A volume and taper prediction system for bald cypress

BERNARD R. PARRESOL
Southern Forest Experiment Station, United States Department of Agriculture, Forest Service, Room T-10210, U.S Postal Service Bldg., 701 Loyola Ave., New Orleans, LA, U.S.A. 70113

AND

JAMES E. HOTVEDT AND QUANG V. CAO
School Forestry, Wildlife, and Fisheries, Louisiana Agricultural Experiment Station, Louisiana State University Agricultural Center, Baton Rouge, LA, U.S.A. 70803

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A volume and taper prediction system based on d_{10} and consisting of a total volume equation, two volume ratio equations (one for diameter limits, the other for height limits), and a taper equation was developed for bald cypress using sample tree data collected in Louisiana. Normal diameter (dn), a subjective variable-height measure applied to bald cypress (*Taxodium distichum* (L.) Rich.) in place of diameter at breast height (dbh), was found to be inferior to five different fixed-height diameter measurement points in terms of predicting total volume. Diameter measured at 10 ft (3.0 m) above the ground, termed d_{10} is recommended as a better diameter measurement point for bald cypress. A number of "goodness-of-fit" statistics were employed to evaluate alternative functions for predicting volume and taper. Two statistics, bias and sum of squared relative residuals, provided the best discrimination between functions.

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Un système de prédiction du volume et du défilement basé sur d_{10} et consistant en une Cquation du volume total, deux Cquations quotients de volume (l'une pour les diamètres limites, l'autre pour les hauteurs limites) et une equation de défilement sont développés pour le cyprès chauve à partir de mesures effectuées sur des arbres tchantillons en Louisiane. Le diamètre normal (dn) qui est une mesure subjective de hauteur variable appliquée au cyprès chauve (*Taxodium distichum* (L.) Rich.) à la place du dhp, a été trouvé à cinq différentes points de mesures du diamètre à hauteur fixe en terme de prédiction de volume total. Le diambtre mesuré à 10 pieds (3 m) au-dessus du sol, diamtre que l'on nomme d_{10}, est recommandé comme le meilleur point de mesure du diamètre pour le cyprès chauve. Des tests statistiques d'ajustement des données ont été employ& pour évaluer différentes fonctions prédictrices du volume et du dtfilement. Deux paramètres statistiques, le biais et la somme des carrés des résidus relatifs se sont avérés les meilleurs critères discriminants les fonctions.

Introduction

Multiple product inventories require accurate estimates of product sizes (diameters and lengths) and volume. An accurate prediction system for volume and taper of bald cypress (*Taxodium distichum* (L.) Rich.) does not exist, possibly because the nature of bald cypress growth has imposed significant problems to mensurationists trying to estimate taper and volume (Husch et al. 1972).

Bald cypress shows considerable variation in the butt region. Cypress typically grows in flood-prone areas and permanent swamps. As a response to this environment, fluted basal swells are formed (Mattoon 1915; Kurz and Demaree 1934). Thus, the usual practice of measuring diameter at 4.5 ft (1.4 m), called diameter at breast height (dbh), for tree volume estimation is meaningless because buttress dimensions will usually have no consistent relation to the quantity of wood in the tree (Husch et al. 1972).

Though recently researchers have successfully used dbh to predict green and dry weights of bald cypress (Swindel et al. 1982; Conde et al. 1979), these cypress occurred as a minor stand component on a pine flatwood site and were not typical of swamp-grown bald cypress. The present study deals with cypress occurring in pure stands or as a predominant stand species.

As a substitute for dbh, measurement of volume on cypress and other swamp species such as tupelo (Nyssa sp.) is currently based on diameter measured 18 in. (46 cm) above pronounced butt swelling (Forbes 1955; Avery and Burkhart 1983). The underlying assumption for this practice is that stem diameter just above the swell is approximately equivalent to what the diameter would be at breast height if the buttress were not present. Consequently, this diameter measure is termed normal diameter (dn) .

The use of dn as a substitute for dbh is questionable. It is not certain that dn is equivalent to what dbh might be without the buttress. More importantly, individual foresters might disagree on the point where butt swell ceases and, therefore, determine different values for dn. To determine if a constant, fixed-height diameter can better estimate total stem volume, volumes based on diameters measured at 6 ft (d_6) to 11 ft (d_{11}) (1.8 m to 3.4 m) above the ground were compared with volumes based on dn. Three foresters were periodically rotated in measuring dn on the sample trees to minimize the effects of one individual's judgement on dn.

A reliable and flexible system providing tree bole description and capable of calculating volume to any standard of utilization is needed for bald cypress. The objectives of this paper are to determine the best possible equations for estimating volume and taper. A system composed of a total volume equation, two volume ratio equations (one for diameter limits, the other for height limits), and a taper equation was chosen over a compatible taper equation system (as defined by Demaerschalk 1973). Compatible taper equations were found to be less precise in volume and diameter estimation when compared with volume

