

STAND DENSITY, STAND STRUCTURE, AND SPECIES COMPOSITION IN TRANSITION OAK STANDS OF NORTHWESTERN PENNSYLVANIA

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ABSTRACT: Transition stands, those containing species associated with both the northern hardwood and oak-hickory forest types, are important to forest diversity in northwestern Pennsylvania. These stands have high value for a variety of forest uses, including timber production, wildlife habitat, and aesthetics. Diameter distributions are characteristically stratified by species, with the most valuable oak species among the largest trees in the stand. Understories typically consist of a mixture of northern hardwood species with those often found in the understory of true oak-hickory stands, such as dogwood, blackgum, and cucumbertree. In this paper, I characterize diameter distributions for transition stands in northwestern Pennsylvania, and discuss their silvicultural implications. Traditional measures of density tend to overestimate the density of such stands. Three alternatives, one based on tree-area ratio and two based on Reineke's stand-density index, are reported.

INTRODUCTION

The forests of the Allegheny Plateau region of northwestern Pennsylvania are dominated by Allegheny hardwood and northern hardwood forest types. Thus stands of the oak-hickory or oak transition forest types are important for diversity to the wood-using industry, to many wildlife species, and to recreationists and conservationists. Stands of the oak transition forest type represent a challenge to managers because they contain a mix of species somewhat different from the classic central hardwood stand. Often, the oak species are among the largest trees in the stand, dominating one or more subordinate canopy layers composed of a mix of traditional oak associates, such as red maple (*Acer rubrum* L.), dogwood (*Cornus florida* L.), blackgum (*Nyssa sylvatica* L.), and hemlock (*Tsuga canadensis* (L.) Carr), and species less often associated with the oaks, such as sugar maple (*Acer saccharum* Marsh.) and American beech (*Fagus grandifolia* Ehrh.). These species cover a wide range of growth rates and tolerance to shade, and, in mixture with the oaks, achieve levels of density (numbers of trees or basal area per acre) not usually associated with the oaks. Because these stands are highly stratified by species and the oak species are difficult to regenerate, the consequences of management actions in these stands are quite important.

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The United States Forest Service estimates that 20 percent of the land area of the Allegheny National Forest, or about 100,000 acres, is covered by stands in which oak is an important component (Allegheny National Forest 1986). When stands that had experienced severe mortality as a result of the combination of gypsy moth defoliation and drought were sampled in the late summer and early fall of 1988, approximately one-third of the area sampled was typed as oak transition, while the remainder was typed as oak-hickory (Allegheny National Forest 1989). These determinations were made by analyzing the sample data with the SILVAH computer program.² In SILVAH, oak-hickory stands are defined as those having at least 50 percent of their basal area in oak and hickory species and at least 65 percent of their basal area in oak, hickory, or commonly associated species. Transition hardwoods are those in which at least 25 percent of the basal area is in each of oak and northern hardwood species, and at least 65 percent of the basal area is in these species plus common associates of either. In these stands, it is not uncommon for northern red oak (*Quercus rubra* L.) to be the sole oak species.

The origin of stands of the oak-hickory and oak transition types on the Allegheny Plateau is not as well documented as is the origin of stands of the Allegheny hardwood forest type. The northwestern Pennsylvania Allegheny Plateau region was virtually completely harvested during turn-of-the century logging, resulting in an even-aged forest. Records from logging companies active during the railroad logging era from 1890 through about 1930 show that most of the stands included in this paper probably originated from a sequence of cuttings. The first was for hemlock and desirable hardwood sawtimber during the period from 1894-1912, and the second for smaller and poorer quality trees between 1913-1930 (Casler 1976). This combination of cuttings was usually quite complete, and similar to a shelterwood sequence. Thus these stands are essentially even-aged and about 70 years old.

