A STAND DENSITY MANAGEMENT DIAGRAM

FOR SAWTIMBER-SIZED MIXED UPLAND CENTRAL HARDWOODS

John A. Kershaw, Jr. and Burnell C. Fischer[1]

ABSTRACT: Data from 190 CFI plots located in southern and west-central Indiana are used to develop a stand density diagram for sawtimber-sized mixed upland hardwoods in the Central States. The stand density diagram utilizes the concepts of self-thinning to establish a maximum size-density curve, and the stocking standards of Gingrich (1967) to formulate intermediate stocking levels. Like Gingrich's diagram, the stand density diagram can be used to determine thinning needs and other intensive management operations. Additionally the diagram has the advantages of relating volume rather than basal area to stand density and is organized in such a manner as to allow stand development patterns to be followed and predicted. Simulation runs from the Central States version of TWIGS are used to compare stand development trajectories on the density management diagram versus the traditional stocking diagram.

INTRODUCTION

The determination of criterion for the expression of stand density or stocking has been a frequent subject of forestry research. The most widely used stocking standard in the Central States is the tree-area equation developed by Gingrich (1967). Gingrich developed stocking standards for upland hardwood forests by utilizing tree-area ratios (Chisman and Schumacher 1940, Krajicek et al. 1961) for trees grown in natural stands under competition, and grown in open conditions free from competition. The tree- area equations express growing space (i.e. - tree area) as a function of number of trees, sum of diameters, and sum of diameters squared. Gingrich presented his results in the form of stocking guides based on number of stems per acre, basal area per acre, and quadratic mean stand diameter (Fig. 1).

Gingrich's stocking guide, and others of similar format (Solomon and Leak 1969; Roach 1977; Tubbs 1977; Hibbs and Bentley 1983), are widely used by practicing foresters in the East as a basis for deciding thinning needs, residual basal areas, and intensive management practices in even-aged stands (Leak 1981). A major reason for the wide utility is that the standards are not considered to be significantly influenced by site quality, stand age, or stand structure (Gingrich 1967).

[1]The authors are Graduate Research Assistant, College of Forest Resources, AR-10, Univ. of Washington, Seattle, WA 98195, and Indiana State Forester, 613 State Office Bldg., Indianapolis, IN 46204.

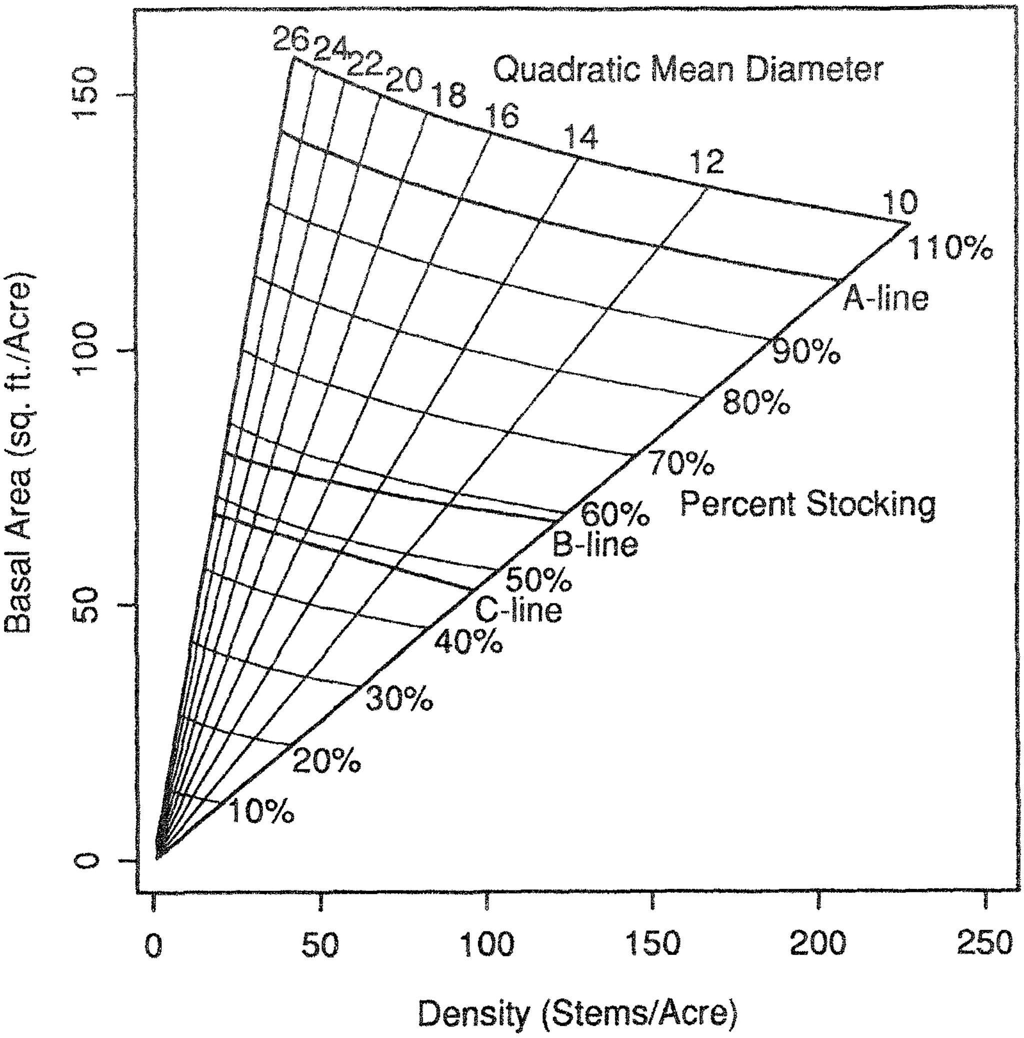

Figure 1. Gingrich's (1967) Upland Central Hardwood stocking guide.