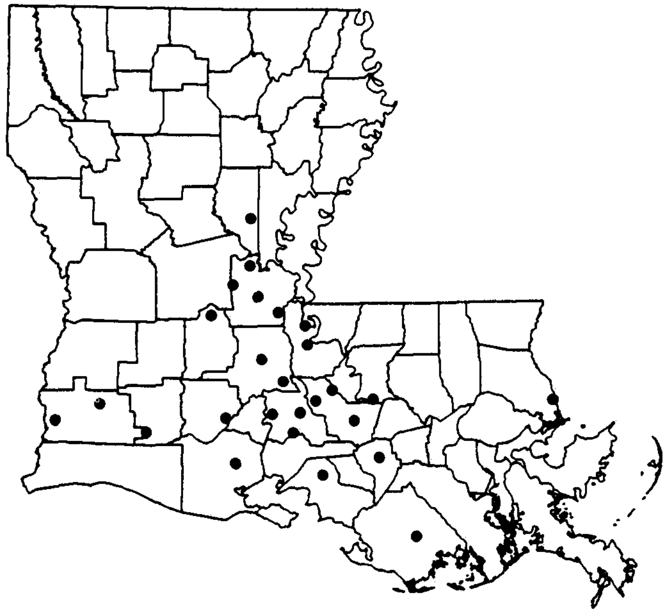

FIG. 1. Locations of the 26 bald cypress sample sites in Louisiana.

ratio equations and noncompatible taper equations (Cao et al. 1980).

Data

Taper data were collected on 157 trees from 26 sites (25 sites with 6 trees and 1 site with 7) located throughout the South Delta region of Louisiana (Fig. 1). Tree normal diameters ranged from 6 to 24 in. (15 to 61 cm), and total tree heights ranged from 46 to 103 ft (14 to 3 1 m) . The sample trees were felled and total height was measured to the nearest 0.1 ft (0.03 m). Diameter and bark thickness measurements to the nearest 0.1 in. (2.5 mm) were measured at dn; at 1, 3, 5, 6, 7, 8, 9, 10, 11, 13, 15, and 17 ft (0.3, 0.9, 1.5, 1.8, 2.1, 2.4, 2.7, 3.0, 3.4, 4.0, 4.6, and 5.2 m); and at every 4ft (1.2 m) thereafter throughout the remainder of the stem. The portion of the bole exhibiting flutes was cross-sectioned at the appropriate heights, and solid-wood diameters were measured by inscribing with an expandable hoop the largest possible circle (including bark) inside the flutes. Inside-bark (ib) and outside-bark (ob) volumes for each bolt were calculated using Smalian's formula. Bolt volumes were summed treating the stump as a cylinder and the last section as a cone to obtain total tree volume (ib and ob). The sample trees were stratified by l-in. (2.5-cm) dn classes and 20% in each class were held out at random to create a validation data set.

Notation

The following notation will be used.

- a_i = join points of the segmented equations
- b_i = regression coefficients
- d = top diameter (ib or ob) in inches (cm) at height h
- D = diameter measure in inches (cm)
- exp = base of the natural logarithm
- h = height above the ground to top diameter d, or height to limit of utilization, in feet (m)
- H = total height in feet (m)
- K = π/576 for converting diameter squared in inches to basal area in square feet, or $\pi/40\ 000$ for converting diameter squared in centimeters to basal area in square meters
- ln = natural logarithm
- p = $H - h$, distance from the tree tip to top diameter d or to limit of utilization
- R = v/V, ratio of merchantable to total volume (ib or ob)
- tan = tangent of an angle
- v = cubic-foot (cubic-metre) volume to some top diameter or height limit
- V = total cubic-foot (cubic-metre) volume
- x = for volume ratio equations, or h/H for taper equations
- z = $(H - h)/H$, relative tree height from the tip to top diameter d, or the proportion of tree height that is unmerchantable

Equations

Diameter measurement points and total volume equations

Various combinations of diameter measurement points and volume prediction functions were tested. Forbes (1955) examined diameter measurement of cypress and stated it should occur as much as 7 to 11 ft (2.1 to 3.4 m) above the ground. Consequently, diameters tested in this study were dn, d_6, d_7, d_8, d_9, d_{10}, and d_{11}. Total cubic volume equations tested were the combined variable, weighted combined variable, comprehensive, Schumacher and Hall's (1933), and Honer's (196.5) equations. These functions are presented in Table 1.

Volume ratio equations

Utilization constraints can be expressed in terms of a diameter limit or a height limit. Therefore, volume ratio functions were fitted for both constraints. Volume ratio equations tested (Table 2) were Burkhart's (1977), Cao and Burkhart's (1980), Van Deusen et al.'s (1981), Cao et al.'s (1980) and Matney and Sullivan's (1980). Function [8] is Van Deusen et al.'s (1981) equation modified by placing a separate exponent on the numerator and denominator of the independent variable *(d/D).* Function [12] is Van Deusen et al.'s (1981) equation modified for height limits by substituting z (relative height) for x (relative diameter). Function [13] is the three-parameter function [8] modified for height limits by substituting *(p/H)* for *(d/D).* Function [15] is Matney and Sullivan's (1980) exponential model modified to handle ratio predictions to any height limit by substituting z for x. A total of 10 volume ratio equations were tested, 5 for diameter limits and 5 for height limits.

Taper equations

Many taper functions, from single equation forms to segmented forms, have been proposed. Functions selected for study (Table 3) were Demaerschalk's (1973) and Ormerod's (197 1) single equation forms, Max and Burkhart's (1976) and Cao et al's (1980) segmented-polynomial equations, and Bennett et al.'s (1978) two-segment taper equation.

An additional function was derived. A cubic-cubic segmented-polynomial function (eq. [21]) was developed by grafting two cubic subfunctions at one join point.

