investment, can be calculated:

1. The discounted value.
2. The percent contribution to total discounted costs or incomes.
3. The effect on the discounted value of (a) a $1.00 change in the future payment, (b) a 1-year change in the time of the future payment, and (c) a 1-percent change in the discount rate.

A multiplier is also calculated that can be used to adjust any discounted value if it is judged appropriate to assume a future infinite series of rotations.

Originally, the routine was written for Control Data Corporation 6400 and 6600 computers, but there should be little difficulty in adapting the routine to other machines.

Linking the IMPACT Routine To an Investment Analysis Program

The IMPACT routine can be called by any standard or "canned" investment analysis program (written in FORTRAN) if a subroutine call statement comparable to the following is supplied as linkage:

Subroutine IMPACT (PAY, NA, NS, INTIME, RATE). The arguments are defined below:

Variable	*Meaning*
RATE	The discount rate.
INTIME	The number of years until the last payment (the investment period or length of rotation).
NS	The number of (nonrecurring) single payments.
NA	The number of (recurring) annual payments.

---

6 *Request for IMPACT source decks or comments on programming details should be addressed to Director, North Central Forest Experiment Station, Folwell Avenue, St. Paul, Minnesota* 55101.

Variable	*Meaning*
PAY (I, J)	The input data array, dimensioned in the subroutine as PAY (100, 2), which includes:
I = 1, NA	The annuities
J = 1	Last year of occurrence
J = 2	Amount of payment
I = (NA + 1), (NA + NS)	The single payments
J = 1	Year of occurrence
J = 2	Amount of payment

Explicit instructions for linking the IMPACT routine to my previously published investment analysis programs, NCRETURN (Schweitzer *et al.* 1967) and NCSUBPR (Schweitzer 1968), are available upon request.

Details of Program Construction

From the array PAY(I, J) supplied by the main program, IMPACT calculates and prints the output array P(I, J) which is defined below. Asterisks indicate that formulas are also given.

P(I, J) I = 1, (NA + NS)	The output data array dimensioned as P(100, 8)
J = 1	Same as PAY(I, 1)
J = 2	Same as PAY(I, 2)
J = 3	*Discounted value of payment
J = 4, 5	If payment is a cost (<0): P(I, 4) = discounted value as a percent of total discounted costs; P(I, 5) = zero. If payment is an income (>0): P(I, 4) = zero; P(I, 5) = discounted value as a percent of total discounted incomes
J = 6	* $\frac{\delta\,(\text{discounted value})}{\delta\,(\text{payment})}$
J = 7	* $\frac{\delta\,(\text{discounted value})}{\delta\,(\text{year of occurrence})}$
J = 8	* $\frac{\delta\,(\text{discounted value})}{\delta\,(\text{discount rate})}$

The following formulas, which are presented in the text, were used in calculating these values.

Annual payments. — Define n as the smaller of P (I, 1) and INTIME

(14)

$$\frac{\delta\ \text{(discounted value)}}{\delta\ \text{(payment)}} = \frac{(1 + \text{RATE})^{n} - 1}{\text{RATE}\ (1 + \text{RATE})^{n}}$$

$$\text{discounted value} = \text{P(I, 2)} \times \left[\frac{\delta\ \text{(discounted value)}}{\delta\ \text{(payment)}}\right]$$

$$\frac{\delta\ \text{(discounted value)}}{\delta\ \text{(year of occurrence)}} = \text{P(I, 2)} \times \left[\frac{1}{\text{RATE}} \times \frac{\ln\ (1 + \text{RATE})}{(1 + \text{RATE})^{n}}\right]$$

$$\frac{\delta\ \text{(discounted value)}}{\delta\ \text{(discount rate)}} = \text{P(I, 2)} \times \left[\frac{1 + \text{RATE} + (n \times \text{RATE})}{\text{RATE}^{2}\ (1 + \text{RATE})^{n+1}} - \frac{1}{\text{RATE}^{2}}\right]$$

Single payments.—Define n as P(I, 1) if P(I, 1) $\leq$ INTIME; otherwise $n = 0$.

(15)

$$\frac{\delta\ \text{(discounted value)}}{\delta\ \text{(payment)}} = \frac{1}{(1 + \text{RATE})^{n}}$$

$$\text{discounted value} = \text{P(I, 2)} \times \left[\frac{\delta\ \text{(discounted value)}}{\delta\ \text{(payment)}}\right]$$

$$\frac{\delta\ \text{(discounted value)}}{\delta\ \text{(year of occurrence)}} = \text{P(I, 2)} \times \left[\frac{\ln\ (1 + \text{RATE})}{(1 + \text{RATE})^{n}}\right]$$

$$\frac{\delta\ \text{(discounted value)}}{\delta\ \text{(discount rate)}} = \text{P(I, 2)} \times \left[\frac{-n}{(1 + \text{RATE})^{n+1}}\right]$$

Other FORTRAN variables used in writing the IMPACT routine are defined below:

