Edge Effects in Line Intersect Sampling With Segmented Transects

David L. R. AFFLECK, Timothy G. GREGOIRE, and Harry T. VALENTINE

Transects consisting of multiple, connected segments with a prescribed configuration are commonly used in ecological applications of line intersect sampling. The transect configuration has implications for the probability with which population elements are selected and for how the selection probabilities can be modified by the boundary of the tract being sampled. As such, the transect configuration also affects the performance of methods designed to eliminate edge-effect bias. We show that the reflection method solves the edge-problem for designs that use symmetric radial transects (e.g., straight-line and X-shaped transects centered at the sample point). This method also applies to designs that use asymmetric radial transects, provided the orientation is selected uniformly at random. Asymmetric radial transects include straight lines emanating from the sample point, and L- and Y-shaped transects. The reflection method does not apply to designs where polygonal transects (e.g., triangular, square, and hexagonal transects) are used, or where the orientations of asymmetric radial transects are fixed. We provide a new method that eliminates edge-effect bias for designs that use asymmetric radial transects with fixed orientation, but a workable solution for polygonal transects remains elusive.

Key Words: Boundary overlap; Inclusion region; Reflection method; Transect configuration.

1. INTRODUCTION

The survey of dead and downed woody material, vegetative cover, and other nontimber forest attributes has become more prevalent in recent years as perceptions of a decline in biodiversity have spread and as needs to assess carbon stocks, driven by the Kyoto Protocol, have increased. In this context, line intersect sampling (LIS) has been extensively applied (Battles, Dushoff, and Fahey 1996; Ståhl, Ringvall, and Fridman 2001) and is now incorporated into many multiresource forest inventory programs (see, e.g., Waddell 2002). Applied to discretely distributed populations, LIS is an unequal-probability sampling

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Journal of Agricultural, Biological, and Environmental Statistics, Volume 10, Number 4, Pages 460–477
DOI: 10.1198/108571105X81355

method in which population elements, such as logs or plant canopies, crossed by a fixed-length transect are selected into the sample.

Early ecological applications of LIS (e.g., Canfield 1941) employed straight-line transects, but since the work of Warren and Olsen (1964), which advocated a model-based approach to inference, an array of transect configurations have been in use. Transects consisting of multiple connected segments arranged in a radial configuration (see Affleck, Gregoire, and Valentine 2005) are now widely used in LIS surveys of coarse woody debris, forest fuels, and logging waste. For example, the British Columbia Vegetation Resources Inventory prescribes the use of randomly oriented L-shaped transects; the USDA Forest Inventory and Analysis (FIA) program establishes Y-shaped transects with a fixed orientation; and the Canadian National Forest Inventory has proposed the use of X-shaped transects. Other agencies have adopted segmented transects with a polygonal configuration: the Nova Scotian forest inventory program employs triangular transects, the National Inventory of Landscapes in Sweden program currently uses square transects, and the U.S. Fire Effects Monitoring and Inventory Protocol (FIREMON) advocates the use of transects with a hexagonal component. Implementation of these and other LIS surveys has resulted in the collection of a large number of samples from segmented transects. The motivation for our research is the need to place estimation on the basis of such samples on a sound statistical foundation.

The potential bias that can result when the boundary of the region being sampled affects the selection probabilities of individual population elements has long been an issue in the design and analysis of field surveys (see Stevens 2002; Ducey, Gove, and Valentine 2004). In LIS surveys of ecological populations, the reflection method (Gregoire and Monkevich 1994) can be used with some designs employing fixed-length transects as a correction for edge-effect bias. Applying this method, any portion of a line transect extending beyond the boundary of the tract is reflected back into the tract, over the existing transect. The reflection method is easily implemented in the field and is effective in eliminating edge-effect bias in classical LIS strategies employing straight-line transects (Gregoire and Monkevich 1994). However, the utility of the reflection method in LIS strategies employing segmented transects has not been established.

The application of segmented as opposed to straight-line transects, and of radial as opposed to polygonal transects, affects the probability with which a population element can be selected into the sample (Affleck, Gregoire, and Valentine 2005). Consequently, the transect configuration is also relevant to the performance of methods designed to eliminate, or minimize, edge-effect bias. This issue has not been previously addressed. As asserted by Affleck, Gregoire, and Valentine (2005), it is important because the composition and abundance of population elements near the edge can differ materially from the composition and abundance in the interior. For example, the abundance and size distribution of coarse woody debris can vary substantially between edge and interior forest conditions (Chen, Franklin, and Spies 1992; Harper and Macdonald 2002). The under-representation of edge conditions implicit in the existence of edge-effect bias is therefore of concern to subject matter specialists.

This article examines the problem of edge-effect bias in the context of replicated LIS designs (see Barabesi and Fattorini 1998) employing fixed-length segmented transects. The problem is approached by focusing on the inclusion probabilities of individual population elements, which in turn are considered in the full generality of Lucas and Seber (1977) and hereafter referred to as particles. We discuss the effectiveness of the reflection method and, in particular, show that it secures design-unbiased inference only for a limited class of LIS strategies when radial transects are used. A new boundary overlap correction technique, the walkback method, is described for LIS strategies employing fixed orientation radial transects. Finally, we discuss why the approaches that work with radial transects do not work when polygonal transects are established, and we conjecture that for LIS strategies employing such transects edge-effect bias can be eliminated only by sampling beyond the tract boundaries.

2. NOTATION AND DEFINITIONS

Define the population of interest as the collection $\mathcal{P} = \{P_1, P_2, \ldots, P_N\}$ of discrete particles distributed over a bounded region whose projection onto the horizontal plane is denoted by $\mathcal{T}$. For the purposes of identifiability, we assume that each particle is connected in the sense that any two points in P_i can be joined by a path contained entirely within the particle. In practical terms this requires that disjoint objects be treated as distinct particles and vice versa. Apart from this requirement, particles can adopt any shape: particles need not be needle-shaped (i.e., approximately conical, with a defined central axis) and can be forked or contain interior holes. Thus, particles can represent pieces of woody debris, plant canopy projections, or regions of homogeneous land cover. Furthermore, no assumptions are made about the distribution or orientations of particles, and the particles can overlap in their respective projections onto the horizontal region $\mathcal{T}$. In the following, we use P_i to denote both the ith particle and its horizontal projection.

In many finite population surveys, interest is directed toward the aggregate value

$$\tau = \sum_{i=1}^{N} y_i \, , \tag{2.1}$$

where y_i is the fixed value of an attribute of P_i (e.g., particle volume or biomass). Alternatively, the population mean $\mu = \tau/N$ or density $\lambda = \tau/|\mathcal{T}|$, where $|\mathcal{T}|$ is the area of $\mathcal{T}$, may be of central interest. In the following, we focus on estimation of τ; estimation of μ and λ has been described elsewhere (e.g., Gregoire and Valentine 2003).

