

INDEPENDENT EFFECTS AND INTERACTIONS OF
STAND DIAMETER, TREE DIAMETER, CROWN CLASS, AND AGE
ON TREE GROWTH IN MIXED-SPECIES, EVEN-AGED HARDWOOD STANDS

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Abstract: Many studies have shown that initial tree diameter is closely correlated with subsequent tree growth. But initial tree diameter is actually a confounded variable, incorporating both competitive position (crown class) and age. A study in hardwood stands in Pennsylvania has shown that the two latter variables were more reliable predictors of tree growth than initial diameter. These two variables lack the confounding of initial diameter, and are biologically more appropriate parameters for inclusion in growth simulators.

Surrogate variables calculated from typical cruise data were found to be nearly as well correlated with growth as the more difficult-to-measure crown class and age. Relative tree diameter (the ratio between actual tree diameter and stand diameter) provided a useful surrogate for crown class, and effective age (calculated from the average diameter of the major species groups present in each stand) provided a useful surrogate for actual age in these even-aged cherry-maple forests.

INTRODUCTION

Many studies have shown that initial tree diameter is closely correlated with subsequent tree growth (Hilt 1983, Hough and Taylor 1946, Marquis 1969, Trimble 1969). Often, other factors studied have added little to the correlation after initial tree diameter was fitted. As a result, initial tree diameter is a good predictor of future growth. It is often used as one of the major independent variables in tree growth-prediction equations (Hilt 1983, Miner and others 1988, Solomon and others 1987). This is probably unfortunate because it tends to mask the biological relationships involved. Initial tree diameter is a variable that actually expresses two more basic factors: competitive position and age.

Large trees in even-aged stands grow faster in large part because of their competitive advantage over their neighbors. In a single stand of trees of the same age, use of initial tree diameter to predict growth causes no problems, because diameter is an expression of competitive position in this case.

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But old trees grow slower than young trees of the same species and competitive position. Individual-tree-diameter growth peaks at an early age, and even trees that retain dominant crown positions decline in growth as they get older and, therefore, larger in diameter.

So, initial tree diameter is confounded as an indicator of future tree growth: large trees grow faster than small trees when comparing trees within an even-aged stand because tree size is a measure of competitive position. But large trees grow slower than small trees when trees of similar species and competitive position are compared across stands of different ages. In this situation, tree size is a measure of age.

To determine how important this confounding might be in growth predictions, and to test other parameters that express competitive position and age more directly, the long term growth records of a group of cherry-maple stands were examined in northwestern Pennsylvania; the records included direct measurements of age and competitive position (crown class). Because crown class and age are difficult to measure accurately and are seldom included in inventory data, several surrogates of these parameters that can be calculated from typical inventory data were tested. This is a report on the results of that investigation.

STUDY METHODS

Individual-tree growth records from three unrelated studies were used in this investigation. The three studies were:

1. Wolf Run Crop Tree Plots--consisted of individual-tree diameter records spanning 37 years, with remeasurements at approximately 5-yr intervals. Trees in this study were open-grown trees left after an overstory removal cut. Records included only d.b.h at each measurement period, plus age from ring counts of stumps at the time of harvest cut. These data are from a single stand on the former TG Forest Industries property adjacent to the Allegheny National Forest. The stand was 44 years old at the first measurement. There were 83 black cherry, 101 sugar maple, and 5 red maple trees available in the Wolf Run Plots. These data were used only for correlations of diameter growth and age.
2. Kane Weeding Study--consisted of stand and individual-tree-diameter records spanning a 35-year period with remeasurements every 5 years. Records included d.b.h. and crown class of each tree at each measurement, plus stand age derived from records of stand origin. Only data from the control plots were used. Each plot was 1/2-a in size. There were four separate stands in this study. Stand ages varied from 13 to 18 years at the start of the study, and from 48 to 53 years at the last measurement. All four stands were located on the Kane Experimental Forest, a part of the Allegheny National Forest.

