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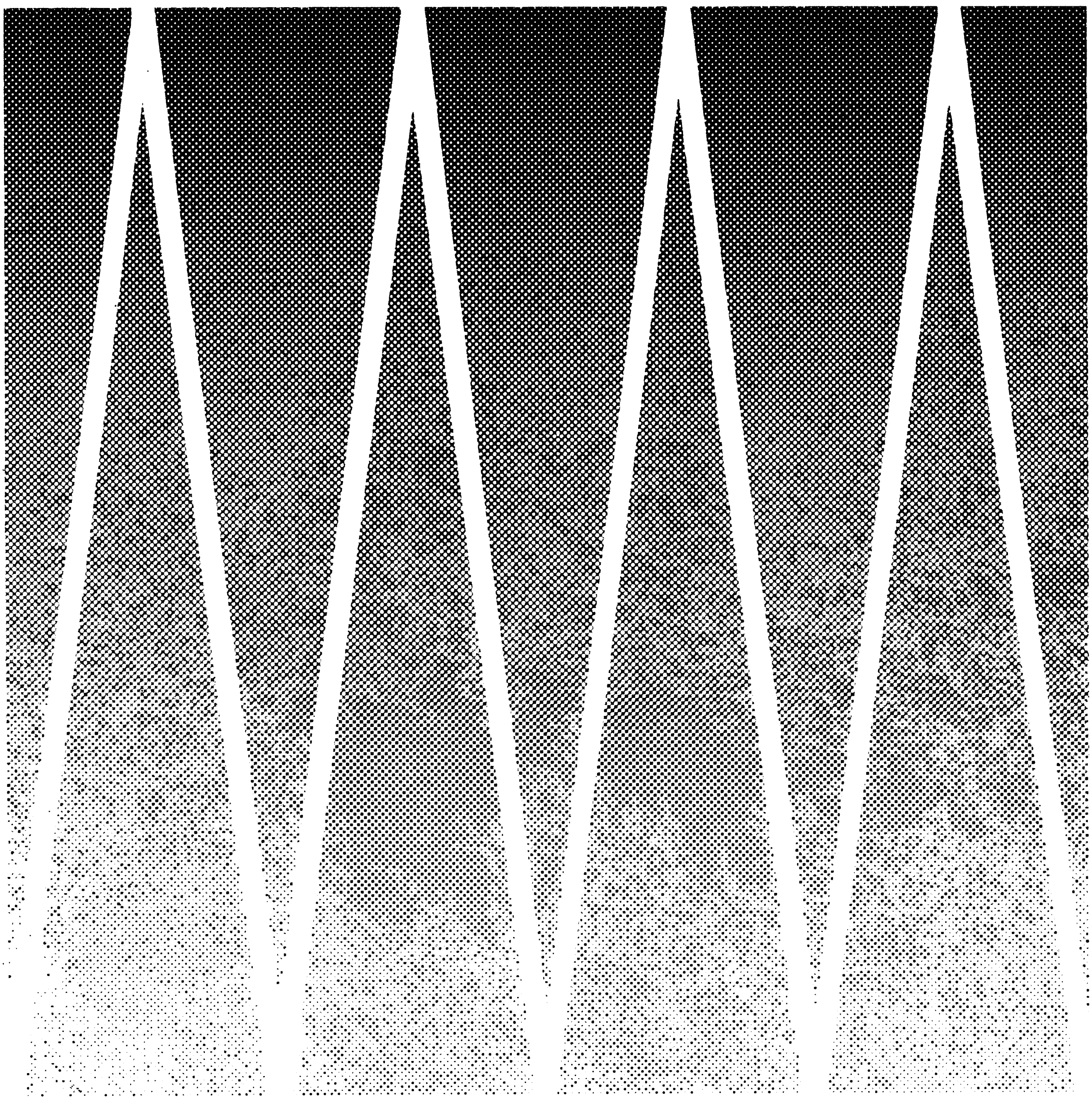
The effect of initial number of trees per acre and thinning densities on timber yields from RED PINE PLANTATIONS in the Lake States



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Contents

THE EFFECT OF INITIAL NUMBER OF TREES PER ACRE AND THINNING DENSITIES ON TIMBER YIELDS FROM RED PINE PLANTATIONS IN THE LAKE STATES

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In red pine (*Pinus resinosa* Ait.) stands the initial density (number of established trees per acre), timing of thinnings, and stand density left after thinning (basal area per acre) greatly affect tree diameter growth, and thus the salable timber output. Planting fewer trees reduces establishment costs and accelerates diameter growth of individual trees, resulting in larger, but fewer, trees at any given age. Thinning to low densities also results in fewer but larger trees. But the effects of initial density and subsequent thinning on the quantity and quality of timber product yields are not immediately obvious. A careful evaluation of stand density alternatives is needed to determine the best course of action to meet management goals.

A wide range of initial tree spacings have been recommended for red pine (Evert 1973). Generally, recommendations based primarily on physical yield considerations tend to favor a higher initial density than those based on economic considerations. Wambach (1967), in his analysis of initial spacing in red pine plantations, concluded that a wide spacing, with 400 established trees per acre, is economically preferable to a higher number of trees per unit area, and suggested that under some conditions even fewer trees may be more desirable. Wambach's findings were not published, and his suggestion has not been followed by an evaluation of lower initial densities. It has become apparent that an analysis of such low initial densities is needed.

This paper summarizes the effects of stand density on tree and stand characteristics and timber yields in red pine plantations in the Lake States region of the United States for a wide range of initial numbers of trees and subsequent thinning densities. Simulations of growth and yield in red pine plantations were made using REDPINE, an unpublished growth and yield computer program developed by the author from growth models by Buckman (1962) and Wambach (1967), along with other data.

The initial densities analyzed ranged from 50 to 1,600 established trees per acre; the thinning densities ranged from 60 to 180 square feet of basal area per acre left after thinning. This information should help forest landowners and managers who must decide which initial stocking level and subsequent program of thinnings will best meet their objectives in growing red pine for timber.

SUMMARY

Volume Yields

Several graphs have been prepared to summarize and better illustrate the effects of initial density, thinning density, and site index on volume yields from red pine plantations. These graphs display contour lines of equal annual volume production for a range of initial and residual thinning densities for each site index. They show major differences in production over the range of sites, initial densities, and thinning densities considered.

Total cubic-foot volume production (mean annual increment) drops rapidly on all sites with fewer than 200 established trees per acre (figs. 1-3). As initial density increases above 200 trees per acre, production rises gradually. As initial densities approach 1,200 to 1,600 trees per acre, production levels off, rising very little with additional trees per acre. Production is highest with a thinning density of from 120 square feet on site index 50 to 140 square feet on site index 70. Production falls off for thinning densities lower or higher than these.

Merchantable cubic-foot volume production displays a different pattern (figs. 4-6). Again, a deep trough of low merchantable volume production is noticeable with fewer than 200 trees per acre, but a peak of maximum production is evident at about 800 trees and 120 square feet of basal area on site index

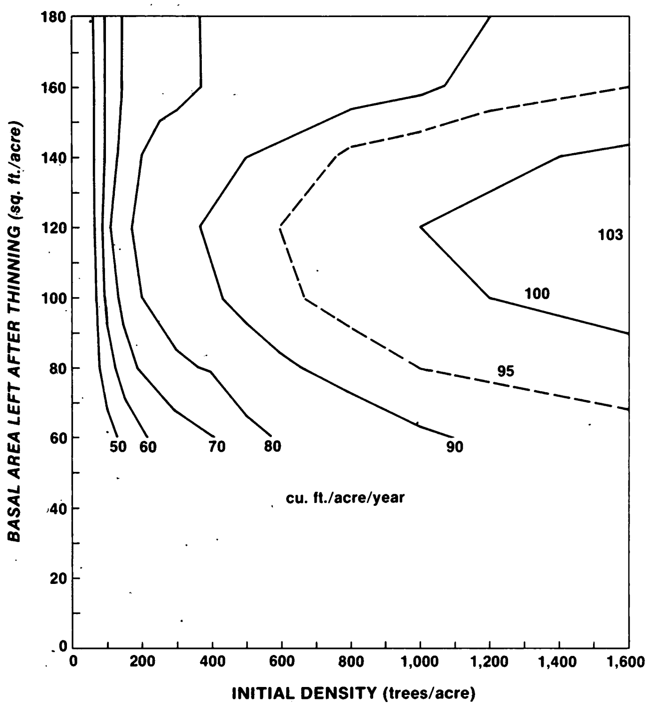

Figure 1.—*The effect of initial and thinning densities on maximum total cubic-foot mean annual increment in red pine plantations on site index 50.*

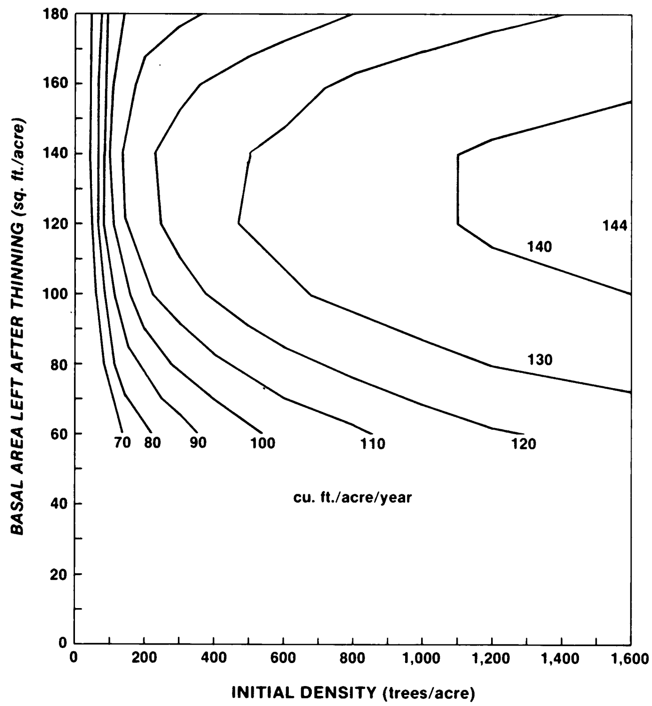

Figure 2.—*The effect of initial and thinning densities on maximum total cubic-foot mean annual increment in red pine plantations on site index 60.*

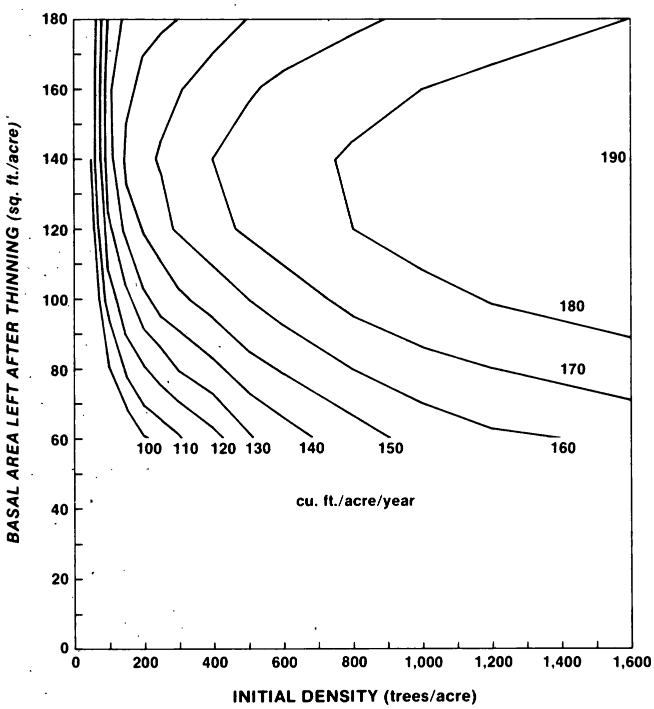

Figure 3.—*The effect of initial and thinning densities on maximum total cubic-foot mean annual increment in red pine plantations on site index 70.*

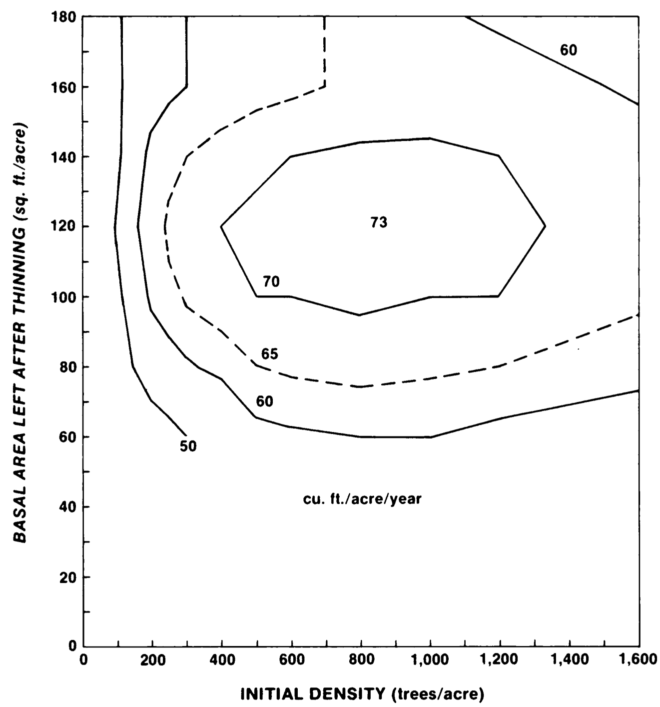

Figure 4.—*The effect of initial and thinning densities on maximum merchantable cubic-foot mean annual increment in red pine plantations on site index 50.*

50, and 1,000 trees and 140 square feet on site index 70. Production drops off at thinning densities above and below these levels.

Board-foot volume production displays an even more noticeable peak in production (figs. 7-9). Once again, there is a distinct depression in production below 200 trees per acre. A sharp peak in production occurs with 200 trees per acre on all sites in stands thinned to 120 square feet of basal area on site index 50, and to 140 square feet on site index 70. Production falls off as numbers of trees per acre increase beyond 200. The choice of thinning density becomes critical at higher numbers of trees per acre, with rapid decreases in production resulting from thinning either too much or too little.