In an alternative approach to the expression of optimum stand density and stocking Drew and Flewelling (1977) utilized concepts developed by Japanese scientists to determine size-density and yield-density relationships. By recognizing the relationship between a given density (stems per unit area) and the maximum mean tree size (weight or volume) a maximum size-density relationship utilizing the -3/2 power rule of self- thinning (Yoda et al. 1963) was developed. Further recognizing there are regions of both random and competition induced mortality, the zone of imminent competition mortality was identified (approximately .51 maximum size-density). Then locating the point of crown closure (.15 maximum size-density) Drew and Flewelling developed a density diagram for Radiata Pine based on density and mean tree volume (cubic feet), thus providing foresters with a simple biological model relating stand density and yield at any stage of stand development. This concept of stand density management has also been applied to Douglas-fir (Drew and Flewelling 1979), western hemlock (Flewelling, Wiley, and Drew 1980), western redcedar (Smith 1989), and to natural mixed deciduous forests in Japan (Kikuzawa 1982).

Like the stocking guides developed in the East the stand density diagrams developed in the West apply to all stages of stand development (Drew and Flewelling 1977), and have the additional advantage, through the use of volume, of providing a forester with a simple biological model relating density and stand development to yield (Drew and Flewelling 1977). In this paper we develop a stand density diagram for sawtimber-sized mixed upland hardwoods in the Central States and use simulations from the Central States version of TWIGS to compare stand development trajectories on the traditional stocking diagram versus the density management diagram.

DATABASE

Data from 190 0.2-acre Continuous Forest Inventory (CFI) plots located throughout southern and west-central Indiana were utilized to formulate the density relationships. The woodlands are typical of the unmanaged or under- managed mixed species woodlands found throughout the Central States. In the early 1950's, when many of the CFI plots were established the woods were understocked, often the result of previous high-grading. Since then only limited improvement cutting or timber stand improvement (TSI) have been done.

The data collected on the CFI plots at each measurement include plot characteristics as well as tree characteristics. Trees are divided into two classes: sawtimber ($\geq$ 9.0 inches dbh) and poletimber (3.0 inches dbh to 8.9 inches dbh). The poletimber data were not collected during all measurement periods and therefore are not used in this study (thus restricting the scope of this paper to sawtimber-sized stands). The span of measurement ranges from 9 to 30 years. The measurement intervals range between 3 and 14 years with an average of 5 years. A total of 919 plot measurements are utilized in this study. For a complete description of data collection and the woodlands see Schroering and Fischer (1982), Fischer and Kershaw (1985), and Fischer (1986).

The species composition of these plots was fairly typical of central hardwood stands (Table 1). White oak (*Quercus alba* L.) average 15% of the stems per plot. Yellow poplar (*Liriodendron tulipifera* L.) averaged 12%. American beech (*Fagus grandifolia* Ehrh.), sugar maple (*Acer saccharum* Marsh.), and hickory (*Carya* spp.) each represented an average of 11% of the stems. Black oak (*Quercus nigra* L.), and northern red oak (*Quercus rubra* L.) represented 10% and 8% respectively. White ash (*Fraxinus americana* L.) averaged 5% abd black walnut (*Juglans nigra* L.) averaged 3%. The remaining species included American basswood (*Tilia americana* L.), eastern cottonwood (*Populus deltoides* Bartr. ex Marsh.), black cherry (*Prunus serotina* Ehrh.), elm (*Ulmus* spp.), black locust (*Robinia pseudoacacia* L.), chinquapin oak (*Quercus muhlenbergii* Engelm.), pin oak (*Quercus palustris* Muenchh.), sassafras (*Sassafras albidum* (Nutt.) Nees), and red maple (*Acer rubrum* L.). All nomenclature follows Kartesz and Kartesz (1980).

Table 1.--Average percent distribution of stems/plot by species and diameter class for mixed upland central hardwood CFI plots located in southern and west-central Indiana.

Species	Diameter Class									
	10	12	14	16	18	20	22	24	26	28+	Total
White Ash	1.95	1.56	0.79	0.28	0.20	0.18	0.18	0.02	0.06		5.23
Am Beech	1.50	1.38	1.22	0.89	1.20	1.60	0.75	0.77	0.67	0.99	10.97
Hickory	3.88	3.22	2.23	1.01	0.54	0.43	0.06	0.06	0.02		11.45
Su. Maple	2.84	2.72	1.56	0.97	0.79	0.57	0.73	0.30	0.12		10.60
N Red Oak	2.13	1.52	1.20	0.89	0.51	0.65	0.36	0.28	0.10	0.16	7.80
Black Oak	1.60	2.78	2.09	0.97	0.49	0.73	0.39	0.32	0.24	0.44	10.05
White Oak	2.37	2.53	2.55	1.82	1.40	1.16	0.51	0.59	0.36	0.88	14.19
Y. Poplar	3.75	3.63	2.09	1.16	0.71	0.30	0.26	0.06	0.06	0.12	12.14
Bl. Walnut	1.03	0.81	0.41	0.53	0.12	0.14	0.08			0.02	3.14
Misc.	3.64	3.47	2.85	1.09	0.68	0.36	0.10	0.04	0.04	0.04	14.41
Total	25.68	24.63	17.01	9.63	6.65	6.12	3.43	2.49	1.68	2.67	100.00

DERIVATION OF THE RELATIVE DENSITY INDEX

As stated above the maximum size-density relationship utilizes the -3/2 power law of self-thinning to identify the maximum average tree size obtainable at a given density (Drew and Flewelling 1977). The maximum size- density relationship is defined as the natural log-log relationship between mean tree volume and trees per acre:

$$\ln(\overline{V_{max}})=\ln(C)-\frac{3}{2}*\ln(N) \tag{1}$$

where,

ln = natural log,

V_{max} = Maximum mean tree volume,

C = a constant,

and N = Number of trees per acre.