Three of the seven growth periods available lacked data on crown class, so these three periods were omitted from investigations of the effect of crown class on diameter growth, and on tests of surrogate measures of crown class. In the summaries that follow, data from the four stands and four measurement periods on each stand are presented in groups based on stand age at the start of the period, as follows: 10-20 yr, 30-40 yr, and 40-50 yr. There were approximately 500 trees in this data set on which individual tree growth records were available, in four species: black cherry (*Prunus serotina* Ehrh.), sugar maple (*Acer saccharum* Marsh.), red maple (*Acer rubrum* L.), and American beech (*Fagus grandifolia* Ehrh.).

All seven growth periods were used for investigation of the effect of age on diameter growth.

3. Fertilizer Study--consisted of stand and individual-tree diameter records spanning a 6-yr period, with remeasurements each year. Records included d.b.h. and crown class of each tree at each measurement, plus stand age derived from increment cores of ten dominant black cherry trees in each stand. There were six separate stands in this study. Stand ages ranged from 55 to 65 yr at the start of the study, and data from these stands are presented in the 55-65-yr age group in the summaries that follow. Only data from the control plots were used. Each plot was 1/2-a. All six stands were located on the Allegheny National Forest. There were approximately 2,000 trees in this data set on which individual growth records were available, in four species: black cherry, sugar maple, red maple, and American beech. These data were used for correlations of diameter growth with competitive position, and for testing of surrogate measures of crown class and age.

Descriptive data on stands from the Kane Weeding Study and Fertilizer Study are presented in Table 1. These data characterize the stands used for investigations of crown class, relative diameter, and tree growth and provide a summary of the distribution of trees into crown classes.

The study reported in this paper was conducted in several separate parts, each designed to test or illustrate a specific point. Investigation 1 illustrates the point that initial tree diameter expresses both stand age and the competitive position of individual trees within a stand, and that these two factors affect subsequent tree growth in opposite directions. Investigation 2 tested the hypothesis that competitive position (crown class) and age together provide better growth predictions than initial tree diameter alone when applied to stands that span a wide range of ages. Investigation 3 tests relative tree diameter as a surrogate measure of crown class or competitive position. And Investigation 4 tests effective age as a surrogate measure for actual age.

Table 1.--Characteristics of stands used in the investigation of the effect of crown class and relative diameter on tree growth.

Variable	Age class			
	10-20 yr	30-40 yr	40-50 yr	55-65 yr
Entire Stand				
Age (yr)	14	36	44	58
Basal area (sq ft/acre)	54	110	133	147
Trees (no./acre)	3599	1569	1071	658
Relative density (%)	68	76	99	99
Medial diameter (in.)	2.7	6.3	8.0	9.5
Quadratic diameter (in.)	1.7	3.7	4.8	5.9
Dominants and codominants				
% of no. trees	29	14	22	14
% of basal area	55	55	67	51
Average diameter (in)	2.4	7.0	8.6	11.6
Dominants, codominants, intermediates				
% of no. trees	92	23	30	31
% of basal area	92	70	76	81
Average diameter (in)	2.0	6.2	7.9	9.7

The basic calculations were made as follows: diameter growth was calculated for each individual tree for each growth period; stand parameters (medial stand diameter, quadratic stand diameter, relative density or stocking, basal area/a, numbers of trees/a.) were calculated for each plot for each measurement period using the computer program SILVAH (Marquis 1986); then, diameter growth averages were calculated by crown and age classes.

In several investigations, simple and multiple regressions were run to relate individual-tree growth (by species) to various tree or stand parameters involved in the investigation. Only those regressions with an F-test that exceeded the 0.05 level were considered significant, and only significant regressions are shown in this paper. Coefficients of determination (r^2) for the individual regressions are shown where appropriate. In other investigations, differences among crown class means were tested with analysis of variance. Only those F-tests that exceeded the 0.05 level were considered significant.

Relative diameter was calculated for each tree as a possible surrogate expression of crown class or competitive position. Relative diameter is simply the diameter of the individual tree divided by the average stand diameter. Thus, a 12-in tree in a stand with an average diameter of 12 in would have a relative diameter of 1.0. A 6-in tree in that same stand would have a relative diameter of 0.5. Using this expression, dominant trees in this study had a relative diameter of over 1.0, while suppressed trees had a relative diameter of 0.5 or less.

Codominant and intermediate trees had intermediate values. Thus, relative diameter is a continuous variable that can be related to the four discrete crown classes actually observed. In the calculation of relative diameter, both medial diameter (diameter of a tree at the mid-point of the basal area distribution) and the quadratic diameter (tree of average basal area) were used as measures of stand diameter.

