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A Simple Model for Estimating Total and Merchantable Tree Heights[1]

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ABSTRACT.—A model is described for estimating total and merchantable tree heights for Lake States tree species. It is intended to be used for compiling forest survey data and in conjunction with growth models for developing projections of tree product yield. Model coefficients are given for 25 species along with fit statistics. Supporting data sets are also described.

KEY WORDS: Tree height estimation equations, regression, survey, growth and yield projection.

A model has been developed for estimating total and merchantable tree height (see Ek *et al.* 1981). The model facilitates the use of volume tables and equations based on tree height for forest survey compilations and forest growth and yield projections. It has also been incorporated in several versions of STEMS, the North Central stand and tree evaluation and modeling system (see Belcher (1981) for a description of STEMS). This note describes the height estimation model and presents coefficients for 25 Lake States tree species. A description of the data upon which the coefficients are based and an assessment of model performance are also given.

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THE MODEL FORM AND COEFFICIENTS

The model form is:

$$H = 4.5 + b_1 (1-e^{-b_2 D})^{b_3} S^{b_4} T^{b_5} B^{b_6}$$

where H is tree height (ft), D is diameter at breast height (inches), S is site index for the species, B is basal area per acre (ft^2) and T = (1.00001 - d/D). The merchantable top diameter outside bark, d, determines whether merchantable or total height is being estimated. When d = 0, the model estimates *total* tree height. When d equals some other value, the model estimates the *merchantable* height to that top diameter. T raised to a power is a simple upper-stem taper expression.

Model coefficients for 25 Lake States tree species were developed from forest survey plot records for Minnesota (1976-78), Wisconsin (1967-68), and Michigan (1979-80). Two separate data sets were used. The first provided total tree height along with age, diameter, and associated stand characteristics for site index trees. The second data set provided merchantable length above a 1.0 ft. stump along with diameter, top diameter outside bark, and associated stand characteristics for trees on the various survey plots throughout the region. An exception was that total length rather than merchantable length was given for trees less than 5.0 inches in diameter.

Pulpwood and saw log top diameters and merchantable lengths were recorded with minimum top diameters of 4.0 and 7.0 inches for pulpwood and sawtimber, respectively (table 1).

Species coefficients are given in table 2 along with the standard errors of the model fits. Detailed height estimates are shown for white pine in table 3. The pattern of the height over diameter relation is illustrated in figure 1. Details of model development and fitting can be found in Ek *et al.* (1981).

DISCUSSION

A special feature of this model is that it provides an estimate of both total and merchantable heights. A consistent relation between these two heights is provided by the stem taper term. The predictions are sensitive to site quality and stand density, but it is less clear whether they are sensitive to crown class. However, dominant trees tend to have the largest diameter stems in a stand. Thus as the model suggests, they would be taller. The converse is true for lower crown classes. Introducing a crown class or relative tree size term in the model did not reduce residual errors in model fitting.

Inserting an individual tree age term could have improved fits. However, age was included only in the site tree data and was not expected to be commonly available for application. Consequently, an age term was not considered for the model. Standard errors for the model fits are close to the usual error encountered when measuring tree heights in the field with hypsometers. Thus, further model refinements probably would not substantially reduce standard errors.

Site quality was an important predictor for 23 species (see table 2). However, for red pine and white