Users of these results should keep in mind that the term "initial density" refers to the number of trees per acre that became established following early mortality the first few years after planting, and assumes reasonably even spacing of the established trees.

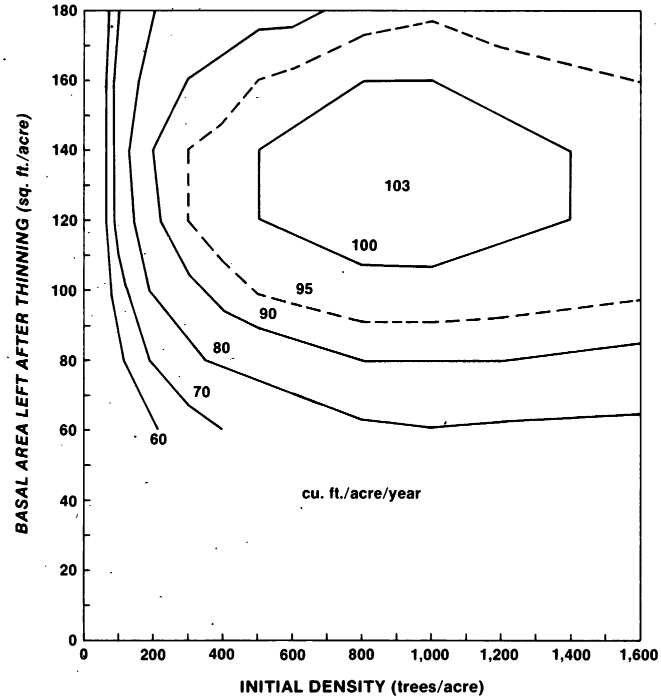

Figure 5.—*The effect of initial and thinning densities on maximum merchantable cubic-foot mean annual increment in red pine plantations on site index 60.*

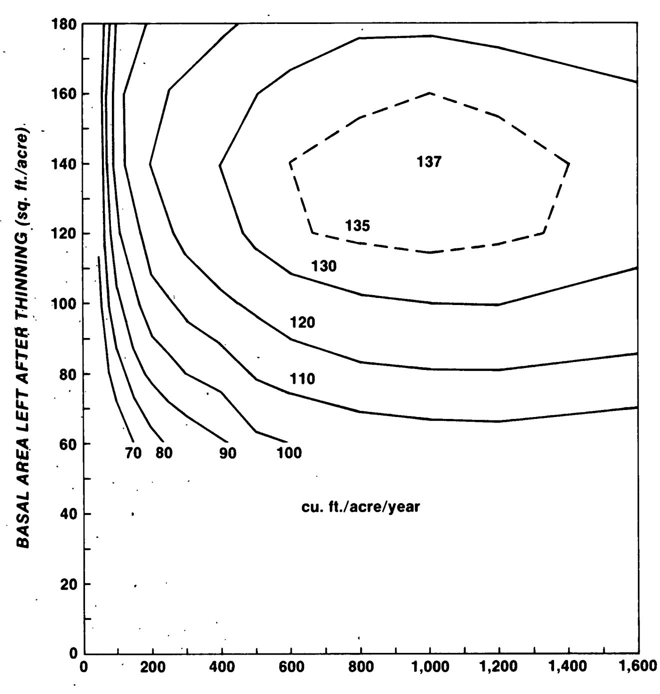

Figure 6.—*The effect of initial and thinning densities on maximum merchantable cubic-foot mean annual increment in red pine plantations on site index 70.*

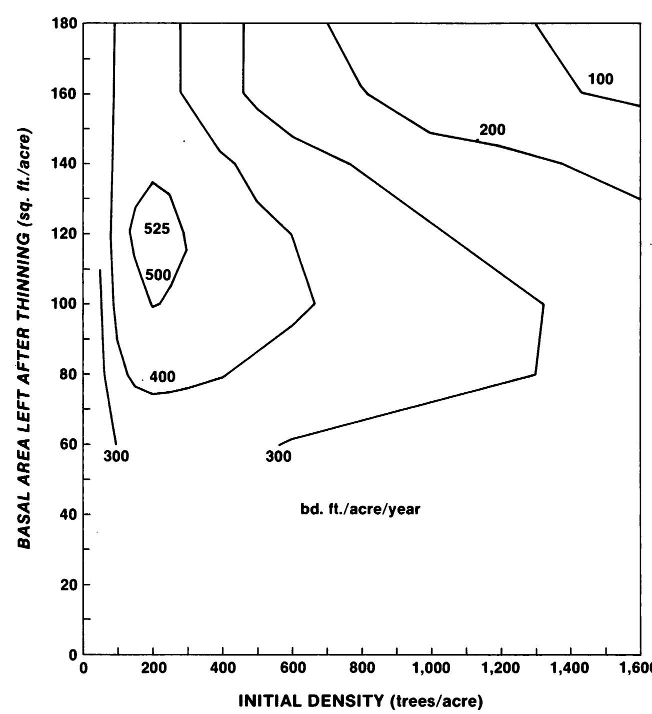

Figure 7.—*The effect of initial and thinning densities on maximum board-foot mean annual increment in red pine plantations on site index 50.*

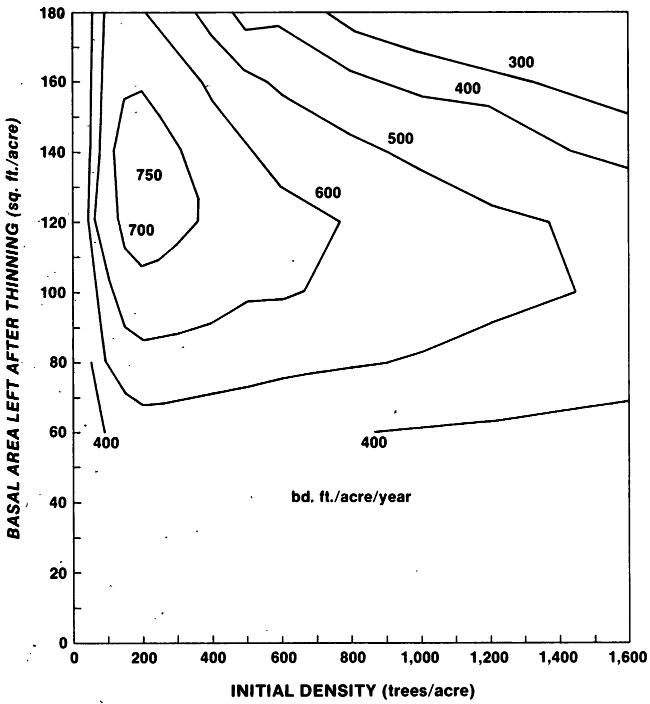

Figure 8.—*The effect of initial and thinning densities on maximum board-foot mean annual increment in red pine plantations on site index 60.*

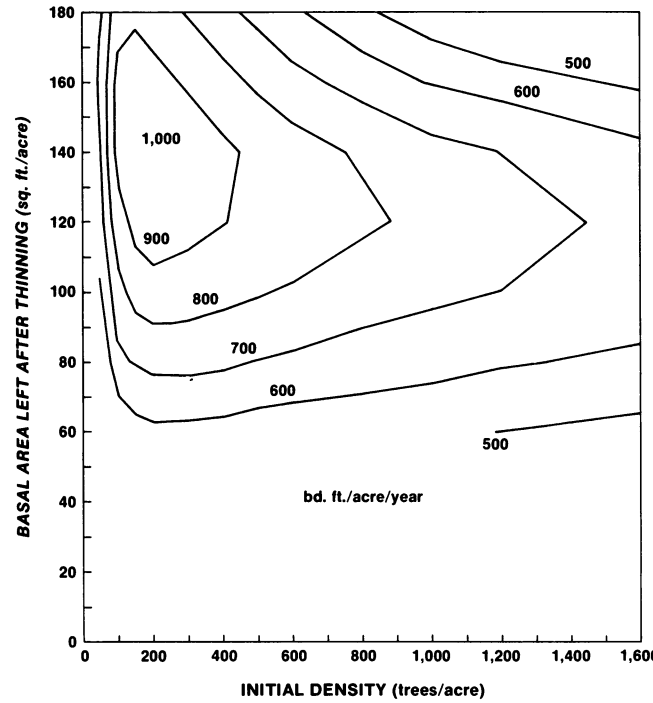

Figure 9.—*The effect of initial and thinning densities on maximum board-foot mean annual increment in red pine plantations on site index 70.*

Tree and Stand Characteristics

Diameter growth is strongly affected by both the initial number of trees per acre and by the stand density left after thinning. Lower initial numbers of trees and lower densities result in fewer but larger trees at a given age, and the effect is great. One of the lowest initial and residual density combinations evaluated here (100 initial trees/acre thinned to 60 square feet of basal area/acre) produced trees 6 inches larger at age 40 and 14 inches larger at age 80 than the highest combination (1,600 initial trees/acre thinned to 180 square feet).

Wood produced with a low initial number of trees/acre has a slightly lower specific gravity than wood produced with a high initial number of trees, varying on site index 60 from 0.34 for 100 trees/acre to 0.35 for 1,600 trees per acre. The average diameter of dead branches at a height of 17 feet was larger for low initial densities, but the difference in branch size between 100 and 1,600 initial trees per acre was less than 3/4 inch.

The initial number of trees per acre has a major impact on the size of trees harvested over a given rotation. With fewer initial trees, fewer trees are harvested over a rotation, but the trees harvested are larger in diameter. On site index 60 stands thinned regularly to 140 square feet, with only 200 established trees per acre, almost all trees harvested for timber over an 80-year rotation were larger than 12 inches d.b.h. and they averaged about 16 3/4 inches. In contrast, almost all trees harvested in the same stands over the same rotation with 1,200 established trees were smaller than 12 inches, and averaged only 7 3/4 inches.

For a given established number of trees per acre, leaving lower basal areas after thinning decreases the average size of trees harvested. Although individual trees grow more rapidly in diameter in stands thinned to low densities, more trees must be cut at an earlier age to reduce the stand to this lower density, and this reduces the average diameter of trees cut. In stands thinned to low basal area densities, more trees have to be harvested to get a thousand cubic feet of total volume than in stands thinned to high basal areas. However, the reverse is true for board-foot volume. Fewer trees have to be cut to get a thousand board feet from stands thinned to low basal areas. These findings, and the analyses from which they are derived, are explained more fully in the sections that follow.

PLANTATION MANAGEMENT REGIMES

Through choices of the initial spacing, the timing, kind, and intensity of thinning, and the final rotation age, a manager can greatly influence the kind, quantity, quality, and timing of salable products harvested from a red pine plantation. An almost endless array of management alternatives exists. Only a limited number are considered here, but these encompass a wide range of choices, making it possible to define regimes that can accomplish a variety of silvicultural and economic objectives.

Initial density is defined as the number of trees successfully established about 5 years after planting; it ranges from 50 to 1,600 trees per acre (T/A) in this analysis, a range in spacing from 29.5 to 5.2 feet between trees.

Trees per acre	Area per tree (sq. ft.)	Square spacing (ft.)
50	871	29.5
100	436	20.9
150	290	17.0
200	218	14.8
300	145	12.0
400	109	10.4
500	87	9.3
600	73	8.5
800	54	7.4
1,000	44	6.6
1,200	36	6.0
1,600	27	5.2

Trees are considered to be 3-year-old seedlings at the time of planting, and spacing is assumed to be approximately square.The young red pine trees will be released from competing vegetation at 3 and 6 years after planting, and at 15 years if necessary because of low initial density.

Thinnings are to be made at not less than 10-year intervals. The first thinning is to be made 20 years after planting, but only if: (1) height to a 3-inch top d.i.b. is at least 17 feet; (2) the average stand d.b.h. is at least 5 inches; and (3) the potential cut is at least 25 percent of the stand basal area per acre. If the plantation does not meet the above criteria at age 20, the thinning is postponed 5 years, and the stand is checked again. After a thinning, the stand must wait 10 years and have a potential cut of at least 25 percent of the stand basal area per acre to qualify for another thinning. If it does not, thinning is postponed for 5 years, at which time the stand is rechecked to see if it qualifies for a thinning. If it does, it is thinned; if not, it is rechecked each 5 years until it qualifies. This process continues throughout the life of the stand until final harvest.

In this analysis a fixed residual basal area level after thinning is assigned to each stand. This basal area density ranges from 60 to 180 square feet per acre (SF/A), and at each thinning the stand is cut back to the assigned density. The management option of varying residual densities throughout the rotation is not considered. Since basal area growth generally declines with age after the first thinning, the basal area level in each stand follows a declining saw-tooth pattern throughout the rotation (fig. 10). It is assumed that in all thinnings the proportion of trees cut is the same in all diameter classes, so that the diameter of the tree of average basal area does not change. At the end of the assigned rotation the stand is clearcut, and an identical management cycle is repeated. Rotation ages range from 20 to 150 years.

This analysis focuses on only one of many kinds of thinning options: that of choosing a fixed level of residual basal area to which the stand is thinned at a fixed interval of 10 years, and thinning from above and below so that the tree of average basal area is not changed by thinning. Many other thinning options could be selected by the manager, including varying stand densities throughout the rotation, varying intervals between thinnings, and thinning from below or above. Although the findings reported in this study cover only a selected group of management practices, they do cover a broad range of typical recommendations and should help managers in selecting regimes for red pine plantations.

