FIELD NOTE

Height–Diameter Equations for 12 Upland Species in the Missouri Ozark Highlands

James R. Lootens, David R. Larsen, and Stephen R. Shifley

ABSTRACT

We calibrated a model predicting total tree height as a function of tree diameter for nine tree species common to the Missouri Ozarks. Model coefficients were derived from nearly 10,000 observed trees. The calibrated model did a good job predicting the mean height–diameter trend for each species (pseudo-R^2 values ranged from 0.56 to 0.88), but for a given tree diameter observed tree heights were highly variable. We also present a technique for incorporating the observed variation in tree heights in the predicted values.

Keywords: height–diameter equations, Missouri, Ozark Highlands, allometric equations

Total tree heights are costly and difficult to accurately obtain. However, they provide a great deal of information about tree volume, site productivity, and stand size structure. In the absence of observed tree heights, total tree heights can be estimated by the use of a height–diameter equation. These equations predict a tree's total height based on its dbh (4.5 ft aboveground), and in many situations, they provide a valuable alternative to measuring the heights in the field. Several such models exist for different species and geographic regions, e.g., models by Monserud (1975), Ek et al. (1984), Larsen and Hann (1987), Parresol (1992), Flewelling and de Jong (1994), and Colbert et al. (2002). This article presents equations for predicting total tree height for 12 species in the southeastern Missouri Ozarks.

Methods

Data used in model development came from the Missouri Ozark Forest Ecosystem Project (Brookshire and Shifley 1997, Shifley and Brookshire 2000) and the Missouri Ecological Classification System Project (Becker 1999, Grabner 2002). Study sites were located in Missouri counties of Shannon, Reynolds, and Carter, in the Ozark Highlands Section of the Eastern Broadleaf Forest (Continental) Province (McNab and Avers 1994). Trees included in the database were from all crown classes and spanned a wide range of diameters (0.1–36.0 in.). Individual trees with signs of damage or broken tops were removed from the data set. In all cases, dbh's were measured with a diameter tape or caliper to the nearest 0.1 in. Heights were measured to the nearest foot with a telescoping height pole or a clinometer.

After preliminary examination of height–diameter models by Monserud (1975), Ek et al. (1984), Larsen and Hann (1987), Parresol (1992), and Flewelling and de Jong (1994), we settled on Monserud's (1975) model:

$$H = 4.5 + \exp(b_0 + b_1 D^{b_2}), \tag{1}$$

where H is total tree height in feet, 4.5 corresponds to breast height (ft), D is dbh (in.), and the b_i are regression coefficients. The flexible model form is easy to fit and has worked effectively for species in the Midwest and elsewhere (e.g., Larsen and Hann [1987 and Colbert et al. [2002]). This equation enforces the constraint that as D approaches zero, H approaches 4.5 ft, given b_1 and b_2 are negative.

We fit separate equations for species with at least 75 observations (Table 1). We also fit composite equations for upland oaks and hickories. Blackjack oak, chinkapin oak, and bitternut hickory were included in their respective pooled groups but did not have sufficient numbers for individual models.

We used nonlinear regression to fit models for all species and species groups. We evaluated model fit using the residual standard error, graphics, and a coefficient of multiple determination for the nonlinear regression (Kvålseth 1985). The pseudocoefficient of multiple determination is analogous to the R^2 in linear regression and is computed

$$R^2 = 1 - \frac{\sum(Y_j - \hat{Y}_j)^2}{\sum(Y_j - \bar{Y})^2}, \tag{2}$$

where $\hat{Y}_j$ is the model estimate for the jth estimate, $\bar{Y}$ is the sample mean, and Y_j is the jth observation.

Results and Discussion

Equation 1 did a good job of defining the height–diameter relationship for the Ozark species. Pseudo-R^2 values ranged from 0.56 to 0.88 (Table 2). The fitted equations closely followed the trends in the data (Figure 1) and residual analyses revealed no patterns indicating a need for remedial measures.

Oaks were the most abundant species in the dataset; however, they also had the largest residual standard errors (RSE) and the lowest pseudo-R^2. White oak had a slightly lower RSE and a higher pseudo-R^2 than the other two individual oak species, scarlet and

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Table 1. Summary of dbh and height statistics for each species group.

