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The tall shrub stratum is an important component of upland forests in the Lake States, especially of deciduous forest communities. The prevalence of tall shrubs has produced a recurring research interest in this stratum both in ecological studies (Ohmann and Ream 1971, Ohmann *et al.* 1973, Tappeiner and John 1973, Ohmann 1982) and wildlife habitat surveys (Rogers and McRoberts 1992) within the region. For such purposes, estimates of the biomass of tall shrubs or their component parts are often needed.

Equations are available for predicting biomass of components of various shrub species from Minnesota (Smith and Brand 1983, Brand and Smith 1985). However, reports differ in many respects, such as in choice of independent variable for estimation, components, species, and region. We needed equations that minimized field time, estimated bark and green weight biomass (in addition to the standard components), and provided standard errors for biomass estimates. Such equations were not available.

So, in this paper we examine goodness of fit of two predictor variables (shrub height and stem diameter class) in three relations. We provide equations that use stem diameter class or stem diameter class and height to estimate biomass of six components of five shrub species, as well as generalized equations derived from a composite of all five species. Finally, we provide the information needed to construct standard errors for biomass estimates.

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METHODS

Field and Laboratory Procedures

Biomass was determined for five shrubs common to northeastern Minnesota [n in brackets]: mountain maple (*Acer spicatum* Lam.) [60], green alder (*Alnus crispa* (Ait.) Pursh.) [95], speckled alder (*Alnus rugosa* (Du Roi) Spreng.) [124], juneberry (*Amelanchier* spp. Medic.) [61], and beaked hazel (*Corylus cornuta* Marsh.) [84]. Shrubs were collected during August and early September of 1979 and 1980 from stands 21 km southeast of Ely in Lake County, Minnesota, and processed as described by Buech and Rugg (1989).

We measured the following components {component number in brackets}: green weight (gwt) of total aboveground biomass {10} (to nearest g); stem diameter class at 15 cm above ground (in 12 quarter-cm diameter classes, e.g., class 2 = stems 0.26 to 0.50 cm); and total length of stem (to nearest 0.1 m). Shrubs were then divided into the following components: leaves {1}, current annual twig {2}, twig sections from current annual twig to 5 mm outside diameter (od) {3}, bark {4} and wood {5} components of twig sections with >5 mm but ≤ 10 mm od, bark {6} and wood {7} components of twig sections with >10 mm od, deadwood {8}, and fruit {9}.

Data Analysis

The independent variables used were stem diameter class and height of stem. The six dependent variables {component numbers in brackets} were dry weight (dwt) of: leaves {1}, current annual twig {2}, bark {4,6}, total wood {2 to 8}, total aboveground biomass {1 to 9}, and gwt of total aboveground biomass {10}.

Biomass-diameter class relations of shrub components were expressed in the raw scale using the allometric equation:

$$Y = \alpha_0 D^{\beta_1} H^{\beta_2} \qquad [1]$$

where Y is biomass (g), D is stem diameter class (1 to 12), and H is stem height. Heterogeneity of variance was substantial, so the analyses were conducted and are reported here in the log-transformed scale (all references to "log-transform" indicate log base e):

$$\log Y = \beta_0 + \beta_1 * \log D + \beta_2 * \log H \qquad [2]$$

$$\log Y = \beta_0 + \beta_1 * \log D \qquad [3]$$

$$\log Y = \beta_0 + \beta_2 * \log H \qquad [4]$$

where $\beta_0 = \log(\alpha_0)$. These linear models were used to assess choice of independent variables and models; coefficients of determination reported are adjusted R^2 values.

RESULTS

Choice of Predictor Variable

Table 1 shows that, as measured by R^2, model [4] (e.g., model 4 - height only) was poorest, model [3] (e.g., model 3 - diameter only) was intermediate, and model [2] (e.g., model 2 - diameter and height) was best. R^2 averaged 0.83 for [4] across all components in the combined species model (range 0.50 to 0.93), relation [3] averaged 0.88 (range 0.60 to 0.97), and [2] averaged 0.89 (range 0.60 to 0.98). In all relations, R^2 values were lowest for current annual twig and highest for total biomass components.