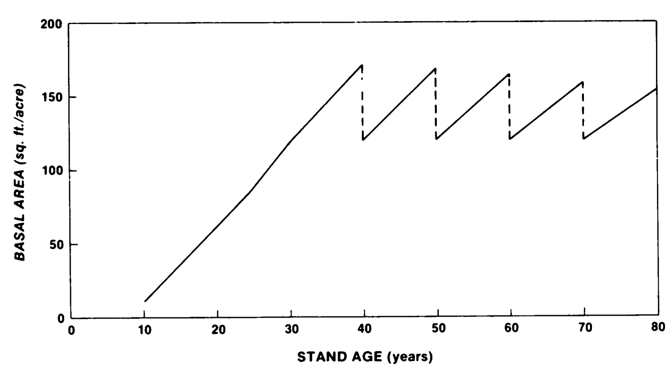

Figure 10.—*Basal area in a red pine plantation with an initial density of 400 established established trees per acre on site index 60, thinned at 10-year intervals to 120 square feet of basal area per acre.*

GROWTH AND YIELD MODEL

A growth and yield model for even-aged red pine plantations and natural stands in the Lake States was developed by modifying, combining, and enlarging the growth and yield models reported by Buckman (1962) and Wambach (1967). Only a brief description of the growth and yield model is given here. Details will be published elsewhere.

Wambach's (1967) reported basal area equation is a relatively simple function that assumes a constant basal area growth regardless of age for a given site and initial number of trees per acre. The original equation (corrected for a misprint of one of the coefficients, and using different symbols for the variables) is:

$$B = 32.278 + 0.001518\,(N)\,(BHA) + 0.6915\,(I)\,(BHA)$$

$R^2 = 0.654$ $\quad SE = 29.9\ ft^2$

where B = basal area in square feet per acre,
N = trees per acre,
BHA = breast-height age of the trees, and
I = site index expressed as a 5-year intercept above breast height, in feet.

A more flexible nonlinear function fit to Wambach's original data provided a considerable improvement in fit:

$$B = 6.565\,(S)\,[1-e^{-.04018A}]^{1.1677}\,[1-e^{-.001885N}]$$

$R^2 = .751$ $\quad SE = 25.0\ ft^2$

where S = site index (total height of dominants and codominants at 50 years), and
A = age of the trees in years.

This equation was used to estimate basal area development in stands up to age 25. In practice, basal area growth was determined each year by using the first derivative of the above equation with respect to age. From age 25 on, Buckman's basal area growth equation was used to project stand development:

$$\Delta B = 1.689 + 0.04107B - 0.000163B^2 - 0.07696A + 0.0002274A^2 + 0.06441S,$$

where ΔB = annual basal area growth in square feet per acre.

Wambach's data were from unthinned, fully stocked plantations. When applied to young thinned stands, or to stands with low numbers of trees per acre, the diameter growths implied by the equation derived from his data appeared under some conditions to be too high. To overcome this deficiency, a nonlinear maximum annual diameter growth constraint was imposed on the basal area growth equations:

$$\Delta Dmax = .007\ S\ e^{-.01BHA}.$$

This closely approximates the linear constraint used by Wambach (1967) to overcome this problem. At best this is only an approximation of the complex relation that determines the maximum potential diameter growth of a tree in the absence of competition, but it does impose a needed constraint when applying the model to extremes of spacing. In practice, basal area growth estimated from the equations was reduced when necessary so that this maximum diameter growth was not exceeded.

Starting with an initial number of established trees at a given age below 25 years on a specified site, the revised equation fit to Wambach's data projects stand basal area at future ages. Knowing the site, age, and basal area, Buckman's equation is then used to project basal area growth beyond 25 years. The diameter of the tree of average basal area (D) was computed from stand basal area and number of trees per acre. To determine the proportion of the stand in poletimber and sawtimber size classes, stands were assumed to have a standard deviation (SD) of tree diameters expressed by the following function fit to data reported by Stiell and Berry (1973):

$$SD = .37628\ \bar{D}\ e^{-.093346\bar{D}},$$

where $\bar{D}$ = mean stand diameter.

Heights of dominant and codominant trees (H) were estimated using the nonlinear function reported by Lundgren and Dolid (1970):

$$H = 1.890\ S(1-e^{-.01979A})^{1.3892}.$$

Total cubic-foot volumes (CF) in poletimber trees (from 5 to 9 inches d.b.h.), in sawtimber trees (9 inches d.b.h. and larger), and in all trees were estimated using the ratio volume equation developed by Buckman (1961):

$$CF = 0.4085\ BH.$$

Merchantable cords were computed as a constant proportion for the total cubic-foot volumes in trees greater than 5 inches d.b.h.:

$$Cords = .0097\ (CF_{5+}).$$

A special function to estimate the board feet (International ¼-inch) per cubic-foot (total) ratio in trees of specified diameter and height was developed from data in Gevorkiantz and Olsen (1955):

$$BdFt/CF = -8.76 + 1.985D - 0.07253D^2 + 0.0008421D^3 + 0.04951H - .00892DH + 0.0003169D^2H - 0.000002786D^3H.$$

This ratio was applied only to the total cubic-foot volume in trees larger than 9 inches d.b.h.

Mortality was approximated by a nonlinear function that limited basal area per acre to a maximum accumulation depending upon site:

$$\text{Basal Area Mortality} = B e^{-20S/B}.$$

The number of trees that died to make up this calculated mortality was estimated by assuming that the dead trees had an average diameter equal to the average stand diameter less one standard deviation.

The final equations were used in a computer simulation program to forecast yields from various sites, initial densities, and thinning schedules. Test runs closely simulated red pine growth and yield reported in stands not used in developing the equations. Subsequent use and comparisons of projected versus observed growth and yield have reaffirmed the results of these earlier tests.

VOLUME YIELD RESPONSE TO STAND DENSITY

The impacts of site, initial density, density after thinning, and rotation age on volume yields in red pine plantations are great. Red pine is a versatile tree and is sold for many products in the Lake States, including posts, pulpwood, poles, piling, and saw logs. Volume outputs can be measured in many ways, but this analysis will consider only three contrasting measures: total cubic feet per acre of the entire stem under bark, including stump and tip for all trees in the stand; merchantable cubic-foot volume per acre in trees 5 inches d.b.h. and larger to a 3-inch top diameter inside bark; and board feet per acre (International ¼-inch rule) in trees 9 inches and larger above a 1-foot stump to a variable top diameter inside bark of not less than 6 inches.

The total-cubic-foot measure reflects total utilization of wood in the central stem. It was determined using Buckman's (1961) stand volume ratio equation, $V = 0.4085BH$,

where V = volume per acre in cubic feet,
B = basal area per acre in square feet, and
H = mean total height in feet of dominant and codominant trees.

Thus, cubic-foot volume is determined solely by the basal area and height of the stand; the diameters of individual trees play no part.

The merchantable cubic-foot volume equation was derived by multiplying Buckman's (1961) cordwood equation, $V = 0.003958BH$, by 79 cubic feet per cord to give:

$$Vm = 0.3127BH,$$

where Vm = merchantable volume per acre in cubic feet.

In effect, this ratio equation assumes a constant 103 cubic feet of total peeled volume of the central stem per merchantable cord, for all merchantable trees. Because only trees 5 inches d.b.h. and larger are considered merchantable, tree diameters do affect this volume estimate.

In contrast, the board-foot volume of a stand is determined by multiplying the total cubic volume by a board-foot recovery factor that is a nonlinear function of average tree diameter and total height. The larger the tree diameter, the higher the board-foot volume per cubic foot of total volume. Thus, the board-foot volume measure is affected by basal area and height, but also is greatly affected by the diameters of trees in a stand.

Comparing volume outputs over time for different sites, initial densities, and thinning schedules can be complex, because the kind, quantity, and quality of outputs vary over time. With the large number of alternatives considered here, some simplification is necessary. This analysis will ignore the timing of outputs and use mean annual increment[1] (MAI) over a rotation as the measure of volume output, expressed as cubic feet per acre per year (CF/A/Y) and board feet per acre per year (BF/A/Y). Most of the examples will be for site index (SI) 60, which is an average site for red pine. Although the details are not included herein, the general conclusions regarding initial density and thinning density appear to hold for a broad range of sites.

[1]*The MAI used here was determined by summing the volume of all prior thinnings (if any), adding the potential volume available for harvest at a given age, and dividing this sum by the total age of the stand since establishment.*

Total Cubic-foot Volume

The pattern of total cubic-foot MAI over a rotation is similar for all stand conditions. A rapid increase for the first 40 to 50 years is followed by a more gradual increase to a maximum MAI at about 60-100 years for most sites, initial numbers of trees per acre, and basal area densities left after thinning. After reaching a maximum, the MAI declines gradually through 150 years. Site index (SI) has a pronounced effect on the level of MAI (fig. 11). For an initial density of 400 established trees per acre (T/A) thinned to 120 square feet per acre (SF/A), for example, maximum MAI ranges from 91 CF/A/Y for SI 50 to 166 CF/A/Y for SI 70, culminating at about 80 to 90 years. Shortening or lengthening the rotation age by 10 years has only a small effect on MAI, usually reducing it by less than 1 percent.

Increasing the initial density also increases cubic-foot MAI, but the effect is less pronounced, varying from 116 CF/A/Y for 200 T/A to 141 CF/A/Y for 1,200 T/A when stands are thinned every 10 years to 120 SF/A of basal area (fig. 12). Quadrupling stocking from 200 to 800 T/A increases total cubic-foot volume production per acre by only 17 percent over the rotation.

Cubic-foot MAI increases with increasing basal area left after thinning, up to a maximum attained with 120-140 SF/A (fig. 13). But with 400 T/A initial density, leaving 120 rather than 80 SF/A (a 50-percent increase in residual stocking) increases cubic-foot volume production by less than 20 percent. With higher initial numbers of trees the percent increase in MAI is less.

In general, the higher the initial and residual stand densities, the higher the total cubic-foot volume production (table 1). It is important to note, however, that although total cubic-foot volume production increases as density increases, it does so at a declining rate. For example, the first 200 red pine trees per acre, thinned to 120 SF/A of basal area, produces 116 CF/A/Y; the second 200 trees (bringing the total to 400 T/A) adds only 12 CF/A/Y to this total MAI production; the third 200 trees increases the total MAI by only 5 CF/A/Y; and the fourth 200 trees adds only 4 CF/A/Y. This decreasing marginal productivity becomes especially important in determining optimum initial densities.

The effect of initial density and density after thinning on MAI is more easily seen when it is graphed (fig. 14). Here it is readily apparent that the

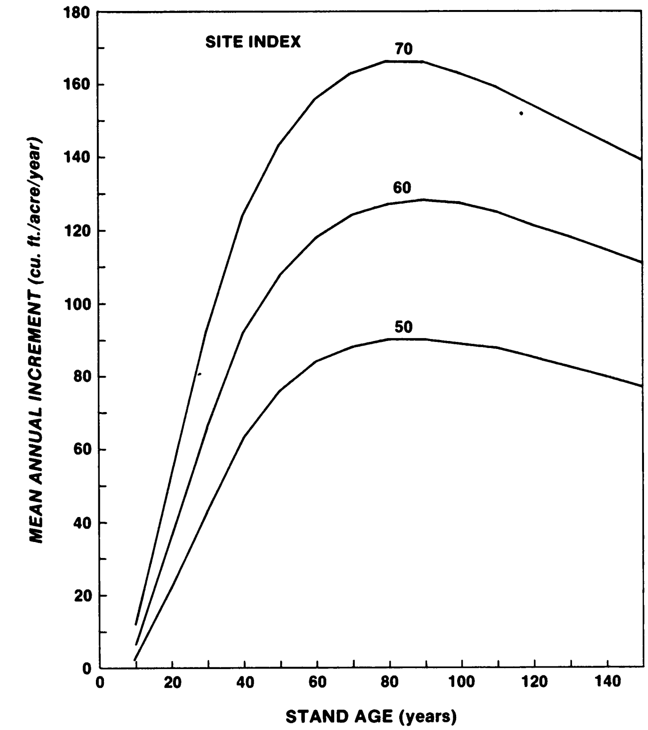

Figure 11.—*The effect of site index on total cubic-foot mean annual increment over a rotation in red pine plantations with an initial density of 400 established trees per acre, thinned to 120 square feet of basal area per acre.*

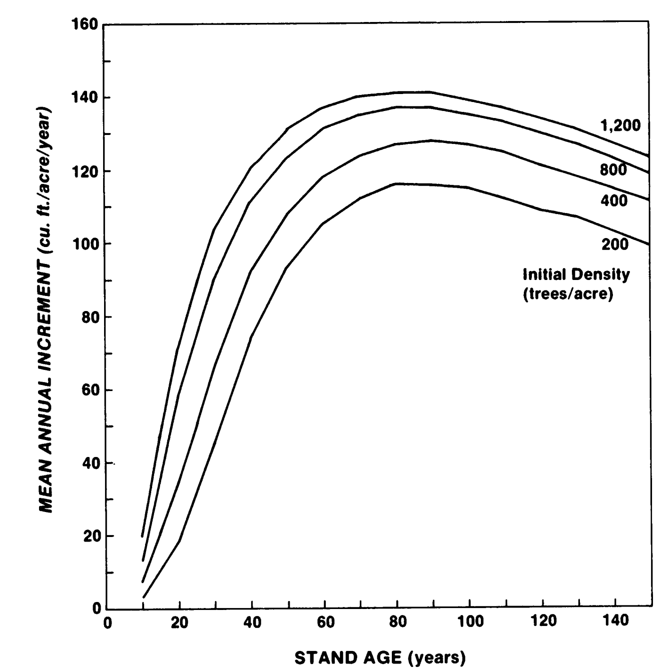

Figure 12.—*The effect of initial number of established trees per acre on total cubic-foot mean annual increment over a rotation in red pine plantations on site index 60, thinned to 120 square feet of basal area per acre.*