We consider LIS designs in which M sample points $\mathbf{s}_m$ $(m = 1, 2, \ldots, M)$ are located on $\mathcal{T}$ either independently and uniformly at random, or systematically with a random start. At each sample point, a transect consisting of $K \geq 1$ directed line segments of equal length L is established. These transect segments are connected in a prescribed radial or polygonal configuration. As defined by Affleck, Gregoire, and Valentine (2005), the K segments in a radial transect extend outwards from a common vertex coinciding with the sample point, as in a L- or Y-shaped transect. When a polygonal transect is employed, segments are established in succession so that the transect forms a closed figure such as an equilateral triangle or square; the sample point thus marks the initiation point of segment 1 and the termination point of segment K. For sake of specificity, we assume in the ensuing discussion that the segments of a polygonal transect are laid out (and indexed) in a counter-clockwise sequence.

Denote the vector of segment orientations for the mth transect by $\boldsymbol{\theta}_m = [\theta_{m1}, \theta_{m2}, \ldots, \theta_{mK}]$, where these orientations are measured with respect to a fixed baseline angle. We assume that only the orientation of the initial segment, θ_{m1}, is free to vary over $(0, 2\pi)$: the orientations of the other segments are dictated by the transect configuration. For example, given θ_{m1}, $\boldsymbol{\theta}_m = [\theta_{m1}, \theta_{m1} + \frac{\pi}{2}, \theta_{m1} + \pi, \theta_{m1} + \frac{3\pi}{2}]$ for either square or X-shaped transects. The orientations of the initial segments can be selected at random or fixed in some objective manner. But when $\boldsymbol{\theta}_m$ is fixed, inference must be conditioned accordingly.

The replicated nature of the sampling design as described above permits the construction of M estimates of τ that can be combined to obtain a single overall estimate,

$$\widehat{\tau} = \frac{1}{M} \sum_{m=1}^{M} \widehat{\tau}_m ,$$

where $\widehat{\tau}_m$ is the estimate of τ based only on the sample from the mth transect. Also, when $M > 1$ sample points are distributed independently at random over $\mathcal{T}$, a design-unbiased estimator of the variance of $\widehat{\tau}$ can be obtained from

$$\widehat{\operatorname{var}}(\widehat{\tau}) = \frac{1}{M(M-1)} \sum_{m=1}^{M} (\widehat{\tau}_m - \widehat{\tau})^2 . \tag{2.2}$$

When a systematic sampling design is employed, design-unbiased estimators of $\operatorname{var}(\widehat{\tau})$ are not available: in particular, Equation (2.2) will generally overestimate this variance (D'Orazio 2003), though it is commonly used nonetheless. Because edge-effect bias is impervious to the number of transects used however, we consider below only the case $M = 1$ and suppress the corresponding subscript (m). Results for LIS strategies with $M > 1$ sample points follow in a straightforward manner.

As noted by Kaiser (1983), design-based inference in LIS can proceed conditionally on the transect orientations used, or, if the orientations are selected at random from a defined distribution, on the basis of both the random location and orientation of transects. Let t_{ik} denote the number of times P_i is selected by segment k ($t_{ik} \in \{0, 1, 2, \ldots\}$) and let $t_{i\bullet} = \sum_{k=1}^{K} t_{ik}$ (protocols for selection are described below). Given the transect orientation, a conditionally design-unbiased estimator of τ can be derived from:

$$\widehat{\tau}^{\,c} = \sum_{i=1}^{N} \frac{t_{i\bullet}\, y_i}{\mathrm{E}[t_{i\bullet} | \boldsymbol{\theta}]} . \tag{2.3}$$

Alternatively, if the orientation of the initial segment is selected at random, one can opt for an unconditionally design-unbiased estimator based on:

$$\widehat{\tau}^{\,u} = \sum_{i=1}^{N} \frac{t_{i\bullet}\, y_i}{\mathrm{E}[t_{i\bullet}]} . \tag{2.4}$$

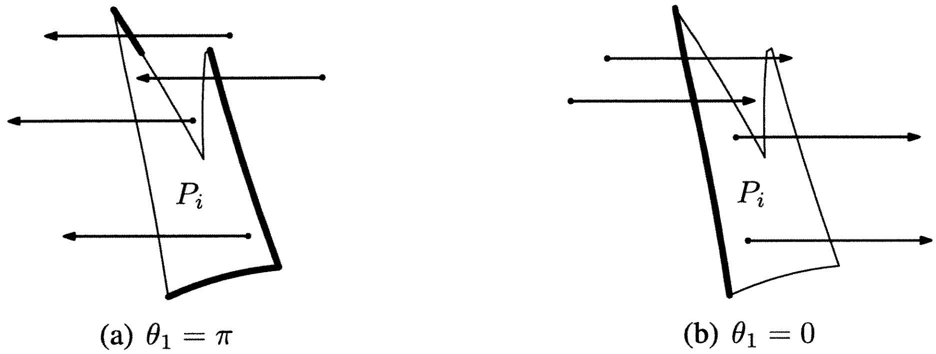

Figure 1. Intersection of a nonconvex particle: the particle is selected only by the upper two transect segments in both (a) and (b). The leading edge of P_i for each θ_1 is indicated by the heavy line.

Equations (2.3) and (2.4) are generalizations of Kaiser's formulas that account for multiply-segmented transects. Expressions for $\mathrm{E}[t_{i\bullet}|\boldsymbol{\theta}]$ and $\mathrm{E}[t_{i\bullet}]$ leading to specific estimators used in ecological and forestry LIS surveys are derived in the following.

As usual, we regard a sampling strategy as the coupling of a particular sample design and estimator and refer to LIS strategies employing estimators based on (2.3) and (2.4) as conditional and unconditional, respectively. The relative merits of conditional and unconditional LIS strategies were discussed by Kaiser (1983), Gregoire and Valentine (2003), and Gregoire, Affleck, and Valentine (2005).

3. PARTICLE INCLUSION REGIONS

Numerous particle selection protocols have been described for LIS (e.g., de Vries 1979; Kaiser 1983; Marshall, Davis, and LeMay 2000; Waddell 2002). Some of these protocols are strictly appropriate only for convex or needle-shaped particles, while others apply only when straight-line transects are used. To overcome such limitations, we adopt the protocol described by Affleck, Gregoire, and Valentine (2005). This protocol stipulates that P_i is selected by segment k only if that segment intersects the particle's "leading edge," where the leading edge is defined with respect to the orientation of the segment under consideration (see Figure 1). That is, $t_{ik} = 1$ if segment k intersects P_i and no portion of P_i is located behind the point at which segment k was initiated. This LIS protocol is applicable to populations composed of discrete particles of any shape and does not require either the delineation of central axes or the subdivision of nonconvex, connected particles. Moreover, the protocol ensures that multiple intersections of a single particle by a single segment are not treated as independent events: particles in the interior of $\mathcal{T}$ (as defined below) can be selected at most once along a single segment, but up to K times by a transect.