By solving (1) for ln(N) the following is obtained:

$$\ln(N)=\frac{2}{3}*\ln(C)-\frac{2}{3}*\ln(V_{max}) \tag{2}$$

or (by taking the antilog):

$$N=C^{\frac{2}{3}}*\overline{V_{max}^{-\frac{2}{3}}}. \tag{3}$$

By substituting the current mean tree volume, V, into (3) an expression for the maximum number of trees that can support the current mean tree volume is thus obtained. The relative

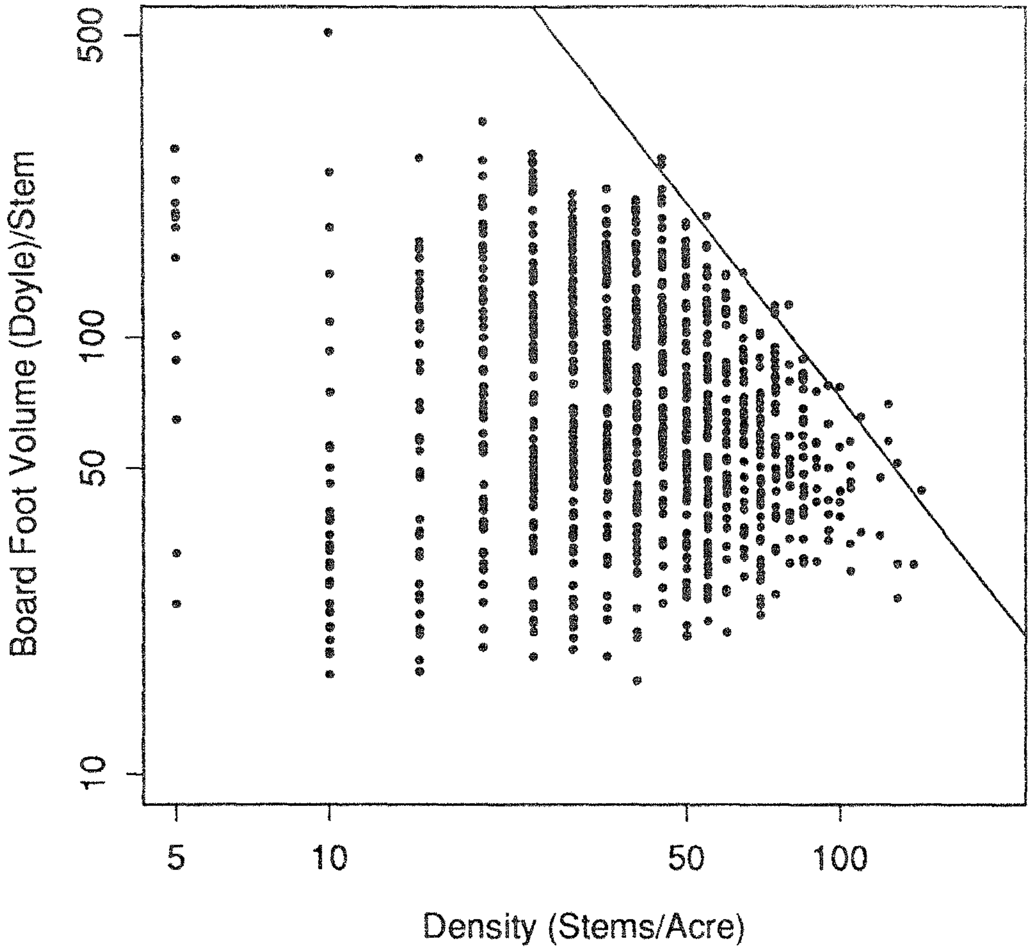

Figure 2. Natural log-log plot of board foot volume (Doyle)/stem over sensity (stems/acre) showing the location of the maximum size-density line. Based on 190 CFI plots located in southern and central Indiana.

density index (RDI) is calculated as the ratio of current number of trees per acre, N, to the maximum number of trees per acre, N_{max}, that can support the current mean tree volume:

$$RDI=\frac{N}{N_{max}}. \tag{4}$$

Since mean tree volume, V, is simply V/N, where V is total volume per acre, (3) can be substituted into (4) and simplified to obtain:

$$RDI=\left(\frac{V}{C}\right)^{\frac{2}{3}}*N^{\frac{1}{3}}. \tag{5}$$

For convenience RDI can be multiplied by 100 so that relative density is expressed as a percentage of maximum density rather than a fraction. In the diagram developed here RDI is expressed in this manner.

Since there is no statistical procedure to estimate maximum relationships, the constant, C, is typically found by placing (1) across the observed maxima on a natural log-log graph of mean tree size (volume or weight) over number of trees per acre, and interpolating (Drew and Flewelling 1977). For each plot, mean board foot volume over number of trees per acre was plotted on a natural log-log scale (Fig. 2). As can be seen from figure 2 there appears to be a maximum mean tree size (board foot volume Doyle) obtainable for a given density (number of trees/acre). Quantification of the maximum size-density relationship was accomplished by constructing lines with slopes of -3/2 at various intercepts until it appeared to represent the observed maximum (Fig. 2). The line placed across this maximum was interpreted to be:

$$\ln(\overline{V}_{max})=11.2-\frac{3}{2}*\ln(N). \tag{6}$$

The line selected was thought to be a more obvious maximum because of the developmental pattern of the plots rather than across the highest point as done in previous studies (Drew and Flewelling 1977, 1979; Flewelling, Wiley, and Drew 1980). Placing the maximum size-density line to allow stands to exceed the maximum size-density relationship temporarily, and thus becoming "overstocked" reflects the thinking of Gingrich (1967) and appears reasonable.