Variable	*Meaning*
ADJ	Infinite rotations adjustment factor, calculated as: $\left[\frac{(1 + \text{RATE})^{\text{INTIME}}}{(1 + \text{RATE})^{\text{INTIME}} - 1}\right]$
DC	Discounted costs
DI	Discounted incomes
N	n (as defined in the preceding formulas)
NAP1	NA + 1
NPAY	NA + NS
PW	Present worth
RATELN	ln(1 + RATE)
RN	$(1 + \text{RATE})^{n}$

Required library functions

ABS	Take absolute value
ALOG	Take natural logarithm
FLOAT	Float a fixed variable
IFIX	Fix a floating variable
MIN1	Take the smaller value

A listing of the IMPACT routine source deck is presented as figure 9.

```
      SUBROUTINE IMPACT (PAY, NA, NS, INTIME, RATE)
      DIMENSION PAY(100, 2), P(100, 8)
C
C         SUBROUTINE PARAMETERS -
C           PAY IS INPUT ARRAY PAY(I,J) WHERE I NUMBERS PAYMENTS,
C               J=1 IS YEAR AND J=2 IS AMOUNT
C           NA IS NUMBER OF ANNUITIES
C           NS IS NUMBER OF SINGLES
C           INTIME IS INVESTMENT PERIOD
C           RATE IS DISCOUNT RATE
C
      WRITE ( 3, 5)
    5 FORMAT ( 1H1,50X, *SUB-PROGRAM IMPACT*  /// )
      RATE = RATE * 100.
      WRITE ( 3, 10)  INTIME, RATE
      RATE = RATE / 100.
   10 FORMAT ( 10X, *INVESTMENT PERIOD =*, I7,
     1      1X, *YEARS* / 10X, *DISCOUNT RATE      = *,F6.2,* PERCENT*//
     2 4X, *CHANGE IN DISCOUNTED VALUE DUE TO ONE PERCENT CHANGE IN*
     3      * DISCOUNT RATE APPLIED TO FUTURE PAYMENT......* /
     4 4X, *CHANGE IN DISCOUNTED VALUE DUE TO ONE YEAR CHANGE IN*
     5      * TIMING OF FUTURE PAYMENT......*, 17X, *.* /
     6 4X, *CHANGE IN DISCOUNTED VALUE DUE TO ONE DOLLAR CHANGE IN*
     7      * FUTURE PAYMENT*, 13X,*.*, 17X,*.*
     8              5( / 68X, *.*, 17X, *.*, 17X, *.* ) )
      WRITE (3, 20)
   20 FORMAT( 60X, 3( * D (DISC. VALUE)  *) /
     1 1X, *TYPE OF  (LAST)  FUTURE DISCOUNTED  PERCENT OF DISCOUNTED  *
     2      3(* ---------------  *)  /
     3 1X, *PAYMENT   YEAR  PAYMENT    VALUE       COSTS    INCOMES    *
     4    * D (PAYMENT)       D (YEAR)            D (RATE)*//)
C
      NPAY = NA + NS
      RATELN = ALOG ( RATE + 1.0)
C
      IF( NA ) 2000, 2000, 1001


C
C                          CALCULATE OUTPUTS FOR ANNUITIES
C
 1001 DO 1999     I = 1, NA
      N = MIN1( PAY(I,1), FLOAT(INTIME) )
      RN = (RATE + 1.0) ** N
      P(I,6) = (RN - 1.0) / (RATE * RN)
      P(I,3) = P(I,6) *PAY(I,2)
      P(I,7) = ABS( PAY(I,2) / RATE  *  RATELN / RN )
 1999 P(I,8) = ABS( ( PAY(I,2) / RATE**2 * (1.0 + RATE + N * RATE ) /
     1        (1.0 + RATE)**(N + 1) - ( PAY(I,2) / RATE**2)) * .01)
C
 2000 IF( NS ) 3000, 3000, 2001
```

Figure 9.—Listing of IMPACT routine source deck.

