

A WHOLE STAND BASAL AREA PROJECTION MODEL FOR APPALACHIAN HARDWOODS

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Abstract.—Two whole-stand basal area projection models were developed for Appalachian hardwood stands. The proposed equations are an algebraic difference projection form based on existing basal area and the change in age, trees per acre, and/or dominant height. Average equation error was less than 10 square feet per acre and residuals exhibited no irregular trends.

INTRODUCTION

The ability to estimate future yields of Appalachian hardwoods has been and still is of great interest to forest practitioners in this region. Recently, whole-stand cubic volume and board foot volume prediction equations were developed for this area based on stand basal area per acre and average dominant height (Brooks and Wiant 2004, 2005). Volume predictions within 10 percent of actual yields were realized with this system. The cubic yield model can predict current conditions based on measurements of stand basal area and the average dominant height of all trees, regardless of species. To expand the usefulness of this model to a yield projection system, dominant height and basal area projection models are necessary. For future estimates of average dominant height, existing site index models can be employed using curves for the main species in the stand (Carmean and others 1989). For future basal area estimates, a projection model based on existing basal area and commonly measured stand variables is needed. This study evaluates several model formulations to develop such a projection equation.

METHODS

Three data sources were used in this investigation, the yield tables developed for Appalachian hardwoods by Schnur (1937) and two sets of remeasurement data from permanent plots from north central West Virginia. From the Schnur yield tables, basal area per acre, trees per acre, and average oak dominant height were obtained for ages 10 to 100 covering site index values of 40 through 80. The permanent plot data were obtained from the West Virginia University Research Forest located in Monongalia and Preston counties, WV. These data include four 0.5-acre control plots established in the late 1940s and early 1950s and remeasured at approximately 5-year intervals through the early 1990s. The second dataset is based on 40 permanent 0.2-acre continuous forest inventory plots established in 1999 and having at least two remeasurements. The permanent plot data reflect even-aged stands with no history of cutting. Stand age ranged from 17 to 74 years of age. A description of stand parameters is displayed in Table 1.

The permanent sample plots consist of two broad cover types (mesophytic and oak types). The former includes the yellow-poplar-white oak-northern red oak (SAF type 59) and the yellow-poplar type (type 57) (Eyre 1980). The oak type is best described as the white oak/black oak/northern red oak (type 52) and the chestnut oak type (type 44). Although three distinct datasets were combined for this analysis, values for all

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Table 1.—Descriptive statistics for the stand-level growth and yield data used in the basal area projection models

Variable	N	Mean	Std Dev	Min	Max
Age1 (yr)	147	56.9	19.9	10	90
Age2 (yr)	147	62.4	18.8	20	100
TPA1	147	545.3	880.3	95.0	6850.0
TPA2	147	389.8	413.3	95.0	3260.0
DHT1 (ft)	147	71.6	22.1	8.0	107.6
DHT2 (ft)	147	75.9	20.5	17.0	124.7
BA1 (ft ²)	147	127.6	37.8	36.0	213.1
BA2 (ft ²)	147	135.5	34.9	60.0	218.9

Age_i = Current (i = 1) or projected stand age (i = 2)

TPA_i = Current (i = 1) or projected (i = 2) trees per acre

DHT_i = Current (i = 1) or projected (i = 2) stand dominant height

BA_i = Current (i = 1) or projected (i = 2) stand basal area per acre

stand variables overlap with respect to age, trees per acre, basal area per acre, and stand dominant height. The data from the Schnur yield tables for site index 40 provided the lowest range of data while some of the 40 permanent plots extended the upper range of the data. Inclusion of such a wide range of overlapping stand variables was deemed beneficial when attempting to develop a single model form for predicting future values for oak dominated hardwood stands over a wide distribution of current stand conditions.

