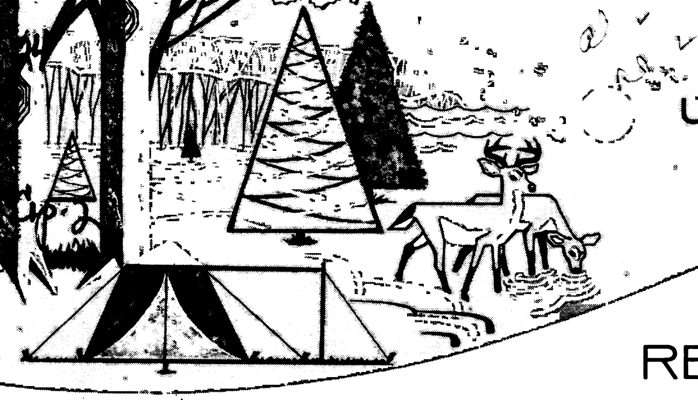

U.S. FOREST SERVICE

RESEARCH NOTE NC-24

NORTH CENTRAL FOREST EXPERIMENT STATION, FOREST SERVICE—U.S. DEPARTMENT OF AGRICULTURE
Folwell Avenue, St. Paul, Minnesota 55101

Stand Volume Equations for Shortleaf Pine In Missouri

ABSTRACT. — Presents stand volume equations and a stand volume table for shortleaf pine in Missouri and describes their development and application.

Stand volume equations provide direct estimates of volume using such simple stand data as basal area per acre and height of dominant trees. This technique is much easier and faster than the traditional sampling methods based on volumes of individual trees. Precision is about the same and adequate for most management purposes. The increasing use of Bitterlich angle-count (point-sampling) cruising has made stand volume equations even more practical for the forest manager.

This paper presents stand volume equations and a stand volume table for shortleaf pine in Missouri and describes their development and application. The equations provide estimates of merchantable cubic-foot and board-foot volume per acre to specified minimum upper-stem diameters inside bark. The equations developed here can be used with point-sampling methods in Missouri and probably in other areas provided the same utilization standards apply.

The composite tree volume tables developed by Gevorkiantz and Olsen (1955) are based on a known relation: Conifers of given diameter and total height tend to have the same taper and hence similar volumes. Total cubic-foot volume of a tree was computed as 42 percent of the product of basal area and height.

Buckman (1961) expanded this concept to develop ratio and linear regression equations for total cubic-foot, cordwood, and board-foot volumes of stands using basal area per acre and the height of dominant trees. The regression of stand volume plotted against the product of basal area and height ($B \cdot H$) was nearly linear or could be approximated by a linear relation.

In a later publication, Buckman (1962) used the independent variables of age, site index, and stand density to predict periodic annual basal-area growth. He then used the ratio of stand volume to $B \cdot H$ in preparing growth and yield tables for red pine. Using a similar approach, Vuokila (1965) found that the use of $B \cdot H^2$ and D, the diameter of the tree of average basal area, in total cubic volume equations greatly improved their accuracy for stands in early stages of development.

Method of Preparation

The stand volume equations are based on data from 57 shortleaf pine stands where volumes have been measured periodically over a period of 15 years. Nearly all the stands were thinned twice during this time. Stand ages ranged from 19 to 85 years and dominant trees were 34 to 85 feet tall. Apparent site indices of the stands ranged from about 55 to 75, but site index was not included as a variable in the stand volume equations.

Data were available for 347 measurements of cubic-foot volume and 297 measurements of board-foot volume. The minimum sizes of trees included were: 4.6 inches d.b.h. to a 3-inch top d.i.b. for cubic-foot volume, and 6.6 inches d.b.h. to a 5-inch top d.i.b. for board-foot volume. Volumes of individual trees in the stands were based on the following equations (Brinkman *et al.* 1965):

$$\text{Cubic-foot volume} = -0.5389 + 0.0023\, D^2H$$
$$\text{Board-foot volume} = -15.24 + 0.013\, D^2H$$

The cubic-foot volumes per acre of individual stands were plotted against the product of their basal area and total height ($B \cdot H$). From these scatter diagrams it was apparent that the ratio method used by Buckman (1962) did not fit the data as well as a linear regression because many small trees have measurable total cubic-foot volume but no merchantable volume.