The inclusion region of P_i is the set of all sample points that lead to its selection by at least one transect segment (i.e., all $\mathbf{s} \in \mathcal{T}$ such that $t_{i\bullet} > 0$). The size and shape of a particle's inclusion region are determined jointly by the dimensions of its horizontal projection onto $\mathcal{T}$, the transect configuration and orientation, and the selection protocol. But the extent of the inclusion region is also a function of the particle's location relative

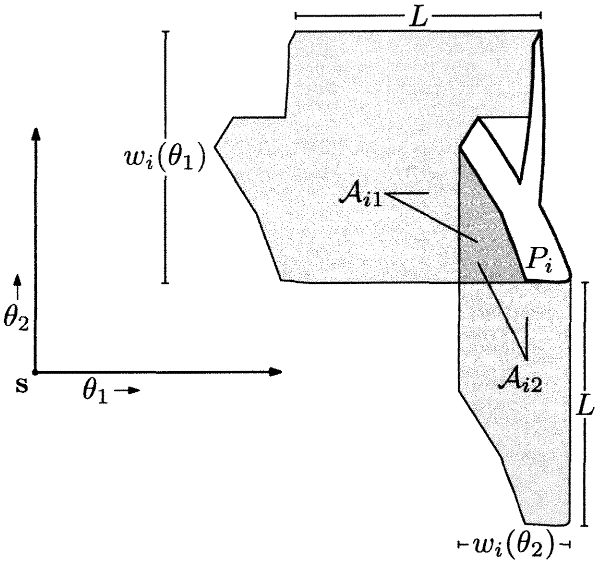

Figure 2. Inclusion region of an interior particle for a L-shaped transect with fixed orientation; P_i is selected once if s *falls in the lightly shaded regions and twice if it falls in the darker shaded region where $\mathcal{A}_{i1}$ and $\mathcal{A}_{i2}$ overlap.*

to the boundaries of $\mathcal{T}$, and it is the failure to recognize or correct for this that leads to edge-effect bias. Hence we discriminate between interior and edge particles: for a given transect configuration, a particle is in the interior of $\mathcal{T}$ if $t_{i\bullet} = 0$ for all s on the boundary of $\mathcal{T}$ and all transect orientations permissible under the design; otherwise, it is an edge particle. Under this definition, an interior particle must be at least L units from the edge if a randomly oriented radial transect is employed; when a randomly oriented polygonal transect is used however, an interior particle may have to be further than L units from the edge (e.g., at least $\sqrt{2}\,L$ if a randomly oriented square transect is used).

For a given transect configuration and orientation, let $\mathcal{A}_{ik}$ denote the set of potential sample points, possibly outside of $\mathcal{T}$, that lead to selection of P_i along segment k (i.e., all points such that $t_{ik} = 1$). As shown in Figures 2 and 3, the area of $\mathcal{A}_{ik}$ for each k is a function of L and $w_i(\theta_k)$, where $w_i(\theta_k)$ is the width of P_i perpendicular to the kth segment (i.e., $w_i(\theta_k)$ is the maximum distance between lines tangent to P_i and parallel to segment k). Note also that the locations of the regions $\mathcal{A}_{ik}$ relative to P_i depend on the transect configuration. In a radial transect all segments are initiated at the sample point and thus $\mathcal{A}_{ik}$ will be adjacent to P_i for each k and will extend back in the direction $\theta_k + \pi$ (Figure 2). By contrast, when polygonal transects are employed some segments are neither initiated nor terminated at the sample point. As a result, when L is large relative to the dimensions of P_i, the segment-specific selection regions $\mathcal{A}_{ik}$ for a polygonal transect will be adjacent to P_i only for $k = 1$ and $k = K$; P_i and $\mathcal{A}_{ik}$ will be disjoint for segments that do not contact the sample point (Figure 3). This difference in the relative locations of the regions $\mathcal{A}_{ik}$ and P_i between radial and polygonal transect designs has material implications with regards to the performance of methods designed to eliminate edge-effect bias, as discussed below.

The inclusion region of an interior particle is the union of the K segment-specific

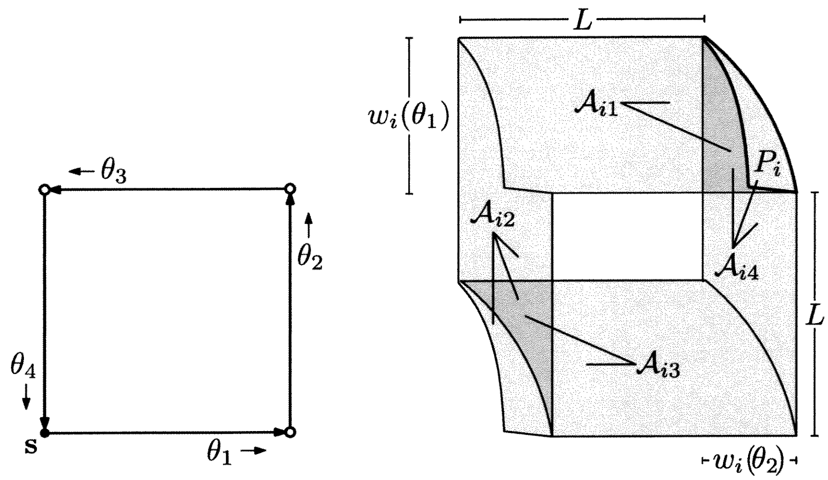

Figure 3. Inclusion region of an interior particle for a square transect with fixed orientation; P_i is selected once if s *falls in the lightly shaded regions and twice if it falls in the darker shaded regions.*

selection regions $\mathcal{A}_{ik}$. Thus, for interior particles it follows that

$$\Pr(t_{ik} = 1|\theta_k) = \Pr(\mathrm{s} \in \mathcal{A}_{ik}|\theta_k) = \frac{L\, w_i(\theta_k)}{|\mathcal{T}|}\,, \tag{3.1}$$

and if θ_1 is selected uniformly at random over $(0, 2\pi)$, then

$$\Pr(t_{ik} = 1) = \frac{L\,\mathrm{E}_\theta[w_i(\theta_k)]}{|\mathcal{T}|} = \frac{L\, c_i}{\pi\, |\mathcal{T}|}\,, \tag{3.2}$$

where c_i is the perimeter of the convex hull of P_i (Kendall and Moran 1963, p. 58). Applying these results, for interior particles we obtain

$$\mathrm{E}[t_{i\bullet}|\boldsymbol{\theta}] = \sum_{k=1}^{K} \Pr(t_{ik} = 1|\theta_k) = \frac{K\, L\, \overline{w}_i(\boldsymbol{\theta})}{|\mathcal{T}|}\,, \tag{3.3}$$

where $\overline{w}_i(\boldsymbol{\theta}) = \sum_{k=1}^{K} w_i(\theta_k)/K$, and

$$\mathrm{E}[t_{i\bullet}] = \frac{K\, L\,\mathrm{E}_\theta[\overline{w}_i(\boldsymbol{\theta})]}{|\mathcal{T}|} = \frac{K\, L\, c_i}{\pi\, |\mathcal{T}|}\,, \tag{3.4}$$

when θ_1 is selected uniformly at random from $(0, 2\pi)$. Substituting expressions (3.3) and (3.4) into Equations (2.3) and (2.4), respectively, provides the linear homogeneous estimators (Hedayat and Sinha 1991, p. 29):

$$\widehat{\tau}^{\,c} = \frac{|\mathcal{T}|}{KL} \sum_{i=1}^{N} \frac{t_{i\bullet}\, y_i}{\overline{w}_i(\boldsymbol{\theta})}\,, \tag{3.5}$$

and

$$\widehat{\tau}^{\,u} = \frac{|\mathcal{T}|\,\pi}{KL} \sum_{i=1}^{N} \frac{t_{i\bullet}\, y_i}{c_i}\,. \tag{3.6}$$

For straight-line transects, $\widehat{\tau}^{\,c}$ and $\widehat{\tau}^{\,u}$ are also the Horvitz-Thompson estimators (Horvitz and Thompson 1952) of τ when estimation is conditioned on the transect orientation, or carried out unconditionally, respectively. Equation (3.6) is commonly used in LIS surveys of woody debris under the assumption that the particles are needle-shaped and that c_i is therefore approximately twice the length of the long axis of P_i (e.g., Marshall, Davis, and LeMay 2000).