Figure 9 Continued

```
C
C                              CALCULATE OUTPUTS FOR SINGLES
C
 2001 NAP1 = NA + 1
      DO 2999     I = NAP1, NPAY
      IF (PAY(I,1) - FLOAT(INTIME) ) 2100, 2100, 2010
 2010 DO 2020     J = 3, 8
 2020 P(I,J) = 0.
      GO TO 2999
 2100 N = IFIX (PAY(I,1))
      RN = (RATE + 1.0) ** N
      P(I,6) = 1.0 / RN
      P(I,3) = P(I,6) *PAY(I,2)
      P(I,7) = ABS(PAY(I,2) * (RATELN / RN)  )
      P(I,8) = ABS(PAY(I,2) * PAY(I,1) / (1.0 + RATE)**(N+1)*.01  )
 2999 CONTINUE
C
C                              CALCULATE ALL PERCENT OUTPUTS
C
 3000 DC = 0.
      DI = 0.
      DO 3030     I = 1, NPAY
      IF ( P(I,3) ) 3010, 3030, 3020
 3010 DC = DC + P(I,3)
      GO TO 3030
 3020 DI = DI + P(I,3)
 3030 CONTINUE
      PW = DC + DI

C
C                              PRINT OUTPUTS
C
      DO 4999     I = 1, NPAY
      P(I,1) = PAY(I,1)
      P(I,2) = PAY(I,2)
      IF (P(I,3) ) 4010, 4030, 4020
 4010 P(I,4) = P(I,3)/DC * 100.
      P(I,5) = 0.
      GO TO 4030
 4020 P(I,4) = 0.
      P(I,5) = P(I,3)/DI* 100.
 4030 IF (I - NA) 4040, 4040, 4050
 4040 WRITE (3, 4041)
 4041 FORMAT (      1X, *ANNUAL*)
      GO TO 4060
 4050 WRITE (3, 4051)
 4051 FORMAT (      1X, *SINGLE*)
 4060 WRITE (3, 4061) (P(I,J), J = 1,8)
 4061 FORMAT(1H+, 9X,F5.0,F9.2, F11.2, 2( 5X, F5.1), 3(7X, F10.5) )
 4999 CONTINUE
      WRITE ( 3, 5010 )  PW
 5010 FORMAT ( / 26X, 9(*-*) // 24X, F11.2, * = PRESENT WORTH* )
C
C                              CALCULATE INFINITE ROTATIONS FACTOR
C
      ADJ = ( 1. + RATE )**INTIME / ( (1. + RATE)**INTIME - 1. )
      WRITE ( 3, 6010 )  ADJ
 6010 FORMAT ( //10X, *INFINITE ROTATIONS ADJUSTMENT FACTOR =*,F10.5)
C
      RETURN
      END
```

SOME RECENT RESEARCH PAPERS
OF THE
NORTH CENTRAL FOREST EXPERIMENT STATION

Durable Rustic Wooden Signboards, by Glenn A. Cooper. USDA Forest Serv. Res. Pap. NC-29, 15 p., illus. 1969.

The Market for Wood Picnic Structures, by Jerry A. Sesco. USDA Forest Serv. Res. Pap. NC-30, 7 p., illus. 1969.

Oak Forests of the Lake States and their Management, by John L. Arend and Harold F. Scholz. USDA Forest Serv. Res. Pap. NC-31, 36 p., illus. 1969.

Wildlife Esthetics & Auto Campers in the Superior National Forest, by David W. Lime and Charles T. Cushwa. USDA Forest Serv. Res. Pap. NC-32, 8 p., illus. 1969.

Standing Timber Coefficients for Indiana Walnut Log Production, by James E. Blyth, Edwin Kallio, and John C. Callahan. USDA Forest Serv. Res. Pap. NC-33, 9 p., illus. 1969.

Climatic Conditions Preceding Historically Great Fires in the North Central Region, by Donald A. Haines and Rodney W. Sando. USDA Forest Serv. Res. Pap. NC-34, 19 p., illus. 1969.

Initial Thinning in Red Pine Plantations, by John H. Cooley. USDA Forest Serv. Res. Pap. NC-35, 6 p., illus. 1969.

Biological Growth Functions Describe Published Site Index Curves for Lake States Timber Species, by Allen L. Lundgren and William A. Dolid. USDA Forest Serv. Res. Pap. NC-36, 9 p., illus. 1970.

Defect Indicators in Sugar Maple–A Photographic Guide, by Richard M. Marden, and Charles L. Stayton. USDA Forest Serv. Res. Pap. NC-37, 29 p., illus. 1970.

Estimating Sugar Maple Bark Thickness and Volume, by Charles L. Stayton and Michael Hoffman. USDA Forest Serv. Res. Pap. NC-38, 8 p., illus. 1970.

ABOUT THE FOREST SERVICE . . .

As our Nation grows, people expect and need more from their forests — more wood; more water, fish, and wildlife; more recreation and natural beauty; more special forest products and forage. The Forest Service of the U.S. Department of Agriculture helps to fulfill these expectations and needs through three major activities:

- Conducting forest and range research at over 75 locations ranging from Puerto Rico to Alaska to Hawaii.
- Participating with all State forestry agencies in cooperative programs to protect, improve, and wisely use our Country's 395 million acres of State, local, and private forest lands.
- Managing and protecting the 187-million acre National Forest System.

The Forest Service does this by encouraging use of the new knowledge that research scientists develop; by setting an example in managing, under sustained yield, the National Forests and Grasslands for multiple use purposes; and by cooperating with all States and with private citizens in their efforts to achieve better management, protection, and use of forest resources.

Traditionally, Forest Service people have been active members of the communities and towns in which they live and work. They strive to secure for all, continuous benefits from the Country's forest resources.

For more than 60 years, the Forest Service has been serving the Nation as a leading natural resource conservation agency.