The analysis of predictor variables and models showed that fit of the allometric model using height as a predictor variable [4] was surprisingly good. Although others have reported plant height to be a relatively poor estimator of biomass (Peek 1970, Ohmann *et al.* 1976, Williams and McClenahen 1984), we found that plant height still provided reasonably good results despite being measured to the nearest 0.1 m. This finding is useful because biomass-height relations can be used in vertical line sampling (Hitchcock 1979). Although height can be a useful predictor, models using diameter [3] produced good results more consistently. This was especially true for the *Amelanchier* spp. and *Corylus cornuta* models. Consequently, if there are sufficient resources to measure only one dimension, diameter class would appear to be a better choice than height.

Adding diameter class to a model with height always resulted in a significantly better model, whether working at the individual ($P < 0.02$) or combined ($P < 0.001$) species level. Similarly, adding height to a model with diameter class always resulted in a significantly better model ($P < 0.004$) when working at the combined species level. However, at the individual species level, height was not a useful additional variable in several cases:

Species	Component	P(height /diameter class)
Alnus crispa	Current annual twig	0.60
Alnus rugosa	Leaves	0.08
	Current annual twig	0.60
Amelanchier spp.	Leaves	0.12
	Bark 5mm dob	0.09
Corylus cornuta	Leaves	0.09
	Current annual twig	0.54
	Bark 5mm dob	0.75
	Total biomass (gwt)	0.24

Clearly the components of general concern are leaves and current annual twig.

The "best" model for small confidence intervals is generally the one with diameter class and height (table 2). The "best" model that balances cost of obtaining data against the accuracy needed will generally be the model with just diameter class (table 3). At times, however, the simple height relationship may be useful (table 4).

Using the Equations

Table 2 provides the information for using the log-transformed version of the allometric equation to estimate biomass of a shrub component from stem diameter class and height. Suppose you are interested in estimating the total biomass (gwt) of an *Acer spicatum* shrub in stem diameter class 4 (0.76 to 1.0 cm) at 15 cm above ground and with a height of 1.1 m. The relevant parameter estimates are $\hat{\beta}_0 = 1.6235$, $\hat{\beta}_1 = 1.7947$, and $\hat{\beta}_2 = 1.1530$. Substituting these values in the allometric relation, [2], we get

$$\log Y = 1.6235 + 1.7947 * 1.3863 + 1.1530 * 0.09531.$$

A hand calculator readily gives an estimate of 4.2214 log g total biomass (gwt) for this shrub.

A simple back-transformation provides the median biomass in the raw scale as 68.13 g. The mean biomass in the raw scale is calculated using the formula $E(Y) = \exp(\mu + 1/2\sigma^2)$ (Lindgren 1976), where μ is estimated by log Y, and the estimate for σ^2 is found in the MSE column of table 2. In this case, E(Y) = exp(4.2214 + 0.5*0.1163) = 72.21 g.

Calculating the variance of this prediction is more involved (Draper and Smith 1981):

$$V(\log \hat{Y}) = \hat{\sigma}^2 + V(\hat{\beta}_0) + (\log D)^2 * V(\hat{\beta}_1) + (\log H)^2 * V(\hat{\beta}_2) + 2*\{(\log D)*Cov(\hat{\beta}_0, \hat{\beta}_1) + (\log H)*Cov(\hat{\beta}_0, \hat{\beta}_2) + (\log D)*(\log H)*Cov(\hat{\beta}_1, \hat{\beta}_2)\} \quad [5]$$

For our example, this results in:

$$V(\log \hat{Y}) = 0.1163 + 0.05161 + (1.3863)^2*0.03651 + (0.09531)^2*0.04803 + 2*\{1.3863*(-.04212) + 0.09531*0.04351 + 1.3863*0.09531*(-0.03970)\} = 0.1195.$$

Given the high R^2 for this model, it should be no surprise that the prediction variance is only 2.8 percent higher than the base MSE. This variance estimate can be used in conjunction with Student's t-tables to generate confidence intervals for predictions, probabilities of biomass being larger than some desired value, etc. Once the interval end-points have been calculated, a simple back-transformation will provide the appropriate interval in the raw scale, if desired. The models can be used with just diameter class or height by simplifying this method and referencing table 3 (diameter class) or 4 (height).