Implicit in opting for $\widehat{\tau}^{\,c}$ or $\widehat{\tau}^{\,u}$ is the choice between conditional and unconditional sampling strategies, with the design-unbiasedness of $\widehat{\tau}^{\,u}$ contingent on uniform random orientation of transects. However, for any particle attribute y_i, estimators $\widehat{\tau}^{\,c}$ and $\widehat{\tau}^{\,u}$ also differ in terms of the field measurements that must be procured. In particular, use of Equation (3.6) requires only c_i for each selected particle, which for P_i lying on $\mathcal{T}$ can be obtained by wrapping a tape about the extremities of the horizontal projection. In contrast, use of Equation (3.5) necessitates the measurement of the width of each selected particle perpendicular to every orientation spanned by the segments of a transect, regardless of which segments actually intersect these particles. Nonetheless, in some applications—for example, if P_i represents a canopy gap and τ the aggregate gap area (see Battles, Dushoff, and Fahey 1996)—the conditional estimator (3.5) might be feasible whereas (3.6) would be problematic.

4. BOUNDARY OVERLAP IN RADIAL TRANSECT DESIGNS

When radial transects are applied, and all particles are in the interior of $\mathcal{T}$, estimators (3.5) and (3.6) are conditionally and unconditionally design-unbiased, respectively. However, when both interior and edge particles are considered, results (3.1) and (3.2) do not hold. To fix ideas, consider a single-segment transect with orientation θ_1. Then an edge particle as defined earlier is located such that part of $\mathcal{A}_{i1}$ overlaps the boundary of $\mathcal{T}$ (see Figure 4). If sample points are restricted to the interior of $\mathcal{T}$ then the boundary overlap is not part of the inclusion region of P_i. In this way, boundary overlap modifies the inclusion probabilities of edge particles relative to interior particles.

For fixed transect configuration and orientation, denote by $\mathcal{S}_{ik}$ the subset of $\mathcal{A}_{ik}$ that overlaps the boundary of $\mathcal{T}$. Then,

$$\begin{aligned}
\Pr(t_{ik} = 1|\theta_k) &= \Pr\left(\mathbf{s} \in \{\mathcal{A}_{ik} \cap \mathcal{T}\}|\theta_1\right) \\
&= \frac{|\mathcal{A}_{ik}| - |\mathcal{S}_{ik}|}{|\mathcal{T}|} \\
&\leq \frac{L\,w_i(\theta_k)}{|\mathcal{T}|}, \qquad (4.1)
\end{aligned}$$

with strict inequality for edge particles. Similarly, for random θ_1,

$$\Pr(t_{ik} = 1) < \frac{L\,c_i}{\pi\,|\mathcal{T}|} \quad , \qquad (4.2)$$

for particles within L units of the edge. Therefore, if estimators (3.5) and (3.6) are used without adjustment for boundary overlap, a bias can accrue, and the estimators will tend to

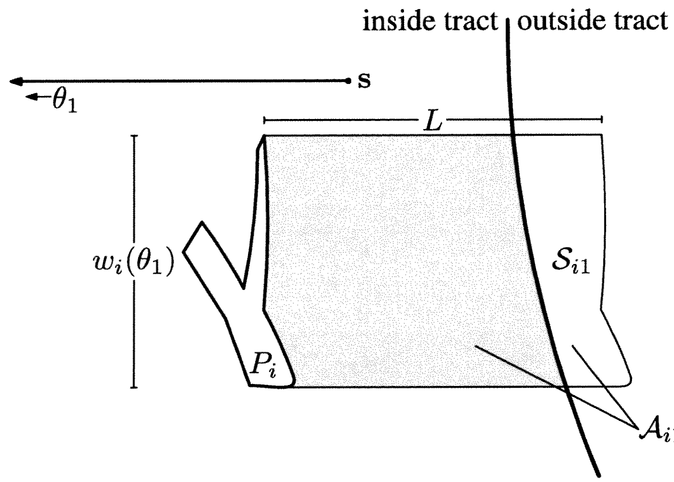

Figure 4. Inclusion region (shaded area) of an edge particle and boundary overlap, $\mathcal{S}_{i1}$, for a single-segment transect with fixed orientation.

underestimate τ. As noted by Gregoire (1982) in a plot sampling context, this edge-effect bias is attributable to the proximity of particles to the boundary of $\mathcal{T}$, and arises regardless of whether or not sample points are located near the boundary in any given survey (see also Gregoire and Scott 2004).

4.1 The Reflection Method

To eliminate edge-effect bias in LIS strategies employing straight-line transects, Gregoire and Monkevich (1994) proposed the reflection method. Applying the reflection method, if a transect segment oriented in the direction θ meets the boundary of $\mathcal{T}$ at a distance $d < L$, the segment is continued for a distance $L - d$ in the direction $\theta + \pi$ over the original segment (and possibly past s; see Figure 5). Any particles encountered along the original or reflected portions of the segment are selected into the sample according to the established protocol. That is, a particle can be selected along both the original and reflected portions of a segment and hence $t_{ik} \in \{0, 1, 2\}$. The reflection method can be viewed as a special case of the edge-correction procedure suggested by Kaiser (1983) wherein the portion of a transect segment overlapping the boundary is reflected after translating it some fixed perpendicular distance along the boundary. For expository purposes, we assume in the following that $\mathcal{T}$ is at least L units wide in the vicinity of all particles; that is, we assume that the reflected portion of a transect segment will not meet another boundary of $\mathcal{T}$. For a generalization of the method to handle narrow $\mathcal{T}$, see Barabesi (1996).