The coefficients in (6) can be rearranged as before to obtain expressions for maximum density (N_{max}) and relative stand density (RDI). The expression for maximum density (3) becomes,

$$N_{max}=(73130)^{\frac{2}{3}}*(V)^{-\frac{2}{3}}, \tag{7}$$

and,

$$RDI=\left(\frac{V}{73130}\right)^{\frac{2}{3}}*(N)^{\frac{1}{3}}. \tag{8}$$

DEVELOPMENT OF THE DENSITY MANAGEMENT DIAGRAM

A relative density measure such as RDI is not very useful unless it is presented diagrammatically relation to easily measured stand parameters. The relationship between mean tree volume, density, and either basal area or quadratic mean diameter was examined in a variety of equation forms, transformations, and weighting schemes. The equations were compared on the basis of Furnival's Index (1961) and residual analyses. The following equation was selected as "best" based on the above criteria:

$$\bar{V}=b_0*N^{b_1}*BA^{b_2} \tag{9}$$

where,

V, N are as defined above,
BA = Basal Area (sq. ft./acre),
and b_i,s are regression parameters.

The parameter estimates, standard errors, and approximate t values are given in table 2. All parameter estimates were significant at the .01 level, the adjusted R^2 was .63 and the root MSE was 41 (bd. ft./stem). This equation will be subsequently used to establish quadratic mean diameter contours on the density management diagram.

Mortality trends were not always as obvious as they were in the data used by Drew and Flewelling (1977, 1979) primarily because of the initial under-stocking and subsequent partial cutting of many of the plots; therefore, another set of relative density criteria needed to be utilized. The stocking standards of Gingrich (1967) was thought to be the most logical criteria to examine.

Table 2.--Parameter estimates, standard errors, and approximate t values for equation 9.

Parameter	Estimate	Standard Error	t-value
b_0	28.69	3.42	8.38
b_1	-1.42	.038	-37.77
b_2	1.52	.044	34.51

Gingrich's stocking standards were translated onto the maximum size- density relationship utilizing the tree-area equations developed by Gingrich and equation (9). Because Gingrich did not provide an equation for the C- line, the equation was derived by interpolating points from the stocking diagram and fitting a simple quadratic equation. The resulting equation was:

$$BA=72.00-0.224*N+0.000254*N^2. \tag{10}$$

The R^2 was .97 and the root MSE was 4.5 (sq. ft.). Figure 3 shows the relationship between the maximum size-density curve and the stocking standards of Gingrich. Based on the equations developed here the maximum size-density curve and the A-line are almost equivalent over the range of data in which the size-density relationship is most evident (45 - 100 trees/acre). As a result of this relationship the density management diagram and the stocking guide are essentially equivalent merely representing a transformation. The final density management diagram is shown in figure 4. The A-line was not included because of

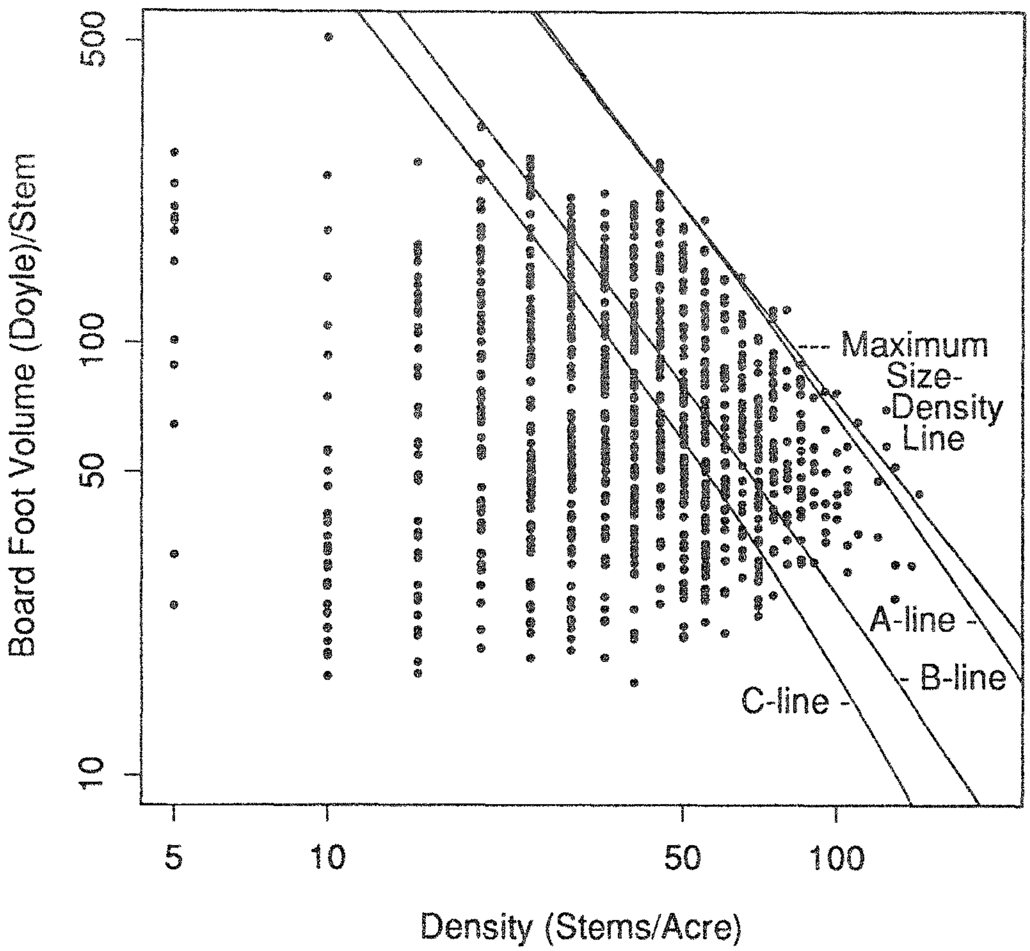

Figure 3. Density management diagram for mixed upland hardwood sawtimber- sized stands (An 8.5 X 11" copy of this diagram is available upon request from the Authors).

it's equivalence to the maximum size-density line (100% relative density line).