The board-foot volume data for individual stands were plotted against both $B \cdot H$ and $B \cdot H^2$. A much better fit of the volume data to a single regression line was indicated when $B \cdot H^2$ was used as the

MAINTAINED IN COOPERATION WITH THE UNIVERSITY OF MINNESOTA

independent variable. Diameter of the tree of average basal area (D) also was included as a variable in the trial equations.

Stand Volume Equations

Plot data were analyzed on an IBM 7040 computer to find which combination of five variables gave the best estimate of stand volume per acre. For both cubic-foot and board-foot volume, the variables tested were: B, H, D, B•H, and $B \cdot H^2$.

For cubic-foot volume estimates, the R^2 value (0.97) with all five variables included was about the same as that for B•H alone, so a linear equation was chosen to estimate stand cubic-foot volume (table 1). Two board-foot volume equations were selected. Their R^2 values were about 0.92, and their standard errors of estimate of individual stand volumes also were similar (table 1). For most stands where board-foot volume is important, the equations produce similar estimates. But trials show that Equation III tends to overestimate the volume of young stands and underestimate volume of mature stands. Although Equation II is more accurate, it is somewhat harder to use.

For this reason, a table was prepared using Equation II showing board-foot volumes for representative combinations of B, H, and D (table 2). By interpolating as necessary in this table the forest manager can make a direct estimate of the present volume of shortleaf pine stands and estimate future volume growth.

Using the Stand Volume Equations

To find the volume of a shortleaf pine stand, substitute the appropriate stand data in the proper equation or use table 2. Small inaccuracies in measuring B and D are likely to produce compensating errors. However, because the value for H is squared in computing board-foot volume, a small error here may produce a large error in the volume estimate.

Basal area per acre should be determined from 10 or more sample points in the stand using a wedge prism or angle gauge with a factor of 10. Include only those trees with diameters larger than the minimum previously specified for the volume measurement involved. When board-foot volume is desired, also count the trees on each sample point so you can compute D. A rangefinder set at 26 - 1/3 feet, the radius of a 1/20-acre plot, can be used to check borderline trees. Knowing total basal area and the number of trees per acre, D to the nearest inch can be computed with the aid of a standard basal-area table. Measure total heights of at least 5 dominant trees in the stand with an accurate hypsometer.

To estimate stand cubic-foot volume, only basal area per acre and average height of dominant trees need be known. For example, assume a stand has a basal area of 70 square feet per acre and the average height of dominant trees is 60 feet. Merchantable cubic-foot volume per acre then is computed with Equation I as:

$$
\begin{aligned}
V &= 0.323\,(B \cdot H) + 92.18 \\
&= 0.323\,(70 \times 60) + 92.18 \\
&= 1{,}448.8 \text{ cubic feet}
\end{aligned}
$$

Board-foot volume is computed in a similar manner but another variable, D, is needed. Assuming stand data show that this is 12 inches, B is 90 square feet per acre and H is 70 feet, the calculation can be made with Equation III as follows:

$$
\begin{aligned}
V &= 0.019\,(B \cdot H^2) + 337.7\,D - 1{,}439 \\
&= 0.019\,(90 \times 70^2) + 337.7\,(12) - 1{,}439 \\
&= 0.019\,(441{,}000) + 337.7\,(12) - 1{,}439 \\
&= 10{,}992 \text{ board feet}
\end{aligned}
$$

For comparison, the volume per acre computed with Equation II for such a stand is 11,112 board feet. Rounded to the nearest 50 board feet as in table 2, these values become 11,000 and 11,100 board feet, respectively.

In most cases, it will be easier to determine board-foot volume per acre by using table 2. Find the tabular volume of the stand with B, D, and H most like the stand in question and interpolate as necessary.

Limitations of the Equations

Most shortleaf pine stands used in developing the stand volume equations had been thinned several times, but the equations seem applicable to the pine component of stands regardless of stocking density. The basic requirement is that all trees considered in computing B and D must be at least 4.6 inches d.b.h. for cubic-foot volume and 6.6 inches d.b.h. for board-foot volume.