The reflection method compensates for edge-effect bias by augmenting the inclusion regions of edge particles. As indicated by Figure 6, application of the reflection method can lead to selection of P_i along the kth radial transect segment for s $\notin \mathcal{A}_{ik}$. For a given transect orientation, denote by $\mathcal{R}_{ik}$ the set of sample points that lead to selection of P_i along the reflected portion of segment k. As is evident from comparison of Figures 4 and 6, there is a connection between the subsets of a particle's inclusion region that might be truncated by the boundary ($\mathcal{S}_{ik}$) and those that can be added by application of the reflection

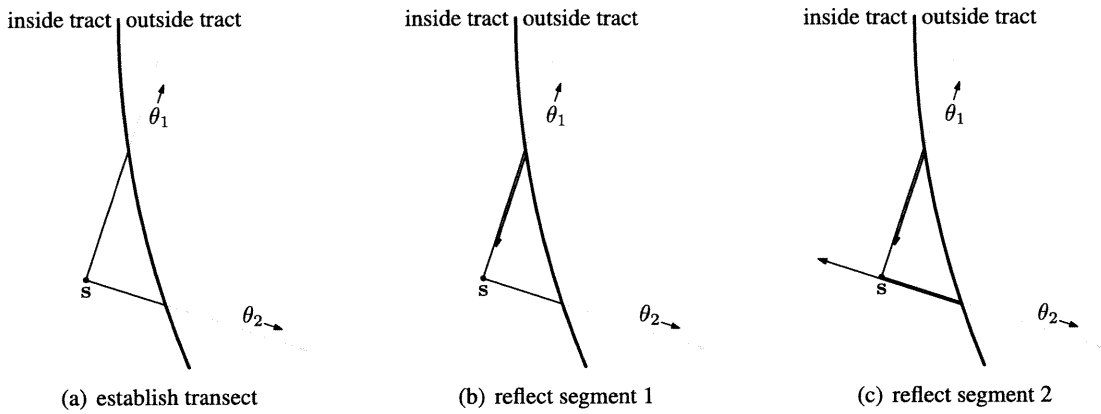

Figure 5. Implementation of the reflection method for an L-shaped transect; the portion of each segment overlapping the boundary is reflected back into the tract toward the sample point.

method ($\mathcal{R}_{ik}$). Specifically, for pairs of transect segments with opposite orientations θ_1 and $\theta_2 = \theta_1 + \pi$, $|\mathcal{S}_{i1}| = |\mathcal{R}_{i2}|$ and vice versa. Conceptually, $\mathcal{R}_{i2}$ can be obtained by rotating each narrow strip of $\mathcal{S}_{i1}$ 180° about the boundary, back into $\mathcal{T}$.

The reflection method was originally proposed for application in LIS strategies employing $K = 2$ straight-line transects (Gregoire and Monkevich 1994). It must be noted, however, that a design employing $K = 2$ straight-line transects differs from a design employing single-segment transects of length $2L$. In particular, the position of the sample point at one end rather than at the midpoint of the transect affects the shape and position of the inclusion regions of individual particles. In turn, this affects the possible extent of boundary overlap and the efficacy of sampling techniques, such as the reflection method, designed to eliminate the resultant bias. Generalizing the reflection method so that it eliminates edge-effect bias when sampling with multiply-segmented radial transects is possible for some, but not all, LIS strategies, as explained henceforth.

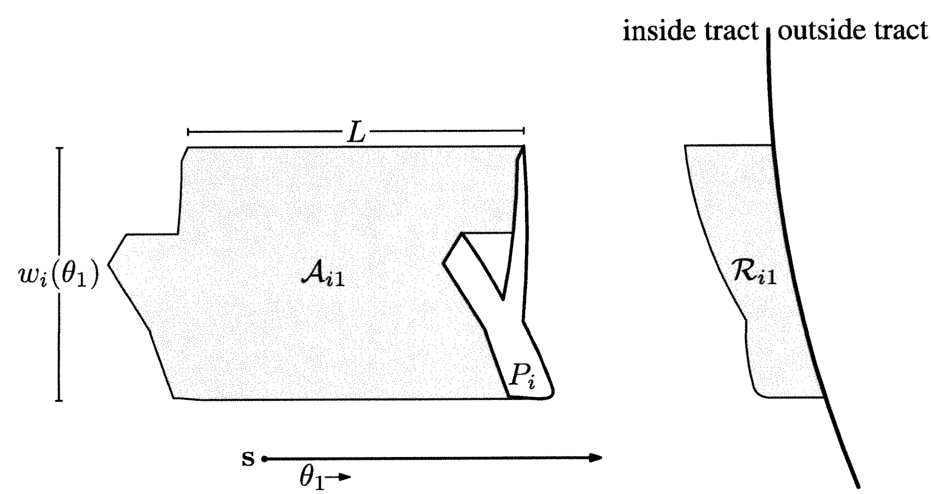

Figure 6. Inclusion region of an edge particle for a single-segment transect with fixed orientation showing additional selection region $\mathcal{R}_{i1}$ generated by the reflection method.

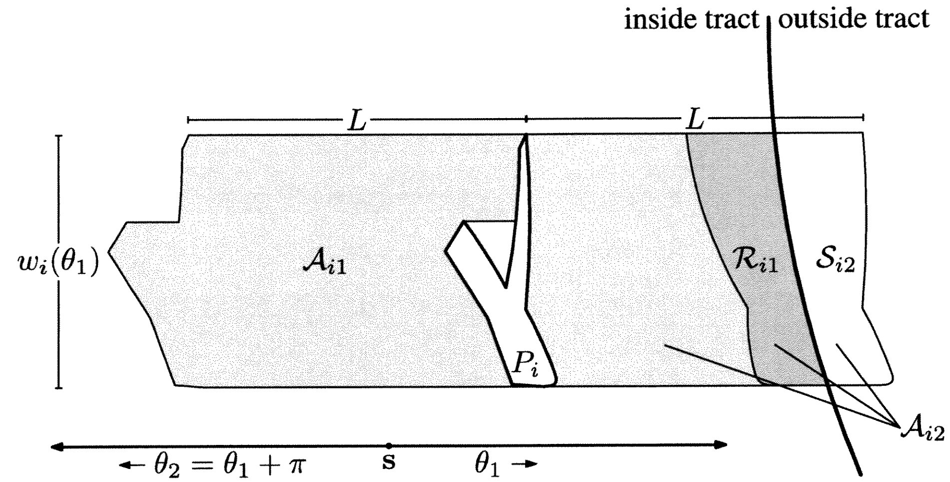

Figure 7. Inclusion region of an edge particle for a $K = 2$, straight-line transect with fixed orientation when the reflection method is applied; P_i is selected once if **s** *falls in the lightly shaded regions and twice if it falls in the darker shaded region.*