Effective age was calculated as a possible surrogate for actual age. Age for each of two major species groups in cherry-maple stands was estimated from their medial diameters, using the average increase in medial diameter expected for that group. The two species groups are: a) fast-growing black cherry, white ash (*Fraxinus americana* L.), yellow-poplar (*Liriodendron tulipifera* L.), red maple, and northern red oak (*Quercus rubra* L.); and b) slower growing sugar maple and American beech. The average increase in medial diameter (herein called the growth factor) for these two species groups was derived from approximately 120 Allegheny hardwood stands in northwestern Pennsylvania (Marquis and others 1984). The growth factors are: a) .20 inches/yr for fast growing species group 1, and b) .15 inches/yr for slower growing species group 2). Thus, effective age provides an estimate of actual age based on the diameters of two major species groups. The equation is:

$$Effective\ age = \frac{\frac{(MD1 * BA1)}{GF1} + \frac{(MD2 * BA2)}{GF2}}{Total\ BA} \quad (1)$$

where:

BA = basal area in trees 1" dbh and larger

MD = medial stand diameter of trees 1" dbh and larger

GF = growth factor

1 = species group 1

2 = species group 2

RESULTS AND DISCUSSION

Investigation 1 -- Is initial dbh a reliable predictor of tree growth?

A common way to express the competitive position of a tree is by the use of crown class -- a subjective grouping of trees into dominant, codominant, intermediate, and suppressed classes. These crown classes reflect the tree's height relative to its competing neighbors in the crown canopy, and they also indicate the proportion of the tree's crown that receives direct sunlight for tree growth.

Data from both the Kane Weeding Study and the Fertilizer Study were used to calculate the average diameter growth for trees of various crown classes. Average initial dbh was also calculated for these same crown classes.

Differences among crown classes in both average initial dbh and average diameter growth rate were highly significant within any of the 10-year age ranges tested (Table 2). Thus, the dominant trees are the largest trees in their stands, and they grow faster than trees of lower crown class or smaller initial dbh. Within a crown class, older trees are also larger in dbh.

All trees in the Kane Weeding Study that remained in the dominant or codominant crown class throughout the 35-year period were selected for analysis of the effect of age on diameter growth rate. Average growth at various ages between 13 and 48 years were plotted. There were 124 black cherry and 41 sugar maple trees in this sample. There were too few trees of other species in the dominant and codominant classes to provide reliable results.

Table 2.--Average dbh and average diameter growth for trees of different crown class, by age ranges.

Crown class	Average initial dbh	Average diameter growth (in/5 yrs)
Ages 10-20 years (Kane Weeding Study)		
Dominant	2.7	1.24
Codominant	2.2	0.86
Intermediate	1.5	0.55
Suppressed	1.2	0.52
Ages 30-40 years (Kane Weeding Study)		
Dominant	8.4	1.02
Codominant	6.3	0.64
Intermediate	4.4	0.36
Suppressed	2.7	0.13
Ages 40-50 years (Kane Weeding Study)		
Dominant	10.7	0.73
Codominant	7.8	0.46
Intermediate	5.1	0.20
Suppressed	3.2	0.12
Ages 55-65 years (Fertilizer Study)		
Dominant	13.0	1.10
Codominant	10.7	0.67
Intermediate	8.2	0.36
Suppressed	2.9	0.07

Diameter growth rates of dominant and codominant black cherry trees had peaked by age 18. Sugar maple growth was erratic up to approximately age 30, but it declined at a steady rate thereafter (Figure 1).