Evaluating equation performance

Total cubic volume equations

These functions were evaluated over the selected fixed-height diameters and dn to determine the "best" combination for diameter measurement point and volume equation. The "fit" statistics used to determine how well the regression functions fit the sample data were (1) coefficient of determination (R^2), (2) root mean square error ($S_{y \cdot x}$), and (3) the sum of squared relative residuals (SSRR). The "validation" statistics used to determine how well the regression functions perform on the independent data, which represents the population, were (1) bias ($\bar{D}$), (2) mean absolute deviation ($|\bar{D}|$), (3) standard deviation of differences (So), and (4) SSRR. Computational formulas for the above fit and validation statistics are given in Table 4.

Because absolute variation of volume per tree increases with tree size, measures of precision such as $S_{y \cdot x}$ and So are not reliable indicators of variation across size classes. The SSRR statistic is used to relate the size of each residual to its observed value. If variance is stable within each size class, the squared ratios developed for each observation should tend towards uniformity of size within each size class. This will result in lower sums of squared ratios being associated with functions producing more uniform variances within and across each size class.

Values of the fit and validation statistics were compared and a rank was assigned to each equation under each criterion. Rank No. 1 corresponded to the "best" value for each statistic, rank No. 2 to the

TABLE 1. Total cubic volume equations selected for bald cypress study

	Eq. No.	Equation
Combined variable	[1]	$b_0 + b_1D^2H$
Weighted combined variable	[2]	$V/D^2H = b_0/D^2H + b_1$
Schumacher and Hall 1933	[3]	$\ln(V) = \ln(b_1) + b_2\ln(D) + b_3\ln(H)$
Comprehensive	[4]	$V = b_0 + b_1D + b_2DH + b_3D^2$
		$+ b_4H + b_5D^2H$
Honer 1965	[5]	$D^2/V = b_0 + b_1/H$

TABLE 2. Volume ratio equations selected for bald cypress study

	Eq. No.	Equation
	Diameter limit equations	
Burkhart 1977	[6]	$R = 1 + b_1(d^{b2}/D^{b3})$
Van Deusen et al. 1981	[7]	$R = \exp(b_1x^{b2})$
Modified Van Deusen et al. 1981	[8]	$R = \exp[b_1(d^{b2}/D^{b3})]$
Cao et al. 1980	[9]	$R = 1 + b_1x + b_2x^2 + b_2x^3 + b_4x^4 + b_5x^5 + b_6x^6$
Matney and Sullivan 1980	[10]	$R = 1 - [1 \quad \exp(-b_1\tan(b_2H^{b3}x))]^{b4}$
	Height limit equations	
Cao and Burkhart 1980	[11]	$R = 1 + b_1(p^{b2}/H^{b3})$
Modified Van Deusen et al. 1981	[12]	$R = \exp(b_1z^{b2})$
Modified Van Deusen et al. 198 1	[13]	$R = \exp[b_1(p^{b2}/H^{b3})]$
Cao et al. 1980	[14]	$R = 1 \quad z + b_2(z^2 - z) + b_3(z^3 - z) + b_4(z^4 - z)$
		$+ b_5(z^5 - z) + b_6(z^6 - z)$
Modified Matney and Sullivan 1980	[15]	$R = 1 - [1 - \exp(-b_1\tan(b_2H^{b3}z))]^{b4}$

TABLE 3. Taper equations selected for bald cypress study

	Eq. No.	Equation
Max and Burkhart 1976	[16]	$d^2/D^2 = b_1(x - 1) + b_2(x^2 - 1) + b_3(a_1 - x)^2I_1 + b_4(a_2 - x)^2I_2$
		where $I_i = \begin{cases} 1, \text{ if } x \le a_i \\ 0, \text{ if } x > a_i \end{cases} \quad i = 1, 2$
Demaerschalk 1973	[17]	$d = b_1D^{b2}p^{b3}H^{b4}$
Ormerod 197 1	[18]	$d = D[(H - h)/(H - i)]^{b1}$
		where i = height to D
		D = Dob, if d is diameter ob, Dib, if d is diameter ib,
		Dib = $b_0 + b_1$Dob
Cao et al. 1980	[19]	$d^2KH/V - 2z = b_1(3z^2 - 2z) + b_2(z - a_1)^2I_1 + b_3(z - a_2)^2I_2$
		where $I_i = \begin{cases} 1, \text{ if } z \ge a_i \\ 0, \text{ if } z < a_i \end{cases} \quad i = 1, 2$
		$D\left[\frac{h}{i}\right]^{b1}$, if $h \le i$,
Bennett et al. 1978	[20]	$d = D\left[\frac{H-h}{H-i}\right] + b_2\left[\frac{(H-h)(h-i)}{H^2}\right] + b_3\left[\frac{D(H-h)(h-i)}{H^2}\right]$
		$+ b_4\left[\frac{D^2(H-h)(h-i)}{H^2}\right] + b_5\left[\frac{(H-h)(h-i)(2H-h-i)}{H^3}\right]$, if $i \le h \le H$
		where i = height to D
		D = Dob, if d is diameter ob, Dib, if d is diameter ib,
		Dib = $b_0 + b_1$Dob
Cubic-cubic	[21]	$d^2/D^2 = z^2(b_1 + b_2z) + (z - a)^2[b_3 + b_4(z + 2a)]I$
		where $I = \begin{cases} 1, \text{ if } z \geqq a \\ 0, \text{ if } z < a \end{cases}$