Table 1.—*Characteristics of the basic tree height data sets*

Species Group code[1]	Species Name	Trees	Diameter range	Maximum top diameter outside bark	Mean height[2]
		No.	*Inches*	*Inches*	*Feet*
1	JACK PINE	1,089	1.0 - 18.0	13.8	36.4
2	RED PINE	1,049	1.0 - 22.5	12.5	41.9
3	WHITE PINE	883	1.4 - 35.9	15.0	50.8
4	WHITE SPRUCE	462	1.0 - 25.3	9.8	40.6
5	BALSAM FIR	1,315	1.0 - 27.9	9.9	35.3
6	BLACK SPRUCE	1,135	1.0 - 15.2	10.2	33.4
7	TAMARACK	1,035	1.1 - 16.0	11.0	35.9
8	NORTHERN WHITE CEDAR	1,180	1.0 - 20.5	12.2	27.9
9	HEMLOCK	105	5.2 - 27.3	10.9	45.3
11	BLACK AND GREEN ASH	1,197	1.0 - 27.0	15.0	41.4
12	COTTONWOOD	97	3.4 - 29.8	14.0	50.6
13	SILVER MAPLE	300	3.0 - 50.4	17.0	44.6
14	RED MAPLE	1,062	1.0 - 33.7	16.4	40.1
15	ELM (AMERICAN, RED, ROCK)	1,081	1.0 - 37.8	21.0	41.2
16	YELLOW BIRCH	825	1.2 - 36.8	22.0	39.8
17	BASSWOOD	1,167	1.1 - 29.3	20.0	46.9
18	SUGAR MAPLE, BLACK MAPLE	1,370	1.0 - 28.2	22.0	45.3
19	WHITE ASH	625	3.4 - 39.1	18.0	44.5
20	WHITE OAK	638	3.0 - 42.1	20.0	43.2
21	SELECT RED OAK	1,329	1.3 - 30.2	18.2	47.3
22	OTHER RED OAK	681	1.0 - 30.7	16.0	39.2
23	HICKORY	109	1.1 - 16.7	10.0	46.6
24	BIGTOOTH ASPEN	1,103	1.0 - 24.8	18.0	50.0
25	QUAKING ASPEN, BALSAM, POPLAR	,488	1.0 - 25.5	18.0	46.5
26	PAPER BIRCH	1,355	1.2 - 25.7	15.4	41.7

[1]Species group codes from Belcher, 1981.

[2]Average height above ground to top diameter, including trees with total and merchantable height.

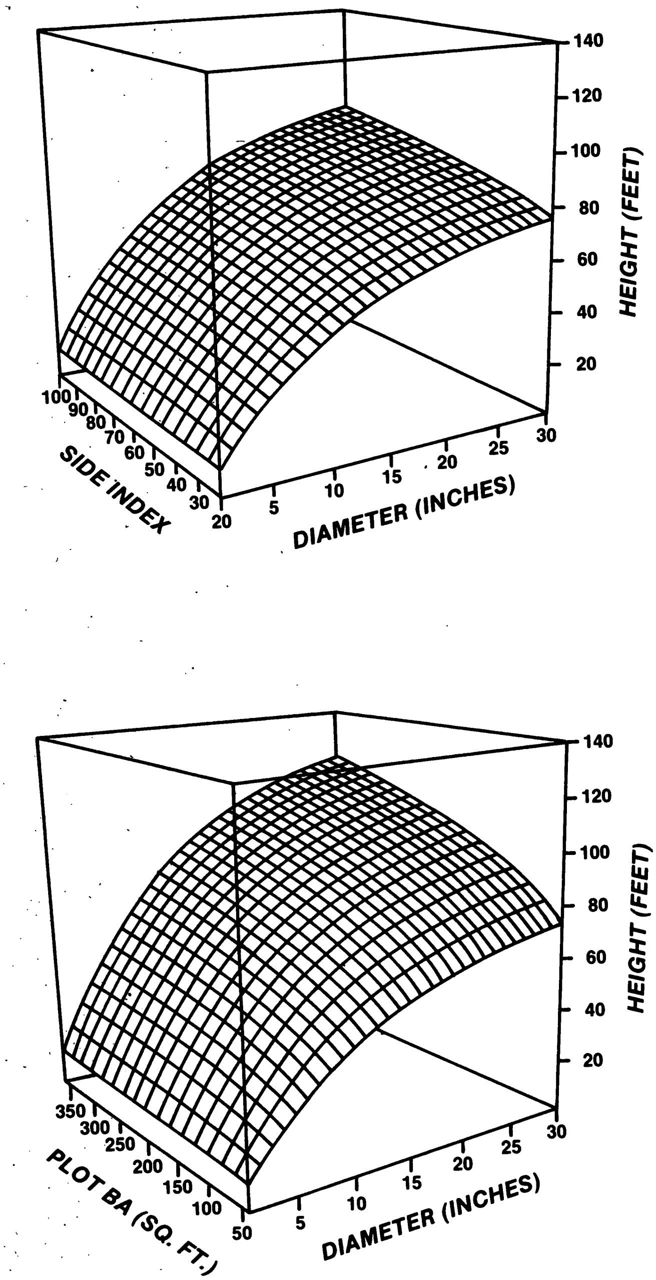

Figure 1.—*Graph of tree height estimates for white pine. Top.—Total height based on site index and diameter with basal area set at 100 ft² per acre. Bottom.—Total height based on basal area per acre and diameter with site index set at 60.*

spruce the b_4 coefficient is near zero. Apparently for these species the site quality effect is reflected in both increased height and diameter development but the relationship of height to diameter remains unchanged. In effect, if diameter is known, site quality adds little information about height for these species.