In the Allegheny Plateau region, oak-hickory and oak transition stands tend to occur near the river basins, where humans have been in residence, and using fire, for many hundreds of years. The influence of humans and fire is believed to have favored the establishment of oak along waterways, due to the tolerance of oak seedlings for fire. Many of these stands still contain old chestnut (*Castanea dentata* (Marsh.) Borkh.) stumps, suggesting that the chestnut blight may have played a role in releasing oak to greater dominance. Transition stands often are on high-quality sites, where the regeneration of oak species suffers from the competition of fast-growing associates like red maple and black cherry (*Prunus serotina* Ehrh.).

In this paper, I report on the diameter distribution and species composition of 21 research plots representative of the mixed oak-northern hardwood stands of northwestern Pennsylvania, and on the changes that occur as a result of various silvicultural strategies. I also report on efforts to assess relative stand density in these stands.

² Marquis, David A., and Richard L. Ernst. in preparation. User's guide to SILVAH: A stand analysis, prescription, and management simulator program for hardwood stands of the Alleghenies.

METHODS

Twenty-one research plots currently under study by staff of the Northeastern Forest Experiment Station's Forestry Sciences Laboratory at Warren in northwestern Pennsylvania were selected for this study on the basis of species composition. These plots were used in a variety of studies focusing on the management of developing stands for timber and aesthetic objectives, but the data reported in this paper are from pretreatment tallies. Plot size ranged from 0.6 to 4.9 acres. Data collected in each stand included a complete tally of all trees 1.0 inches d.b.h. and larger on each plot by species and 1-inch diameter class. All plots had been free of human disturbance but many showed evidence of natural disturbance, especially chestnut stumps. Northern red oak is the principal oak species on these plots; white oak (*Quercus alba* L.) is the other major oak species. All stands met the criteria for stands at average maximum density (Ernst and Knapp 1985), as they showed no evidence of past cutting, little understory development, and mortality was readily observed among the smallest trees.

Data from each plot were summarized into per acre values of numbers of trees, basal area, and sum of diameters and diameters squared. For each plot, both the quadratic mean diameter (QMD, or diameter of the tree of average basal area) and the medial diameter (MD, or diameter of the tree at the mid-point of the basal area distribution), were also calculated. MD is calculated as a weighted average diameter. Each 1-inch diameter is multiplied by the basal area in that diameter class, and the sum divided by the stand basal area. Composite diameter distributions were calculated by averaging per acre values of numbers of trees per diameter class. Statistical analyses were conducted with SYSTAT statistical analysis software (Wilkinson 1988), and a 5% significance level was accepted in all tests.

DIAMETER DISTRIBUTIONS

The characteristics of the individual stands are summarized in Table 1. MD in these stands ranged from 10.3 to 17.0 inches, and the percent of basal area in oak species ranged from 25 to 75.

The frequency distribution of diameters for all of these stands has a roughly inverse J shape. Figure 1 shows the averaged frequency distributions of diameter for each of three groups of these stands, using medial diameter as the grouping variable. Figure 1d is the frequency distribution of diameter from Schnur's (1937) representative stand table for an age 60 oak stand on a site index 60 site, shown here for contrast. Frequently, the inverse J-shaped diameter distribution is associated with uneven-aged stands, but in the even-aged forests of the Allegheny plateau region, many stands have inverse J-shaped diameter distributions because of the persistence of a dense lower canopy layer of shade-tolerant species. In species composition and diameter distribution, the sapling class in these transition oak stands is similar to the shade tolerant understory layer in even-aged Allegheny and northern hardwood stands.

Table 1. Selected characteristics of sample Pennsylvania oak transition stands.

