AN INDIVIDUAL-TREE DBH-TOTAL HEIGHT MODEL WITH RANDOM PLOT EFFECTS FOR SHORTLEAF PINE

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EXTENDED ABSTRACT

Individual tree measurements were available from over 200 permanent plots established during 1985-1987 and later remeasured in naturally regenerated stands of shortleaf pine (*Pinus echinata* Mill.) in western Arkansas and eastern Oklahoma. The objective of this study was to model shortleaf pine growth in natural stands for the region. As a major component of the shortleaf modeling effort, an individual-tree diameter at breast height (DBH)-total height model was developed in which random parameters were estimated for plots. The model predicts total tree height based on DBH and dominant stand height (which could be obtained from a site index model). The mixed-effects model approach was found to predict the total height better than similar models developed previously for this species. Moreover, such a model has the appeal of generalization of the results over a region from which the plots were sampled; and also of calibration of parameters for stands with minimal measurements.

The following DBH-total height model was developed by Lynch and Murphy (1995) and provides a starting point for development of a random effects model:

$$(H_i - 4.5) = b_0(H_D - 4.5)^{b_1} \exp(-b_2 D_i^{-b_3}) \qquad (1)$$

where H_i is the total height of tree i in feet; H_D is the dominant height of the stand in feet; D_i is the DBH of tree i in inches; and b_0, b_1, and b_3 are parameter estimates which Lynch and Murphy (1995) obtained using generalized least squares. For predictive applications, H_D can be obtained using site index relationships for shortleaf pine developed by Graney and Burkhart (1973). This equation is used to obtain individual tree total heights in the Shortleaf Pine Stand Simulator, an individual-tree growth simulator for even-aged shortleaf pine stands (Lynch et al. 1999).

The parameters in equation 1 were estimated using the first two measurements on shortleaf pine trees in the shortleaf pine growth study discussed above. The availability of a third measurement of the permanently established plots in this study provided the opportunity to apply mixed-model or random parameter methods of estimation to develop an improved DBH-total height model. A random parameter model is able to represent the situation in which the parameters in Equation 1 may change in different forest stands. When a stand is selected at random from the total population of stands, the parameters representing this change can be termed random parameters. Lappi (1997) provided an early application of mixed-model estimation with random parameters for tree height models. Budhathoki's (2006) analysis of these shortleaf pine data indicated that there was a significant random component associated with the b_2 parameter estimate in equation 1. This finding leads to the following equation:

$$(H_i - 4.5) = b_0(H_D - 4.5)^{b_1} \exp(-(a_{2j} + b_2) D_i^{-b_3}) \qquad (2)$$

where a_{2j} is a plot-level random effect associated with plot j and fixed-effects parameter estimates are b_0, b_1, and b_2. Conceptually, fixed-effects parameters remain the same for all forest stands while random-effects parameters vary by stand; in our data, stands are represented by plots. Mixed-effects estimation techniques can be used to estimate fixed and random effects simultaneously. SAS PROC NLMIXED (SAS 2000-2006) was used to estimate parameters in equation 2 for our data. Statistics of fit including the Akaike Information Criterion (AIC), the Bayesian Information Criterion (BIC), and the log-likelihood indicated that inclusion of a random parameter associated with b_2 improves the fit of the model to our DBH-total height data for shortleaf pine natural stands. Additionally, a variety of residual plots from the model indicated that the model performs well for these data since no pronounced biases or trends were evident.

Mixed-effects modeling improved the performance of a DBH-total height model for naturally-occurring shortleaf pine forests by including a random parameter that reflects changes in the equation for different forest stands (represented by plots). This approach also provides the opportunity to calibrate the model for a particular stand by collecting a small amount of data used to estimate the random parameter associated with that forest stand. Lappi (1991) indicated how random parameter models can be used to calibrate height and volume equations. Recently Lynch et al. (2005) fitted a random parameter DBH-height model for cherrybark oak (*Quercus pagoda* Raf.) and demonstrated the procedure used to calibrate the model for particular forest stands of interest.

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