Suppose you want to compute the variance of a predicted mean based on *m* new observations at X=x, then the appropriate formula is (Draper and Smith 1981):

$$V(\hat{\mu}) = \hat{\sigma}^2/m + V(\hat{\beta}_0) + x^2*V(\hat{\beta}_1) + 2*x*Cov(\hat{\beta}_0, \hat{\beta}_1) \quad [6]$$

DISCUSSION

Fit of the allometric model was good for most components and species, and comparable to reports by others on the same or similar species (Telfer 1969, Parker and Schneider 1975, Brown 1976, Ohmann *et al.* 1976, Roussopoulos and Loomis 1979, Connolly and Grigal 1983, Smith and Brand 1983). R^2 values ranged from 0.49 to 0.96, but for most components were typically >0.90. The good fits we obtained were somewhat surprising given that we used diameter class rather than actual diameter as the independent variable. This result is useful because it allows the use of ocular estimates and templates (Ohmann 1973) for rapidly assigning stems to diameter class instead of using the more precise, but more time intensive, method of measuring actual stem diameter.

Goodness of fit was not equal among components. Regressions for components related to annual production (especially current annual twig) fit noticeably poorer than those for components approaching total aboveground biomass of the shrub (e.g., wood and total aboveground biomass-dwt and gwt). This result was noted previously (Alemdag and Horton 1981, Schmitt and Grigal 1981, Alemdag 1984, Grigal and Kernik 1984). The comparative lack of precision for components related to annual production suggests that, if the primary interest lies with these components, more caution should be used when considering the application of models derived from other areas or even other species (Buech and Rugg 1989). Site and other environmental factors appear to affect these components, but these factors are not accounted for in the models reported here.

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Table 1.—*Adjusted R^2 values for the three model forms, {[2], [3], [4]}, used to estimate biomass of six components of five shrub species and all species combined.*

		R^2			
Species	**Component**	**Diam + Ht**	**Diameter**	**Height**	**n[1]**
Acer spicatum	Leaves	0.92	0.89	0.90	60
	Current annual twig	0.79	0.77	0.77	60
	Bark 5mm dob	0.93	0.92	0.90	60
	Total wood	0.98	0.97	0.94	60
	Total biomass	0.97	0.96	0.93	60
	Total biomass (gwt)	0.97	0.96	0.93	60
Alnus crispa	Leaves	0.88	0.85	0.86	94
	Current annual twig	0.65	0.65	0.57	94
	Bark 5mm dob	0.95	0.94	0.91	95
	Total wood	0.98	0.96	0.95	95
	Total biomass	0.97	0.96	0.94	94
	Total biomass (gwt)	0.98	0.96	0.93	92
Alnus rugosa	Leaves	0.88	0.88	0.84	121
	Current annual twig	0.57	0.57	0.55	119
	Bark 5mm dob	0.93	0.91	0.93	122
	Total wood	0.98	0.97	0.97	119
	Total biomass	0.98	0.97	0.96	117
	Total biomass (gwt)	0.98	0.98	0.94	124
Amelanchier spp.	Leaves	0.88	0.88	0.69	50
	Current annual twig	0.61	0.57	0.38	61
	Bark 5mm dob	0.95	0.95	0.86	61
	Total wood	0.99	0.99	0.90	61
	Total biomass	0.99	0.98	0.87	50
	Total biomass (gwt)	0.98	0.98	0.87	61
Corylus cornuta	Leaves	0.90	0.90	0.86	83
	Current annual twig	0.66	0.67	0.63	81
	Bark 5mm dob	0.87	0.87	0.80	82
	Total wood	0.98	0.98	0.94	79
	Total biomass	0.98	0.98	0.94	79
	Total biomass (gwt)	0.96	0.96	0.90	83
All species combined	Leaves	0.88	0.87	0.82	408
	Current annual twig	0.60	0.60	0.50	415
	Bark 5mm dob	0.92	0.91	0.87	420
	Total wood	0.98	0.97	0.93	414
	Total biomass	0.97	0.97	0.93	400
	Total biomass (gwt)	0.97	0.97	0.91	420

[1] *Discrepancies in sample sizes (n) are due to missing data and deletion of outliers.*

Table 2.—*Regression statistics for predicting biomass of six shrub components of five tall shrubs:* Acer spicatum, Alnus crispa, Alnus rugosa, Amelanchier *spp.*, Corylus cornuta, *and all species combined. Parameters are from the relationship* $E(\log Y) = \beta_0 + \beta_1*(\log D) + \beta_2*(\log H)$, *where Y = biomass in g, D = stem diameter class at 15 cm above ground, and H = stem height to nearest 0.1 m; logarithms are base e. All weights are dry weight (dwt), except total biomass was measured in both dwt and green weight (gwt). The 12 stem diameter classes were 2.5 mm wide and covered the diameter range of 0 to 3 cm, e.g., size class 5 = stems > 1.0 and ≤ 1.25 cm.*