After some initial evaluation, two prospective model forms were selected for testing. The first is based on an algebraic difference equation developed by Pienaar and Shiver (1986) involving current and future estimates of stand trees per acre (TPA) and stand average dominant height:

$$BA_2 = \text{Exp} \left[\text{Ln}(BA_1) + \beta_1 (\text{Ln}(DHT_2) - \text{Ln}(DHT_1)) + \beta_2 (\text{Ln}(TPA_2) - \text{Ln}(TPA_1)) \right] \quad [1]$$

where BA_i is stand basal area per acre (ft²) at time period i (1 = current, 2 = projected), DHT_i is the average stand dominant height at time period i, TPA_i is stand trees per acre at time period i, Ln is the natural logarithm, and β_i are the parameters estimated with the data (i = 1...2).

This model form has an advantage in that stand age is not explicitly required, although some estimate of future stand density (TPA₂) is needed. The second equation is a modified form of an equation developed

by Pienaar and others (1988) that involves the initial basal area and the change in stand average dominant height, trees per acre, and stand age:

$$BA_2 = \text{Exp} \left[\begin{aligned} & \text{Ln}(BA_1) + \beta_1 \left(\frac{1}{A_2} - \frac{1}{A_1} \right) + \beta_2 (\text{Ln}(DHT_2) - \text{Ln}(DHT_1)) + \\ & \beta_3 (\text{Ln}(TPA_2) - \text{Ln}(TPA_1)) + \beta_4 \left(\left(\frac{\text{Ln}(DHT_2)}{A_2} \right) - \left(\frac{\text{Ln}(DHT_1)}{A_1} \right) \right) + \\ & \beta_5 \left(\left(\frac{\text{Ln}(TPA_2)}{A_2} \right) - \left(\frac{\text{Ln}(TPA_1)}{A_1} \right) \right) \end{aligned} \right] \quad [2]$$

where A_i is the stand age at time period i ($1 = \text{current}$, $2 = \text{projected}$) and all other variables and parameters are as previously defined.

Both equation forms have been commonly applied to southern pine growth and yield systems, but no record has been found of their application in Appalachian hardwoods.

The full form of each of the projection models was fit to the hardwood growth data utilizing all non-overlapping growth intervals. Models were fit using SAS Proc NLIN (SAS 2002). Evaluation of model goodness of fit was based on residual analysis, model root mean squared error (RMSE), and average bias (BIAS) as defined as:

$$BIAS = \frac{\sum_{i=1}^n \hat{BA} - BA_i}{n} \quad [3]$$

RESULTS

Equation 1 and 2 were fit to the hardwood growth and yield data. Non-significant parameters were removed from the full equation form and refit to the original dataset. The model and all parameter estimates in equation 1 were significant ($p < 0.0001$), and so the final model form is the same as the full model:

$$BA_2 = \text{Exp} \left[\text{Ln}(BA_1) + 0.4879 * (\text{Ln}(DHT_2) - \text{Ln}(DHT_1)) - 0.1462 * (\text{Ln}(TPA_2) - \text{Ln}(TPA_1)) \right] \quad [4]$$

For equation 2, β_1 , β_2 , and β_3 were not significant and were sequentially removed from the equation. Final equation parameter estimates and fit statistics are displayed in Table 2. This equation is of the form:

$$BA_2 = \text{Exp} \left[\begin{aligned} & \text{Ln}(BA_1) + 0.2139 * (\text{Ln}(DHT_2) - \text{Ln}(DHT_1)) - \\ & 3.5966 * \left(\left(\frac{\text{Ln}(DHT_2)}{A_2} \right) - \left(\frac{\text{Ln}(DHT_1)}{A_1} \right) \right) \end{aligned} \right] \quad [5]$$

EXAMPLE

Table 2.—Parameter estimates and fit statistics for the stand-level hardwood basal area projection models.