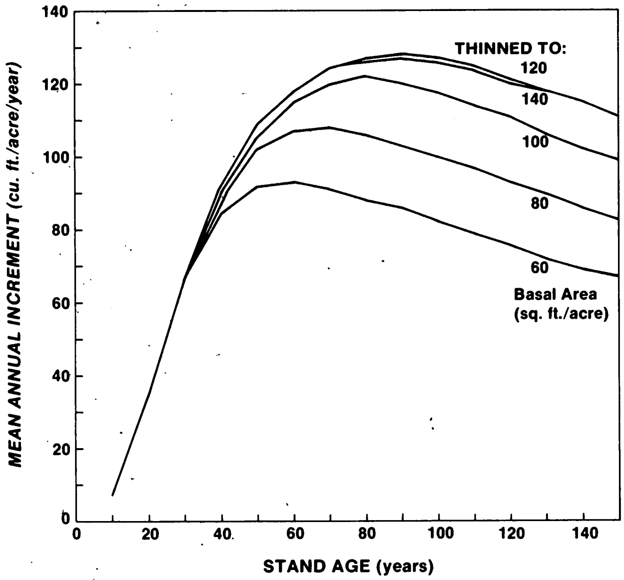

Figure 13.—*The effect of residual basal area left after thinning on total cubic-foot mean annual increment over a rotation in red pine plantations on site index 60, with an initial density of 400 established trees per acre.*

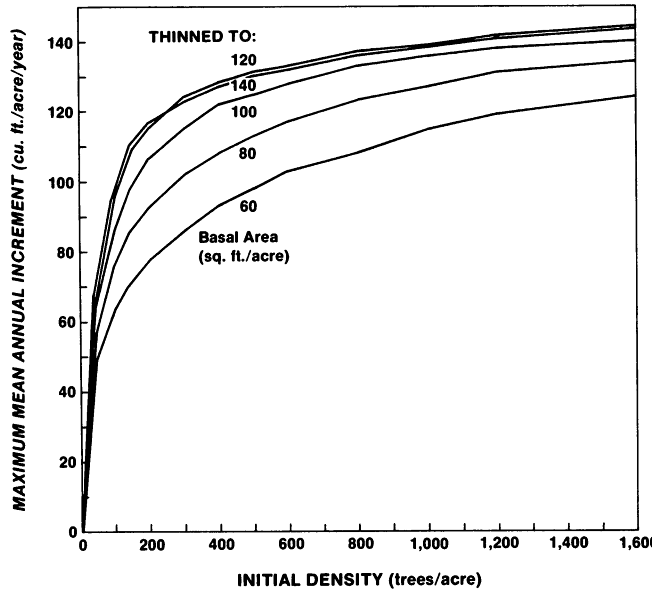

Figure 14.—*The effect of initial and thinning densities on maximum total cubic-foot mean annual increment in red pine plantations on site index 60.*

Table 1.—*Maximum mean annual increment in thinned red pine stands on site index 60, for selected initial numbers of trees and basal areas left after thinning*

Initial stand density (trees/acre)	Basal area left after thinning (sq. ft./acre) 60	80	100	120	140	160
	TOTAL STEM VOLUME UNDER BARK (cu. ft./acre/year)					
100	64	76	87	96	99	97
150	72	87	99	110	112	107
200	78	93	107	116	117	113
300	86	102	115	124	123	118
400	93	108	122	128	127	121
500	98	113	125	131	130	124
600	103	117	128	133	132	126
800	108	123	133	137	136	132
1,000	115	127	136	139	139	134
1,200	119	131	138	141	141	136
1,600	124	134	140	144	143	139
	MERCHANTABLE CUBIC-FOOT VOLUME (cu. ft./acre/year)					
100	49	58	67	73	76	74
150	55	66	76	84	86	82
200	59	71	82	89	90	87
300	66	78	88	95	95	90
400	70	82	93	98	97	92
500	73	85	96	100	100	95
600	75	87	97	101	101	96
800	78	90	99	102	103	100
1,000	79	90	99	102	103	100
1,200	78	90	98	101	102	98
1,600	77	88	96	99	98	95
	BOARD-FOOT VOLUME[1] (bd. ft./acre/year)					
100	417	508	588	656	681	649
150	439	555	647	729	736	687
200	456	569	673	743	741	694
300	446	562	657	718	704	645
400	437	551	643	687	656	583
500	427	535	615	659	608	526
600	421	523	612	634	567	485
800	404	508	573	593	524	424
1,000	393	493	551	557	479	380
1,200	386	467	527	521	436	334
1,600	367	445	485	471	375	228

[1]International ¼-inch rule.

first 200 T/A achieve a relatively high MAI, especially if the stand is thinned to 120-140 SF/A. The increase in MAI from additional trees after the first 200 T/A is relatively minor.

The manager can considerably influence the total cubic-foot output from a red pine plantation by the choice of initial number of trees and the basal area left after thinning. A combination of low initial density, say 200 established T/A, and a schedule of thinnings to low basal areas, say 60 SF/A, will result in an uninspiring 78 CF/A/Y. At the other extreme, a choice of high initial and thinning densities (for example, 1,200 T/A and 140 SF/A) could increase production to 141 CF/A/Y, a sizable gain. However, production from a plantation with low initial density can be increased considerably by choosing a high density after thinning. With 200 T/A, thinning to 140 rather than 60 SF/A will increase production from 78 to 117 CF/A/Y, a substantial increase.

Merchantable Cubic-foot Volume

The relations of merchantable cubic-foot MAI to rotation age, initial density, and thinning density are similar to the total cubic-foot volume relations, with a few significant differences. Merchantable cubic-foot MAI begins at a later stand age, but culminates at about the same age as total cubic-foot MAI, or perhaps only 5 years or so later in some instances. Rotation ages are similar over the range of densities considered here. Merchantable cubic-foot MAI is, of course, less than total cubic-foot MAI at each stand age.

The highest MAI is produced by an initial density of 800-1,000 T/A (fig. 15). Increasing the initial density beyond 1,000 T/A reduces total stand production of merchantable wood over the rotation, in contrast to an increase in total cubic-foot volume production. However, the effect of initial density on merchantable wood production is small over a wide range of initial densities. Production varies by only about 4 percent from 400 to 1,600 T/A in stands thinned to 120-140 SF/A of basal area.

For a given initial number of trees per acre the manager can influence merchantable volume yields by his choice of thinning densities, but over a wide range of basal area densities the effect is not large (fig. 16). With 800 T/A, volume yields vary only 4 percent for thinning densities ranging from 100 to 160 SF/A of basal area. Within this range, the choice of thinning density has only a slight effect on merchantable yields from red pine plantations, but the highest yields are produced from stands thinned to 120-140 SF/A of basal area.

As with total cubic-foot volume, the manager can substantially influence merchantable cubic-foot volume yields from red pine plantations by his choice of initial and thinning densities (table 1). A low initial density and low basal area left after thinning—for example, 200 T/A thinned to 60 SF/A—would produce only 59 CF/A/Y. In contrast, 800 T/A thinned to 140 SF/A would produce 103 CF/A/Y, a 75-percent increase over the low-density choice.

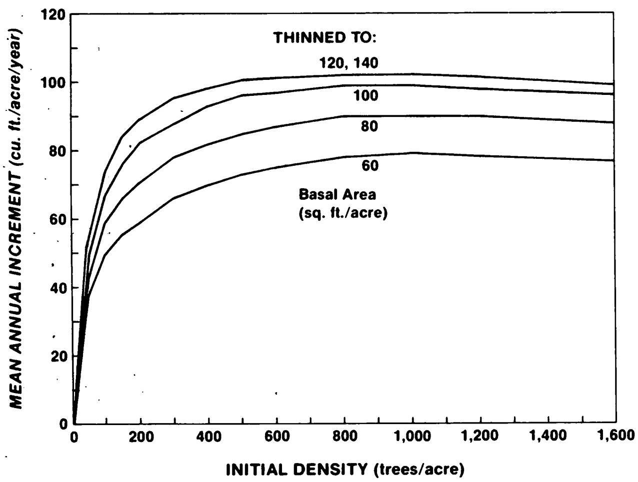

Figure 15.—*The effect of initial density on maximum merchantable cubic-foot mean annual increment for selected basal area densities left after thinning in red pine plantations on site index 60.*

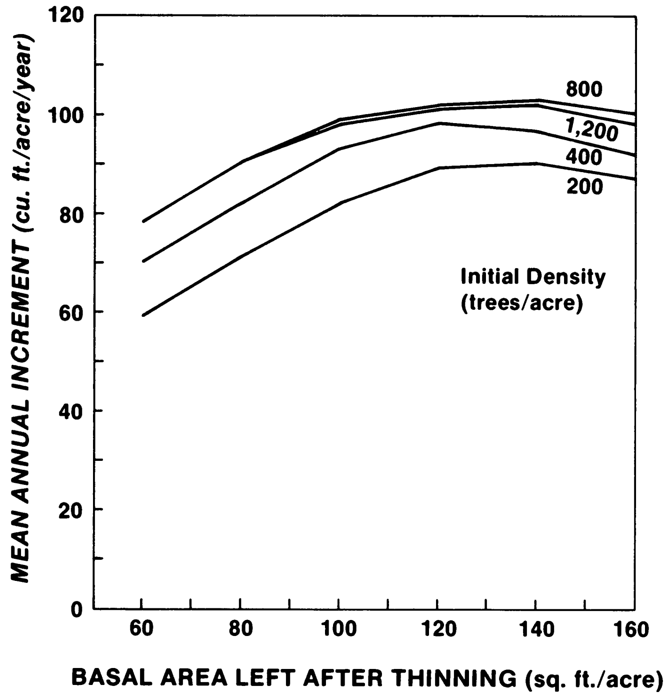

Figure 16.—*The effect of basal area density left after thinning on maximum merchantable cubic-foot mean annual increment for selected initial numbers of established trees per acre.*

Board-Foot Volume

A different picture emerges if we look at board-foot production in relation to stand density. First, although MAI curves for board feet show the same general pattern as those for cubic feet (a rapid initial increase followed by several decades of only slight change), they start at a later date and culminate later than the cubic-foot curves (fig. 17). In general, the better the site, the earlier the initiation of board-foot production.

But the big difference between cubic-foot and board-foot production in relation to stand density shows up when we examine MAI for a range of initial densities (fig. 18): whereas cubic-foot MAI increases as initial density increases from 200 to 1,200 T/A, board-foot MAI shows a reverse pattern, decreasing from the highest MAI with 200 T/A to the lowest with 1,200 T/A. In addition, although the age at which cubic-foot MAI culminates is only slightly affected by initial density (fig. 12), the age at which board-foot MAI culminates is strongly affected by initial density, particularly if the stand is thinned to higher densities (100 SF/A and above). For SI 60 stands thinned to 120 SF/A, board-foot MAI culminates at age 100 with 200 T/A, compared with age 140 with 1,200 T/A (fig. 18).

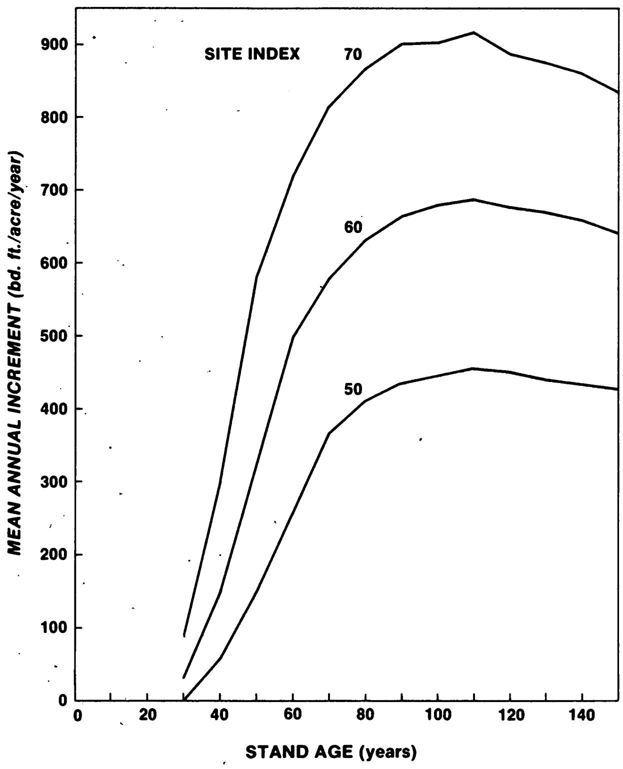

Figure 17.—*The effect of site index on board-foot mean annual increment over a rotation in red pine plantations with an initial density of 400 established trees per acre, thinned to 120 square feet of basal area per acre.*

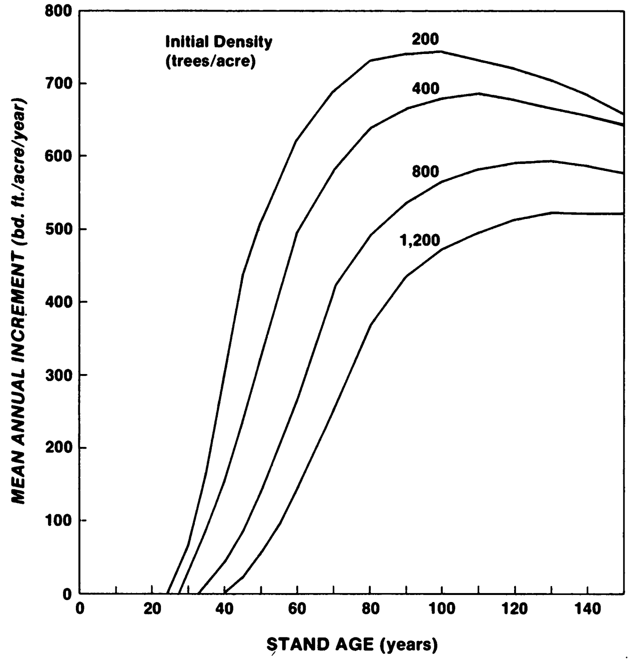

Figure 18.—*The effect of initial number of established trees per acre on board-foot mean annual increment over a rotation in red pine plantations on site index 60, thinned to 120 square feet of basal area per acre.*

Stand density after thinning strongly affects the pattern of board-foot MAI over stand age. In general, the higher the basal area left after thinning (up to about 120 SF/A), the higher the production and the later the culmination of MAI. With 400 T/A, thinning a stand to 120 SF/A rather than 60 SF/A increases production from 437 to 687 BF/A/Y, a 57-percent increase (fig. 19). However, at the same time, the age maximizing MAI increases from 85 to 110 years.