Species	Component	Parameter	Estimate	Var(β_i)	Cov(β_0,β_i)	Cov(β_1,β_2)	MSE
Acer spicatum	Leaves	β_0	0.6580	8.471E-02			0.1909
		β_1	0.9955	5.991E-02	-6.913E-02		
		β_2	1.1938	7.883E-02	7.140E-02	-6.515E-02	
	Current annual twig	β_0	0.2042	1.769E-01			0.3987
		β_1	0.8576	1.251E-01	-1.444E-01		
		β_2	0.9845	1.646E-01	1.491E-01	-1.361E-01	
	Bark 5mm dob	β_0	-0.0078	9.099E-02			0.2051
		β_1	1.4154	6.436E-02	-7.425E-02		
		β_2	1.0208	8.468E-02	7.669E-02	-6.998E-02	
	Total wood	β_0	0.2135	3.766E-02			0.0849
		β_1	2.0184	2.664E-02	-3.073E-02		
		β_2	1.1014	3.505E-02	3.175E-02	-2.896E-02	
	Total biomass	β_0	1.0791	4.453E-02			0.1004
		β_1	1.6924	3.150E-02	-3.634E-02		
		β_2	1.1037	4.144E-02	3.753E-02	-3.425E-02	
	Total biomass (gwt)	β_0	1.6235	5.161E-02			0.1163
		β_1	1.7947	3.651E-02	-4.212E-02		
		β_2	1.1530	4.803E-02	4.351E-02	-3.970E-02	
Alnus crispa	Leaves	β_0	0.5295	9.010E-02			0.1918
		β_1	0.9706	6.464E-02	-7.461E-02		
		β_2	1.1464	6.827E-02	6.924E-02	-6.310E-02	
	Current annual twig	β_0	-1.4649	2.093E-01			0.4455
		β_1	1.7821	1.502E-01	-1.734E-01		
		β_2	-0.2104	1.586E-01	1.609E-01	-1.466E-01	
	Bark 5mm dob	β_0	-0.4132	5.361E-02			0.1178
		β_1	1.6743	3.753E-02	-4.389E-02		
		β_2	1.0205	3.812E-02	3.996E-02	-3.588E-02	
	Total wood	β_0	0.6608	3.097E-02			0.0681
		β_1	1.7095	2.168E-02	-2.535E-02		
		β_2	1.4567	2.202E-02	2.308E-02	-2.072E-02	
	Total biomass	β_0	1.0853	3.370E-02			0.0717
		β_1	1.6576	2.418E-02	-2.791E-02		
		β_2	1.2775	2.554E-02	2.590E-02	-2.360E-02	
	Total biomass (gwt)	β_0	1.4377	3.334E-02			0.0661
		β_1	1.9062	2.344E-02	-2.734E-02		
		β_2	1.0633	2.418E-02	2.501E-02	-2.259E-02	

(table 2 continued on next page)

(table 2 continued)