The reflection method can be used to eliminate edge-effect bias in conditional LIS strategies when symmetric radial transects are employed, where a symmetric radial transect is defined as one in which for every segment there is a segment extending an equal length in the opposite direction. Without loss of generality, take a $K = 2$, straight-line transect as illustrated in Figure 7. Applying the reflection method,

$$\begin{aligned}
\mathrm{E}[t_{i\bullet}|\boldsymbol{\theta}] &= \mathrm{E}[t_{i1}|\theta_1] + \mathrm{E}[t_{i2}|\theta_2 = \theta_1 + \pi] \\
&= \left[\Pr(\mathbf{s} \in \{\mathcal{A}_{i1} \cap \mathcal{T}\}|\theta_1) + \Pr(\mathbf{s} \in \mathcal{R}_{i1}|\theta_1)\right] \\
&\quad + \left[\Pr(\mathbf{s} \in \{\mathcal{A}_{i2} \cap \mathcal{T}\}|\theta_1) + \Pr(\mathbf{s} \in \mathcal{R}_{i2}|\theta_1)\right] \\
&= \left[\frac{|\mathcal{A}_{i1}| - |\mathcal{S}_{i1}|}{|\mathcal{T}|} + \frac{|\mathcal{R}_{i1}|}{|\mathcal{T}|}\right] + \left[\frac{|\mathcal{A}_{i2}| - |\mathcal{S}_{i2}|}{|\mathcal{T}|} + \frac{|\mathcal{R}_{i2}|}{|\mathcal{T}|}\right] \\
&= \frac{|\mathcal{A}_{i1}|}{|\mathcal{T}|} + \frac{|\mathcal{A}_{i2}|}{|\mathcal{T}|} \\
&= \frac{2\,L\,w_i(\theta_1)}{|\mathcal{T}|},
\end{aligned}$$

where the next-to-last equality follows from $|\mathcal{S}_{i1}| = |\mathcal{R}_{i2}|$ and $|\mathcal{S}_{i2}| = |\mathcal{R}_{i1}|$. These equalities ensure that by application of the reflection method Equation (3.5) will be conditionally design-unbiased provided a symmetric radial transect is employed.

By contrast, when asymmetric radial transects (e.g., L- or Y-shaped transects) are employed, the reflection method will not negate edge-effect bias in $\widehat{\tau}^c$ when inference is conditioned on the transect orientation. In fact, for fixed orientation LIS designs, application of the reflection method can impart an additional bias to Equation (3.5): the inclusion regions of edge particles on one side of $\mathcal{T}$ will be truncated by the boundary, while the inclusion regions of particles at the opposite side of $\mathcal{T}$ will be augmented (compare Figures 4 and 6). For asymmetric transect designs, the reflection method can eliminate edge-effect bias in Equation (3.5) only when $\Pr(\boldsymbol{\theta} = \boldsymbol{\theta}_0) = \Pr(\boldsymbol{\theta} = \boldsymbol{\theta}_0 + \pi\mathbf{1}_K)$; that is, when every transect orientation permissible under the design and its opposite occur with equal probability. In effect, this requirement induces in expectation a transect symmetry of the type described above. Consider again a single-segment radial transect, but with random orientation θ where $\Pr(\theta = \theta_1) = \Pr(\theta = \theta_1 + \pi) = 1/2$. Applying the reflection method,

$$\begin{aligned}
\mathrm{E}[t_{i1}] &= \mathrm{E}_\theta\big[\,\mathrm{E}[t_{i1}|\theta]\,\big] \\
&= \mathrm{E}_\theta\big[\,\Pr(\mathrm{s} \in \{\mathcal{A}_{i1} \cap \mathcal{T}\}|\theta) + \Pr(\mathrm{s} \in \mathcal{R}_{i1}|\theta)\big] \\
&= \frac{1}{2}\big[\,\Pr(\mathrm{s} \in \{\mathcal{A}_{i1} \cap \mathcal{T}\}|\theta_1) + \Pr(\mathrm{s} \in \mathcal{R}_{i1}|\theta_1)\big] \\
&\quad + \frac{1}{2}\big[\,\Pr(\mathrm{s} \in \{\mathcal{A}_{i1} \cap \mathcal{T}\}|\theta_1 + \pi) + \Pr(\mathrm{s} \in \mathcal{R}_{i1}|\theta_1 + \pi)\big] \\
&= \frac{L\,w_i(\theta_1)}{2\,|\mathcal{T}|} + \frac{L\,w_i(\theta_1 + \pi)}{2\,|\mathcal{T}|} \\
&= \frac{L\,w_i(\theta_1)}{|\mathcal{T}|}\,.
\end{aligned}$$

Thus, under restricted random orientation, $\widehat{\tau}^{\,c}$ is conditionally design-unbiased. More generally, the conditional estimator (3.5) will be design-unbiased for any radial transect configuration when inference is conditioned on the set of transect orientations referenced by the pair $(\theta_1,\ \theta_1 + \pi)$, provided each is permitted an equal probability of selection. For example, if the design specifies the use of L-shaped transects with the initial segment running north-south, then the orientations $\boldsymbol{\theta} = [0°, 90°]$ and $\boldsymbol{\theta} = [180°, 270°]$ must be selected with equal probability.

In partial summary, the reflection method eliminates edge-effect bias in Equation (3.5) for radial transect LIS designs when the transects are symmetric, either in configuration or in expectation. In the latter case, it must be recognized that conditional inference on τ is based on the randomization distribution of $\widehat{\tau}^{\,c}$ induced by the random location of sample points and the restricted randomization of transect orientations. Finally, because $\widehat{\tau}^{\,c}$ is conditionally unbiased for asymmetric radial transect designs for any pair of transect orientations ($\boldsymbol{\theta}$, $\boldsymbol{\theta} + \pi\mathbf{1}_K$), it follows that the unconditional estimator (3.6) is design-unbiased for any radial transect configuration provided θ_1 is selected uniformly at random over $(0, 2\pi)$. That is, the reflection method compensates for boundary overlap in all unconditional LIS strategies employing radial transects.

4.2 THE WALKBACK METHOD

Kaiser (1983) originally motivated conditional LIS strategies employing straight-line transects as a means to reduce among-transect variation when sampling populations exhibiting a marked orientation bias. It is not apparent to what extent this rationale would apply when sampling with transects whose segments span multiple orientations. Nonetheless, asymmetric radial transects are commonly used and in certain applications it may be neither desirable nor possible to randomize the transect orientation. For example, the FIA prescribes the use of $K = 3$ asymmetric radial transects with fixed orientation $\boldsymbol{\theta} = [0°, 135°, 225°]$ (USDA Forest Service 2002). When asymmetric radial transects are employed with prefixed orientation, or if inference is otherwise to proceed conditionally on the transect orientations

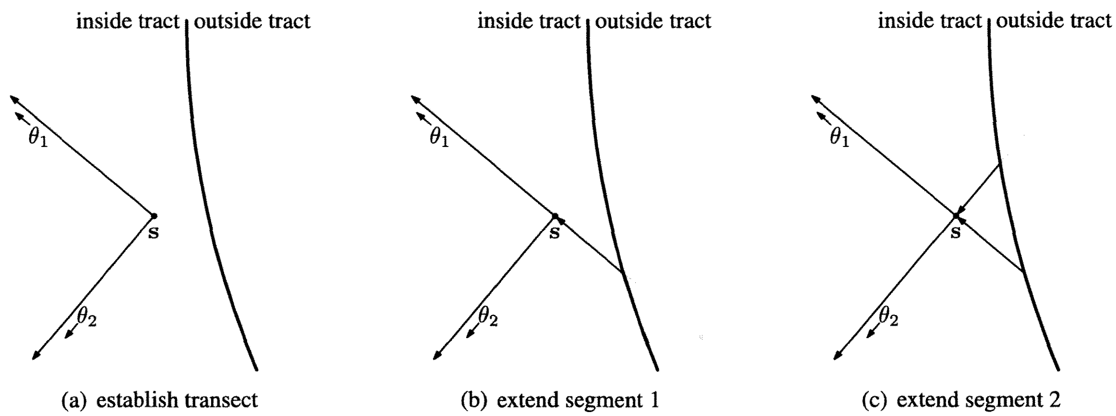

Figure 8. Implementation of the walkback method for an L-shaped transect; each segment is extended up to L units in the opposite direction if this extension intersects the boundary of $\mathcal{T}$.

used, the reflection method will not compensate for boundary overlap or eliminate edge-effect bias, and a different protocol is needed.