Species group	n	dbh (in.)				Height (ft)			
		Mean	SD	Min	Max	Mean	SD	Min	Max
Flowering dogwood (*Cornus florida* L.)	647	1.3	1.13	0.1	6.1	12	6.36	5	43
Shortleaf pine (*Pinus echinata* Mill)	990	11.2	3.72	4.0	28.9	66	14.25	21	113
Red maple (*Acer rubrum* L.)	186	0.9	0.99	0.1	6.0	12	9.23	5	61
Hickory (Composite)	279	4.0	3.67	0.1	25.6	29	20.38	5	91
Pignut hickory (*Carya glabra* [Mill.] Sweet)	91	3.3	2.88	0.1	16.1	26	16.40	5	76
Black hickory (*Carya texana* Buckl.)	88	3.7	4.20	0.1	25.6	25	16.95	5	65
Mockernut hickory (*Carya tomentosa* Nutt.)	81	3.1	2.74	0.1	14.7	23	16.50	5	80
Bitternut hickory[a] (*Carya cordiformis* [Wangenh.] K. Koch)	19	8.7	2.06	5.8	13.1	65	6.93	57	78
Upland oak (Composite)	7,758	10.2	4.42	0.1	35.8	64	17.51	5	128
White oak (*Quercus alba* L.)	2,881	8.6	4.31	0.1	35.8	56	17.50	5	109
Scarlet oak (*Quercus coccinea* Muenchh.)	2,516	11.0	4.19	0.1	33.0	70	16.32	5	116
Black oak (*Quercus velutina* Lam.)	2,288	11.5	4.18	0.1	27.7	68	14.21	5	114
Blackjack oak[a] (*Quercus marilandica* Muenchh.)	28	9.2	2.79	5.3	15.7	43	8.93	26	62
Chinkapin oak[a] (*Quercus muehlenbergii* Engelm.)	45	6.9	1.86	4.6	12.0	448	12.38	21	67

[a] Too few observations for a separate species-specific equation. Use composite equation.
Max, maximum; Min, minimum.

Table 2. Coefficients for the fitted Equation 1 to predict height (ft) from dbh (in.) for each species group.

Species or group	n	b_0	b_1	b_2	RSE	R^2
Flowering dogwood	647	2.9876	−1.0111	−1.2462	3.2	0.75
Shortleaf pine	990	4.6189	−5.9256	−1.0645	8.9	0.62
Red maple	186	8.0535	−5.9141	−0.2000[a]	3.2	0.88
Pignut hickory	91	4.3756	−3.0539	−0.8358	6.2	0.86
Black hickory	88	4.2136	−2.8050	−0.8743	4.8	0.92
Mockernut hickory	81	4.5333	−3.2153	−0.7138	6.6	0.84
Hickory	279	4.5456	−3.3358	−0.7915	7.1	0.88
White oak	2,881	4.5024	−5.0009	−1.0845	8.1	0.75
Scarlet oak	2,516	4.5004	−9.1643	−1.4756	8.9	0.66
Black oak	2,288	4.3702	−13.0002	−1.8022	9.4	0.56
Upland oak	7,758	4.5409	−6.7095	−1.2405	9.5	0.70

[a] The b_2 was constrained at −0.2 for red maple as Equation 1 failed to converge.
RSE is the residual standard error (ft) and the r^2 is a puedo-R^2 as described in the text.

black oak. Less accurate prediction in these species may be related to the wide variety of sites on which these species exist. Better fit statistics for white oak may be related to white oak being more of a site generalist—being able to thrive on a wide variety of sites. The pooled oak group, including all of the aforementioned oaks, as well as chinkapin and blackjack oak, had a higher RSE and lower pseudo-R^2 than the white and scarlet oak species, but it may provide a useful tool for determining height for aggregated groups of oaks.

Fit statistics for the hickories are much better than the oak species. However, with many fewer observations than the oaks, caution should be taken when comparing the fits of these two species groups. Pseudo-R^2 values for the hickories are all relatively high, over 0.84 for each hickory species and for the aggregate hickory group, including all listed hickories and bitternut.