	Minimum	Maximum	Mean +/- Std. Dev.
No. trees/acre	265	920	449 +/- 163
Basal area/acre (ft. ²)	122.3	200.4	154.0 +/- 18.7
Medial Diameter	10.3	17.0	13.5 +/- 1.8
Quadratic Mean Diameter	5.4	11.0	8.2 +/- 1.4
% northern red oak (in basal area)	0	75	40 +/- 18
% oaks (in basal area)	25	75	48 +/- 14

In the smallest diameter stands (Fig. 1a), 46 percent of the sapling (one-inch through five-inch diameter classes) basal area consists of American beech trees, 40 percent of red maple trees, and the remainder consists of birch (*Betula* L.), cucumbertrees (*Magnolia acuminata* L.), sugar maple, and noncommercial species. Average sapling basal area is 17 square feet. The smallest oaks in these stands are 7 inches in diameter, but the pole class (6-inch through 11-inch) is dominated by red maple, which averages 48 percent of the 53 square feet of poletimber. No other species represents more than 11 percent of the poletimber basal area. Among the sawtimber classes (those 12 inches and larger), the oaks dominate, with northern red oak representing 64 percent of the sawtimber basal area.

The pattern for the other two groups is similar. In the medium-diameter stands (Fig. 1b), beech and red maple dominate the sapling class, red maple dominates the pole-size class, and northern red oak dominates the sawtimber-size classes. In the largest diameter stands (Fig. 1c), eastern hemlock dominates the sapling class. Red maple and eastern hemlock are important in the pole-size classes, and northern red oak, red maple, and white oak dominate the sawtimber sizes.

If we look closely at these inverse J curves, we can see them as the curves of three species groups superimposed. The tail of the curve is formed by the inverse J shape of the diameter distribution of shade-tolerant species, while the red maple/intermediate-tolerance species and oak groups each have bellshaped distributions with different means.

If we contrast these patterns with the diameter distribution of Schnur's classic oak stand (Fig. 1d), we see that the overall form as well as the distribution by species is different. This frequency distribution of diameters has a bell shape rather than an inverse J shape, and the distribution for each species group is bellshaped as well. In particular, the shade-tolerant understory species are not as numerous as in northwestern Pennsylvania oak stands, nor does their distribution in the small size classes have an inverse J shape.