The effect of stand density on board-foot volume production (as measured by maximum board-foot MAI) is dramatically evident if MAI is plotted over initial density for a range of basal area densities left after thinning (fig. 20). On SI 60, the highest board-foot MAI's are obtained with 120 SF/A left after thinning for initial densities below 1,200 T/A, and with 100 SF/A for initial densities above 1,200 T/A. The most striking feature about these curves is the rapid increase in MAI from the first 100 T/A, followed by a further increase to a peak MAI with 200 T/A, followed by a decline in board-foot volume production over a rotation as initial stocking is increased beyond

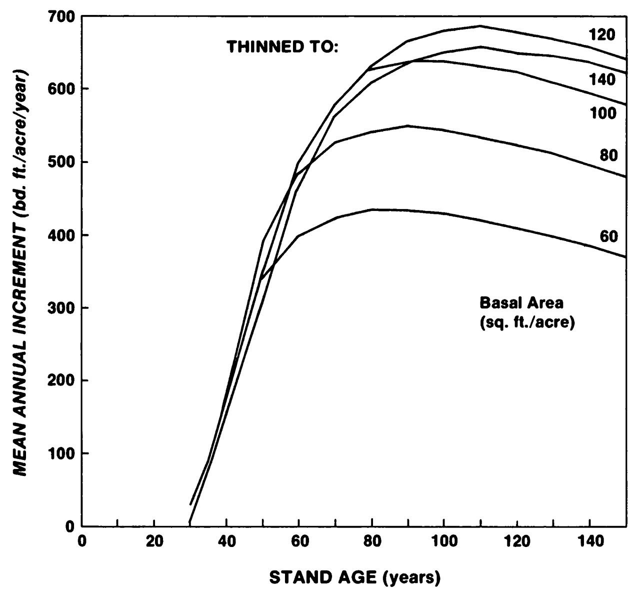

Figure 19.—*The effect of residual basal area density left after thinning on board-foot mean annual increment over a rotation in red pine plantations on site index 60, with an initial density of 400 established trees per acre.*

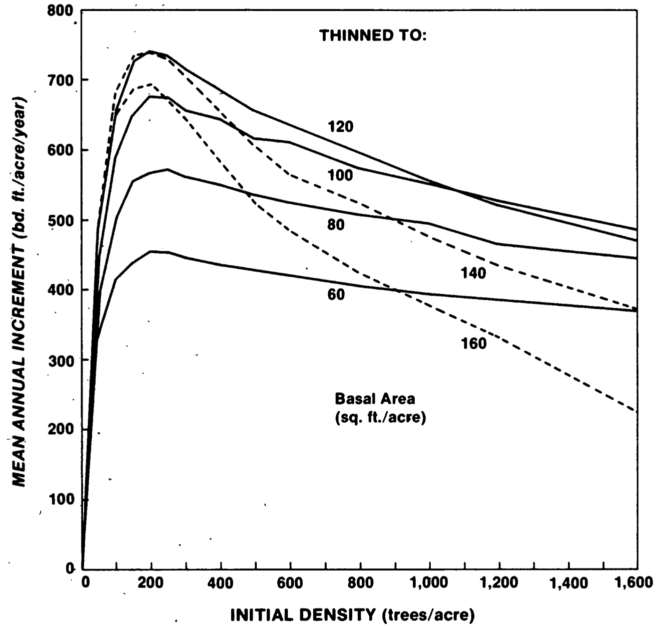

Figure 20.—*The effect of initial number of established trees per acre on maximum board-foot mean annual increment in red pine plantations on site index 60, for selected basal area densities left after thinning.*

200 T/A. It is evident from this figure that the maximum board-foot volume production occurs with an initial density of about 200 T/A thinned to 120 SF/A of basal area every 10 years. In such a stand the first thinning would be made at age 45 and the final harvest would be at age 100.

The combined effect of initial number of trees and density after thinning is large. A choice of 1,600 T/A thinned to 60 SF/A produces less than 370 BF/A/Y. In contrast, choosing 200 T/A thinned to 120 SF/A doubles production, to about 740 BF/A/Y. Thus, through the choice of initial density and densities after thinning, the forest manager can greatly alter the production of board-foot volume from his stand. Once a stand is established with a given number of trees, and has grown for 20-40 years with no thinnings, the manager is somewhat restricted in the effects he can achieve with thinning prescriptions.

For initial densities of 200 to 400 T/A and thinning densities of 80 to 160 SF/A, board-foot volume production is maximized at rotation ages ranging from 80 to 110 years. For initial densities of 500 to 1,000 T/A and the same range of thinning densities, production is usually maximized at rotation ages ranging from 105 to 135 years. Generally, the more established trees per acre, the longer the rotation needed to maximize board-foot MAI.

Timber Production in Relation to Site

To avoid presenting too much detail, most of the previous information was given only for SI 60. However, analyses of timber production for a wide range of SI's (40-80) indicate that the same general conclusions hold for all sites. Total cubic-foot volume production is highest with a high number of trees per acre, but the first 200 T/A accounts for about 80 percent of the MAI obtained with 1,200 T/A. Generally, the highest cubic-foot MAI for any initial number of trees is obtained by thinning to basal areas of about 120 SF/A on poor sites to 140 or more SF/A on good sites. Site quality has a substantial effect on volume production. An initial density of 800 T/A thinned to 120 SF/A on SI 50 produces 98 CF/A/Y over an 80-year rotation. The same number of trees per acre thinned to 140 SF/A on SI 70 will produce 181 CF/A/Y over the same rotation, an increase of 85 percent. In general, maximum MAI in merchantable cubic feet is obtained with slightly higher densities on better sites, ranging from 73 CF/A/Y with 800 trees thinned to 120 SF/A on SI 50, to 137 CF/A/Y with 1,000 trees thinned to 140 SF/A on SI 70.

Board-foot volume production is highest on all sites with an initial density of about 200 T/A. With 200 T/A optimal basal area densities after thinning are lowest (about 120 SF/A) on poor sites. Better sites appear able to support higher basal area densities throughout a rotation.

EFFECT OF STAND DENSITY ON TREE AND STAND CHARACTERISTICS

Volume yields are greatly affected by initial number of trees and thinning density. But stand density also affects other tree and stand characteristics important in choosing a management regime for red pine. In the following discussion, all examples are for SI 60 unless otherwise stated.

Tree Size and Form

In the growth and yield simulation model, height growth of trees in the stand was assumed to follow published site index curves (Gevorkiantz 1957, Lundgren and Dolid 1970) regardless of stand density, and to apply to all trees in the stand on a given site at a given age regardless of diameter (fig. 21).

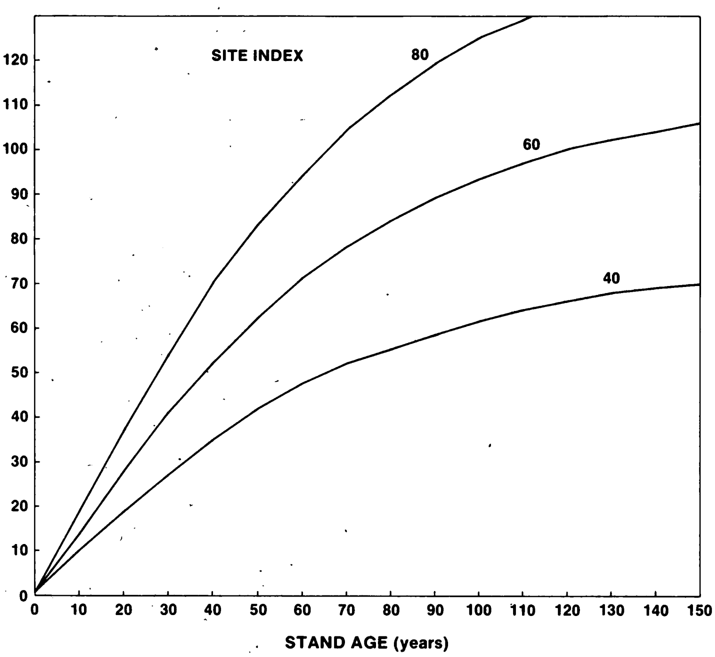

Figure 21.—*Average height of dominant and codominant red pine trees at different ages.*

Although Stiell and Berry (1977) reported a stand density effect on height for a SI 80 red pine plantation, it amounted to only a 2-foot difference at age 20 between extremes of density. Thus, the effect of stand density on height will be ignored in this analysis. Because heights apply to dominant and codominant trees, heights of intermediate and suppressed trees will be overestimated. The exact effect of this is not known, but is expected to be small in all but the densest stands.

In unthinned stands, diameter growth (change in diameter of the tree of average basal area) on a given site is strongly affected by the initial stand density (fig. 22). With 1,200 T/A, for example, diameter growth appears to slow down when the stand is about 11-13 years old, and falls below that in less dense stands. With 800 T/A, diameter growth falls behind at about 12-15 years; with 400 trees, at about 17-20 years; and with 200 trees, at about 25 years. These densities correspond to basal areas of about 40 to 60 SF/A, and mark the onset of competition. This decline in diameter growth occurs earlier on better sites and later on poorer sites.

Average tree diameters in thinned stands are affected by both the initial number of trees and the basal area density left after thinning (Lundgren and Wambach 1963). For example, with 400 T/A, thinning every 10 years to 80 SF/A rather than 120 SF/A results in larger trees throughout the rotation (trees average 3 inches larger in d.b.h. by the end of an 80-year rotation) (fig. 23). Initial density has a similar

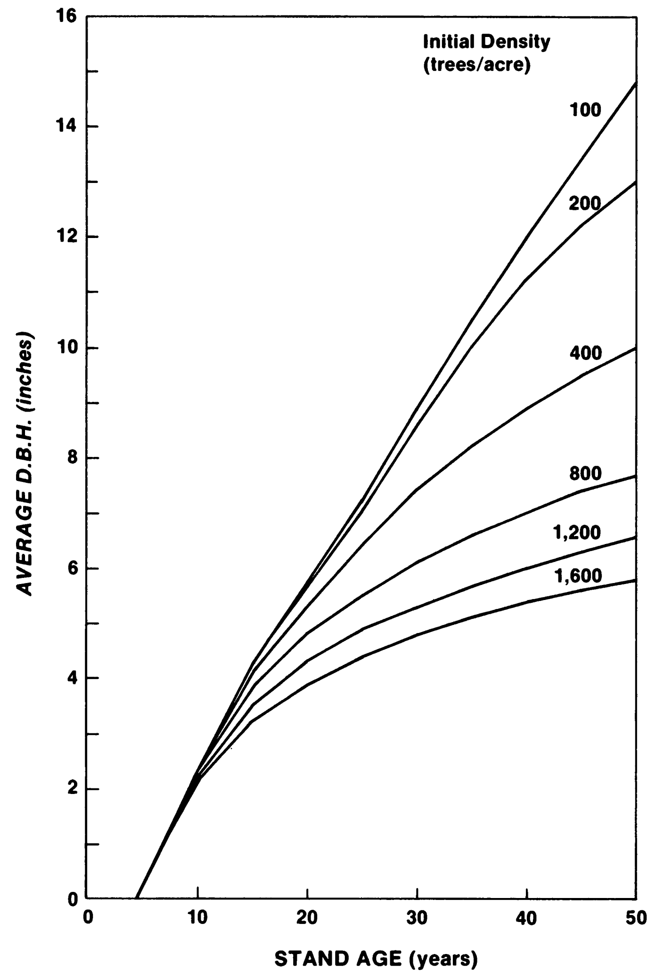

Figure 22.—*Average stand diameters for different numbers of initial trees per acre in unthinned red pine plantations on site index 60.*

effect on tree diameters. Establishing 400 T/A rather than 800 results in larger trees throughout the rotation, by about 3 inches towards the end of an 80-year rotation when thinning to 120 SF/A (fig. 24). Establishing only 200 T/A rather than 800 produces trees averaging more than 6 inches larger at the end of an 80 year-rotation. In summary, lower initial numbers of trees and lower basal areas left after thinning both lead to increased tree diameters at a given age, and the effect is substantial.

The bole form of a tree can influence the product output. For example, less taper in a tree of a given d.b.h. and height indicates not only more total cubic volume, but also more board-foot volume output.