APPLICATIONS OF THE DENSITY MANAGEMENT DIAGRAM

TWIGS (The Woodsman's Ideal Growth Simulator) was used to develop a series of examples with which to compare the density management diagram with the stocking guide in relationship to stand development patterns. A stand of average composition was developed from the Feldun Purdue Agricultural Center (FELPAC) CFI database (Fischer et al. 1984). Three initial stand density levels were selected: low - 25 trees per acre; average - 50 trees per acre; and high - 100 trees per acre. The low and high densities represent the range for most of the CFI plots.

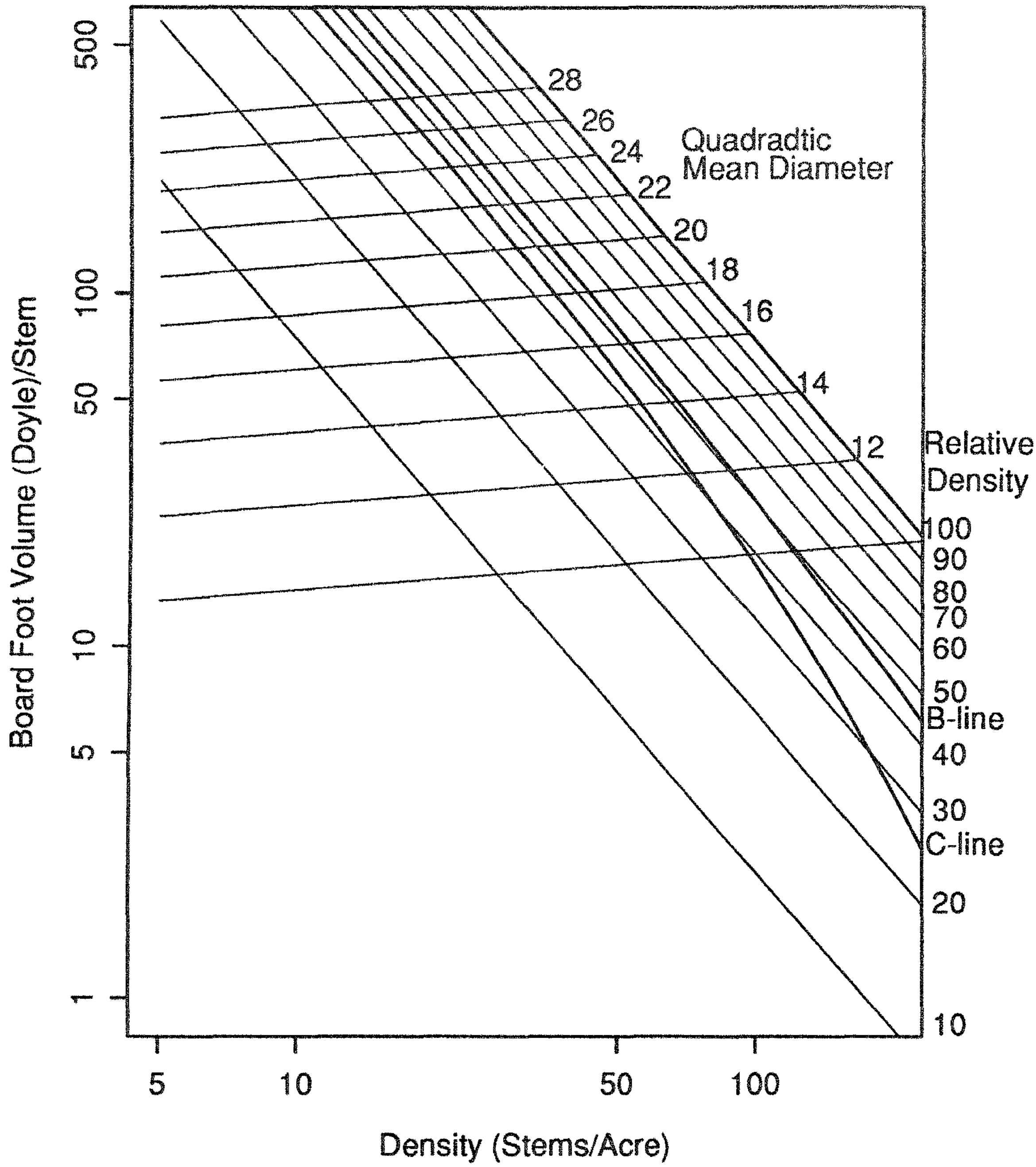

Figure 4. Plot of the maximum size-density relationship showing the location of the stocking criteria of Gingrich (1967).

In the first set of growth simulations (Fig. 5) the stands were allowed to develop without the influence of ingrowth. As can be seen from figure 5a the development of the low density stand is somewhat erratic, while the development of the high density stand is a rather smooth trajectory along the maximum size-density curve. This agrees with the theories of Drew and