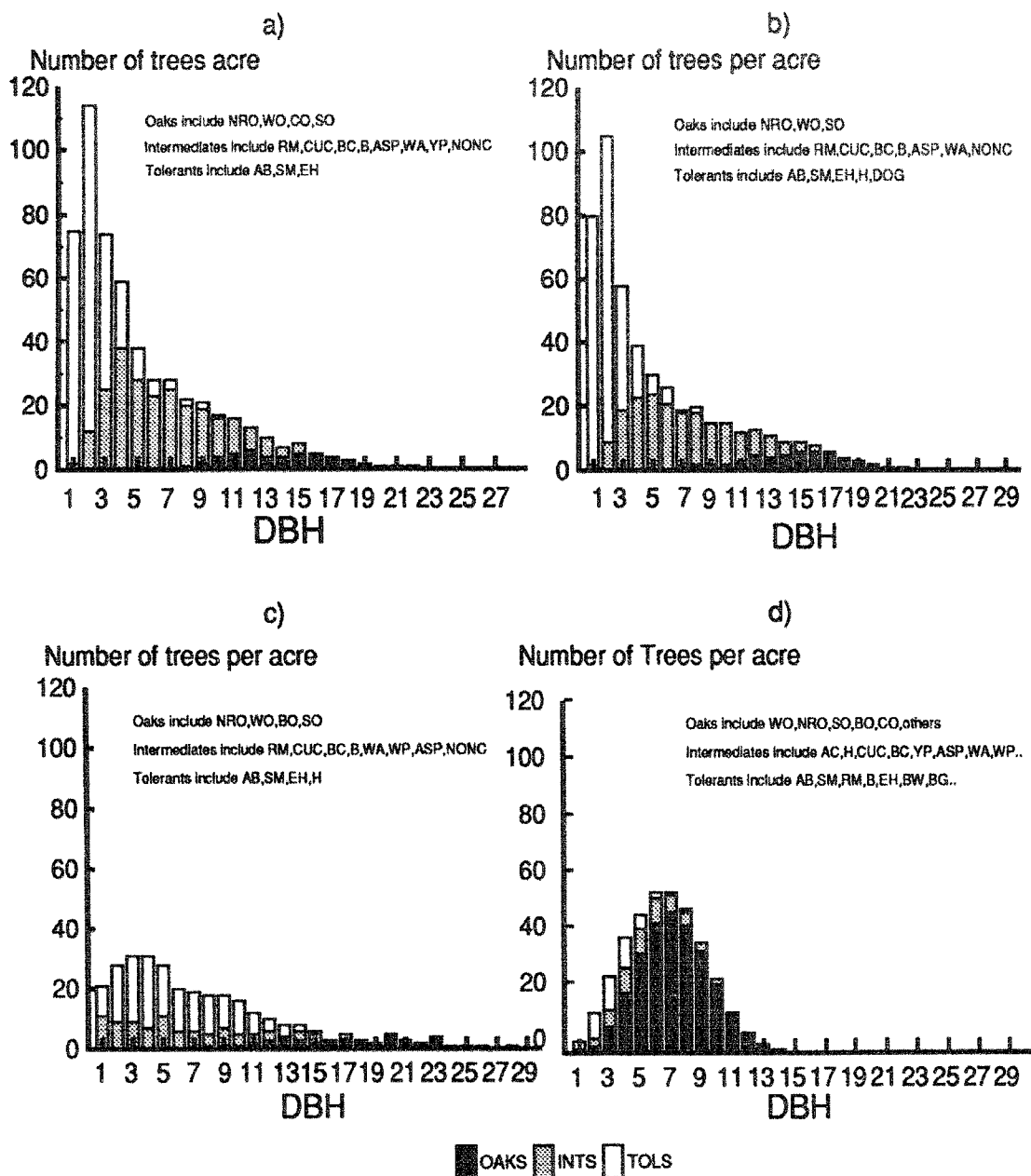


Figure 1. Diameter distributions of composites of the study plots, grouped by medial diameter, and showing species composition. a) Medial diameters from 10. - 12.5 inches. b) Medial diameters from 12.5 - 14.5 inches. c) Medial diameters from 14.5 - 17.0 inches. d) Schnur's (1937) SI 60, age 60 oak stand.

SILVICULTURAL IMPLICATIONS

In the face of serious difficulties in obtaining natural regeneration of oak species, the stratification by species described represents a serious challenge to forest managers who wish to preserve oak. Partial cuttings are desirable to increase the rate of growth of the best trees for the management objective. For wood products, thinnings provide faster diameter growth of the residual trees, capture some potential mortality, and shorten rotation lengths. For wildlife-habitat management, partial cuttings stimulate the growth of tree crowns and increase mast production. But to achieve multiresource objectives in these stands, managers must remain sensitive to a variety of special concerns. Guidelines to minimize gypsy moth impacts (Gansner and others 1987) are important, as is an understanding of the likely impact of common silvicultural practices on species composition. Figure 2 shows residual diameter frequency distributions of the composite, medium diameter class stand shown in Figure 1b after 2 common partial cuts.

Uneven-age management practices like single-tree selection have been shown to favor regeneration of species more shade tolerant than the oaks (Schlesinger 1976). In northwestern Pennsylvania oak stands, however, application of uneven-age practices may reduce the proportion of oak by the cutting alone. The theory of uneven-age silvicultural systems is the creation and preservation of a balanced inverse J diameter distribution in which the largest diameter trees are the oldest and the smaller diameter trees are the youngest. The inverse J shape of diameter distributions in northwestern Pennsylvania oak stands and the shade tolerance of the northern hardwood species in the smaller age classes will ease the transition to uneven-age management in these even-aged stands. At each cutting cycle, the oldest or largest trees are removed, the younger or smaller trees are thinned, and space is created for regeneration. If density is reduced to the levels recommended to obtain a good growth response in the residual trees (Marquis and others 1984, Roach and Gingrich 1968),³ and no cutting is done in the sapling class, the effect will be to reduce the proportion of oak. The higher the minimum cutting diameter employed, the more pronounced this effect will be. But even if cutting is uniform throughout the retained diameter classes, removing trees from the large end of the diameter distribution, above the selected maximum diameter, will reduce the proportion of oaks over time. Figure 2a shows the species composition of the residual stand after hypothetical selection cutting of the stand in Figure 1b to a Q of 1.5 and a residual density 65 percent that of the original stand, with a maximum retained tree size of 23 inches. No trees smaller than 5.5 inches d.b.h. were cut. The cutting removed 55 square feet of basal area in 73 trees per acre. In this stand, with an original maximum tree diameter of 23 inches, the proportion of oak was unchanged at 47 percent. Little or no replacement of the oaks can be expected to occur under this silvicultural system.