Fit statistics for shortleaf pine, the only conifer in the group, were similar to the oak species listed, with an RSE of 8.9 ft and a pseudo-R^2 of 0.62. Flowering dogwood had the smallest RSE, 3.2 ft. However, this is caused by the fact that dogwood also has the smallest range of diameters and heights. Flowering dogwood does not typically reach the canopy in mature Missouri Ozark forests. The nonlinear regression of red maple failed to converge for Equation 1 because coefficient b_1 and b_2 were highly correlated. When b_2 was constrained to −0.2 for red maple, the remaining parameters were readily obtained, and the resulting model had a pseudo-R^2 of 0.88 and an RSE of 3.2 ft. For all species, the fitted model coefficients are similar in sign and magnitude to those developed for other species and geographical regions (Larsen and Hann 1987, Colbert et al. 2002).

Although the model does a good job of predicting the mean height–diameter trend for each species, tree heights for a given diameter were highly variable in the data. For example, white oak trees with diameters of approximately 20 in. ranged from about 50 to 90 ft in height. The model, which predicts the mean tree height (e.g., 80 ft for a 20-in. white oak), tends to obscure this variability. In situations where it is desirable to retain the variation in estimated heights, a random component based on the root mean square error can be added to the model estimate so estimates vary within the 95% confidence bounds:

$$H \pm 1.96\ (\text{RSE}). \tag{3}$$

Generally, it is advisable to apply these equations only over the range of the observed diameters (Table 1). With the exception of red maple, all species approach reasonable maximum tree heights for diameters greater than 30 in. At large diameters the oak tree heights level out at 80–95 ft, the hickories reach 75–90 ft, the shortleaf pine reaches nearly 100 ft, and dogwood tops out at 24 ft. The equation for red maple is based only on trees less than 6 in. in diameter, and when extrapolated to larger diameters modeled red maple heights are unreasonably large (Figure 1). Although in other mesic and hydric ecosystems red maples can reach much larger diameters and heights, in these upland Ozark ecosystems red maples typically remain small trees that are restricted in size by droughty conditions.

Conclusions

The height–diameter equations presented here can be used to obtain height estimates for trees for which height was not measured.