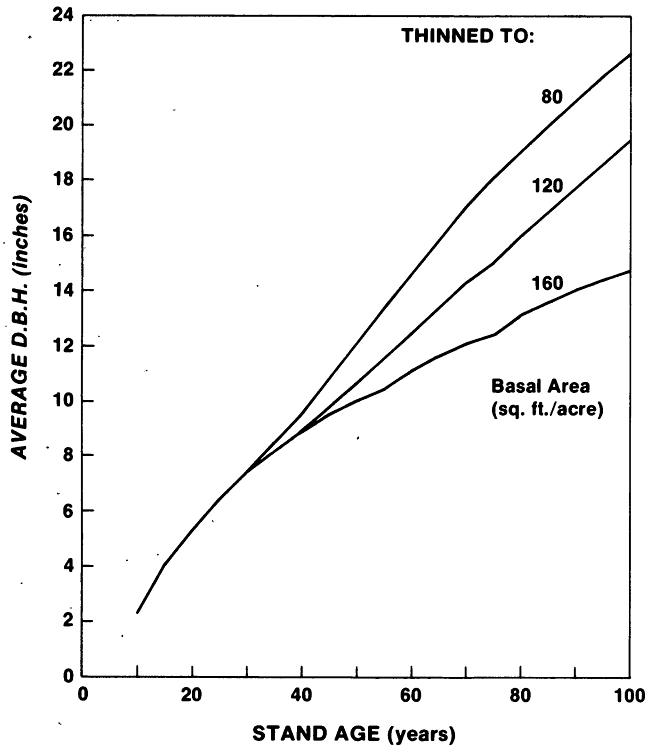

Figure 23.—*Diameter of the tree of average basal area in red pine plantations thinned to different basal area densities on site index 60, with an initial density of 400 established trees per acre.*

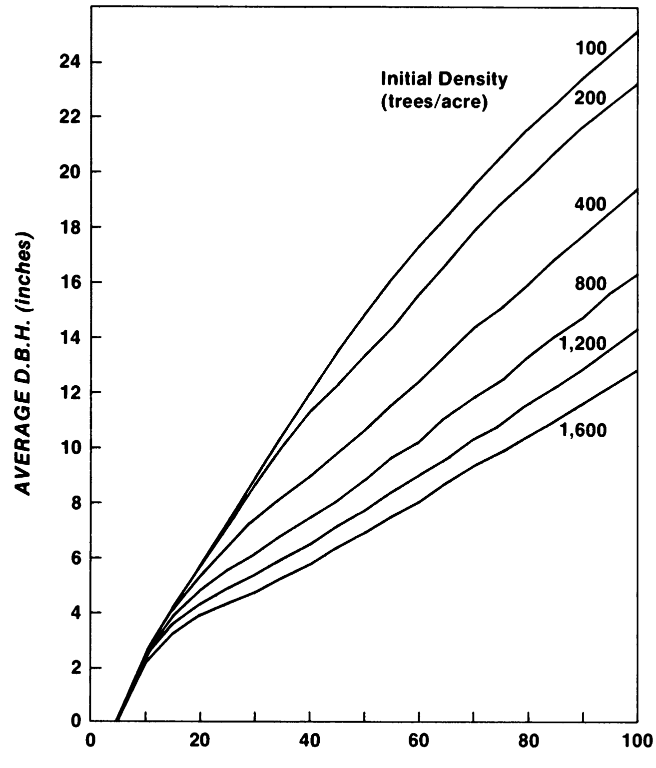

Figure 24.—*Diameter of the tree of average basal area in red pine plantations thinned to 120 square feet of basal area per acre on site index 60, for selected initial numbers of established trees per acre.*

In estimating the product output from red pine stands for this analysis, no allowance was made for possible changes in tree form due to changes in age, site, or stand density. Wambach (1967), after studying form quotients[2] in Lake States' red pine plantations, reported: "No significant relationship was found between form quotient and either stocking, site, or age." He concluded that "...the experience gained in the course of this study lends support to the conclusions drawn by Berry, Eyre and Zehngraff, and others...that spacing does affect tree form, with close spacing leading to slightly better form. But the effect is small and temporary. The influence on yield and quality over the course of a full rotation will not be very important." Apparently, ignoring tree form will have little effect on the outcome of silvicultural and economic analyses.

[2]*Form quotient is defined as the ratio between the diameter of a tree at one-half its total height and at breast height.*

Timber Quality

Earlier we saw that the highest board-foot volume production per acre is achieved with a low initial density (about 200 T/A thinned to 140 SF/A). But what about the quality of wood produced at such wide initial spacing? Larson (1962) has shown that the size and distribution of the living crown affect the amount and quality of wood produced. Crown development is in turn governed to a large extent by stand structure. Larson has pointed out that stem wood produced within crowns of conifers has a higher proportion of juvenile wood (which is of low quality) than stem wood produced below crowns. Open-grown trees have larger, deeper crowns than more closely spaced trees where crown competition has begun. As a result, the quality of wood from open-grown trees is expected to be lower than that from trees in denser stands. Baker (1969) has documented this for red pine plantations for a range of spacings.

Unfortunately, data were not available to accurately model crown size development in red pine stands in relation to site, age, initial density, and thinning schedules and to relate this in turn to wood quality. Instead, three indirect indicators of the impact of spacing on wood quality were used: diameter growth rate (expressed as rings per inch), number and size of branches, and specific gravity.

Previous studies have reported that conifer trees with more than 6 to 8 rings per inch (a d.b.h. growth rate of from ⅓- to ¼-inch per year) are acceptable for saw logs (Lundgren 1965). In stands thinned to 120 SF/A, only at the earliest ages does tree growth exceed ⅓-inch in diameter per year and produce fewer than six rings per inch. Diameter growth rates of six or more rings per inch are produced after stand age 15 even in stands with only 200 T/A. Even with this low initial density, stands thinned to 120 SF/A produced at least eight rings per inch from age 40 on (table 2).

Diameter growth rates on better sites are higher, prolonging the production period of wood with fewer than six rings per inch. But even on SI 80, with 200 T/A thinned to 120 SF/A, diameter growth has slowed by stand age 30 to produce more than six rings per inch. Thus, it appears that except for a juvenile core of wood, red pine plantations with low initial densities (200 T/A) will produce wood of acceptable quality for saw logs throughout most of a rotation.

Table 2.—*Diameter growth throughout a 100-year rotation of red pine plantations on site index 60, for a range of initial trees per acre thinned to 120 sq. ft. of basal area per acre*

(In inches)

Stand age (years)	Initial density (trees/acre)								
	200			400			800		
	D[1]	ΔD[2]	RPI[3]	D	ΔD	RPI	D	ΔD	RPI
10	2.3	—	—	2.3	—	—	2.3	—	—
		0.33	6.1		0.30	6.7		0.25	8.0
20	5.6			5.3			4.8		
		.30	6.7		.21	9.5		.13	15.4
30	8.6			7.4			6.1		
		.26	7.7		.15	13.3		.13	15.4
40	11.2			8.9			7.4		
		.21	9.5		.17	11.8		.14	14.3
50	13.3			10.6			8.8		
		.22	9.1		.18	11.1		.14	14.3
60	15.5			12.4			10.2		
		.23	8.7		.19	10.5		.16	12.5
70	17.8			14.3			11.8		
		.20	10.0		.17	11.8		.15	13.3
80	19.8			16.0			13.3		
		.19	10.5		.17	11.8		.14	14.3
90	21.7			17.7			14.7		
		.16	12.5		.18	11.1		.16	12.5
100	23.3	—	—	19.5	—	—	16.3	—	—

[1]D.b.h. (outside bark).
[2]Average annual growth in d.b.h. over the decade.
[3]Rings per inch growth in d.b.h.

The number and size of branches on a tree affect the number and size of knots in the saw logs from that tree, which in turn affect the grade of lumber output from the tree. Wambach (1967) studied the number of branches per whorl from breast height to 20 feet above the ground. He found that the average number of branches per whorl was related to site, increasing from 4.3 to 5.9 branches per whorl as SI increased from 40 to 70. However, he reported that initial stand densities from 400 to over 2,000 T/A had no significant effect on number of branches per whorl.

If we accept Wambach's conclusion, the manager's choice of initial density on a given site has no effect on the number of branches per whorl. Since height growth is essentially unaffected over the range of stand densities considered here, the number of whorls for a given lineal length of stem (say the butt 16-foot log) will be controlled by the site. A rough estimate indicates the number of whorls in a 16-foot butt log ranges from 16 on SI 40 to 9 on SI 70 (table 3). Thus, even though the average number of branches per whorl increases on better sites, the number of branches per foot of stem decreases because there are fewer whorls due to greater height growth.

Wambach (1967) found that the average diameter of dead branches (measured 1 inch from the stem) was affected by initial stand density. Branch diameter increased as initial density decreased, as site quality increased, and as distance from the ground increased. But the differences in average branch diameter were not large. On SI 40, with 1,200 trees per acre, dead branch diameter at breast height averaged 0.5 inch (table 4). In contrast, on SI 70 with

Table 3.—*Estimated number of branches per butt log in relation to site index in red pine plantations in the Lake States*

Site index	Years to grow to 17 feet[1]	Whorls per butt log[2]	Branches per whorl[3]	Branches per butt log	Branches per foot of butt log
40	21	16	4.3	69	4.3
50	17	13	4.7	61	3.8
60	15	11	5.2	57	3.6
70	13	9	5.9	53	3.3

[1]Computed from Lundgren and Dolid (1970). Based on total years from seed.
[2]Determined by subtracting an estimated 5 years from seed to reach a 1-foot stump height on SI 40 and 4 years for SI 50-70, from total years to reach 17 feet.
[3]Computed from Wambach (1967).

Table 4.—*Average diameter of dead branches in red pine plantations at selected heights in relation to site index and initial stand density*[1]

(In inches)

Initial trees per acre	Average dead branch diameter at 4.5 feet whorl height on site index:				Average dead branch diameter at 17.0 feet whorl height on site index:			
	40	50	60	70	40	50	60	70
200	0.7	0.7	0.8	0.8	1.0	1.0	1.1	1.1
400	.7	.7	.7	.8	.9	1.0	1.0	1.1
600	.6	.7	.7	.8	.9	.9	.9	1.0
800	.6	.6	.7	.7	.8	.9	.9	1.0
1,000	.6	.6	.6	.7	.8	.8	.8	.9
1,200	.5	.6	.6	.7	.7	.8	.8	.9

[1]From Wambach (1967).

200 T/A, at 17 feet (the top of the butt log), dead branch diameter averaged 1.1 inches. Although on a given site at a given distance up the stem initial density did affect dead branch diameter, the effect was not large. An increase in initial density of 1,000 T/A would reduce average branch diameter only by about ¼ inch.

If we accept Wambach's findings we can conclude that the choice of initial stand density has no effect on the number of branches (and thus the number of potential knots) in the butt log, and has only a slight effect on the average size of dead branches over the range of initial densities considered in this study. Laidly and Barse[3] reported that dead branch diameters in the butt log of trees from a SI 70 red pine plantation ranged from 0.7 inches at 5-foot spacing (1,742 T/A) to 1.2 inches at 11-foot spacing (360 T/A). This closely parallels Wambach's findings. Persson (1977) reported that the largest branches in young Scotch pine plantations in Sweden ranged from 0.7 inch at 4,000 T/A to 1.1 inches at 454 T/A, findings similar to the red pine results.

Wambach (1967) estimated specific gravity of trees in the red pine plantations he studied from measurements of large-diameter increment cores. He found that the average specific gravity of cores at breast height decreased with increasing site quality and increased with increasing initial density, although site had considerably more effect than stocking. An increase in SI from 40 to 70 reduced specific gravity from about 0.36 to 0.33, approximately 10 percent, while a change in initial density from 200 to 1,200 T/A increased specific gravity by only about 2 percent (table 5). Wambach found that adding tree age as a variable did not improve the estimates, an unexpected result that he thought might be due to the small range of ages sampled (from 14 to 49 years). Baker and Shottafer (1970) reported similar results in plantation red pine, where an increase in stand density from 440 T/A (10-foot spacing) to 1,210 T/A (6-foot spacing) resulted in an increase in specific gravity of only 2 percent.

Although the specific gravity of an increment core is not necessarily the average specific gravity of the tree, if we accept Wambach's results as indicative of the relative differences in specific gravity due to site and initial density, we must conclude that on a given site the choice of initial density has only a small impact on the specific gravity of the wood produced. It is likely that over an entire rotation the effect of initial spacing on the specific gravity of the trees will be slight. This is in line with Larson's (1972) conclusion that wide spacing has relatively little effect on wood specific gravity in young pines.

[3]*Personal communication, 1978.*

Crown Width and Site Occupancy

We saw earlier that the first 200 T/A account for a large amount of the potential stand productivity. This point is so important to this analysis that it bears further explanation. In the following exercise we assume SI 60 with a rotation age of 80 years (the age maximizing MAI of total cubic-foot volume under bark of the entire stem for stands thinned to about 120 SF/A).

Table 5.—*Specific gravity of increment cores at breast height in red pine plantations, in relation to site index and initial stand density*[1]

Initial trees per acre	Site index			
	40	50	60	70
200	0.355	0.347	0.337	0.323
400	.357	.349	.339	.325
600	.358	.350	.340	.326
800	.359	.351	.341	.327
1,000	.361	.353	.343	.329
1,200	.362	.354	.344	.330

[1]From Wambach (1967).

Ek (1971) developed a relation between tree diameter and crown width based on 95 red pine trees on 76 different sites in Michigan, Minnesota, and Wisconsin. If we assume that his equation holds for square-spaced red pine plantations, the crown width (CW) in feet just before the crowns touch and competition sets in can be predicted from tree d.b.h. in inches (D) by his equation:

$$CW = 4.2334 + 1.4616D.$$

Using the average stand diameters predicted over time by the red pine growth simulation model for the various initial spacings, and assuming all trees have the diameter of the tree of average basal area, it is possible to predict the age at which the crown width just equals the tree spacing, and the crowns just touch. At this age, assuming a full, circular crown and perfectly square spacing of equal-diameter trees, the crowns in a stand would occupy about 79 percent of the available area. Any growth beyond this point would result in overlapping crowns and increasing crown competition.

Knowing the average stand diameter at each age, one can compute the area per acre occupied by tree crowns and thus determine the percent of the total area covered by crowns at each age until the crowns touch (fig. 25). For example, 400 T/A will average 4.1 inches d.b.h. at stand age 15. A tree this size has an estimated crown width of 10.2 feet, or a crown area of 82 square feet. With 400 trees per acre, crowns would occupy 32,860 SF/A, 75 percent of the total area. In such a stand crowns should begin touching about 1 year later, when the stand is 16 years old and crown width equals tree spacing (10.4 feet). With 200 T/A, crowns would touch about 26 years after planting; with 800 trees, about 10 years after planting; and with 1,200 trees, about 8 years after planting.

As we have seen (fig. 22), projected diameters in simulated stands with 200 T/A begin to slow in potential growth at about 25 years; with 400 T/A, at 17-20 years; with 800 T/A, at 12-15 years; and with 1,200 T/A, at 10-13 years. If we assume that competition among trees begins about the time the crowns touch, the ages at which this occurs agree closely with our estimates of the onset of competition based on reduction in diameter growth. This comparison substantiates the suitability of the simulation model underlying these analyses. Newnham's (1966) earlier model, $CW = 3.417 + 1.609D$, based on a 15-year-old natural stand of red pine in Ontario, predicts almost identical stand ages for crown closure, further substantiating our results.