TABLE 4. Criteria for evaluating total cubic volume functions

Fit statistics-sample data
(1) Coefficient of determination: $R^2 = 1 - \Sigma(V_i - \hat{V}_i)^2/\Sigma(V_i - \bar{V})^2$
(2) Root mean square error: $S_{y \cdot x} = \sqrt{\Sigma \mathrm{Diff}_i^2/(N - P)}$
(3) Sum of squared relative residuals: $\mathrm{SSRR} = \Sigma(\mathrm{Diff}_i/V_i)^2$
Validation statistics-independent data
(1) Bias: $\bar{D} = \Sigma \mathrm{Diff}_i/N$
(2) Mean absolute deviation: $\lvert\bar{D}\rvert = \Sigma \lvert \mathrm{Diff}_i \rvert /N$
(3) Standard deviation of differences: $S_D = \sqrt{\Sigma(\mathrm{Diff}_i - \bar{D})^2/(N - 1)}$
(4) Sum of squared relative residuals: defined as above

NOTE: $\mathrm{Diff}_i = V_i - \hat{V}_i$, observed minus predicted volume on the ith tree; N = number of observations (trees); P = number of parameters for each model.

TABLE 5. Criteria for evaluating volume ratio equations and taper equations on the sample and independent data

Fit and validation statistics
(1) Bias: $\bar{D} = \Sigma\Sigma \mathrm{Diff}_{ij}/N$
(2) Mean absolute deviation: $\lvert\bar{D}\rvert = \Sigma\Sigma \lvert \mathrm{Diff}_{ij}\rvert/N$
(3) Standard deviation of differences: $S_D = \sqrt{\Sigma\Sigma(\mathrm{Diff}_{ij} - \bar{D})^2/(N - 1)}$
(4) Sum of squared relative residuals: $\mathrm{SSRR} = \Sigma\Sigma(\mathrm{Diff}_{ij}/v_{ij})^2$

NOTE: $\mathrm{Diff}_{ij} = v_{ij} - \hat{v}_{ij}$, volume deviation on *ith* tree to top of jth bolt, or $\mathrm{Diff}_{ij} = d_{ij} - \hat{d}_{ij}$, diameter deviation on ith tree at top of jth bolt; N = total number of observations (bolts).

next best value of each statistic and so on. If under a criterion values were equal, the mean of the corresponding ranks was assigned.

Rank sum ib and rank sum ob were computed for each of the volume equations by adding the sum of the ranks under the fit and under the validation statistics. The lower the rank sums the better the equation. The lowest rank sum corresponded to the "best" function. Additionally, plots of the function values against the independent data were employed to evaluate function performance.

Volume ratio equations

Identical fit and validation statistics were used to evaluate the inside and outside bark volume ratio equations on the sample and independent data sets. They were (1) bias, (2) mean absolute deviation, (3) standard deviation of differences, and (4) sum of squared relative residuals. These statistics (Table 5) were computed using the predicted ($\hat{v} = R\hat{V}$) and the observed merchantable volumes (v) to the top of each successive bolt for each tree. Predicted total cubic volume ($\hat{V}$) was calculated from the diameter measurement point-volume function combination selected in the previous step. A ranking procedure identical to that used in evaluating the volume equations was employed in the analysis of the volume ratio equations in addition to plots against the independent data.

Taper equations

To evaluate the taper functions, observed and predicted diameters at the top of each bolt were compared. Statistics used in the evaluation were identical to the ones used in the evaluation of the volume ratio equations (Table 5). A ranking procedure identical to that used on the total cubic volume equations was employed, as well as plots against the independent data.

TABLE 6. Values of the statistics on five bald cypress total volume equations using diameter measured at 10 ft (3.0 m)

Statistic	Equation No. [1]	[2]	[3]	[4]	[5]
	Sample data-inside bark				
R^2	0.978	0.968	0.981	0.981	0.973
$S_{y \cdot x}$[a]	3.823	4.621	3.638	3.647	4.304
	(0.108)	(0.131)	(0.103)	(0.103)	(0.122)
SSRR	4.544	1.653	1.438	1.745	1.868
	Independent data-inside bark				
$\bar{D}$[a]	0.553	-0.229	0.737	0.727	-0.006
	(0.016)	(-0.006)	(0.021)	(0.020)	(-0.0002)
$\lvert\bar{D}\rvert$[a]	3.154	3.093	2.785	2.888	3.084
	(0.089)	(0.087)	(0.079)	(0.082)	(0.087)
S_D[a]	4.480	4.812	4.371	4.507	4.498
	(0.127)	(0.136)	(0.124)	(0.128)	(0.127)
SSRR	1.412	0.429	0.399	0.566	0.519

[a]Values in cubic feet (cubic metres).

Results and discussion

Total volume equations

The "best" total volume equation, regardless of diameter measurement point used, was Schumacher and Hall's (1933) logarithmic function (given as eq. [3] in Table 1). This function had the lowest rank sum (ib and ob) of all volume equations tested over all diameter measurement points tested. To illustrate, values of the fit and validation statistics using d_{10} are listed for each volume ib function in Table 6. Rank sums and overall ranks are presented in Table 11. Response patterns were similar using the other diameter measurement points.

Table 7 lists values of the statistics for function [3] for each diameter measurement point. The fit and validation statistics, except for bias ($\bar{D}$), steadily improved as diameter measurement point was higher on the tree and were better using d_7 through d_{11} than they were using dn.

The objective is to find the lowest possible fixed-height diameter measurement point and still achieve good accuracy and precision. Approximately 95% of the sample trees had butt swells ending at 9.5 ft (2.9 m) or less, with the average occurring at 5.7 ft (1.7 m). This is an important consideration in choosing a fixed-height diameter measurement point for bald cypress because buttress flutes prevent accurate measure of solid-wood diameter in standing trees.