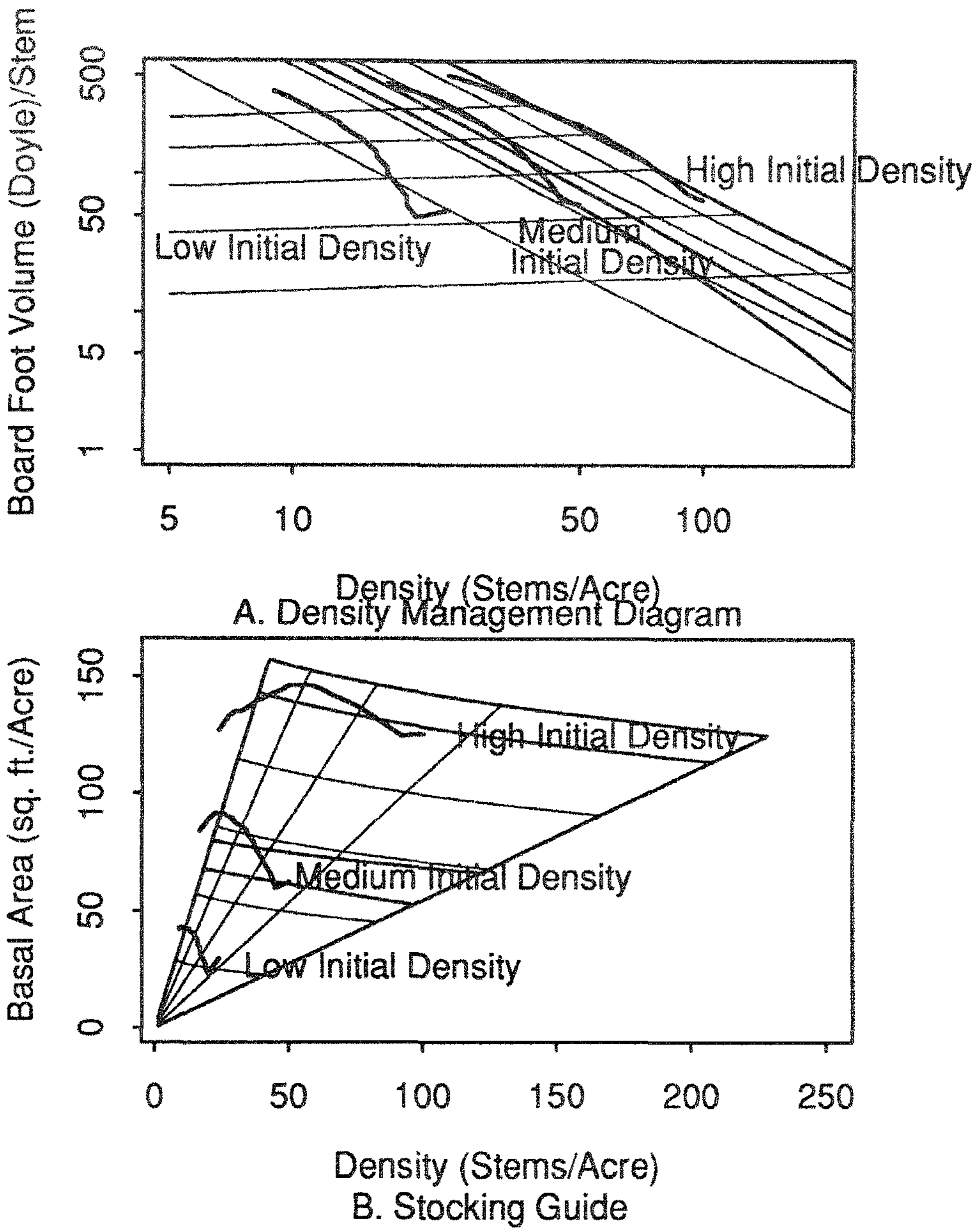

Figure 5. Comparisons of stand development trajectories on the stocking guide versus the density management diagram using simulation results from TWIGS and not accounting for ingrowth.

Flewelling (1977) in which stands below the B-line are subject to "random" mortality, and, therefore, have more erratic development pattern; while stands above the B-line experience "competition-induced" mortality, and have more predictable patterns. Comparing the same data plotted on the stocking guide (Fig. 5b) the trajectories are not nearly as obvious and it is very difficult to discern exactly what stand development processes are occurring especially with the high density stand.

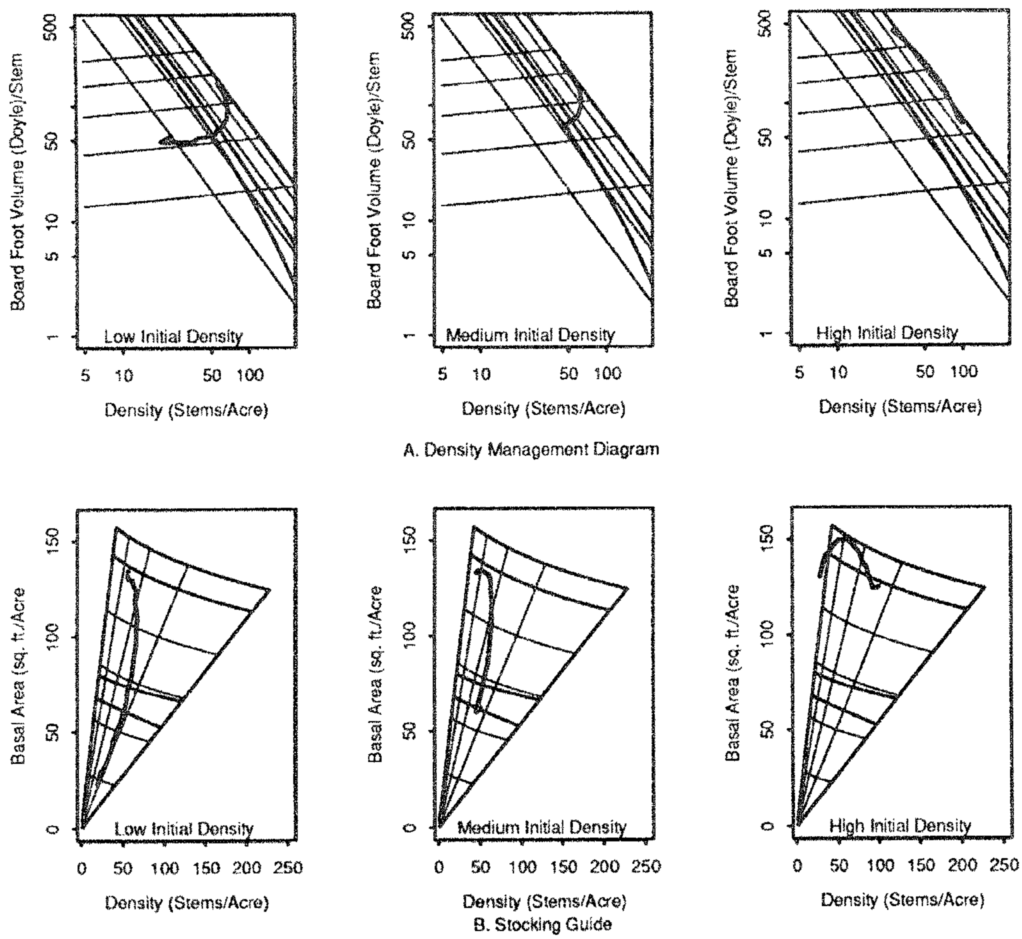

Figure 6. Comparisons of stand development trajectories on the stocking guide versus the density management diagram using simulation results from TWIGS adding ingrowth at an average rate by species and density.