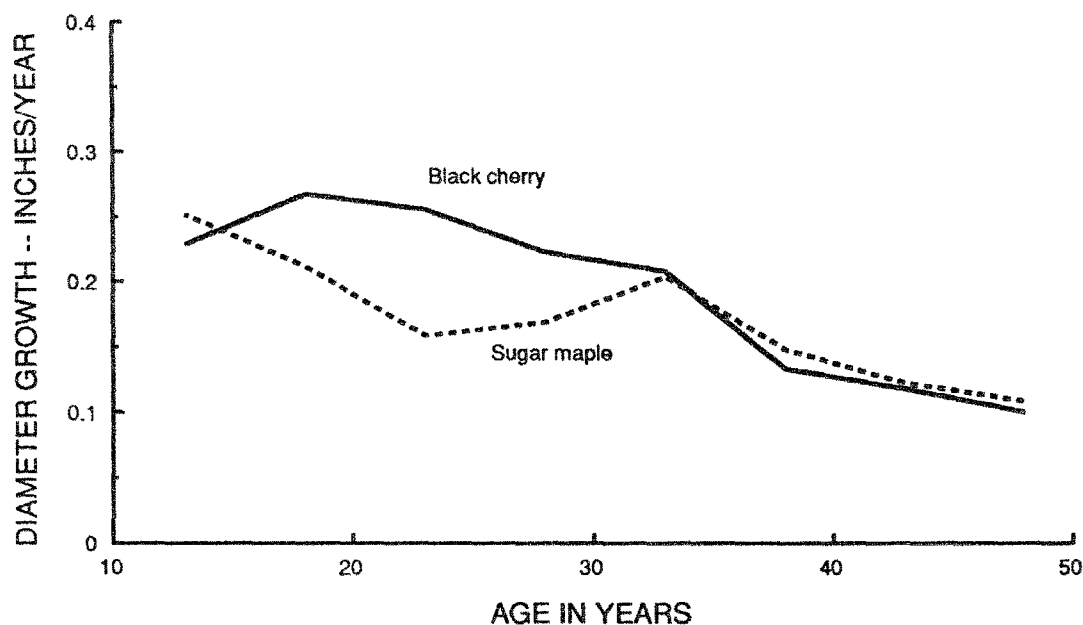


Figure 1. The effect of age on diameter growth (dbh) of individual trees (age 13 to 48).

Data from the Wolf Run plots provide data from older trees than available in the Kane Weeding Study. These data were analyzed in the same manner as above. Diameter growth rate of all three species (black cherry, sugar maple, and red maple) had peaked before the start of measurements (44 yr), and declined steadily throughout the 37-yr growth period (Figure 2).

These several sets of data illustrate that initial tree dbh affects individual tree diameter growth in two conflicting ways. Within stands of similar age, initial dbh is an indicator of competitive position, and large trees grow faster than smaller ones. But across a range of stand ages, initial dbh is an indication of age; large trees grow slower than small trees of the same crown class because the large trees are older. Thus, initial dbh is not a desirable predictor of individual tree growth when used across a range of stand ages and crown classes. Initial diameter is a confounded expression of two other variables that affect tree growth in different directions.

Investigation 2 -- Are crown class and age better predictors than initial dbh?

Using individual tree data from both the Kane Weeding Study and Fertilizer Study, regression equations were calculated to predict individual tree growth based either on: a) initial dbh, or