Based on the improvement in estimates with increasingly higher fixed-height diameters and the 95% swell height criterion, d_{10} and d_{11} were considered preferable over d_7 through d_9. Individual tree volume predictions were similar using d_{10} and d_{11}; hence d_{10} was chosen as the most appropriate fixed-height diameter measurement point since it provided similar results and should be somewhat easier to measure in practice.

Finally, function [3] was evaluated using diameters measured at d_{10} and dn. Relative to dn, use of d_{10} resulted in substantial improvements in many of the statistical values. Review of Table 7 shows R^2 increasing from 0.93 to 0.98, $S_{y \cdot x}$ dropping from 6.7 ft^3 (0.19 m^3) to 3.6 ft^3 (0.10 m^3), S_D dropping from 5.3 ft^3 (0.15 m^3) to 4.4 ft^3 (0.12 m^3), and $\lvert\bar{D}\rvert$ dropping from 4.06 ft^3

TABLE 7. Values of statistics computed for different diameter measurement points using the logarithmic volume equation

Statistic	Diameter measurement point						
	d_6	d_7	d_8	d_9	d_{10}	d_{11}	dn
			Sample data-inside bark				
R^2	0.914	0.943	0.964	0.975	0.981	0.983	0.934
$S_{y \cdot x}$[a]	7.659 (0.217)	6.219 (0.176)	4.916 (0.139)	4.121 (0.117)	3.638 (0.103)	3.443 (0.097)	6.709 (0.190)
SSRR	2.598	2.326	1.977	1.616	1.438	1.255	3.210
			Independent data-inside bark				
$\bar{D}$[a]	0.889 (0.025)	1.258 (0.036)	1.410 (0.040)	1.494 (0.042)	0.737 (0.021)	0.279 (0.008)	-0.172 (-0.005)
$\lvert\bar{D}\rvert$[a]	4.316 (0.122)	3.758 (0.106)	3.309 (0.094)	3.172 (0.090)	2.785 (0.079)	2.323 (0.066)	4.059 (0.115)
S_D[a]	6.564 (0.186)	6.203 (0.176)	5.854 (0.166)	5.772 (0.163)	4.371 (0.124)	3.735 (0.106)	5.263 (0.149)
SSRR	0.727	0.615	0.477	0.472	0.399	0.251	0.874

[a]Values in cubic feet (cubic metres).

TABLE 8. Values of the statistics on five bald cypress volume ratio equations to top diameter limits

Statistic	Equation No.					
	[6]	[7]	[8]	[9]	[10]	Mean[a]
			Sample data-inside bark			
$\bar{D}$[b]	0.663 (0.019)	-0.395 (-0.011)	0.377 (0.011)	-0.657 (-0.019)	-0.213 (-0.006)	0.461 0.013
$\lvert\bar{D}\rvert$[b]	4.305 (0.122)	3.613 (0.102)	3.382 (0.096)	3.525 (0.100)	3.357 (0.095)	3.636 0.103
S_D[b]	6.467 (0.183)	5.521 (0.156)	4.944 (0.140)	5.331 (0.151)	4.972 (0.141)	5.447 0.154
SSRR	16479.2	13199.3	10948.1	12739.6	11431.4	12960
			Independent data-inside bark			
$\bar{D}$	0.822 (0.023)	-0.012 (-0.001)	0.656 (0.019)	-0.258 (-0.007)	-0.054 (-0.002)	0.360 0.010
$\lvert\bar{D}\rvert$	4.063 (0.115)	3.300 (0.093)	3.216 (0.091)	3.244 (0.092)	3.044 (0.086)	3.373 0.095
S_D	6.004 (0.170)	5.077 (0.144)	4.865 (0.138)	5.001 (0.142)	4.683 (0.133)	5.126 0.145
SSRR	2774.4	1705.7	1470.6	1734.5	1683.4	1874

[a]Computed on absolute values.
[b]Values in cubic feet (cubic metres).

(0.12 m^3) to 2.78 ft^3 (0.08 m^3). Similar improvements occurred for the outside bark data. These analyses indicate that use of d_{10} in total volume estimation provides volume estimates superior to those from dn. Consequently, d_{10} is recommended as the "best" diameter measurement point for bald cypress. For all subsequent functions tested, d_{10} was the input for diameter. For trees with swell heights over 9.5 ft (2.9 m), dn was used.

Volume ratio equations

Values of the fit and validation statistics on the volume ratio ib equations are presented in Tables 8 and 9. These tables show that the $\bar{D}$ statistic provides a strong discrimination between equations, as does SSRR. This can be more easily seen by comparing the individual values against the mean value for each statistic. For example, under the independent data section in Table 8, the bias for eq. [6] is 2.3 times greater than the mean value whereas the bias for eq. [10] is 1/6 the size of the mean value. Under the sample data section in Table 9, SSRR for eq. [12] is 2.2 times greater than the mean but SSRR for eq. [11] is 1/3 the size of the mean SSRR. A clear distinction between

TABLE 9. Values of the statistics on five bald cypress volume ratio equations to height limits