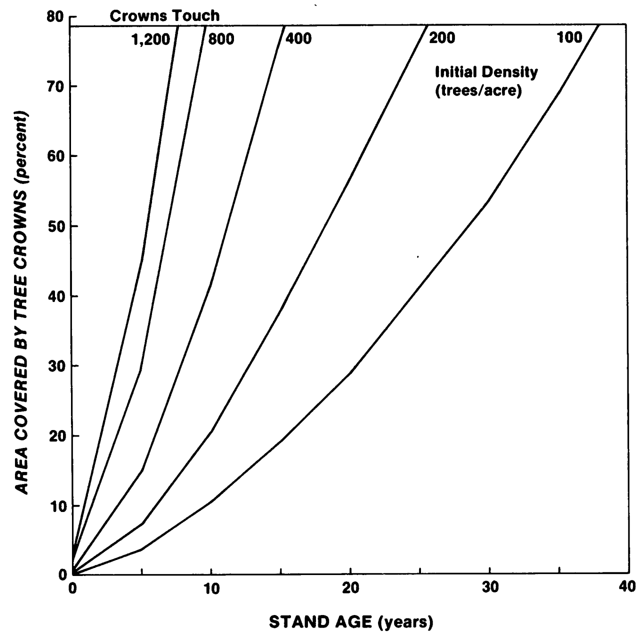

Figure 25.—*Estimated percentages of land area covered by tree crowns in unthinned red pine plantations on site index 60, for a range of initial numbers of established trees per acre.*

The important thing to note is that on SI 60 a decline in potential diameter growth has set in as early as 17-20 years with only 400 T/A, indicating the onset of competition at this age. With only half that many trees, the onset of competition is delayed from 5 to 10 years at most, until about stand age 25. If we use the age at which trees begin to compete with one another as an indicator of full site occupancy (only approximately), we see that even with only 200 T/A, the site would be fully occupied at age 25 for the remaining two-thirds of a 75-year rotation. If we double this initial density to 400 T/A, we get full site occupancy at 17-20 years, but with a 75-year rotation this increases full occupancy only from about 67 percent to perhaps 77 percent of the rotation, at most. The first 200 trees are able to capture a large portion of potential site productivity over the rotation lengths considered here. This leaves only a relatively small amount of unused potential to be captured by increasing the initial density.

Stand Characteristics

Mortality in regularly thinned stands of red pine in the Lake States is relatively small after establishment, except in high-density stands where diameter growth rates are very low (Buchman 1979). In this analysis very few trees died from competition. Essentially all changes in numbers of trees per acre after initial establishment were due to timber harvesting. On a given site, number of trees is controlled by the choice of initial density, thinning schedules, and rotation age (table 6). In general, following a prescribed thinning schedule throughout a rotation on a poor site results in leaving more trees per acre after thinnings than following the same schedule on a good site.

Basal area growth in young plantations is affected by site index, tree age, and initial number of trees per acre, and at later ages by site index, tree age, and basal area density. Establishing more trees per acre allows faster accumulation of basal area per acre; for example, on SI 60, plantations reach 120 SF/A at 38 years with 200 T/A, at 31 years with 400 T/A, at 23 years with 800 T/A, and at 20 years with 1,200 T/A (fig. 26). The effect of increased initial stand density is to shorten the time to full occupancy, when the first thinning can be made.

Buckman's (1962) basal area growth equation[4] specifies that on a given site at a given age, all stands with a specified basal area will have the same basal area growth, regardless of number of trees or past stand history. According to our model, an unthinned plantation with 400 T/A will have about 85 SF/A at age 25; an unthinned plantation with 800 trees will have about 130 SF/A at this age (fig. 26). If, at age 25, the denser stand is thinned from 130 to 85 SF/A (removing about 35 percent of the basal area and thus about 35 percent of the trees), the two stands would be projected to grow the same amounts of basal area during the ensuing years.

Table 6.—*Number of trees per acre in red pine plantations before thinning, for selected initial stand densities and basal areas left after thinning on site index 60*

	Initial density (trees per acre)			
	200		1,200	
Stand age	60 sq. ft./ acre	120 sq. ft./ acre	60 sq. ft./ acre	120 sq. ft./ acre
Years				
0	200	200	1,200	1,200
10	200	200	1,200	1,200
20	200	200	1,200	1,200
30	200	200	1,190	1,190
40	148	200	386	769
50	80	149	206	526
60	52	108	116	373
70	38	79	73	273
80	26	79	52	186
90	26	56	35	185
100	19	56	35	133
110	19	40	25	132
120	19	39	25	93
130	14	39	25	92
140	14	29	18	69
150	14	29	18	69

Harvest Factors

Initial density and the timing and intensity of thinning can greatly affect harvesting operations, and thus indirectly affect the value of volume yields to the timber purchaser. For example, the number of trees per acre affects tree spacing, which can in turn affect future timber harvest options. Access may be difficult in closely spaced plantations, particularly for mechanized harvest operations. Assuming square spacing and full survival, an initial planting of 1,200 T/A results in rows about 6 feet apart, with trees within rows 6 feet apart. By the time of the first thinning the crowns would be touching. Removal of a complete row would leave an access strip at most 6 feet wide between crowns, 12 feet between stems. With only 200 T/A the rows would be spaced about 15 feet apart, leaving a 15-foot access strip with removal of a row. Generally, establishing fewer trees per acre provides more working space for tree harvesting.

Of course, spacing need not be square. For a given number of trees per acre, spacing between rows can be increased by spacing trees within rows closer together. With 1,200 trees per acre, rows could be 8 feet apart with trees spaced 4.5 feet within rows. Although this spacing increases the width of the access strip, it leaves less space for tree felling within rows.

[4] $\Delta B = 1.6889 + 0.041066B - 0.00016303B^2 - 0.076958A + 0.00022741A^2 + 0.06441S$,

where: ΔB = *periodic net annual basal area increment,*
B = *basal area in square feet per acre,*
A = *tree age in years, and*
S = *site index (50-year base).*

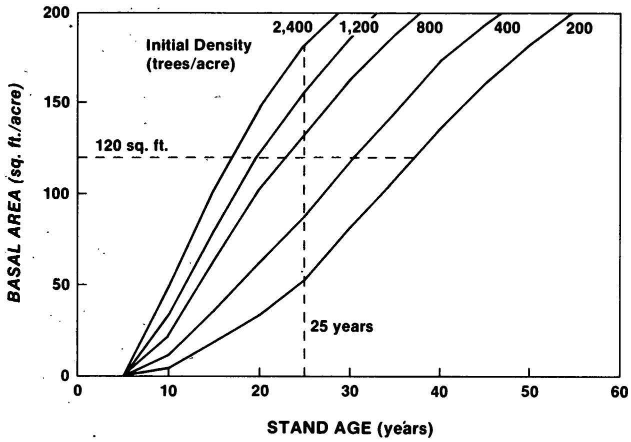

Figure 26.—*Basal area growth in unthinned red pine plantations on site index 60, for a range of initial numbers of established trees per acre.*

In stands with fewer trees per acre and higher residual basal area levels, fewer thinnings would be scheduled than in stands with more trees per acre and lower residual basal area levels. In stands with low initial numbers of trees to be thinned to high basal area levels, first thinnings generally are delayed until age 40 to 50. With higher initial densities and lower basal area levels, first thinnings may be scheduled as early as 20 to 30 years. The timing and thus the number of thinnings can be controlled to some extent by the manager, but generally lower initial densities result in one or two fewer thinnings over a rotation than higher initial densities. Higher numbers of trees per acre may produce more cubic-foot volume over a given rotation, but that also may require more harvest entries, thus increasing timber sale and harvesting costs.

Studies of how tree size affects harvest costs generally have shown a sharp decline in costs as diameters increase immediately above the minimum merchantable limit, followed by a more gradual decline as diameters increase further. For example, Manthy *et al.* (1967) recorded 4.8, 3.5, 2.8, 2.5, and 2.4 man-hours per cord to fell-limb-buck red pine trees in plantations for trees 5, 6, 7, 8, and 9 inches in d.b.h., respectively. Hannula (1971) showed a sharp decline in logging costs in pine and spruce stands in Canada from 5 to 7 inches, with a more gradual decline thereafter.

The initial number of trees per acre has a major impact on the size of trees harvested over a given rotation age. In red pine plantations thinned to 100 SF/A every 10 years over an 80-year rotation, the average diameter of all trees harvested varied from 16 inches for plantations with 200 initial T/A to only 7 inches for plantations with 1,200 T/A (table 7).

The diameter distributions of trees cut over an 80-year rotation in stands thinned to 140 SF/A every 10 years were estimated for extremes of initial stand density (fig. 27).[5] Of the 1,137 trees harvested from the stand with 1,200 trees initial density, 12 percent were less than 5 inches d.b.h. and thus were considered nonmerchantable. More than 40 percent of all the trees cut were less than 7 inches d.b.h. Only 30 percent of those cut were greater than 9 inches d.b.h., the minimum diameter for sawtimber, and only eight trees per acre were larger than 13 inches d.b.h.

Table 7.—*Average diameter of all trees harvested over an 80-year rotation in red pine stands on site index 60, for a range of initial and thinning densities*
(In inches)

Initial stand density	Basal area left after thinning (sq. ft./acre)					
	60	80	100	120	140	160
Trees/acre						
200	13	15	16	16	16	16
400	10	11	12	12	12	12
600	9	10	10	10	10	10
800	8	9	9	9	9	9
1,000	7	8	8	8	8	8
1,200	7	7	7	8	8	8
1,600	6	6	7	7	7	7

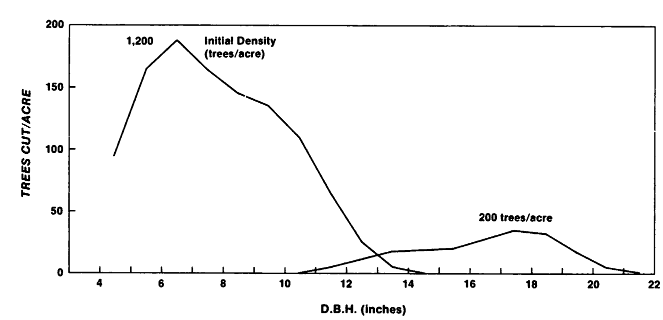

Figure 27.—*Number of cut trees by 1-inch diameter classes in red pine plantations on site index 60 over an 80-year rotation, for initial densities of 200 and 1,200 trees per acre, thinned to 140 square feet of basal area per acre.*

[5]*Assumes a normal distribution of trees around the average diameter of trees cut, with a standard deviation calculated from the formula given earlier.*

In contrast, all of the 195 trees harvested from the stand with 200 trees initial density were greater than 10 inches d.b.h., and more than half were larger than 16 inches d.b.h.

Densities left after thinning did affect the size of trees harvested, but the effect was not as great as the effect of initial density. Furthermore, the effect was the opposite of what would be expected intuitively. Since trees in stands managed with a low basal area density grow faster, one would expect the average size of trees harvested from these stands to be larger. But the reverse is true.

With a given initial density, leaving lower basal areas after thinning tends to decrease the average size of trees harvested and produce less volume in larger trees. Individual trees grow more rapidly in diameter if a stand is kept at low density by thinning, but more trees are cut in the first thinning to reduce the stand to this level than would be cut if a higher density were maintained (table 8). This heavy first cut, which consists of the smallest trees that will be cut during the rotation, may remove up to half the trees in the stand. The effect of such early heavy thinnings is to reduce the number of trees available for harvest at a later date when they are larger.

The initial number of trees and the thinning density level interact to affect tree size and volume production from the stand. This interaction is reflected in a measure that can affect timber harvest costs—the number of trees that must be cut to produce a unit of volume.

If we establish 200 T/A, and mortality is light, then over the rotation we can expect to harvest almost 200 trees. If we establish 1,200 T/A we can expect to harvest nearly 1,200 trees, or almost six times as many. We have already seen (fig. 14) that 1,200 T/A does not produce six times the cubic-foot volume, but only about 20 percent more than is produced by 200 T/A. So, we should expect that many more trees would have to be harvested to get a unit of volume output from stands with high initial densities, and such is the case. In stands thinned to 120 SF/A every 10 years, we would have to harvest about 20 trees over the rotation for every thousand cubic feet of total tree volume if initial density is only 200 trees (fig. 28). We would have to harvest more than 100 trees for every thousand cubic feet if initial density is 1,200 trees. The effect is even more pronounced for board-foot volumes (fig. 29).

Table 8.—*Diameters and numbers of trees cut per acre over an 80-year rotation in red pine plantations on site index 60, for a range of initial and thinning densities*

Stand age	Basal area left after thinning (sq. ft./acre): 60		100		140	
	Ave. d.b.h.	No. of trees cut	Ave. d.b.h.	No. of trees cut	Ave. d.b.h.	No. of trees cut
200 TREES PER ACRE INITIAL DENSITY						
30	8.6	52	—	—	—	—
35	—	—	—	—	—	—
40	11.7	67	11.2	53	—	—
45	—	—	—	—	—	—
50	14.5	28	13.7	49	—	—
55	—	—	—	—	13.7	60
60	17.1	14	16.2	28	—	—
65	—	—	—	—	—	—
70	—	—	—	—	16.3	38
75	20.5	11	19.6	22	—	—
80	21.5	26	20.6	48	18.1	97
Entire rotation	13.4	198	15.7	200	16.4	195
1,200 TREES PER ACRE INITIAL DENSITY						
30	5.3	804	5.3	546	—	—
35	—	—	—	—	5.7	380
40	7.3	180	6.7	229	—	—
45	—	—	—	—	6.7	202
50	9.7	90	8.1	137	—	—
55	—	—	—	—	—	—
60	12.3	43	9.8	85	8.1	170
65	—	—	—	—	—	—
70	14.6	21	11.6	54	—	—
75	—	—	—	—	9.5	104
80	16.7	52	13.4	137	10.1	283
Entire rotation	6.2	1,190	7.4	1,188	7.7	1,139

RISKS IN DENSITY CHOICES

In the preceding analyses we have assumed that future yields can be predicted with certainty, that the unexpected will not happen, and that there are no risks to consider in choosing initial number of trees or thinning densities. This section decribes some of the risks that should be kept in mind when planning for red pine plantation establishment and management.