Management guidelines for even-age silviculture in many eastern hardwood stands recommend intermediate thinnings that concentrate removals in the smaller merchantable

³ Stout, Susan L. in preparation. Uneven-age management. In: Quantitative silviculture for hardwood stands of the Alleghenies. Available from the author.

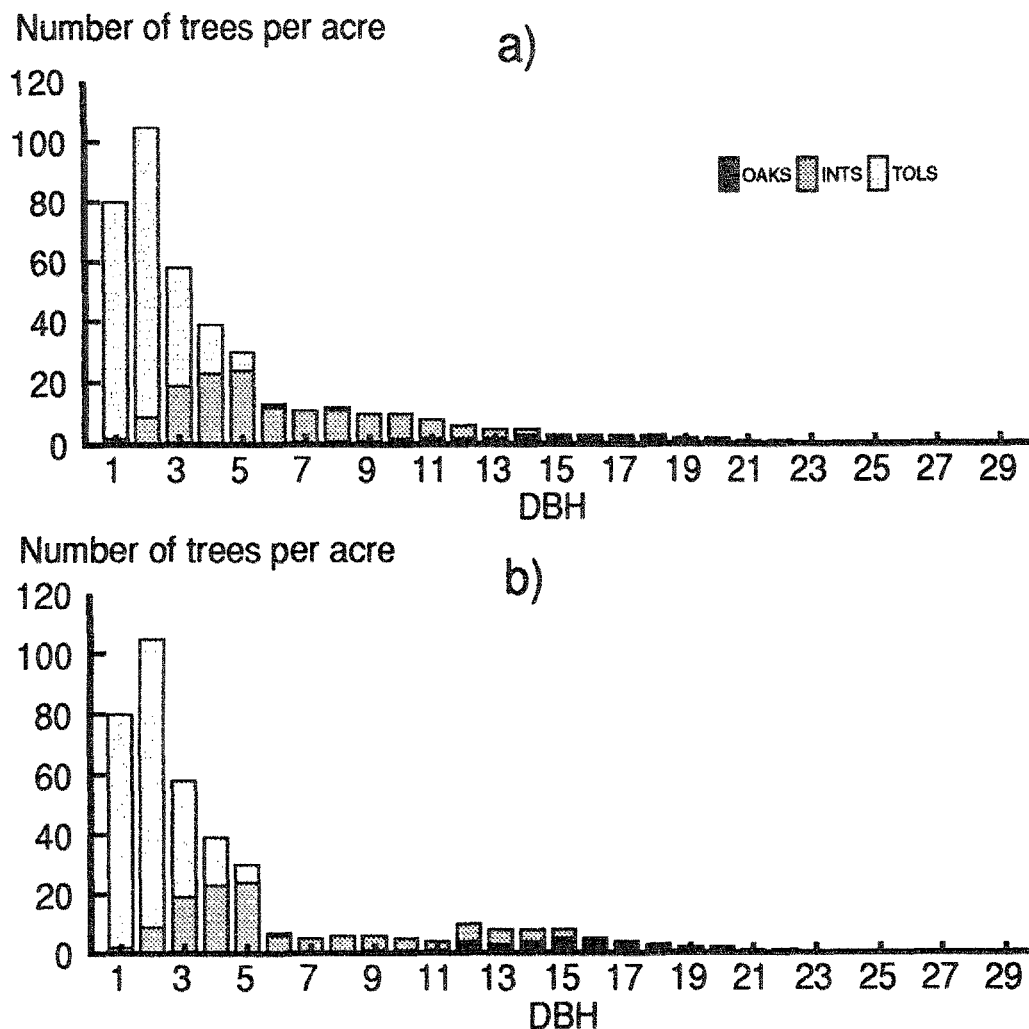


Figure 2. Changes in diameter distribution in the composite stand from Figure 1b. a) after a selection system partial cut. b) after an even-age commercial thinning.

stems, with only enough removals in the largest diameter and tallest trees to free some room in the main canopy (Marquis and others 1984, Roach and Gingrich 1968). Because such treatments concentrate removals in the size classes dominated by non-oak species, they tend to increase the proportion of oak (Figure 2b). In the composite stand shown in Figure 2, the commercial thinning to 65 percent of the original density, primarily from below, increased the proportion of oak from 47 percent to 54 percent of the basal area. This cutting removed 51 square feet in 92 trees per acre. Control of density in these treatments may be difficult,

however, as none of the currently available measures of relative density gives desirable results in northwestern Pennsylvania transition oak stands.

ASSESSING RELATIVE DENSITY

Measures of relative stand density allow foresters to assess the crowding in forest stands as a function of average tree size and, in some cases, species composition. Relative density is expressed as a percent of the average maximum density observed in undisturbed stands of similar average tree size and species composition (Ernst and Knapp 1985). In theory, most undisturbed stands should have relative density values close to 100 percent; in years of optimum growing conditions, tree growth and survival may be high, and relative-density estimates may exceed 100 percent by small amounts, while episodes of poor growing conditions will result in mortality and lower relative-density estimates.