Species	Component	Parameter	Estimate	Var(β_i)	Cov(β_0,β_i)	Cov(β_1,β_2)	MSE
Alnus rugosa	Leaves	β_0	-0.1046	9.276E-02			0.2538
		β_1	1.4631	6.278E-02	-7.503E-02		
		β_2	0.4079	5.331E-02	6.495E-02	-5.592E-02	
	Current annual twig	β_0	-0.2584	1.545E-01			0.3999
		β_1	0.8458	1.063E-01	-1.260E-01		
		β_2	0.1607	9.166E-02	1.103E-01	-9.561E-02	
	Bark 5mm dob	β_0	0.7385	6.853E-02			0.1824
		β_1	0.8180	4.686E-02	-5.573E-02		
		β_2	1.3127	4.003E-02	4.848E-02	-4.192E-02	
	Total wood	β_0	0.6008	2.780E-02			0.0698
		β_1	1.6508	1.892E-02	-2.256E-02		
		β_2	1.2531	1.602E-02	1.951E-02	-1.685E-02	
	Total biomass	β_0	1.0566	2.526E-02			0.0634
		β_1	1.6066	1.719E-02	-2.049E-02		
		β_2	1.0374	1.456E-02	1.773E-02	-1.531E-02	
	Total biomass (gwt)	β_0	0.8126	2.790E-02			0.0765
		β_1	2.3119	1.889E-02	-2.257E-02		
		β_2	0.5384	1.604E-02	1.954E-02	-1.683E-02	
Amelanchier spp.	Leaves	β_0	-1.3314	9.597E-02			0.2912
		β_1	2.3967	7.196E-02	-7.944E-02		
		β_2	-0.4852	8.984E-02	7.598E-02	-7.406E-02	
	Current annual twig	β_0	-1.7108	1.775E-01			0.5606
		β_1	2.1424	1.320E-01	-1.461E-01		
		β_2	-1.0682	1.585E-01	1.373E-01	-1.347E-01	
	Bark 5mm dob	β_0	-0.4127	4.493E-02			0.1419
		β_1	2.0079	3.341E-02	-3.698E-02		
		β_2	0.3480	4.012E-02	3.477E-02	-3.410E-02	
	Total wood	β_0	0.3504	1.346E-02			0.0425
		β_1	2.3149	1.001E-02	-1.108E-02		
		β_2	0.5386	1.202E-02	1.041E-02	-1.021E-02	
	Total biomass	β_0	0.5825	1.708E-02			0.0518
		β_1	2.3066	1.281E-02	-1.414E-02		
		β_2	0.4217	1.599E-02	1.352E-02	-1.318E-02	
	Total biomass (gwt)	β_0	1.1063	2.777E-02			0.0877
		β_1	2.3362	2.065E-02	-2.286E-02		
		β_2	0.3572	2.480E-02	2.149E-02	-2.108E-02	

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(table 2 continued)

Species	Component	Parameter	Estimate	Var(β_i)	Cov(β_0,β_i)	Cov(β_1,β_2)	MSE
Corylus cornuta	Leaves	β_0	-0.5193	1.123E-01			0.2732
		β_1	1.8063	9.711E-02	-1.026E-01		
		β_2	0.5216	8.672E-02	9.108E-02	-8.817E-02	
	Current annual twig	β_0	-1.4323	2.289E-01			0.5039
		β_1	1.2646	1.969E-01	-2.087E-01		
		β_2	0.2572	1.737E-01	1.842E-01	-1.779E-01	
	Bark 5mm dob	β_0	-0.8587	1.207E-01			0.2912
		β_1	2.1707	1.041E-01	-1.101E-01		
		β_2	-0.1004	9.344E-02	9.805E-02	-9.478E-02	
	Total wood	β_0	-0.0745	3.311E-02			0.0721
		β_1	2.3573	2.839E-02	-3.014E-02		
		β_2	0.7975	2.516E-02	2.667E-02	-2.571E-02	
	Total biomass	β_0	0.3285	3.116E-02			0.0678
		β_1	2.3367	2.671E-02	-2.836E-02		
		β_2	0.5720	2.367E-02	2.510E-02	-2.419E-02	
	Total biomass (gwt)	β_0	0.5925	6.232E-02			0.1494
		β_1	2.6630	5.361E-02	-5.678E-02		
		β_2	0.2603	4.759E-02	5.024E-02	-4.855E-02	
All species combined	Leaves	β_0	-0.2957	1.998E-02			0.2802
		β_1	1.6413	1.455E-02	-1.662E-02		
		β_2	0.4143	1.481E-02	1.522E-02	-1.393E-02	
	Current annual twig	β_0	-1.5839	4.540E-02			0.6227
		β_1	1.9147	3.315E-02	-3.780E-02		
		β_2	-0.5351	3.380E-02	3.468E-02	-3.183E-02	
	Bark 5mm dob	β_0	-0.3443	1.618E-02			0.2277
		β_1	1.7423	1.176E-02	-1.345E-02		
		β_2	0.5757	1.192E-02	1.231E-02	-1.126E-02	
	Total wood	β_0	0.3277	7.265E-03			0.0993
		β_1	1.9979	5.262E-03	-6.024E-03		
		β_2	1.0280	5.314E-03	5.494E-03	-5.024E-03	
	Total biomass	β_0	0.7874	6.816E-03			0.0918
		β_1	1.9277	4.965E-03	-5.668E-03		
		β_2	0.8434	5.049E-03	5.187E-03	-4.753E-03	
	Total biomass (gwt)	β_0	1.0596	7.869E-03			0.1099
		β_1	2.2231	5.697E-03	-6.523E-03		
		β_2	0.6327	5.755E-03	5.951E-03	-5.440E-03	