Statistic	Equation No. [11]	[12]	[13]	[14]	[15]	Mean[a]
			Sample data-inside bark			
$\bar{D}$[b]	-0.087	-0.607	-0.349	-0.241	0.018	0.260
	(-0.002)	(-0.017)	(-0.010)	(-0.007)	(0.001)	0.007
$\lvert\bar{D}\rvert$[b]	2.108	2.480	2.412	2.139	2.129	2.254
	(0.060)	(0.070)	(0.068)	(0.061)	(0.060)	0.064
S_D[b]	2.849	3.341	3.241	2.920	2.891	3.048
	(0.081)	(0.095)	(0.092)	(0.083)	(0.082)	0.087
SSRR	326.4	2142.6	1975.8	114.2	324.4	976.7
			Independent data—inside bark			
$\bar{D}$	-0.036	-0.446	-0.282	-0.122	0.066	0.190
	(-0.001)	(-0.013)	(-0.008)	(-0.003)	(0.002)	0.005
$\lvert\bar{D}\rvert$	1.998	2.385	2.286	2.043	2.012	2.145
	(0.057)	(0.067)	(0.065)	(0.058)	(0.057)	0.061
S_D	3.114	3.500	3.393	3.198	3.147	3.270
	(0.088)	(0.099)	(0.096)	(0.091)	(0.089)	0.093
SSRR	82.8	492.2	479.1	24.4	78.1	231.3

[a]Computed on absolute values.
[b]Values in cubic feet (cubic metres).

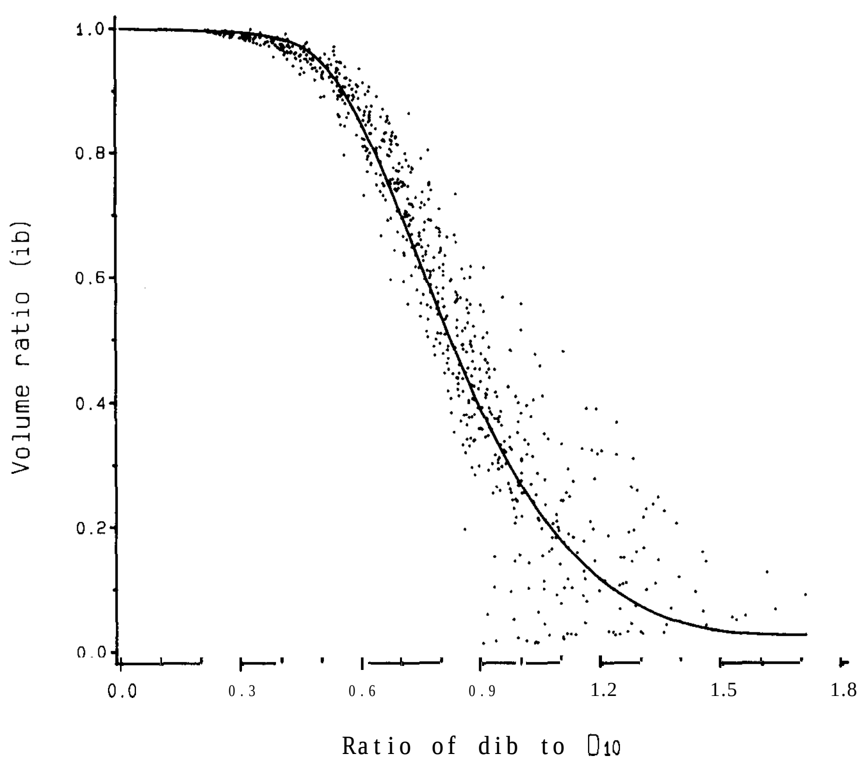

FIG. 2. Observed and predicted volume ratio values to top diameter limits for the independent data set.

equations based on $\bar{D}$ and SSRR can be seen in these examples and elsewhere in the tables. The magnitudes of differences between $\lvert\bar{D}\rvert$ values across equations and S_D values across equations tended to be small in Tables 8 and 9. It should be noted that the ob patterns mimicked the ib patterns displayed in Tables 8 and 9.

Overall ranks of the volume ratio equations are given in Table 11. Matney and Sullivan's (1980) exponential function (given as eq. [10] in Table 2) gave the best merchantable volume predictions to top diameter limits. Cao and Burkhart's (1980) power function (given as eq. [11] in Table 2) gave the best merchantable volume predictions to height limits.

TABLE 10. Values of the statistics on six bald cypress taper equations

Statistic	Equation No. [16]	[17]	[18]	[19]	[20]	[21]	Mean"
	Sample data-inside bark						
$\bar{D}^b$	-0.174 (-0.442)	-0.012 (-0.030)	0.213 (0.541)	-0.127 (-0.323)	0.085 (0.216)	-0.045 (-0.114)	0.109 0.278
$\lvert\bar{D}\rvert^b$	0.685 (1.740)	0.808 (2.052)	0.790 (2.007)	0.717 (1.821)	0.730 (1.854)	0.706 (1.793)	0.739 1.878
$S_D{}^b$	1.116 (2.835)	1.160 (2.946	1.201 (3.050)	1.074 (2.728)	1.145 (2.908)	1.129 (2.868)	1.137 2.889
SSRR	1117.4	230.1	290.1	1308.1	165.5	91.4	367.1
	Independent data-inside bark						
ii	-0.113 (-0.287)	0.039 (0.099)	0.235 (0.597)	-0.073 (-0.185)	0.109 (0.277)	0.012 (0.030)	0.097 0.246
$\lvert\bar{D}\rvert$	0.664 (1.687)	0.783 (1.989)	0.771 (1.958)	0.688 (1.748)	0.698 (1.773)	0.687 (1.745)	0.715 1.817
S_D	1.003 (2.548)	1.111 (2.822)	1.134 (2.880)	0.998 (2.535)	1.032 (2.621)	1.031 (2.619)	1.052 2.671
SSRR	37.4	60.1	76.5	362.7	45.9	26.0	101.4

'Computed on absolute values.
[b]Values in inches (centimetres).