Correct assessment of relative density is key to understanding and managing mixed-species stands. In stands of the same average tree size, but different species composition, absolute measures of density, such as numbers of trees or basal area per acre, vary with species composition, sometimes by as much as 50 percent. Basal-area values that are unrealistically high in one species mix may be average in another. Managers who underestimate the relative density of these complex stands may miss opportunities to increase individual tree growth and crown development, while managers who overestimate relative density risk loss of productivity or encouragement of undesirable plants. Yet assessment of relative density can be particularly difficult in stratified mixed stands. The stands included in this study were specifically chosen to represent the conditions of average maximum density. Thus, one criteria for choosing a density measure for use in these stands is the nearness of the mean estimate of density to 100 percent, as well as the proportion of these stands whose density is estimated as close to 100 percent. Table 2 shows the density of these northwestern Pennsylvania oak stands as assessed by five different measures.

The oak stocking chart in Roach and Gingrich (1968) expressed average maximum density as a curve that showed the average maximum basal area and numbers of trees per acre in upland central hardwood stands as their QMD increased. This chart was extended and used to determine percent oak stocking shown in the first row of Table 2. The oak stocking chart was developed partly with the stand tables in Schnur (1937), and was intended for use in these typical, white oak dominated stands with bellshaped diameter distributions. When applied to northwestern Pennsylvania stands such as those studied here, the charts yield estimates of density well above 100 percent. The mean value is 140 percent, and the estimates range from 111 to 169 percent. Densities this high suggest stagnation and severe mortality among the smallest trees in these stands, which is inconsistent with observation in these stands.

Table 2.--Relative density of study plots as calculated by five density measures.

	Minimum	Maximum	Mean +/- Std. Dev.
Oak Stocking Chart ¹	111	169	140 +/- 14
SILVAH Relative Density ²	85	143	105 +/- 14
NE-TWIGS Generalized Stocking Percent ³	60	97	83 +/- 8
PA Oak tree-area ratio ⁴	79	125	105 +/- 11
PA Oak Stand Density Index ⁴	74	121	96 +/- 11

¹ Roach and Gingrich (1968)

² Marquis, David A., and Richard L. Ernst. in preparation. User's guide to SILVAH: a stand analysis, prescription, and management simulator program for hardwood stands of the Alleghenies. Available from the author.