Equation	Parameter	Estimate	RMSE (ft ² /ac)	BIAS (ft ² /ac)
1	β1	0.488	9.786	-2.181
1	β2	-0.146		
2	β2	0.214	8.131	-2.203
2	β4	-3.597		

Residual analysis across all model variables did not indicate any irregular model behavior for either equation. Residuals by projected basal area for both equations and by data source are shown in Figure 1. Equation 1 had an average BIAS of -2.181 square feet per acre and a RMSE of 9.786 square feet per acre. Equation 2 had an average BIAS of -2.203 square feet per acre and a RMSE of 8.131 square feet per acre. For equation 2, the largest residuals were associated with three of the 77 growth intervals for the 40 permanent sample plots, but, 95 percent of the basal area residuals were within +/- 15 square feet per acre, indicating good prediction capability. Since these three plots represent actual stand values, they are not considered outliers and were included in the model fitting process. For equation 2, none of the basal area residuals from the Schnur yield tables were greater than +/- 10 square feet per acre which is likely due in part to the fact that these data represent average yield curve points thus minimizing natural plot-to-plot variation. Over all datasets, 94 percent of the basal area residuals for equation 2 were less than +/- 11 square feet per acre (Fig. 1).

CONCLUSIONS

A whole stand basal area projection model was developed for unthinned hardwood stands in the central Appalachian region. Since Schnur's (1937) data were based on predominately oak-dominated stands, these models are recommended for oak-dominated mixed Appalachian hardwood forests. Two model forms are presented, each with some limitations in application. Equation 1 is based on existing basal area per acre and an estimate of change in average dominant height and trees per acre. Current basal area per acre can be obtained through typical inventory procedures, while projected dominant height can be estimated using currently published site index/dominant height equations. Projected survival may be more difficult to estimate based on the lack of stand-level survival models for this region. Additional research is underway to rectify this limitation. Equation 2, having the smaller RMSE, requires estimates of existing basal area per acre as well as current and projected dominant height. These values, too, can be estimated based on current inventory data and existing site index/dominant height functions. This model form also requires current and projected stand age. In hardwood regions forest managers historically have been reluctant to estimate stand age during typical inventory procedures. However, since these models are based on even-aged stands, current stand age should be easily acquired through harvesting records. Thus equation 2 may be the easiest avenue for estimating future basal area and hence future cubic foot volume yields utilizing the whole-stand cubic volume equations by Brooks and Wiant (2004).