Table 3.—*Regression statistics for predicting biomass of six shrub components of five tall shrubs:* Acer spicatum, Alnus crispa, Alnus rugosa, Amelanchier *spp.,* Corylus cornuta, *and all species combined. Parameters are from the relationship* $E(\log Y) = \beta_0 + \beta_1*(\log D)$, *where* Y = *biomass in g, and* D = *stem diameter class at 15 cm above ground; logarithms are base e. All weights are dry weight (dwt), except total biomass was measured in both dwt and green weight (gwt). The 12 stem diameter classes were 2.5 mm wide and covered the diameter range of 0 to 3 cm, e.g., size class 5 = stems > 1.0 and ≤ 1.25 cm.*

Species	Component	Parameter	Estimate	Var(β_i)	Cov(β_0,β_1)	MSE
Acer	Leaves	β_0	-0.4233	2.594E-02		0.2471
spicatum		β_1	1.9820	7.865E-03	-1.310E-02	
	Current	β_0	-0.6875	4.537E-02		0.4323
	annual twig	β_1	1.6712	1.376E-02	-2.292E-02	
	Bark	β_0	-0.9324	2.572E-02		0.2450
	5mm dob	β_1	2.2590	7.799E-03	-1.299E-02	
	Total wood	β_0	-0.7840	1.407E-02		0.1341
		β_1	2.9286	4.267E-03	-7.107E-03	
	Total biomass	β_0	0.0795	1.569E-02		0.1495
		β_1	2.6045	4.758E-03	-7.924E-03	
	Total biomass	β_0	0.5792	1.783E-02		0.1698
	(gwt)	β_1	2.7475	5.405E-03	-9.002E-03	
Alnus	Leaves	β_0	-0.6333	2.380E-02		0.2298
crispa		β_1	2.0302	7.565E-03	-1.271E-02	
	Current	β_0	-1.2515	4.579E-02		0.4420
	annual twig	β_1	1.5877	1.455E-02	-2.445E-02	
	Bark	β_0	-1.4829	1.504E-02		0.1512
	5mm dob	β_1	2.6347	4.824E-03	-8.056E-03	
	Total wood	β_0	-0.8662	1.372E-02		0.1379
		β_1	3.0806	4.399E-03	-7.346E-03	
	Total biomass	β_0	-0.2104	1.251E-02		0.1208
		β_1	2.8385	3.976E-03	-6.681E-03	
	Total biomass	β_0	0.3375	1.125E-02		0.0997
	(gwt)	β_1	2.8998	3.511E-03	-5.976E-03	

(table 3 continued on next page)

(table 3 continued)

Species	Component	Parameter	Estimate	Var(β_i)	Cov(β_0,β_i)	MSE
Alnus rugosa	Leaves	β_0	-0.6015	1.387E-02		0.2583
		β_1	1.8909	4.209E-03	-7.027E-03	
	Current annual twig	β_0	-0.4518	2.177E-02		0.3974
		β_1	1.0135	6.494E-03	-1.094E-02	
	Bark 5mm dob	β_0	-0.8512	1.327E-02		0.2463
		β_1	2.1927	3.996E-03	-6.707E-03	
	Total wood	β_0	-0.9252	7.399E-03		0.1278
		β_1	2.9690	2.190E-03	-3.722E-03	
	Total biomass	β_0	-0.2067	6.000E-03		0.1035
		β_1	2.6974	1.784E-03	-3.021E-03	
	Total biomass (gwt)	β_0	0.1565	4.665E-03		0.0872
		β_1	2.8769	1.401E-03	-2.356E-03	
Amelanchier spp.	Leaves	β_0	-0.9211	3.279E-02		0.3011
		β_1	1.9968	1.128E-02	-1.737E-02	
	Current annual twig	β_0	-0.7851	6.459E-02		0.6195
		β_1	1.2345	1.931E-02	-3.242E-02	
	Bark 5mm dob	β_0	-0.7143	1.530E-02		0.1468
		β_1	2.3037	4.575E-03	-7.681E-03	
	Total wood	β_0	-0.1164	6.171E-03		0.0592
		β_1	2.7727	1.845E-03	-3.097E-03	
	Total biomass	β_0	0.2258	6.835E-03		0.0628
		β_1	2.6542	2.351E-03	-3.622E-03	
	Total biomass (gwt)	β_0	0.7967	9.790E-03		0.0939
		β_1	2.6398	2.927E-03	-4.914E-03	
Corylus cornuta	Leaves	β_0	-1.0671	1.712E-02		0.2804
		β_1	2.3366	7.671E-03	-1.027E-02	
	Current annual twig	β_0	-1.7051	3.324E-02		0.4999
		β_1	1.5280	1.457E-02	-1.986E-02	
	Bark 5mm dob	β_0	-0.7533	1.758E-02		0.2879
		β_1	2.0689	7.921E-03	-1.056E-02	
	Total wood	β_0	-0.9200	6.358E-03		0.0948
		β_1	3.1723	2.785E-03	-3.790E-03	
	Total biomass	β_0	-0.2780	5.306E-03		0.0791
		β_1	2.9213	2.324E-03	-3.163E-03	
	Total biomass (gwt)	β_0	0.3176	9.332E-03		0.1502
		β_1	2.9286	4.096E-03	-5.551E-03	