In the second set of simulations (Fig. 6) ingrowth was allowed to influence stand development. Ingrowth was added at an average rate (percentage) by species and density level as computed from the FELPAC database (Fischer et al. 1984). In this set of simulations the stand development processes are again more clearly reflected in the density management trajectories. The stocking guide trajectories for the low and average initial densities do make sense; however, on the density management diagram the process of self-thinning is more evident. Above a relative density of 80 self-thinning is the major stand development process occurring in these simulated stands. On the density management diagram the results of self-thinning are illustrated by the "bend" in trajectory and the movement along the maximum size-density curve. On the stocking guide the "bend" occurs but is not as pronounced and the stand development processes are not as easily interpreted.

Finally, the last set of simulations introduce partial harvesting into the stand development history. Using the high density stand a series of 1, 2, or 3 partial harvests are simulated to illustrate the use of the diagrams for making management decisions. Cuts are made initial and at 25 year intervals. All harvests attempt to reduce the relative density to the B-line. In this set of simulations both diagrams do an excellent job at depicting the stand development processes and management activities (Fig. 7). In this case the major advantage that the density management diagram has over the stocking guide is the depiction of relative density in terms of volume and stems/acre. Since forest managers typically manage for volume being able to view management decisions and stand development in terms of volume is extremely advantageous.

DISCUSSION AND CONCLUSIONS

The relative density diagram presented in figure 4 provides practicing foresters with a valuable management tool relating volume yield per tree to stand density. The significance of this work to the problem of forest stocking evaluation cannot be underestimated. Results of intermediate operations can now be examined in the units of interest to foresters managing timber: volume and trees per acre rather than in basal area.

The density management diagram integrates relative stocking and stand development theory providing foresters with a tool that not only allows point estimation of stocking but also serves as a stand monitoring system in which stand development and treatment can be traced throughout the stand management history. As the landbase available for commodity production continues to decrease foresters are going to need to intensify management efforts on the remaining acreage. This effort is going to require the development and implementation of innovative management techniques and plans. Utilizing this diagram as a diagnostic and monitoring tool should enable foresters to better evaluate management activities and develop a more adaptive approach to stand management.

The relative density measures presented here do not depart from the measures developed by Gingrich (1967) but are a transformation of Gingrich's standards and an incorporation of the ideas of self-thinning and maximum size-density. The density management diagram is a simple biological model relating yield and density at any stage of stand development, and, although the theoretical constancy of the self-thinning slope (-3/2) has come under criticism (Weller 1987, Zeide 1987), the utility of these diagrams for monitoring stand development and making management decisions should not be overlooked (Smith 1989). Additionally, the criticisms do not reject the concept of self-thinning, but rather question whether the slope of the maximum size-density curve should be constant for all species.

A potential shortcoming of this diagram is the lack of information about trees less than 9.0 inches dbh. It needs to be emphasized that this diagram is for sawtimber-sized stands.