When developing the coefficients, we also compared fits for the entire region versus separate fits within individual survey units (13 survey units in total). Results indicated that standard errors did not appreciably increase with the larger data sets. Examination of residuals within survey units for the regionwide coefficients also failed to indicate lack of fit or geographic trends.

We also checked the equations by computing total heights for site indices 30, 50, 70, and 90 for diameters up to 24 inches and stand basal areas from 50 to 300 ft^2 per acre. The computed heights for species 1, 2, 3, 9, 18, 23, 24, 25, and 26 (see table 1 for species codes) equaled or exceeded the site index values—i.e., the 50 year expected height—for all of these site indices. The remaining species, except silver maple, fell short of expected height for only site index 90. Silver maple fell short of expected height for both site index 70 and 90. However, most of the differences between expected 50 year height and computed height were within the standard errors for the model fits. Also, the expected 50 year height was less likely to be achieved at a basal area of 50 ft^2 as compared to basal areas of 200 ft^2 or more. The failure to achieve a height of, say, 90 feet on a site index 90 is a limitation of the model for some species, in part a function of the sample. However, the model attempts to predict average height for a given diameter, and no distinction is made for the tree's dominance class.

Table 3 indicates how height is affected by stand density. Note that the lack of a density effect for hemlock (table 2, b_6 =0) was probably due more to an inadequate sample of stand conditions than the lack of any real effect.

Another limitation appears for small top diameters on very large trees (trees 40 inches in diameter). Here the difference between predicted total and merchantable tree heights may be unrealistically small. This is partly due to the small d/D ratios for such situations. Also, such trees did not occur frequently in the data set; those found may have been irregular in form with damaged tops. Consequently, caution is suggested when applying these equations to large trees with small top diameters.

APPLICATIONS

The equations given here may be useful in forest inventory compilations when heights have not been measured or when considering a range of utilization standards and associated top diameters.

Height growth may be estimated with these equations by taking the difference of predictions between two sets of diameter and basal area inputs corresponding to different ages. However, caution is recommended when the basal area has been reduced

Table 2.—*Species coefficients and standard errors of height estimation model*

Species Code	Species Name	b_1	b_2	b_3	b_4	b_5	b_6	Standard error
								Feet
1	JACK PINE	16.9340	0.12972	1.0000	0.20854	0.77792	0.12902	7.2
2	RED PINE	36.8510	.08298	1.0000	.00001	.63884	.18231	8.5
3	WHITE PINE	16.2810	.08621	1.0000	.16220	.86833	.23316	10.7
4	WHITE SPRUCE	31.9570	.18511	1.7020	.00000	.68967	.16200	8.2
5	BALSAM FIR	14.3040	.19894	1.4195	.23349	.76878	.12399	5.9
6	BLACK SPRUCE	20.0380	.18981	1.2909	.17836	.57343	.10159	5.5
7	TAMARACK	13.6200	.24255	1.2885	.25831	.68128	.10771	5.6
8	NORTHERN WHITE CEDAR	8.2079	.19672	1.3112	.33978	.76173	.11666	5.4
9	HEMLOCK	5.3117	.10357	1.0000	.68454	.71410	.00000	7.2
11	BLACK AND GREEN ASH	11.2910	.25250	1.5466	.35711	.75060	.06859	7.2
12	COTTONWOOD	13.6250	.28668	1.6124	.30651	1.02920	.07460	8.1
13	SILVER MAPLE	6.9572	.26564	1.0000	.48660	.76954	.01618	9.1
14	RED MAPLE	6.8600	.27725	1.4287	.40115	.85299	.12403	7.2
15	ELM (AMERICAN, RED, ROCK)	8.4580	.27527	1.9602	.34894	.89213	.12594	8.1
16	YELLOW BIRCH	7.1852	.28384	1.4417	.38884	.82157	.11411	9.3
17	BASSWOOD	6.3628	.27859	1.8677	.49589	.76169	.05841	7.7
18	SUGAR MAPLE, BLACK MAPLE	5.3416	.23044	1.1529	.54194	.83440	.06372	7.7
19	WHITE ASH	8.1782	.27316	1.7250	.38694	.75822	.10847	8.3
20	WHITE OAK	9.2078	.22208	1.0000	.31723	.83560	.13465	8.2
21	SELECT RED OAK	6.6844	.19049	1.0000	.43972	.82962	.10806	8.2
22	OTHER RED OAK	3.8011	.39213	2.9053	.55634	.84317	.09593	7.7
23	HICKORY	6.1034	.17368	1.0000	.44725	1.02370	.14610	7.3
24	BIGTOOTH ASPEN	5.5346	.22637	1.0000	.46918	.72456	.11782	7.2
25	QUAKING ASPEN, BALSAM POPLAR	6.4301	.23545	1.3380	.47370	.73385	.08228	6.7
26	PAPER BIRCH	7.2773	.22721	1.0000	.41179	.76498	.11046	7.1