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(table 3 continued)

Species	Component	Parameter	Estimate	Var(β_i)	Cov(β_0,β_i)	MSE
All species combined	Leaves	β_0	-0.7216	4.449E-03		0.2875
		β_1	2.0311	1.484E-03	-2.358E-03	
	Current annual twig	β_0	-1.0348	9.992E-03		0.6340
		β_1	1.4108	3.236E-03	-5.234E-03	
	Bark 5mm dob	β_0	-0.9387	3.698E-03		0.2422
		β_1	2.2857	1.208E-03	-1.942E-03	
	Total wood	β_0	-0.7351	2.346E-03		0.1471
		β_1	2.9698	7.585E-04	-1.229E-03	
	Total biomass	β_0	-0.0791	2.011E-03		0.1241
		β_1	2.7216	6.632E-04	-1.062E-03	
	Total biomass (gwt)	β_0	0.4054	1.997E-03		0.1279
		β_1	2.8212	6.458E-04	-1.045E-03	

Table 4.—*Regression statistics for predicting biomass of six shrub components of five tall shrubs:* Acer spicatum, Alnus crispa, Alnus rugosa, Amelanchier *spp.*, Corylus cornuta, *and all species combined. Parameters are from the relationship* $E(\log Y) = \beta_0 + \beta_1{*}(\log H)$, *where* Y *= biomass in g, and* H *= stem height to nearest 0.1 m; logarithms are base e. All weights are dry weight (dwt), except total biomass was measured in both dwt and green weight (gwt). The 12 stem diameter classes were 2.5 mm wide and covered the diameter range of 0 to 3 cm, e.g., size class 5 = stems > 1.0 and ≤ 1.25 cm*

Species	Component	Parameter	Estimate	Var(β_i)	Cov(β_0,β_1)	MSE
Acer spicatum	Leaves	β_0	1.8066	6.280E-03		0.2421
		β_1	2.2762	1.014E-02	-4.771E-03	
	Current annual twig	β_0	1.1936	1.121E-02		0.4322
		β_1	1.9170	1.810E-02	-8.520E-03	
	Bark 5mm dob	β_0	1.6252	8.084E-03		0.3116
		β_1	2.5598	1.305E-02	-6.142E-03	
	Total wood	β_0	2.5423	7.971E-03		0.3072
		β_1	3.2961	1.287E-02	-6.056E-03	
	Total biomass	β_0	3.0317	6.642E-03		0.2560
		β_1	2.9439	1.072E-02	-5.046E-03	
	Total biomass (gwt)	β_0	3.6941	7.557E-03		0.2913
		β_1	3.1044	1.220E-02	-5.741E-03	
Alnus crispa	Leaves	β_0	1.6497	4.563E-03		0.2200
		β_1	2.0939	7.651E-03	-4.123E-03	
	Current annual twig	β_0	0.5921	1.126E-02		0.5431
		β_1	1.5294	1.888E-02	-1.018E-02	
	Bark 5mm dob	β_0	1.5447	4.099E-03		0.2112
		β_1	2.6211	6.845E-03	-3.584E-03	
	Total wood	β_0	2.6599	3.222E-03		0.1660
		β_1	3.0910	5.380E-03	-2.817E-03	
	Total biomass	β_0	2.9986	3.308E-03		0.1596
		β_1	2.8958	5.548E-03	-2.989E-03	
	Total biomass (gwt)	β_0	3.6607	3.957E-03		0.1791
		β_1	2.9007	6.510E-03	-3.617E-03	