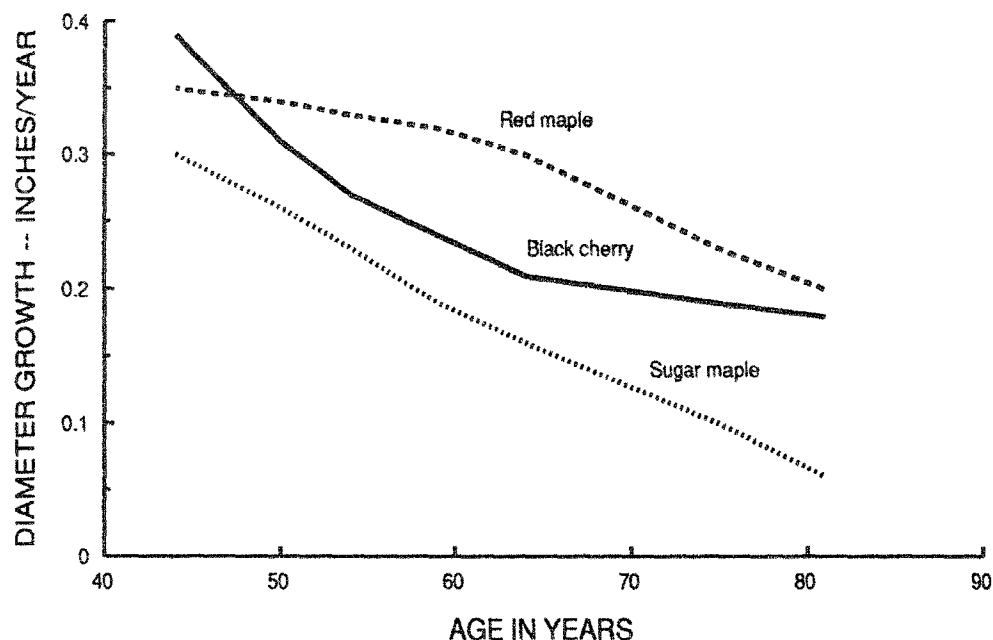


Figure 2. The effect of age on diameter growth (dbh) of individual trees (age 44 to 81).

b) crown class plus age. Crown class was entered into the equations as a set of dummy variables. Separate equations were calculated for black cherry and sugar maple.

Coefficients of determination showed that equations including two independent variables to express both competitive position and age were much better (had higher r^2 values) than equations including only initial dbh as an independent variable (Table 3). In fact, initial dbh was only poorly correlated with diameter growth across the range of ages included in these equations, and regressions with dbh were not statistically significant.

Table 3.--Coefficients of determination (r^2) between individual tree diameter (dbh) growth and either initial dbh or age plus crown class.

Species	Independent variable(s)	
	Initial dbh	Age + Crown Class
Black cherry	0.00	0.41
Sugar maple	0.08	0.62

Investigation 3 -- Relative diameter as a measure of competitive position

Crown class is a subjective measure of competitive position, and therefore not evaluated equally by all observers. It provides estimates of competitive position in discrete classes, and is therefore inconvenient to include in prediction equations. Furthermore, it is an observation not commonly included in inventories made on an operational basis. Therefore, crown class has limitations as an independent variable in growth prediction equations.

It would be very desirable to find a continuous variable that provides a measure of competitive position that can be objectively estimated from commonly available inventory data. Relative diameter (diameter of the tree relative to the average stand diameter) is a variable that expresses the size of a tree relative to its competitors in the same stand, and therefore appears promising as a measure of competitive position and a surrogate for crown class.

The Kane Weeding Study and Fertilizer Study data were used to calculate the average diameter growth of all species stratified into four crown classes and five relative-diameter classes. Both crown classes and relative-diameter classes had a strong influence on diameter growth, with a clear trend of best growth from trees in the better competitive positions (Table 4).

Table 4.--Diameter growth rates (inches/5 yrs) as affected by competitive position for stands of four age groups (all species together).

Variable	Age class			
	10-20 yr	30-40 yr	40-50 yr	55-65 yr
Crown class				
Dominant	1.24	1.02	0.73	1.10
Codominant	0.86	0.64	0.46	0.67
Intermediate	0.55	0.36	0.20	0.36
Suppressed	0.52	0.13	0.12	0.07
Relative diameter class (based on medial diameter)				
1.25+	1.45	0.97	0.72	0.92
1.00 - 1.25	1.15	0.78	0.59	0.63
0.75 - 1.00	0.92	0.53	0.39	0.38
0.50 - 0.75	0.67	0.29	0.25	0.18
0.00 - 0.50	0.43	0.01	0.07	0.05

When diameter growth is predicted using regressions of diameter growth as a function of relative diameter and compared to predictions of diameter growth based on crown class averages, the two measures perform about equally. There was a slight tendency for crown class to predict growth better than relative diameter, but the differences were minor (Table 5). This indicates that relative diameter can be used in place of crown class as a predictor of diameter growth.

Table 5.--Standard errors for diameter growth estimates based on crown class averages versus those based on regressions with relative diameter.

Age range	Crown class	Relative diameter
	30-40 years	
black cherry	.32	.32
sugar maple	.18	.18
	40-50 years	
black cherry	.22	.21
sugar maple	.13	.12
	55-65 years	
black cherry	.25	.26
sugar maple	.11	.08

Data from both the Kane Weeding Study and the Fertilizer Study were then used to calculate regressions of diameter growth as a function of initial diameter and as a function of two different relative diameters. Relative diameter was calculated using both the medial diameter and the quadratic diameter. The regressions were run within single stands (where age was constant), and across all stands in the data set (where age varied from 10 to 65 years).