Here we introduce a new edge-correction protocol, termed the walkback method. The walkback method compensates for truncation of the segment-specific selection regions $\mathcal{A}_{ik}$ directly, rather than indirectly via selections along oppositely oriented segments. The method amounts to an elongation of certain segments under particular edge conditions and, like the reflection method, is implemented separately for each transect segment. When the sample point is near the boundary of $\mathcal{T}$ and segment k is oriented away from this boundary, we extend this segment in the opposite direction (i.e., $\theta_k + \pi$) by as much as L units. If, and only if, this extension intersects the boundary of $\mathcal{T}$, then we treat this intersection point as the initial point of the segment and "walk back" toward and past the original sample point, measuring all particles that are intersected by the original segment and its extension. If, on the other hand, the extension of the segment fails to reach the boundary, then the extended portion of the segment and all the particles it intersects are ignored and the sampling is conducted in the usual manner from the original sample point. Field implementation of the method is illustrated in Figure 8.

To see the implications of adopting the walkback method consider again the case of a single-segment transect with fixed orientation θ_1. Application of the walkback method augments the inclusion regions of edge particles because these particles can now be selected in cases where the sample point falls outside of $\mathcal{A}_{i1}$. Specifically, an edge particle P_i will be selected if the sample point is located (1) inside $\mathcal{A}_{i1}$, as before; or (2) outside $\mathcal{A}_{i1}$, if an extension of the segment of length L in the direction $\theta_1 + \pi$ intersects both P_i and the boundary of $\mathcal{T}$ (see Figure 9). In effect, the walkback method amounts to the translation of $\mathcal{S}_{ik}$ a distance L in the direction θ_k, creating a region $\mathcal{W}_{ik} \in \mathcal{T}$ of equal area. Note that applying this method, no particles are selected more than once by any segment of a transect because, when s $\in \mathcal{W}_{ik}$, P_i is not selected by the original segment (i.e., $t_{ik} \in \{0, 1\}$).

Assuming as before that $\mathcal{T}$ is at least L units wide in the vicinity of all particles, $|\mathcal{S}_{ik}| = |\mathcal{W}_{ik}|$ and the walkback method will secure the conditional design-unbiasedness of Equation (3.5) under asymmetric (or symmetric) radial transect designs. Inasmuch as

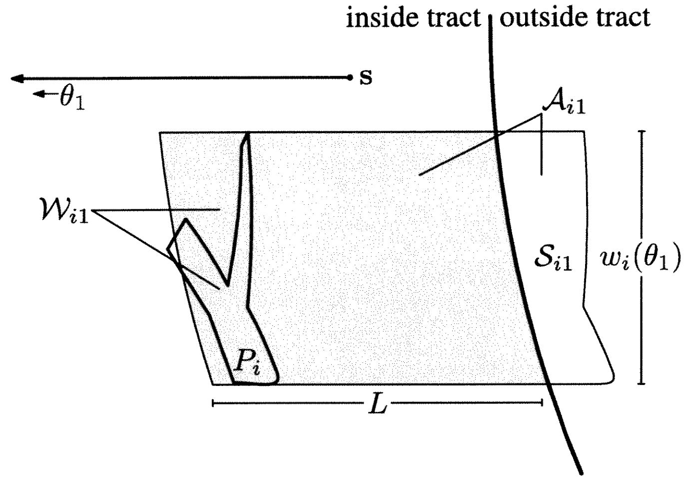

Figure 9. Inclusion region (shaded area) of an edge particle for a single-segment transect with fixed orientation when the walkback method is applied.

estimator (3.5) will be conditionally design-unbiased for any transect orientation vector $\boldsymbol{\theta}$, the walkback method will also secure the design-unbiasedness of the unconditional estimator (3.6) if θ_1 is selected uniformly at random over $(0, 2\pi)$.

5. BOUNDARY OVERLAP IN POLYGONAL TRANSECT DESIGNS

When sampling with polygonal transects, boundary overlap is more insidious and its resultant bias more difficult to eliminate by simple transect-based methods. Because the segments of a polygonal transect are established sequentially, each segment beginning where the previous segment terminates, some segments do not contact the sample point. Hence, it is possible for entire segments of a polygonal transect to fall outside of $\mathcal{T}$ and, more importantly, it is possible for the corresponding segments' selection regions ($\mathcal{A}_{ik}$) to be entirely outside of $\mathcal{T}$ for some edge particles (e.g., $\mathcal{A}_{i3} \equiv \mathcal{S}_{i3}$ in Figure 10). That is, for some edge particles and some transect orientations

$$\Pr(t_{ik} = 1 \,|\, \theta_k) = 0$$

for segments that do not contact the sample point (i.e., for $k = 2, 3, \ldots, K-1$). This limits the types of methods that can be used to mitigate boundary overlap: an impossible selection event cannot be compensated for by weighting its occurrence more heavily or by permitting multiple selections.

The spatial configuration of the inclusion and boundary overlap regions of edge particles under polygonal LIS strategies are such that the reflection and walkback methods are generally ineffective in mitigating edge-effect bias. Consider application of the reflection method. For radial transects, reflection can be carried out on a segment-by-segment basis because all the segments are initiated at the sample point and all the segment-specific selection regions are adjacent to P_i. But when segments are arranged into polygonal configurations, they cannot be individually reflected without breaking the polygonal transect

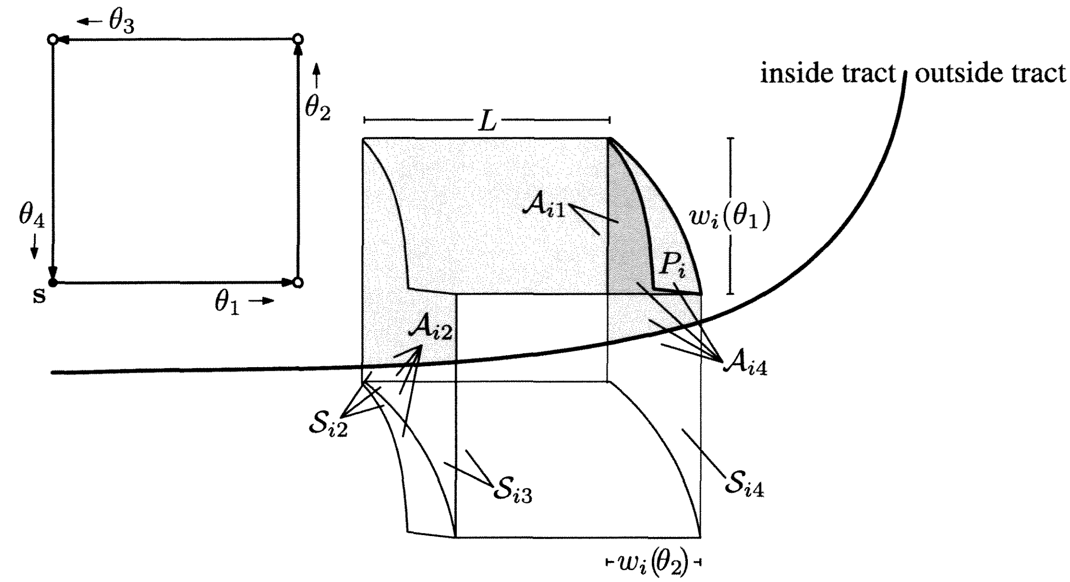

Figure 10. Inclusion region (shaded areas) of an edge particle and boundary overlap for a square transect with fixed orientation.