For Missouri shortleaf pine stands, Equation I provides a realistic estimate of merchantable cubic-foot volume per acre. To test the equation in another area, it was applied to data for a young plantation in Indiana that had been thinned to several stocking densities (Williams 1959). Measured and corresponding computed cubic-foot volumes per acre were as follows:

Basal area per acre	*Measured volume*	*Computed volume*
177	2,562	2,550
170	2,432	2,454
143	2,089	2,079
122	1,837	1,789

Table 1. — *Cubic-foot and board-foot volume equations for shortleaf pine stands in Missouri: — gross volumes to specified minimum top diameters inside bark*

Volume	Equation[1/]	Standard error of estimate of individual stand volumes	Standard error of estimate as a percent of mean volume
Cubic feet[2/]	(I) $V = 0.323\ (B \cdot H) + 92.18$	177.4	9.31
Board feet[3/]	(II) $V = 0.0214\ (B \cdot H^2) - 79.54\ H + 624.16\ D - 247$	1,148.0	13.72
	(III) $V = 0.019\ (B \cdot H^2) + 337.7\ D - 1,439$	1,201.0	14.36

1/ V is volume per acre, B is basal area in square feet per acre, H is mean total height of the dominant stand, and D is diameter of the tree of average basal area.

2/ Cubic-foot volume per acre of all trees 4.6-inches d.b.h. and larger to a 3-inch top (d.i.b.).

3/ Board-foot volume per acre (Int. 1/4-inch rule) of all trees 6.6-inches d.b.h. and larger to a 5-inch top (d.i.b.).

Table 2. — *Board-foot volume per acre of shortleaf pine stands in Missouri in relation to basal area, height of dominant trees, and average tree diameter*[1]

Average diameter (inches)	Height of dominant trees (feet): 40	50	60	70	80	90
70 Square Feet Basal Area Per Acre						
8	3,950	4,500	5,350	6,500	7,950	9,700
9	4,600	5,150	6,000	7,150	8,600	10,350
10	5,200	5,750	6,600	7,750	9,200	10,950
11	5,850	6,400	7,250	8,400	9,850	11,550
12	6,450	7,000	7,850	9,000	10,450	12,200
13	7,100	7,650	8,500	9,650	11,100	12,800
14	7,700	8,250	9,100	10,250	11,700	13,450
15	8,350	8,900	9,750	10,900	12,300	14,050
16	8,950	9,500	10,350	11,500	12,950	14,700
17	9,550	10,150	10,800	12,100	13,550	15,300
18	10,200	10,750	11,600	12,750	14,200	15,950
90 Square Feet Basal Area Per Acre						
8	4,650	5,600	6,900	8,600	10,700	13,150
9	5,250	6,200	7,500	9,200	11,300	13,800
10	5,900	6,800	8,150	9,850	11,950	14,400
11	6,500	7,450	8,750	10,450	12,550	15,050
12	7,150	8,050	9,400	11,100	13,200	15,650
13	7,750	8,700	10,000	11,720	13,800	16,300
14	8,400	9,300	10,650	12,350	14,450	16,900
15	9,000	9,950	11,250	12,950	15,050	17,550
16	9,650	10,550	11,900	13,600	15,700	18,150
17	10,250	11,200	12,500	14,200	16,300	18,800
18	10,900	11,800	13,150	14,850	16,950	19,400
110 Square Feet Basal Area Per Acre						
8	5,300	6,650	8,450	10,700	13,400	16,600
9	5,950	7,250	9,050	11,300	14,050	17,250
10	6,550	7,900	9,700	11,950	14,650	17,850
11	7,200	8,500	10,300	12,550	15,300	18,500
12	7,800	9,150	10,950	13,200	15,900	19,100
13	8,450	9,750	11,550	13,800	16,550	19,750
14	9,050	10,400	12,200	14,450	17,150	20,350
15	9,700	11,000	12,800	15,050	17,800	21,000
16	10,300	11,650	13,450	15,700	18,400	21,600
17	10,950	12,250	14,050	16,300	19,050	22,250
18	11,550	12,900	14,700	16,950	19,650	22,850
130 Square Feet Basal Area Per Acre						
8	6,000	7,700	9,950	12,800	16,150	20,100
9	6,650	8,350	10,600	13,400	16,800	20,700
10	7,250	8,950	11,200	14,050	17,400	21,350
11	7,900	9,550	11,850	14,650	18,050	21,950
12	8,500	10,200	12,450	15,250	18,650	22,600
13	9,150	10,850	13,100	15,900	19,300	23,200
14	9,750	11,450	13,700	16,550	19,900	23,850
15	10,400	12,100	14,350	17,150	20,550	24,450
16	11,000	12,700	14,950	17,800	21,150	25,100
17	11,650	13,350	15,600	18,400	21,800	25,700
18	12,250	13,950	16,200	19,050	22,400	26,350

1/ Based on regression equation: $V = 0.0214\ (B \cdot H^2) - 79.54H + 624.16D - 247$ where V is gross board-foot volume (Int. 1/4" rule) to a 5-inch top (d.i.b.), B is basal area in square feet per acre, H is mean total height of the dominant stand, and D is diameter of the tree of average basal area. Volumes rounded to nearest 50 board feet per acre.