The coefficients of determination (r^2) for three regressions of diameter growth on d.b.h., relative medial diameter, and relative quadratic diameter were identical within each stand. And correlations between d.b.h. and the two relative diameter measures were perfect -- r^2 of 1.00 (Table 6). This is to be expected since the two relative diameters are equivalent to d.b.h. divided by a constant. Within a single stand, where all the trees are the same age, all three measures provide identical results as expressions of competitive position.

However, when the same regressions are calculated across a group of stands of varying ages, r^2 for regressions containing relative diameter are much higher than those for regressions on initial d.b.h. (Table 6). In this situation, the two relative diameter measures continue to provide an expression of the competitive position of each tree's position that is unconfounded with age. But initial d.b.h. is now expressing both competitive position and age, two factors that affect diameter growth in opposite ways as demonstrated earlier. Thus, relative diameter appears promising as a measure of competitive position in place of crown class; and it is a measure that is not confounded by age as is initial dbh.

Table 6.--Coefficients of determination (r^2) for initial diameter and relative diameter (based upon medial stand diameter and quadratic stand diameter) as affected by age range in the data.

Age range	No. of stands	Dependent variables				
		Diameter Growth			D.b.h.	
		Independent variables			Independent variables	
		Dbh ¹	RMD ²	RQD ³	RMD	RQD
Black cherry						
Within a single stand*						
age 10-20	4	0.40	0.40	0.40	1.00	1.00
age 30-40	7	0.51	0.51	0.51	1.00	1.00
age 40-50	3	0.42	0.42	0.42	1.00	1.00
age 55-65	6	0.54	0.54	0.54	1.00	1.00
Across a wide range of stand ages*						
age 10-65	22	0.00	0.22	0.23	0.53	0.49
age 30-65	18	0.28	0.48	0.45	0.66	0.56
Sugar Maple						
Within a single stand*						
10-20	4	0.25	0.25	0.25	1.00	1.00
30-40	7	0.60	0.60	0.60	1.00	1.00
40-50	5	0.55	0.55	0.55	1.00	1.00
55-65	6	0.51	0.51	0.51	1.00	1.00
Across a wide range of stand ages*						
10-65	22	0.08	0.57	0.50	0.42	0.48
30-65	18	0.36	0.58	0.59	0.83	0.70

¹Dbh = diameter at breast height at the beginning of the growth period

²RMD = relative diameter based upon medial stand diameter

³RQD = relative diameter based upon quadratic stand diameter

*Values shown within a single stand are the mean of the r^2 values for each individual stand in the age range shown. Values shown across a wide range of stand ages are the single r^2 value from a regression using all stands in the specified age range.

Differences between the two relative diameter measures were small. The one based on medial diameter gave slightly better results in most cases than the one based on quadratic diameter. The exception was in very young stands where results were erratic (Table 6).

Medial diameter has the added advantage that it provides a much more realistic estimate of tree size than quadratic diameter in these mixed-species hardwood stands. Quadratic diameter is heavily influenced by the large number of suppressed trees, so it gives an estimate of stand diameter that does not adequately reflect the size of the crop trees in the stand.

To illustrate, the quadratic stand diameter and the medial stand diameter are plotted against the arithmetic average diameter of the dominant plus codominant trees and against the arithmetic average diameter of the dominant, codominant, and intermediate trees in the Kane Weeding Study and Fertilizer Study data sets (Figure 3). Medial stand diameter closely approximates the average diameter of the dominants, codominants, and intermediates--the trees that make up the main crown canopy. Quadratic stand diameter is much lower, and does not adequately represent the diameter of the important trees in the stand.