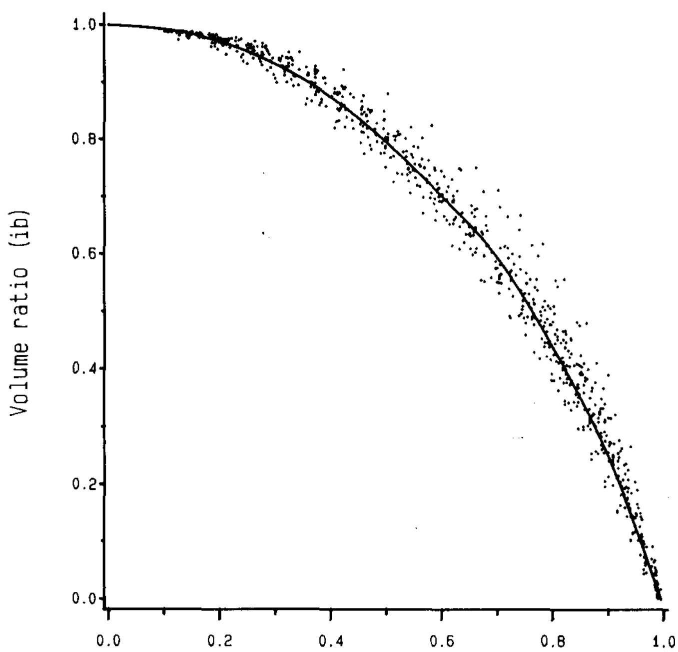

FIG. 3. Observed and predicted volume ratio values to top height limits for the independent data set.

Graphical analyses revealed that the observed volume ratio data assumed different shapes, depending on the merchantable volume basis for which they were plotted (diameter limit or height limit). The observed ratio values plotted over ***d/D*** (for diameter limit equations) displayed an inflection point, so these values were better fitted by the exponential functions. The observed ratio values plotted over relative distance from tree tip,

TABLE 11. Overall rank of the bald cypress total volume, volume ratio, and taper equations

Eq. No.	Equation	Rank sum	Overall rank
	Total cubic volume equations		
[3]	Schumacher and Hall 1933	11.5	1
[4]	Comprehensive	20.5	2
[5]	Honer 1965	22.0	3
[2]	Weighted combined variable	25.0	4
[1]	Combined variable	26.0	5
	Volume ratio - diameter limit equations		
[10]	Matney and Sullivan 1980	12.0	1
[8]	Modified Van Deusen et al. 1981	15.0	2
[9]	Cao et al. 1980	26.0	3
[7]	Van Deusen et al. 1981	27.0	4
[6]	Burkhart 1977	40.0	5
	Volume ratio - height limit equations		
[11]	Cao and Burkhart 1980	13.0	1
[15]	Modified Matney and Sullivan 1980	15.0	2
[14]	Cao et al. 1980	20.0	3
[13]	Modified Van Deusen et al. 1981	32.0	4
[12]	Modified Van Deusen et al. 198 1	40.0	5
	Taper equations		
[21]	Cubic-cubic	16.0	1
[16]	Max and Burkhart 1976	20.0	2
[19]	Cao et al. 1980	26.5	3
[20]	Bennett et al. 1978	28.5	4
[17]	Demaerschalk 1973	33.0	5
[18]	Ormerod 1971	44.0	6

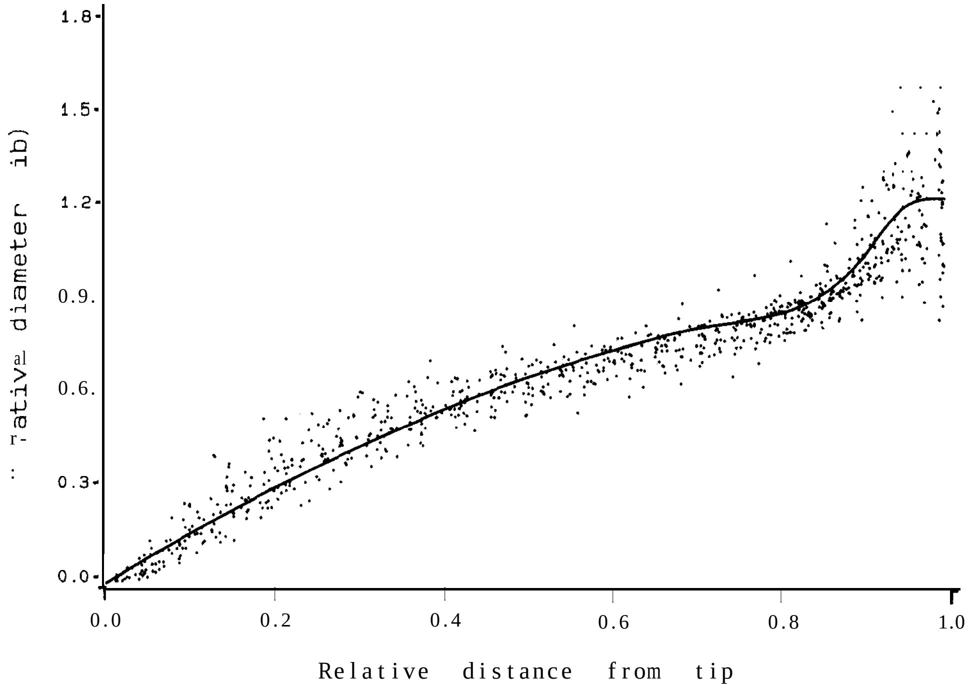

FIG. 4. Observed and predicted stem taper for the independent data set.