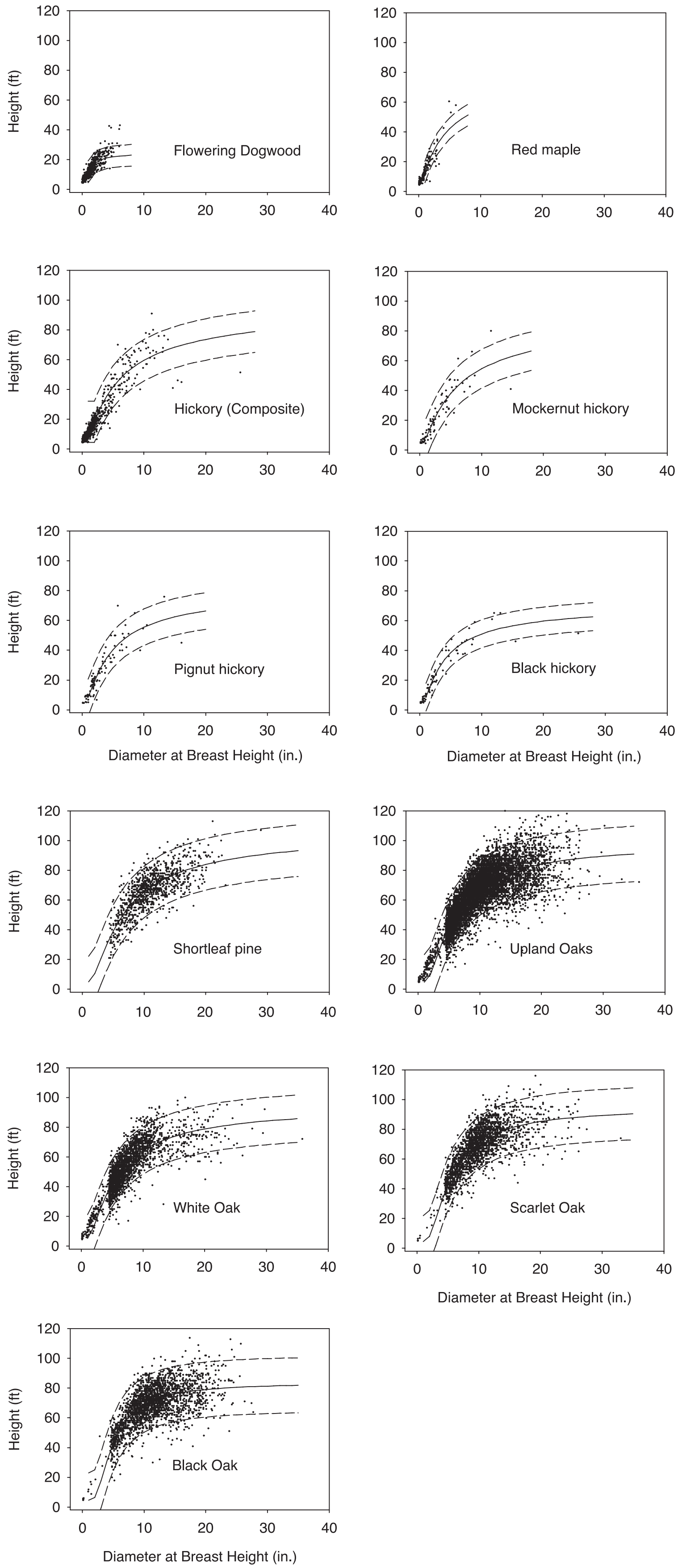

Figure 1. Height–diameter curves. The prediction equations (solid lines) and the 95% confidence limits (dashed lines) are plotted over the observed data.

The equations produced estimates that are consistent with biological expectations for each species group and well constrained for trees with very small and very large diameters. The fitted model is suitable for application within the range of the fitted data for each species and it produces reasonable estimates when extrapolated to various diameters. The model is relatively easy to apply to a wide variety of inventory, modeling, projection, silvicultural, and wildlife settings. With as little information as tree species and dbh, heights can be estimated to provide additional information to practitioners on tree volume and stand characteristics such as vertical structure.

Literature Cited

BECKER, C. 1999. *Multifactor classification of intermittent creeks in southeast Missouri Ozarks.* MS thesis, Univ. of Missouri–Columbia, Columbia, MO. 186 p.

BROOKSHIRE, B.L., AND S.R. SHIFLEY. 1997. An experimental approach to landscape research. In *Proc. of the Missouri Ozark Forest Ecosystem Project Symp.*, St. Louis, MO, June 3–5, 1997. USDA For. Serv. Gen. Tech. Rep. NC-193. 378 p.

COLBERT, K.C., D.R LARSEN, AND J.R. LOOTENS. 2002. Height-diameter equations for thirteen midwestern bottomland hardwood species. *North. J. Appl. For.* 19(4):171–176.

EK, A.R., E.T. BIRDSALL, AND R.J. SPEARS. 1984. *A simple model for estimating total and merchantable tree heights.* USDA For. Serv. Res. Note NC-309. 5 p.

FLEWELLING, J.R., AND R. DE JONG. 1994. Considerations in simultaneous curve fitting for repeated height-diameter measurements. *Can. J. For. Res.* 24: 1408–1414.

GRABNER, J.K. 2002. *Patterns in upland forest vegetation in relation to geology, topography, and soils: An approach to ecological land classification in the southeast Missouri Ozarks.* MS thesis, Univ. of Missouri–Columbia, Columbia, MO. 137 p.

KVÅLSETH, T.O. 1985. Cautionary note about R^2. *Am. Stat.* 39(4):279–285.

LARSEN, D.R., AND D.W. HANN. 1987. *Height-diameter equations for seventeen tree species in southwest Oregon.* Res. Pap. 49, Forest Research Laboratory, Oregon State Univ., Corvalis, OR. 16 p.

MCNAB, W.H., AND P.E. AVERS (COMPS.). 1994. *Ecological subregions of the United States: Section descriptions.* USDA For. Serv. Administrative Publ. WO-WSA-5, Washington, DC. 267 p.

MONSERUD, R. 1975. *Methodology for simulating Wisconsin northern hardwood stand dynamics.* PhD dissertation (Univ. Microfilm No. 7602496), Univ. of Wisconsin, Madison, WI. 156 p.

PARRESOL, B.R. 1992. Baldcypress height-diameter equations and their prediction confidence intervals. *Can. J. For. Res.* 22:1429–1434.

SHIFLEY, S.R., AND B.L. BROOKSHIRE. 2000. *Missouri Ozark Forest Ecosystem Project: Site history, soils, landforms, woody and herbaceous vegetation, down wood, and inventory methods for the landscape experiment*. USDA For. Serv. Gen. Tech. Rep. NC-208. 314 p.