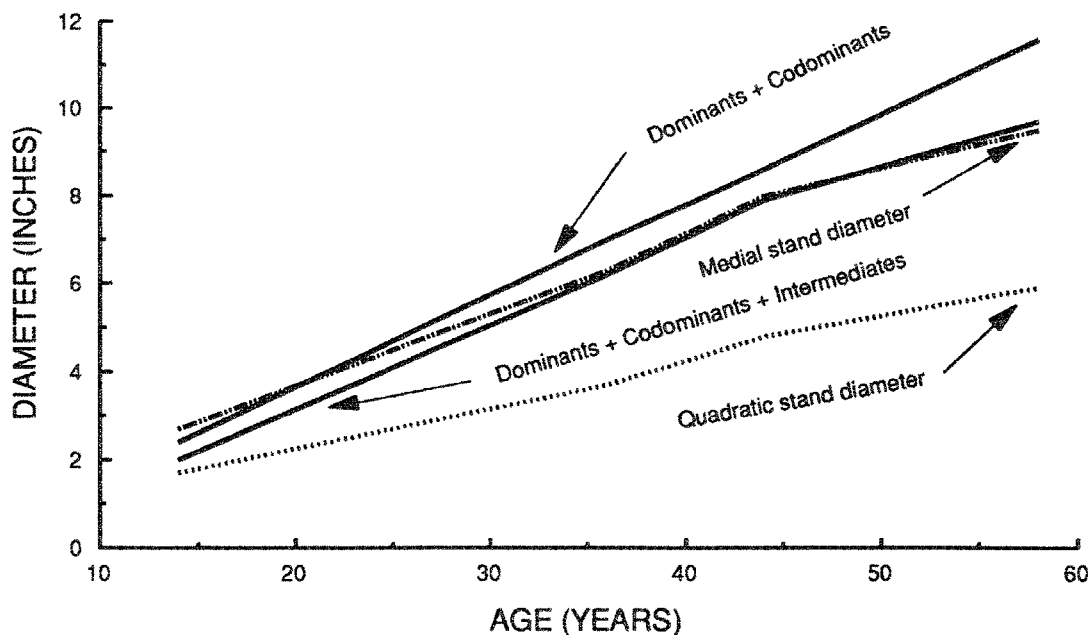


Figure 3. A comparison of medial stand diameter and quadratic stand diameter as expression of average tree dbh of various crown class groups in cherry-maple stands.

So, medial stand diameter clearly seems to be a better measure of stand size class, and is better correlated with diameter growth than quadratic stand diameter. Its use as an expression of average stand diameter is recommended.

Data from the Kane Weeding Study and Fertilizer Study were examined to determine whether or not relative diameter might be used to segregate trees into crown classes when direct measures of crown class are not available. The relative diameter breaking points used to separate one crown class from another were estimated by regressing the relative diameter limits from each age class over age (Figure 4). The results of segregations made on this basis were compared with actual crown class observations (Table 7).

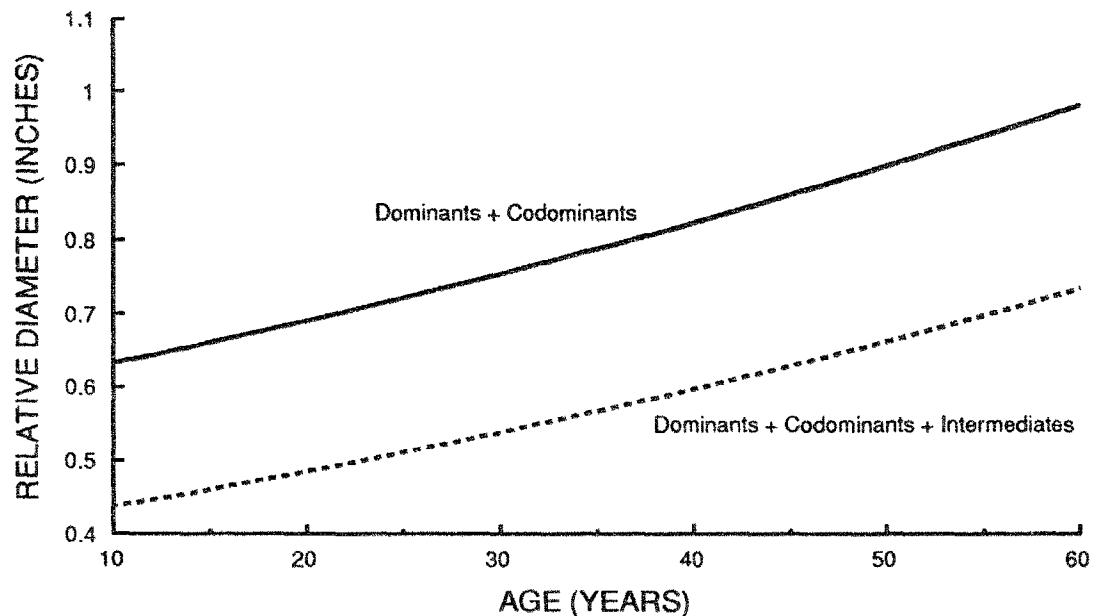


Figure 4. Relative diameter limits used to separate crown classes in cherry-maple stands, as a function of age.