³ Gribko and others (in press).

⁴ Determined using measures developed for this study.

The SILVAH computer programs calculate the density of forest stands using a variant of the measure developed by Roach (1977) and Stout and Nyland (1987). With this procedure, the density contribution of individual trees is estimated as a function of their species and diameter. Species are assigned to one of three groups, based predominantly on their growth rates and tolerance to shade. These include a fast-growing, shade-intolerant group consisting of black cherry, white ash (*Fraxinus americana* L.), and yellow-poplar (*Liriodendron tulipifera* L.); a slow-growing, shade-tolerant group consisting of sugar maple, American beech, and striped maple (*Acer pennsylvanicum* L.); and an intermediate group whose main constituent was red maple, but to which most species were assigned for application purposes. Northwestern Pennsylvania transition hardwood stands were not in the data base used to develop the tree-area coefficients for Allegheny hardwoods. In the SILVAH programs, the density coefficients for the intermediate group are used for northern red oak. The coefficients for the slow-growing shade-tolerant group were so similar to those reported by Gingrich (1968) for the white oaks and hickory that these species were assigned to that group. The densities reported in the second row of Table 2 are the result of calculations made with the SILVAH computer programs. The mean value of relative density calculated by SILVAH is 105 percent, only slightly higher than the value we would prefer in unmanaged stands. But the range of values observed in this small group of stands, from 79 to 125 percent, suggests that this measure does not discriminate well between understocked, fully stocked, and overstocked stands.

A different approach to estimating density in mixed stands was taken by the developers of NE-TWIGS, the northeastern regional variant of the TWIGS forest growth projection system (Teck 1990). NE-TWIGS calculates a generalized stocking percent (GSP) for all stands. This measure is a weighted average relative stand density, calculated using Stand Density Index (Reineke 1933) values for seven different forest types. GSP is influenced by the species composition of the stand, and is a summation of weighted relative stand densities, where each species can only represent a single forest type (personal communication, Richard

Teck). The NE-TWIGS GSP values for these stands are reported in the fourth row of Table 2. The mean value of relative density calculated by NE-TWIGS is 83%, quite low for undisturbed stands. But the range of estimates is the narrowest of any of the measures reported, from 60 to 97 percent.

The values from the 21 undisturbed stands studied here were used to estimate the coefficients for two new measures of relative density for undisturbed stands. The fifth and sixth rows of Table 2 show the density of these stands by two measures developed during this study. Column 5 shows the results of applying a tree-area ratio measure developed from these data. The mean value is close to 105 percent. The range of values calculated for these stands is from 79 percent to 125 percent.

The tree-area ratio equation calculated for these transition stands is:

$$\text{Tree area} = -0.0068718 * N + 0.0167869 * \sum_{i=1}^N D_i + 0.0019797 * \sum_{i=1}^N D_i^2 \quad (1)$$

where N is number of trees per acre and D_i is the diameter of the i th tree.