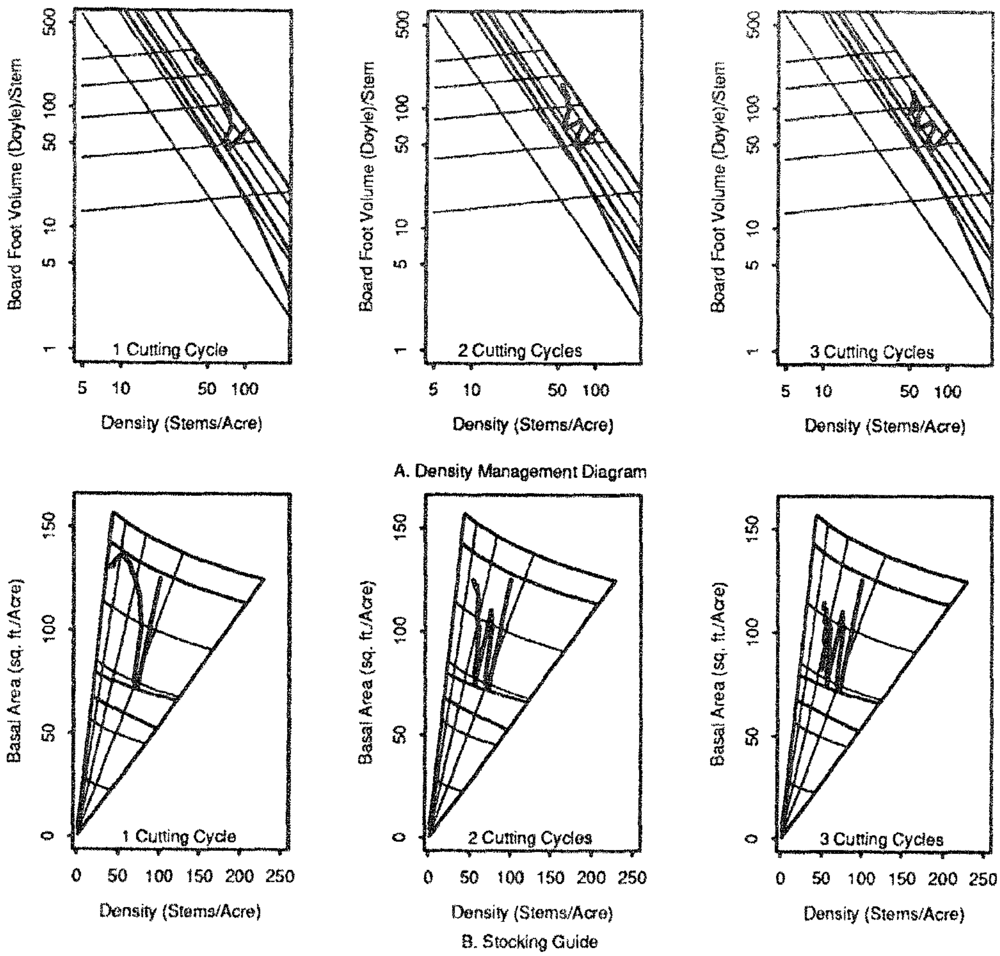

Figure 7. Comparisons of stand development trajectories on the stocking guide versus the density management diagram using simulation results from TWIGS illustrating the influence of management activities on stand development.

Stands which fall below the C-line are considered understocked in sawtimber-sized trees and, therefore, not sawtimber-sized stands. It should also be pointed out that Gingrich's (1967) stocking chart only utilizes trees within the main canopy (i.e. - dominant or codominant). So even though trees down to 2 inches dbh were utilized, very few would be contributing to the overall percent stocking in sawtimber-sized stands. Additionally, the density management diagram is based on merchantable volume; therefore, inclusion of trees below 9.0 inches dbh, which have little or no merchantable volume, would have little or no effect on relative density because mean tree volume will decrease proportionally as the number of trees with no volume increase. Management guidelines for non-sawtimber-sized stands are definitely a gap in our stand management knowledge in the central hardwoods.

Further development of the maximum size-density relationship presented here, including the incorporation of quality related information, will be important future improvements. Much work continues to be needed in the area of optimal stand density and yield in mixed upland hardwoods. This work represents a first step in linking the two together.

LITERATURE CITED

Chisman, H. H., and F. X. Schumacher. 1940. On the tree-area ratio and certain of its applications. J. For. 38:311-317.

Drew, T. J., and J. W. Flewelling. 1977. Some recent Japanese theories of yield-density relationships and their application to Monterey pine plantations. For. Sci. 23:517-534.

Drew, T. J., and J. W. Flewelling. 1979. Stand density management: an alternative approach and its application to Douglas-fir plantations. For Sci. 25:518-532.

Fischer, B. C. 1986. Development and analysis of the Indiana CFI data base. Purdue Univ. Dept. For. and Nat. Res. Unpublished Report. 12p.

Fischer, B. C., J. A. Kershaw Jr., and G. M. Wright. 1984. Growth and yield of upland hardwoods at FELPAC. In: Indiana For. Prod. Marketing and Wood Util. Rep., Purdue Univ. Bull. No. 177: 17-19.

Fischer, B. C., and J. A. Kershaw Jr. 1985. Growth relationships for upland hardwood sawtimber stands in Indiana. Proc., 5th Central Hardwood Forest Conference. 111-117.

Flewelling, J. W., K. N. Wiley, and T. J. Drew. 1980. Stand- density management in western hemlock. Weyerhaeuser For. Res. Tech. Rep. 042- 1417/80/32. Western For. Res. Center. Centralia, WA. 16p.

Furnival, G. M. 1961. An index for comparing equations used in constructing volume tables. For. Sci. 7:337-341.

Gingrich, S. F. 1967. Measuring and evaluation stocking and stand density in upland hardwood forests in the Central States. For. Sci. 13:38-53.

Hibbs, D. E., and W. R. Bentley. 1983. A management guide for oak in New England. Univ. Conn. Coop. Ext. Serv. Bull. 83-12. 12p.

Kartesz, J. T., and R. Kartesz. 1980. A Synomized Checklist of the Vascular Flora of the United States, Canada, and Greenland. Vol. II: The Biota of North America. The Univ. of North Carolina Press. Chapell Hill, NC. 498p.

Kikuzawa, K. 1982. Yield-density diagram for natural deciduous broad-leaved forests stands. For. Ecol. and Mgmt. 4:341-358.

Krajicek, J. E., K. A. Brinkman, and S. F. Gingrich. 1961. Crown competition - a measure of density. For. Sci. 7:35- 42.

Leak, W. B. 1981. Do stocking guides in the Eastern United States relate to growth? J. For. 79:661-664.

Roach, B. A. 1977. A stocking guide for Allegheny hardwoods and its use in controlling intermediate cuttings. USDA For. Serv. Northeast For. Exp. Sta. Res. Pap. Ne-373. 30p.

Schroering, J. D., and B. C. Fischer. 1982. Growth and yield on upland hardwoods in Southern Indiana. Proc. 4th Central Hardwoods Forest Conference. 141-157.

Smith, N. J. 1989. A stand-density control diagram for western red cedar, Thuja plicata For. Ecol. and Manag. 27:235-244.

Solomon, D. S., and W. B. Leak. 1969. Stocking, growth, and yield of birch stands. Birch Symposium Proc. USDA For. Serv. 106-118.

Tubbs, C. H. 1977. Managers handbook for: Northern hardwoods in the North Central States. USDA For. Serv. North Central For. Exp. Sta. Gen. Tech Rep. NC-39. 29p.

Weller, D. E. 1987. A re-evaluation of the -3/2 power rule of self-thinning. Ecol. Monog. 57:23-43.

Yoda, K., T. Kira, H. Ogawa, and K. Hozumi. 1963. Intraspecific competition among higher plants. XI. Self-thinning in overcrowded pure stands under cultivated and natural conditions. J. Biol. Osaka City Univ. 14:107-129.

Zeide, B. 1987. Analysis of the 3/2 power law of self-thinning. For. Sci. 33:517-537.