TABLE 12. Coefficients for English measurement units

Estimate type[a]	Parameter estimates				
	b_1	b_2	b_3	b_4	a
	Total cubic-foot volume equations				
ib-g	0.00437	1.78756	1.00866		
ob-g	0.00641	1.77322	0.95224		
ib-s	0.00163	1.83164	1.18070		
ob-s	0.00253	1.81735	1.11300		
	Volume ratio equations-top diameter limit[b]				
ib-g	13.13753	0.10537	0.24691	22.46406	
ob-g	17.06281	0.07523	0.26902	27.20511	
ib-s	4.11116	0.42629	0.14284	14.67623	
ob-s	4.00147	0.37534	0.16945	15.44679	
	Volume ratio equations-top height limit				
ib-g	0.44459	2.44371	2.26844		
ob-g	0.47960	2.36641	2.20833		
ib-s	1.16685	2.38469	2.40657		
ob-s	1.22985	2.31165	2.34601		
	Taper equations				
ib	2.65881	- 1.86993	1260.06962	-465.49004	0.82880
ob	3.12253	-2.31694	1234.80653	-456.36746	0.82491

[a] ib-g means inside bark, above the ground; ib-s means inside bark, above a 3-ft (0.9-m) stump; and the other abbreviations follow logically.
[b] Ratio values ib are to top diameters ib; ratio values ob are to top diameters ob.

$(H - h)/H$, did not have an inflection point, so the power function and polynomial functions gave a better fit to these values. Figures 2 and 3 show plots of function [10] and function [11], respectively. These figures illustrate why different functional forms fit better on the two different merchantable volume bases.

Taper equations

Table 10 lists values of the statistics on the taper ib equations. The same trends occur in Table 10 as in Tables 8 and 9. That is, differences across equations were large for D and SSRR and tended to be small for $\overline{ID}$ I and S_D. As before, examination of the ob patterns gave the same results. Ranks are given in Table 11.

TABLE 13. Coefficients for metric measurement units

Estimate type[a]	Parameter estimates b_1	b_2	b_3	b_4	
		Total cubic-me&e volume equations			
ib-g	7.75×10^{-5}	1.78756	1.00866		
ob-g	1.08×10^{-4}	1.77322	0.95224		
ib-s	3.41×10^{-5}	1.83164	1.18070		
ob-s	4.93×10^{-5}	1.81735	1.11300		
		Volume ratio equations--top diameter limit[b]			
ib-g	13.13753	0.14129	0.24691	22.46406	
ob-g	17.06281	0.10356	0.26902	27.20511	
ib-s	4.11116	0.50514	0.14284	14.67623	
ob-s	4.00147	0.45905	0.16945	15.44679	
		Volume ratio equations--top height limit			
ib-g	0.54751	2.44371	2.26844		
ob-g	0.57869	2.36641	2.20833		
ib-s	1.13690	2.38469	2.40657		
ob-s	1.18065	2.31165	2.34601		
		Taper equations			
ib	2.65881	−1.86993	1260.06962	−465.49004	0.82880
ob	3.12253	−2.31694	1234.80653	−456.36746	0.82491

[a]ib-g means inside bark, above the ground; ib-s means inside bark, above a 3-ft (0.9-m) stump; and the other abbreviations follow logically.
[b]Ratio values ib are to top diameters ib, ratio values ob are to top diameters ob.

The cubic-cubic segmented polynomial function derived in this study (given as eq. [21] in Table 3) ranked "best" for estimating taper. This new taper equation may prove useful in other studies. Figure 4 contains a plot of the recommended taper equation over the independent data (ib). The graph illustrates excellent conformance to the data.

A significant trend revealed in Table 11 is that the segmented-polynomial functions ranked better than the other functions. Bennett et al.'s (1978) equation ranked behind the segmented-polynomial functions, followed by the single-function equations. Graphical analyses illustrated the flexibility of the segmented functions and their ability to follow the natural inflections in the data.

Tables 12 and 13 list the coefficients for the recommended equations (ib and ob) in English and metric system measurement units, respectively. The total cubic volume equations and volume ratio equations were fitted above a 3-ft (0.9-m) stump height, the average bald cypress stump height in the study area, as well as from ground level. The aboveground volume system was fitted to provide a means for adjusting volume estimates on sites where trees are harvested below a 3-ft (0.9-m) level.

Summary and conclusions

This study considered the effects of buttresses on volume and taper estimation for bald cypress. Solid wood diameters in a buttress were determined by cross sectioning and inscribing a circle inside the flutes. Diameter measured at 10 ft (0.3 m) above the ground, termed d_{10}, proved to be a better diameter measurement point than the currently used normal diameter (dn), measured 18 in. (46 cm) above butt swell. A pole caliper as described by Ferree (1946) can be used to measure d_{10}. The methodology on fixed-height diameters should be applicable to any buttressed species.

Of the various statistical measures used to evaluate different equations, $\bar{D}$ and SSRR provided the best discrimination. Inside bark and outside bark patterns were similar for each equation across the "goodness-of-fit" statistics. Graphical analyses revealed different shapes when plotting volume ratio values for diameter limits or height limits, which explains why different functional forms performed better on the two different merchantable volume bases. The volume and taper prediction system, based on d_{10}, presented above provides flexibility in volume and taper estimation for any specified utilization constraints.

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