Although the Indiana data were not used in preparing the basic equation and the plantation grew much faster than most Missouri stands, Equation I gave similar estimates of merchantable cubic-foot volume.

No comparable data were available for shortleaf pine stands in other areas to test the use of the board-foot volume equations, but they seem to be applicable to Missouri stands. If further experience shows that the equations result in consistently high or low volume estimates, minor adjustments should be made.

Growth Predictions

The stand volume equations and table 2 also can be used to predict volume growth for short periods when expected increments in basal area per acre, height, and mean tree diameter are known. Obviously, the growth predictions will be no more accurate than the estimates of changes in these variables.

In Missouri shortleaf pine stands, annual basal-area growth ranges from 2 to 4 square feet per acre, depending on stocking density and age of trees. On average sites, height growth will average a foot per year between ages 25 and 40, decreasing to 0.8-foot from age 41 to 60, and 0.5-foot in older stands. Mean d.b.h. growth generally is about 2 inches for every 10-foot increase in total height, but this depends on stand density. In unthinned stands, d.b.h. growth may be as low as 1. 3 inches per 10-foot height increase.

To illustrate one method of growth prediction, assume a 40-year-old shortleaf pine stand with 90 square feet of basal area per acre, dominant trees 60 feet tall, and a mean d.b.h. of 10 inches. The present volumes computed with Equations I and III are 1,836 cubic feet and 8,100 board feet per acre. During the next 5 years, it is estimated that the stand will add about 15 feet of basal area, 5 feet in total height, and 1 inch in mean d.b.h. When these values are added to those of the present stand, the expected volumes at age 45 become 2,296 cubic feet and 10,700 board feet per acre. This indicates a growth increment of 460 cubic feet, or 2,600 board feet in 5 years.

An indication of expected board-foot volume growth also can be obtained by using table 2. Interpolation nearly always is necessary. For the stand described in the previous example, the original volume shown in table 2 is 8,150 board feet. In 5 years this would increase to about 10,700 board feet per acre, the same as computed above.

Summary

Stand volume equations were developed for even-aged shortleaf pine stands in Missouri. The equations provide direct estimates of merchantable cubic-foot and board-foot volumes to specified top diameter limits using simple stand data. Expected volume growth also can be estimated with the equations.

Cubic-foot volume was found to be a linear function of the product of basal area per acre (B) and total height of the dominant stand (H) plus a constant. The two board-foot volume equations are based on multiple regression analysis using B and H plus the diameter of the tree of average basal area (D).

A board-foot volume table for shortleaf pine stands was prepared for various combinations of B, H, and D. By interpolating in the table as necessary the forester can obtain direct estimates of present stand volume and an indication of expected volume growth.

The equations should be satisfactory for inventory purposes in shortleaf pine stands in Missouri and probably elsewhere. Where experience shows that the equations give consistently high or low estimates of stand volume, minor adjustments can be made.

Literature Cited

Brinkman, Kenneth A., Rogers, Nelson F., and Gingrich, Samuel F.
1965. Shortleaf pine in Missouri: stand density affects yield. U.S. Forest Serv. Res. Pap. CS-14, 14 pp., illus. Cent. States Forest Exp. Sta., Columbus, Ohio.

Buckman, R. E.
1961. Development and use of three stand volume equations for Minnesota. J. Forest. 59: 573-575.

Buckman, R. E.
1962. Growth and yield of red pine in Minnesota. U.S. Forest Serv. Tech. Bull. 1272, 50 pp.

Gevorkiantz, S. R., and Olsen, L. P.
1955. Composite volume tables for timber and their application in the Lake States. U.S. Dep. Agr. Tech. Bull. 1104, 51 pp.

Vuokila, Yrjo.
1965. Functions for variable density yield tables of pine based on temporary sample plots. Communications Instituti Forrestalis Fenniae 60.4. (Metsantutkimuslaittoksen Julkaisuja 60.4)

Williams, R. A.
1959. Growth and yield of a thinned shortleaf pine plantation. Cent. States Forest Exp. Sta. Tech. Paper 169, 12 pp.

KENNETH A. BRINKMAN
Silviculturist, Columbia, Mo.[1]

[1] *Field office maintained in cooperation with the University of Missouri Agricultural Experiment Station.*