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(table 4 continued)

Species	Component	Parameter	Estimate	Var(β_i)	Cov(β_0,β_1)	MSE
Alnus rugosa	Leaves	β_0	1.6439	3.954E-03		0.3244
		β_1	1.7109	4.488E-03	-2.390E-03	
	Current annual twig	β_0	0.7445	5.342E-03		0.4195
		β_1	0.9217	5.911E-03	-3.278E-03	
	Bark 5mm dob	β_0	1.7113	2.499E-03		0.2026
		β_1	2.0445	2.808E-03	-1.535E-03	
	Total wood	β_0	2.5684	2.036E-03		0.1552
		β_1	2.7231	2.252E-03	-1.284E-03	
	Total biomass	β_0	2.9720	1.916E-03		0.1455
		β_1	2.4682	2.124E-03	-1.195E-03	
	Total biomass (gwt)	β_0	3.5750	3.074E-03		0.2533
		β_1	2.5979	3.454E-03	-1.887E-03	
Amelanchier spp.	Leaves	β_0	1.3144	2.187E-02		0.7695
		β_1	1.9815	3.599E-02	-1.527E-02	
	Current annual twig	β_0	0.6607	2.482E-02		0.8816
		β_1	1.1187	3.300E-02	-1.850E-02	
	Bark 5mm dob	β_0	1.8099	1.210E-02		0.4298
		β_1	2.3976	1.609E-02	-9.018E-03	
	Total wood	β_0	2.9128	1.204E-02		0.4276
		β_1	2.9015	1.601E-02	-8.972E-03	
	Total biomass	β_0	3.1288	1.419E-02		0.4993
		β_1	2.7957	2.336E-02	-9.908E-03	
	Total biomass (gwt)	β_0	3.6923	1.349E-02		0.4792
		β_1	2.7419	1.794E-02	-1.005E-02	

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(table 4 continued)

Species	Component	Parameter	Estimate	Var(β_i)	Cov(β_0,β_1)	MSE
Corylus cornuta	Leaves	β_0	1.3891	5.519E-03		0.3832
		β_1	2.1616	9.360E-03	-2.906E-03	
	Current annual twig	β_0	-0.0919	8.370E-03		0.5493
		β_1	1.4000	1.413E-02	-4.738E-03	
	Bark 5mm dob	β_0	1.4369	6.537E-03		0.4522
		β_1	1.8750	1.116E-02	-3.378E-03	
	Total wood	β_0	2.4280	3.943E-03		0.2544
		β_1	2.9320	6.620E-03	-2.188E-03	
	Total biomass	β_0	2.8092	3.829E-03		0.2470
		β_1	2.6879	6.428E-03	-2.124E-03	
	Total biomass (gwt)	β_0	3.4129	5.731E-03		0.3916
		β_1	2.6722	9.481E-03	-3.098E-03	
All species combined	Leaves	β_0	1.5781	1.464E-03		0.4072
		β_1	1.9854	2.139E-03	-9.979E-04	
	Current annual twig	β_0	0.5992	2.911E-03		0.7879
		β_1	0.3032	4.101E-03	-2.038E-03	
	Bark 5mm dob	β_0	1.6475	1.312E-03		0.3677
		β_1	2.2430	1.859E-03	-9.013E-04	
	Total wood	β_0	2.6151	1.045E-03		0.2820
		β_1	2.9354	1.469E-03	-7.314E-04	
	Total biomass	β_0	2.9882	9.919E-04		0.2643
		β_1	2.6888	1.436E-03	-6.896E-04	
	Total biomass (gwt)	β_0	3.6050	1.228E-03		0.3376
		β_1	2.7555	1.722E-03	-8.546E-04	

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Buech, Richard R.; Rugg, David J.
1995. **Biomass relations for components of five Minnesota shrubs.** Res. Pap. NC-325. St. Paul, MN: U.S. Department of Agriculture, Forest Service, North Central Forest Experiment Station. 14 p.

Presents equations for estimating biomass of six components on five species of shrubs common to northeastern Minnesota. Regression analysis is used to compare the performance of three estimators of biomass.

KEY WORDS: Biomass estimation, tall shrubs, prediction equations, biomass equations, allometric relations.