structure and altering the location and orientation of all successive transect segments. In turn, this would mean that subsets of an edge particle's inclusion region would not result in selection. For example, the within-tract portion of $\mathcal{A}_{i4}$ in Figure 10 would not lead to the selection of P_i along segment 4 if segment 1 were reflected at the boundary. Reflection of polygonal transect segments can partially account for boundary overlap, but also modifies the size and shape of the inclusion region.

For similar reasons, the walkback method is not applicable when polygonal transects are employed. When radial transects are used, only the shape and proximity of the boundary within the immediate neighborhood of the sample point are relevant; when sampling with polygonal transects, each segment defines a selection region whose location, and possible boundary overlap, depends on the location and orientation of the previously established segments. Thus, to account for boundary overlap it is not sufficient to implement walkback from the sample point for the first segment only—one would have to consider different walkback routes from the sample point, one for each segment of the transect. Such a technique would be impractical.

We conjecture that because of the spatial configurations of polygonal transects and of the resultant particle inclusion regions, no simple transect-based method can secure design-unbiased inference for conditional and unconditional polygonal transect LIS strategies. Instead, methods to prevent edge-effect bias for such strategies should be considered at the design stage, for example by allowing sample points to be located in an expanded region around $\mathcal{T}$ (see Masuyama 1953; Flewelling and Iles 2004).

6. DISCUSSION

Due to the variability among and within the different classes of ecological populations to which LIS is applied, it is impossible to quantify the magnitude of bias that can arise from boundary overlap. Nor are we aware of any studies, simulated or empirical, that have been carried out to assess the magnitude of this bias in LIS survey. Simulation studies carried out in the context of fixed-area plot sampling strategies have indicated that edge-effect biases can be as large as 9% of the total, depending on the plot size and attribute under consideration (Gregoire and Scott 1990, 2004; Ducey, Gove, and Valentine 2004). Those studies have also highlighted the need to consider the mean squared error (MSE) of sampling strategies employing edge-correction procedures. In some circumstances, the edge-effect bias associated with a particular sampling strategy may be more than offset by reduced variability relative to an unbiased strategy, resulting in a smaller MSE.

The design-bias caused by boundary overlap is completely eliminated by the reflection method only for LIS strategies employing symmetric radial transects when the tract of interest is sufficiently broad to allow the boundary overlap of any edge particle to be reflected back into the tract without itself overlapping another boundary of $\mathcal{T}$. That is, the tract must be at least L units wide in the vicinity of all edge particles. The same condition is required for the elimination of edge-effect bias by the walkback method. However, implementation of both the reflection and walkback methods will be complicated if $\mathcal{T}$ contains interior holes or if its boundaries have many lobes or pockets. These difficulties of implementation can be avoided altogether by sampling beyond the boundaries of $\mathcal{T}$ using the procedures described by Masuyama (1953) and Flewelling and Iles (2004). Unfortunately, sampling in an expanded region around and including the tract of interest, a subset of which will necessarily have a relatively low or null population attribute density, will lead to an inflation of the MSE. In addition, it is not always possible to sample outside of $\mathcal{T}$ when its edges represent physical boundaries such as water bodies or cliffs.

The reflection and walkback methods do not require sampling outside of $\mathcal{T}$ and are relatively simple to apply under most edge conditions. In terms of the requisite field work, the reflection method will generally be preferred where both methods are applicable: when symmetric radial transects are employed, the two methods demand approximately the same amount of field work; but when randomly oriented, asymmetric radial transects are used, the reflection method will typically require less walking and the measurement of fewer particles. Also, inasmuch as the reflection method permits multiple selections of the same particle along the same segment (i.e., $t_{ik} \geq 1$) while the walkback method results in differential segment lengths, these methods will result in distinct sampling errors for $\widehat{\tau}^{\,c}$ and $\widehat{\tau}^{\,u}$. For the same reasons, both methods may lead to larger MSEs relative to unbiased edge-correction methods that maintain a constant transect length (e.g., Kaiser's 1983 method; Stevens and Urquhart 2000). Research on the relative accuracies and efficiencies of LIS strategies employing different edge-correction procedures is definitely needed.

The fact that the reflection method can eliminate edge-effect bias for both conditional and unconditional LIS strategies employing symmetric radial transects, whereas the application of asymmetric transects (radial or polygonal) requires more involved field procedures, raises the important question of whether the latter type of transect should be used at all. The adoption of asymmetric transects has often been prompted by the observation that in many natural populations the particles exhibit a marked "orientation bias" (van Wagner 1968; Bell, Kerr, McNickle, and Woollons 1996). For example, the distribution of woody debris can be strongly influenced by forest harvesting methods, to the extent that the particles are oriented primarily along the same axes (Warren and Olsen 1964).

When inference is based on a presumed spatial Poisson model for the population, orientation bias is problematic. The use of transects with segments spanning multiple directions may reduce, but cannot eliminate, model-based inferential errors that result from orientation bias (see van Wagner 1968; de Vries 1979; Bell et al. 1996). However, as noted previously, design-based inference does not rely on assumptions concerning the distribution and orientation of particles; only the distribution and orientation of transects is relevant. Particle orientations cannot effect a design-bias, although the perception to the contrary is pervasive. But even under this perception, there is no compelling reason for using polygonal transects: an X-shaped transect can provide the same directional orientations as a square transect, and a Y-shaped transect the same three directions as a triangle or hexagon. Moreover, adopting a design-based approach to inference, information on the orientation of particles can be exploited to increase the precision of the estimated total by sampling with straight-line transects and conditioning on the transect orientation (Kaiser 1983; Muttlak and McDonald 1992; Gregoire, Affleck, and Valentine 2005). From this perspective, it is difficult to see the advantage of using anything other than a straight-line transect centered at the sample point. Nonetheless, if asymmetric transects must be used, then on the basis of the ease with which edge effects can be handled, the segments should be arranged in a radial, rather than polygonal, configuration and randomly oriented.

ACKNOWLEDGMENTS

This research was supported by funds from the USDA Forest Service, Northeastern Research Station, RWU-4104, through a cooperative agreement with the School of Forestry and Environmental Studies at Yale University.

[Received December 2004. Revised July 2005.]

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