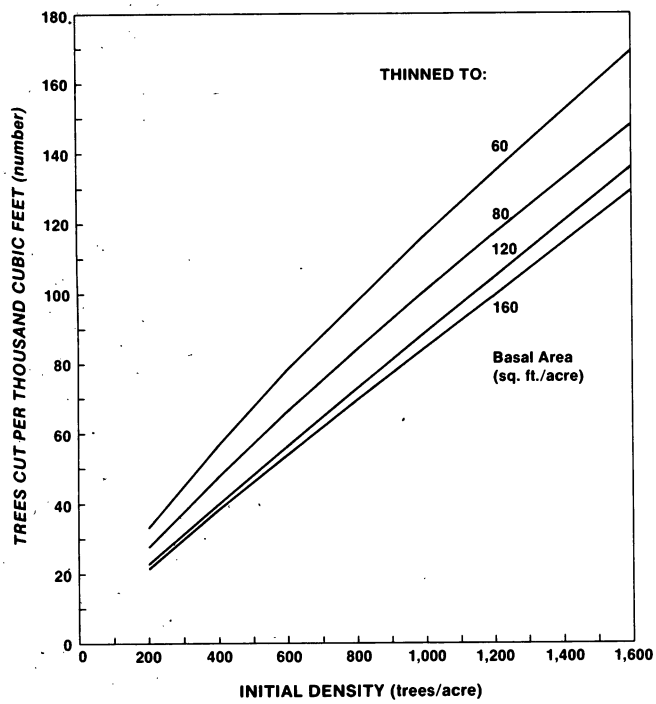

Figure 28.—*Average number of trees cut per thousand cubic feet of total tree volume harvested from red pine plantations over an 80-year rotation on site index 60, for a range of initial and thinning densities.*

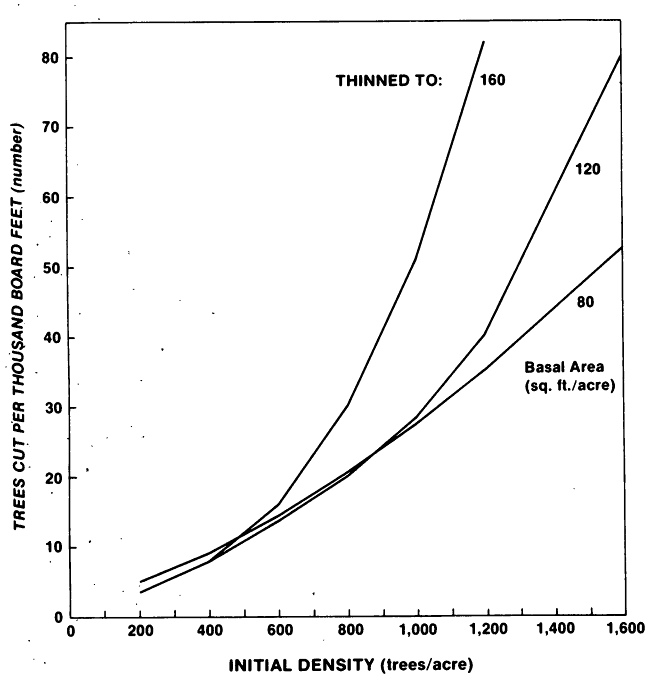

Figure 29.—*Average number of trees cut per thousand board feet of timber harvested from red pine plantations over an 80-year rotation on site index 60, for a range of initial and thinning densites.*

The analyses have assumed that the forest manager or planner can accurately predict the number of established trees per acre. But in truth, the manager may have only limited control over how many trees become established in a plantation. The manager can usually control the selection of areas to be planted, methods and intensity of site preparation, condition of the planting stock, timing of and technique used in planting, spacing and number of trees planted, and subsequent follow-up treatments to improve survival. Yet, at best one can only estimate survival rates, and thus guess at the number and spacing of trees that will result from the planting operation.

Past experience may lead the manager to expect an average tree loss of 25 percent—or a 75-percent survival rate. On this basis, if the objective is to establish 600 T/A, 800 T/A would be planted. However, this 75-percent survival is an average rate, and variations are likely to occur from year to year and from site to site. If conditions are ideal, all planted trees might survive, so the plantation will have 800 established T/A instead of 600. Or, if conditions are bad, half or more of the planted trees might be lost, resulting in a stand with only 400 T/A or fewer. This variation in survival must be considered in choosing the number of trees to plant. How much is the potential loss from a wrong guess?

One approach to answering this question is based on a simple variant of game theory (Luce and Raiffa 1957, Thompson 1968, Halter and Dean 1971). The planting of trees can be viewed as a game against nature. The number of trees to be planted is to be chosen by the forest manager from a range of trees per acre that could be planted. The percentages of planted trees surviving are the states of nature.

We start by asking how many trees per acre would survive and become established for different numbers of trees per acre planted, if we experience different survival rates in a SI 60 red pine stand. We can consider as many planting actions and states of nature as desired, but to keep this example simple we will use only three initial density levels and five survival rates (table 9A). The number of trees surviving (S) for each initial density (N) and survival rate (R) is easily determined by using the formula:

$$S = N \times R.$$

For each number of surviving trees (table 9A) we estimate (in this example, from table 1) the maximum MAI we can expect from a stand with that many surviving trees (assuming that we thin to the basal

Table 9.—*Strategy analysis for deciding how many trees per acre to plant for red pine plantations on site index 60*

Percent of trees surviving	Trees planted per acre 200	300	400
A. NUMBER OF TREES SURVIVING (PER ACRE)			
100	200	300	400
75	150	225	300
50	100	150	200
25	50	75	100
0	0	0	0
B. EXPECTED MAI WITH OPTIMAL THINNING (BD. FT./ACRE/YEAR)			
100	743[1]	718	687
75	736	739	718
50	681	736	743[1]
25	485	580	681
0	0	0	0
C. LOSS IN MAI FROM MAXIMUM (BD. FT./ACRE/YEAR)			
100	0	25	56
75	7	4	25
50	62	7	0
25	258	163	62
0	743	743	743

[1]Maximum possible MAI.

area density that gives the highest MAI for this number of surviving trees). For this example we will use board foot MAI. These board foot MAI's are recorded in table 9B. From table 1 we estimate that the highest board-foot MAI we can get from a red pine stand on SI 60 comes from a plantation with 200 established T/A. We assume that this maximum MAI occurs at one of the initial densities listed in the table, and not a density falling between those listed (for SI 60, this maximum MAI is 743 BF/A/Y). The expected loss (table 9C) is the difference between the maximum MAI, 743 BF/A/Y, and the MAI expected from a given combination of trees planted per acre and percent of trees surviving (from table 9B). For example, if we planted 300 T/A and only 50 percent survived, we would get a MAI of 736 BF/A/Y (table 9B), a loss of 7 BF/A/Y from the site's potential (743 – 736 = 7).

This table of expected losses can be used to explore the consequences of different planting actions if we can assign probabilities of occurrence to each "state of nature", the percentage of trees surviving. For example, suppose past experience indicates that 60 percent of the time we can expect a 75-percent survival rate; 10 percent of the time a 100-percent survival rate; 20 percent of the time a 50-percent survival rate; and 10 percent of the time a 25-percent survival rate; and no complete failures. With these estimates we can calculate the probability of loss in MAI for each number of trees planted.

In each "number of trees planted" column, the loss expected to occur for a given survival rate (table 9C) is multiplied by the corresponding probability that that rate will occur. For 200 trees planted per acre, if 100 percent of the trees survive, we expect a loss of 0 BF/A/Y. Multiplying this by a probability of 0.1 still leaves 0 BF/A/Y as the expected loss.

If 75 percent of the trees survive, the loss is 7 BF/A/Y. Since we expect this to occur 60 percent of the time, the expected loss is 7 BF/A/Y × 0.6 = 4.2 BF/A/Y. If only 50 percent of the trees survive, the expected loss is 62 BF/A/Y × 0.2 = 12.4 BF/A/Y. With 25-percent survival, the expected loss is 258 BF/A/Y × 0.1 = 25.8 BF/A/Y. Finally, if no trees per acre survive, the loss is 743 BF/A/Y, but since there is no chance of this occurring (in our example) the expected loss is 743 BF/A/Y × O = O BF/A/Y. These expected losses for each probability of survival can be summed to get the total expected loss for each planting action (table 10A). For this example, planting 200 T/A results in a total expected loss of 42.4 BF/A/Y. The loss expected from planting 300 T/A is 24.0 BF/A/Y, and from 400 T/A, 26.8 BF/A/Y. To minimize expected losses in board-feet MAI per acre with these probabilities of tree survival, one would choose 300 T/A from among the planting options considered here (200, 300, or 400 T/A).

The effects of other tree survival probabilities on the choice of planting density can be explored (tables 10B and 10C). A slight change in probability of a given survival rate can change the expected loss by a relatively large amount (table 10B), although in this example the density choice is not affected. A fairly large shift in probabilities of survival may have little or no effect on the manager's choice of the number of trees per acre to plant (table 10C).

The exploration of risk examples given here use board-foot MAI, but similar analyses could be done for other product yields. The simple calculations above do not tell the manager what to do, but they help him explore the consequences of an array of actions and so may help him choose a defensible one.

Table 10.—*Expected loss in mean annual increment for varying probabilities of tree survival*
(In bd. ft./acre/yr.)

Percent of trees surviving	Probability of occurrence	Trees planted per acre 200	300	400
A. FIRST PROBABILITY OF OCCURRENCE EXAMPLE				
100	0.10	0	2.5	5.6
75	.60	4.2	2.4	15.0
50	.20	12.4	2.8	0
25	.10	25.8	16.3	6.2
0	0	0	0	0
Total expected loss		42.4	24.0	26.8
B. SECOND PROBABILITY OF OCCURRENCE EXAMPLE				
100	.10	0	2.5	5.6
75	.60	4.2	2.4	15.0
50	.30	18.6	2.1	0
25	0	0	0	0
0	0	0	0	0
Total expected loss		22.8	7.0	20.6
C. THIRD PROBABILITY OF OCCURRENCE EXAMPLE				
100	.333	0	6.2	18.7
75	.333	2.3	1.3	8.3
50	.333	20.7	2.3	0
25	0	0	0	0
0	0	0	0	0
Total expected loss		23.0	9.8	27.0

CONCLUSION

This analysis of product yields expected from red pine stands with various combinations of initial and thinning densities has led to several interesting conclusions.

The initial number of trees per acre has a major impact on the amount and quality of product yields from red pine stands. The more trees established per acre, the higher the total cubic-foot volume yield. However, this increased volume must be harvested from more and smaller trees. Furthermore, the added production drops off sharply after the first 200 T/A. Merchantable cubic-foot yields (pulpwood) are maximized with about 800 to 1,000 established T/A, but increasing the initial density above 500 established T/A increases production by less than 5 percent. Board-foot yields, which are strongly related to individual tree size, are maximized with relatively low levels of initial density—about 200 established T/A.

With low initial densities it is necessary to maintain stands at higher densities throughout the rotation to obtain highest board-foot yields; this can be accomplished by thinning stands to 120-140 SF/A every 10 years. These are higher than the 90 SF/A residual densities recommended by Benzie (1977) for poletimber stands, but close to his recommended thinning density for sawtimber stands. The higher the site index, the higher the basal area density required to achieve maximum yields.

The red pine growth equations used in this analysis do not explicitly include the effect of shrub and other understory competition on tree growth, other than the undocumented effects of whatever conditions existed on the study plots used in developing the original growth models. Fear has been expressed that the yields reported here for low initial densities may not be achieved in practice because other trees, shrubs, and grasses may invade some sites and compete strongly with the widely spaced red pine trees. The yields reported here were developed from Buckman's and Wambach's growth models, but the 200 T/A that produces maximum board-foot growth is just below the lower end of their data. Wambach's study of red pine plantations did include plots with as few as 270 T/A, and he developed diameter-growth constraints for low initial densities. Diameter growth projections by REDPINE, over a wide range of initial and thinning densities, were compared with those made by an updated version of the FREP Tree Growth Projection System (USDA Forest Service 1979). At low initial densities (400 T/A and fewer) both systems projected almost identical diameters for the tree of average basal area during the first 30-40 years, so the diameter-growth constraint equation in REDPINE that governs maximum diameter growth in early years at low densities appears reasonable. Nevertheless, to be conservative, one should assume that to achieve these growth rates, periodic tree release treatments to remove competing vegetation in low-density red pine stands will be required until the first thinning.

The low initial densities indicated by this analysis for maximum board-foot volume production fall below the densities commonly used in establishing red pine plantations in the Lake States. For this reason these results should be considered tentative until more experience has confirmed their applicability. Still, all the available growth and yield evidence points towards establishing plantations with fewer trees per acre than has been common in the past. This will not only reduce initial costs, but increase future board-foot yields.

Our immediate need is for growth and yield studies to test these lower densities, and to document more accurately the development of low-density plantations throughout the region. But until this information becomes available, this analysis can serve as a guide to those wishing to manage red pine plantations with fewer trees per acre.

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Describes an analysis of initial density and subsequent thinning options for red pine (*Pinus resinosa* Ait.) plantations in the Lake States. Results showed that the initial number of established trees per acre has a major impact on the amount and quality of timber product yields, with 200 trees per acre (500/ha) thinned to 120 square feet of basal area per acre (27.5 m^2/ha) maximizing board-foot volume yields.

KEY WORDS: Volume yields, timber management, stand density, *Pinus resinosa*, timber quality.

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