These trials suggest that it is possible to select those trees in a stand which represent any crown class (or group of crown classes) by using relative diameter as a surrogate. The group of trees selected as representing a particular crown class will not correspond exactly to the actual crown classes, but will contain approximately the same number of trees and have approximately the same growth rate and average diameter as that crown class (Table 6). These trials also suggest that the relative diameter that separates one class from another is not a constant, but varies with both age and species. Thus, additional study would be needed to define the relative diameter breaking points appropriate to a wide range of stand compositions and ages. Nevertheless, relative diameter appears to be a useful surrogate for crown class.

Table 7.--Conformity among crown class and relative diameter in selection of the trees in the upper crown canopy over four age classes -- all species.

Competitive position*	Age class			
	10-20 yr	30-40 yr	40-50 yr	55-65 yr
Number of trees/acre				
D & C crown classes	299	319	238	284
D & C via Relative dia.	302	325	234	273
D, C & I crown classes	481	449	294	619
D, C & I via Relative dia.	481	459	287	619
Average diameter (dbh)				
D & C crown classes	2.4	7.0	8.6	11.6
D & C via Relative dia.	2.3	7.2	8.7	11.8
D, C & I crown classes	2.0	6.2	7.9	9.7
D, C & I via Relative dia.	2.0	6.4	8.1	9.8
Diameter growth, inches/5 yrs				
D & C crown classes	0.98	0.76	0.53	0.80
D & C via Relative dia.	0.99	0.74	0.53	0.78
D, C & I crown classes	0.82	0.64	0.47	0.56
D, C & I via Relative dia.	0.82	0.63	0.49	0.56

*D = dominant crown class

*C = codominant crown class

*I = intermediate crown class

Investigation 4 -- Effective age as a measure of actual age

Like crown class, age is difficult to measure in the field, and is seldom available from inventory data. The effective age calculated by the SILVAH program, and described earlier, provides a possible surrogate for actual age.

To test the value of effective age in prediction equations, a series of multiple regressions that included relative diameter plus either actual age or effective age was run. Effective age had r^2 values nearly as high as actual age in most regressions, and slightly better than actual age in one regression. Thus, effective age appears to be a useful surrogate for actual age that can be used in growth prediction equations. The regressions also show that the combination of relative diameter and effective age is considerably better as a predictor of diameter growth than either variable alone (Table 8).

Table 8.--Coefficients of determination (r^2) for regressions of diameter growth (dbh) and various combinations of relative stand diameter, age, and effective age.

Independent variable	Black cherry	Sugar Maple
Relative diameter	0.22	0.57
Age	0.08	0.26
Effective age	0.07	0.23
Rel. dia. + Age	0.41	0.62
Rel. dia. + Eff. age	0.39	0.63

CONCLUSIONS

These investigations show that initial tree diameter may express either a tree's competitive position in stands of similar age, or a tree's age when viewed across stands of many ages. Since a large initial dbh produces fast growth in the former situation, and slow growth in the latter, initial tree dbh is a confounded variable that may provide inaccurate growth predictions when applied to trees of a wide range of ages and competitive positions. Separate measure of age and competitive position provide more accurate growth prediction than use of initial dbh alone.

Since neither age nor competitive position (crown class) are commonly included in the data from operational inventories, easily calculated surrogates of these variables are needed. Relative diameter provides a useful measure of competitive position, and effective age provides a useful measure of actual age in Allegheny hardwood stands. Both of these measures can be calculated from the species and diameter records commonly collected in forest inventories. Predictions of growth based on these surrogates provide estimates with accuracy similar to those based upon crown class and actual stand age.

Medial diameter is a better measure of average stand diameter in Allegheny hardwood stands than quadratic diameter. When used to calculate relative diameter, it also provides better correlation with tree growth than relative diameter calculated from quadratic stand diameter.

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