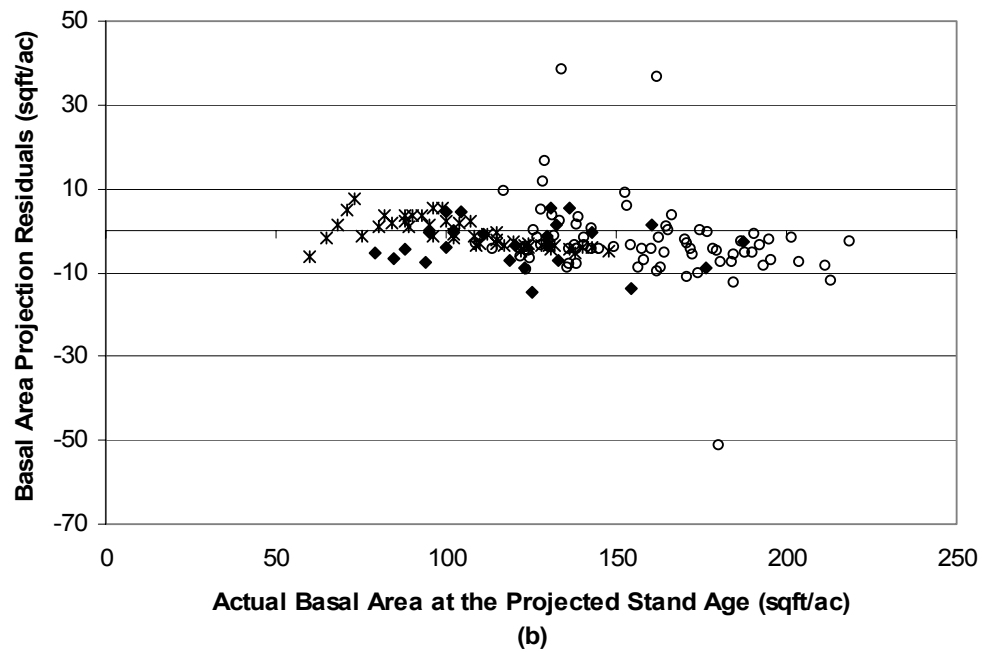
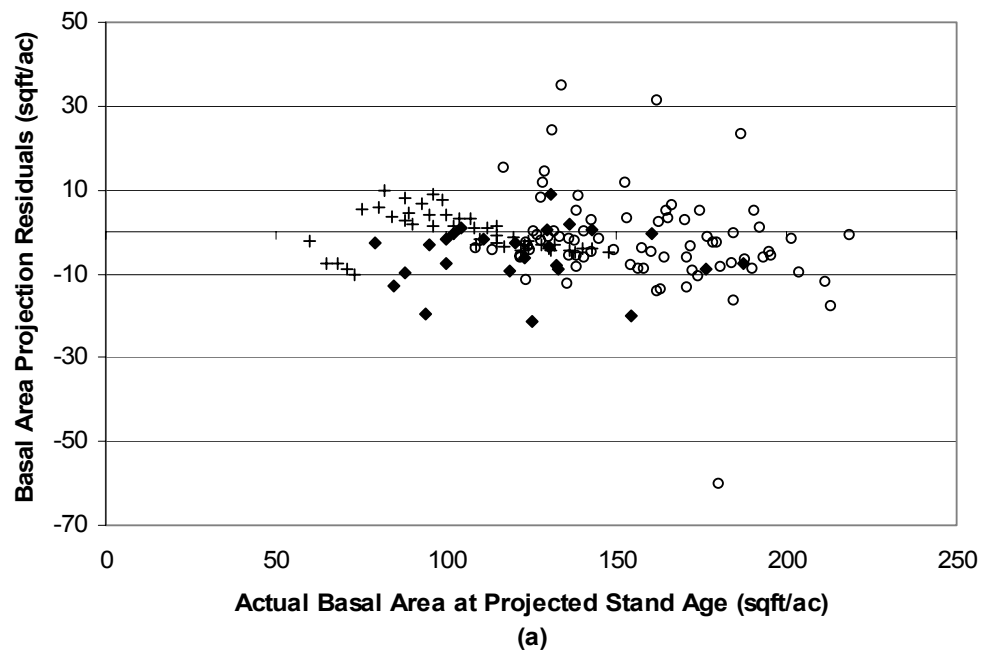


Figure 1.—Basal area projection residuals $\left(\hat{BA} - BA_i \right)$ for equation 1 (a) and equation 2 (b).

EXAMPLE

Inventory data for one of the long-term remeasurement plots at age 34 (A1) show 438 TPA, 102.24 square feet of basal area per acre, and an average dominant height of 54.7 feet. Current yield using equations by Brooks and Wiant (2004) would be:

$$CFAC = 0.40243 * (102.24 * 54.7)$$

$$CFAC = 2,250.6 \frac{ft^3}{ac}$$

Actual cubic foot volume for this stand is 2,355.1 cubic feet per acre.

If the estimated dominant height at age 58 is 72.4 feet, projected basal area for this stand at age 58 (A2) is:

$$BA = Exp \left[Ln(102.24) + 0.2139 * (Ln(72.4) - Ln(54.7)) - 3.5966 * \left(\left(\frac{Ln(72.4)}{58} \right) - \left(\frac{Ln(54.7)}{34} \right) \right) \right]$$

$$BA = 127.11 \frac{ft^2}{ac}$$

Actual basal area per acre for this stand is 130.8 square feet per acre. Our estimated cubic foot volume at age 58 is:

$$CFAC = 0.40243 * (127.12 * 72.4)$$

$$CFAC = 3,703.5 \frac{ft^3}{ac}$$

Actual volume for this stand is 4,096.4 cubic feet per acre, an error of slightly less than 10 percent.

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