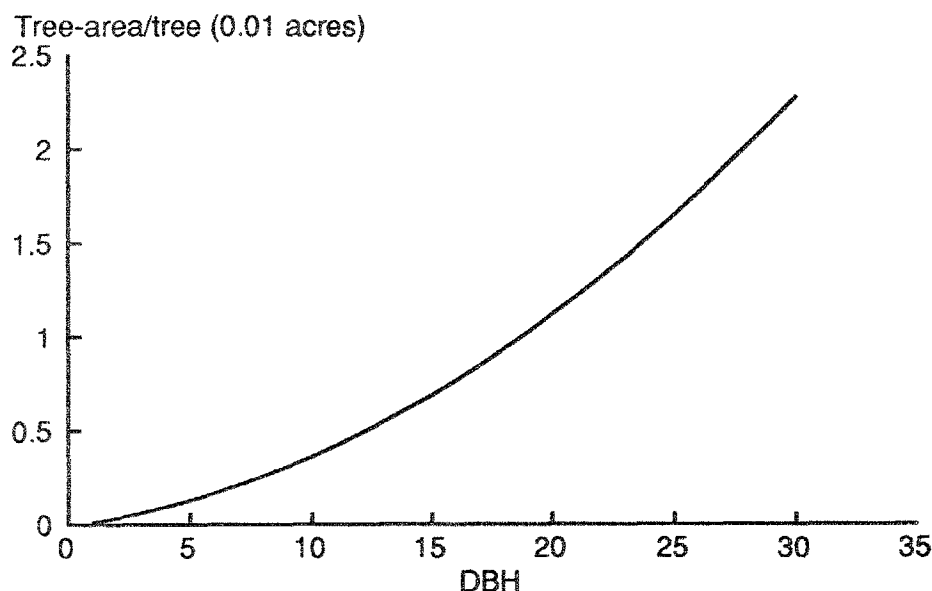


Figure 3. Tree area per tree (in centacres) as estimated by the tree-area ratio equation developed for this data set.

This tree-area curve is shown in Figure 3 as it would be used to estimate the density contribution of an individual tree. With 21 stands in the data used to estimate the coefficients for this equation, separation into species groups did not significantly improve the fit of the equation.

Row 6 of Table 2 contains an estimate of stand density based on the relationship between the number of trees and the quadratic stand diameter, as suggested by Reineke (1933). For the 21 undisturbed stands studied here:

$$\log_{10} N = 4.393 - 1.962 * \log_{10} QMD \quad (2)$$

where N is number of trees per acre and QMD is the quadratic mean diameter.

This equation has an r^2 of .88. The values in row 6 are ratios of the number of trees per acre in each of these stands to the number predicted by equation (2), expressed as a percent. This measure has a mean value of 96 percent in these stands, with a range from 74 percent to 121 percent.

DISCUSSION

The research reported here is part of a larger effort to develop regional tools to assess relative density in mixed hardwood stands. Managers must be able to discriminate between stands whose density is close to the average maximum for a type and those whose density is well below the average maximum, regardless of their management objective. Such discrimination is key to identifying opportunities and projecting changes (Stout and Larson 1987). The work reported here highlights the difficulties of assessing density in complex, stratified mixtures. The structure of these stands, with bellshaped oak distributions in the larger diameters and inverse J distributions of shade-tolerant northern hardwoods in the smaller diameters, contributes to their consistently overstocked ratings by traditional measures.

As our understanding of the interactions of these structures with site, management, and stand development improves, we will develop new and more efficient methods of assessing density in such stands. Differences among density measures reported here may help managers choose which of the existing measures of density they prefer, as well as point directions for future research. The new measures developed using this data provide the mean relative-density estimates closest to 100 percent for these 21 undisturbed stands, but their ranges are quite large, and they are untested outside this data set. The NE-TWIGS measure has the narrowest range, but underestimates the density of these stands, heightening the risk of missed treatment opportunities. Estimates made using the oak stocking guide or SILVAH tend to be high and have wide ranges, but may be acceptable as they minimize the risk of missed treatment opportunities.

CONCLUSIONS

Oak stands in northwestern Pennsylvania are usually stratified mixtures of central and northern hardwood species. In many such stands, northern red oaks are the largest trees in the stand, with red maple, the other oak species, cucumbertree, eastern hemlock, birch, and others in the middle of the diameter distribution, and American beech, sugar maple, and eastern hemlock forming a dense, sapling understory. Managers of such stands must be sensitive to the effects of silvicultural treatments on species composition by cutting alone; uneven-age cuts will tend to decrease the proportion of oak species or leave it unchanged, and even-age cuts will tend to increase it. Traditional measures of relative density are often poor at discriminating among these stratified oak stands with respect to stocking for any management objective. Both tree-area ratio and stand density index procedures offer some promise for assessing density in these stands.

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