between the first and second ages due to management or mortality, because height estimates can decrease with a reduction in basal area. Clearly, height development of a given tree is influenced by stand density; but height should not be reduced by a new basal area level. Users may wish to constrain growth estimates to no less than zero to alleviate the problem in studies of management alternatives.[2]

In summary, this model and coefficients can be used for survey compilations over broad regions within the Lake States with little loss in precision as compared to equations developed for individual survey units. They can also be used in conjunction with growth projection models for assessing growth sensitivity to stand conditions in silvicultural studies.

[2]*The authors attempted to avoid this problem by including a relative diameter term and other expressions in the model development process. However, these terms did not improve the fit. To minimize bias when compiling large inventories for updates or volume projections, the height estimates are best left unconstrained.*

LITERATURE CITED

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Table 3.—*Estimated total and merchantable tree heights for white pine across a range of diameters for selected site indices*

D.b.h. (inches)	Total height: Stand basal area — ft² per acre 50	100	150	200	250	Height to 4 inch top d.o.b.: Stand basal area — ft² per acre 50	100	150	200	250
	(In feet)									
	SITE INDEX 50									
4	26.8	30.7	33.3	35.3	37.0	4.5	4.5	4.5	4.5	4.5
6	35.4	40.8	44.4	47.2	49.4	16.4	18.5	19.9	20.9	21.8
8	42.6	49.3	53.7	57.1	59.9	25.4	29.0	31.5	33.3	34.9
10	48.7	56.4	61.6	65.5	68.8	32.8	37.8	41.1	43.7	45.8
12	53.8	62.4	68.2	72.6	76.2	39.2	45.2	49.3	52.4	54.9
14	58.1	67.5	73.7	78.5	82.5	44.5	51.5	56.2	59.8	62.7
16	61.7	71.7	78.4	83.5	87.8	49.1	56.9	62.1	66.1	69.4
18	64.8	75.3	82.3	87.7	92.2	52.9	61.4	67.1	71.4	75.0
20	67.3	78.3	85.7	91.3	95.9	56.3	65.3	71.4	76.0	79.8
22	69.5	80.9	88.4	94.3	99.1	59.1	68.7	75.0	79.9	83.9
24	71.3	83.0	90.8	96.8	101.7	61.5	71.5	78.2	83.3	87.5
	SITE INDEX 70									
4	28.0	32.2	34.9	37.0	38.8	4.5	4.5	4.5	4.5	4.5
6	37.1	42.8	46.6	49.5	52.0	17.1	19.3	20.7	21.9	22.8
8	44.7	51.8	56.5	60.1	63.0	26.5	30.4	33.0	34.9	36.6
10	51.1	59.3	64.8	68.9	72.4	34.4	39.7	43.2	45.9	48.1
12	56.5	65.7	71.7	76.4	80.2	41.1	47.5	51.8	55.1	57.8
14	61.1	71.0	77.6	82.7	86.9	46.8	54.2	59.1	62.9	66.0
16	64.9	75.5	82.6	88.0	92.4	51.6	59.8	65.3	69.5	73.0
18	68.1	79.3	86.7	92.4	97.1	55.7	64.6	70.6	75.2	79.0
20	70.8	82.5	90.2	96.2	101.1	59.2	68.7	75.1	80.0	84.0
22	73.1	85.2	93.2	99.3	104.4	62.2	72.3	79.0	84.1	88.4
24	75.0	87.4	95.6	102.0	107.2	64.7	75.3	82.3	87.7	92.1