DOMINANT HEIGHT-BASED HEIGHT-DIAMETER EQUATIONS FOR TREES IN SOUTHERN INDIANA

John A. Kershaw, Jr., Robert C. Morrissey, Douglass F. Jacobs, John R. Seifert, and James B. McCarter[1]

Abstract.—Height-diameter equations are developed based on dominant tree data collected in 1986 in 8- to 17-year-old clearcuts and the phase 2 Forest Inventory and Analysis plots on the Hoosier National Forest in south central Indiana. Two equation forms are explored: the basic, three-parameter Chapman-Richards function, and a modification of the three-parameter equation incorporating dominant canopy height as a surrogate variable for stand age and site quality. For most species or species groups, the modified equation provided a better fit to the height-diameter data than the standard form. Data from two follow-up studies in clearcuts collected between 2003 and 2006 were used to assess performance of the height-diameter equations. In general, the equations performed very well in terms of reflecting field assessed canopy class. In terms of predicting dominant canopy height, predicted tree heights overestimated dominant height relative to dominant height measured in the field; however, predicted heights for measured crop trees in the thinning trial were less than field measurements.

INTRODUCTION

Indiana is thought to have some of the most productive forest lands in the Central United States (Schmidt and others 1998). Indiana forests average 110 m^3ha^{-1} standing growing stock volume and have an average annual growth rate of 3.6 $m^3ha^{-1}year^{-1}$, the highest of any State in the former North Central Region (Shifley and Sullivan 2002).

Much of the land that comprises the Hoosier National Forest (HNF) was acquired in the 1930s following land abandonment (Leatherberry 2003). Many current stand structures that characterize the HNF, and much of southern Indiana, are the result of recolonization of abandoned fields and pastures or are heavily influenced by past human-induced disturbances including frequent burning, grazing of livestock, and repeated firewood cutting and/or high-grade lumbering (DenUyl 1954, Jenkins and Parker 2000). Productivity values in these stands often reflect low stocking, poor quality, and degraded stand structures (Schnur 1937), while well stocked stands regenerating following clearcutting, composed primarily of sprouts and seedlings, are considered to be more indicative of production capacity and site quality (Schnur 1937, Standiford and Fischer 1984).

From the mid-1960s to the mid-1980s, the HNF used clearcut harvesting to regenerate maturing stands, with the primary objective of regenerating oak and hickory species. In 1986, a study examining species response to clearcutting of the HNF was initiated (Fischer 1987, Fischer and others 1987, George and Fischer 1989). Stands sampled in 1986 were resampled between 2003 and 2005 (Kershaw and others

[1]Professor of Forest Mensuration (JAK), University of New Brunswick, Faculty of Forestry, 28 Dineen Dr. Fredericton, NB E3B 6C2, Canada; Research Associate (RCM), Purdue University, Department of Forestry and Natural Resources, Pfendler Hall 715 W. State St. West Lafayette, IN 47907; Associate Professor (DFJ), Purdue University, Department of Forestry and Natural Resources; State Forester (JRS), Indiana Department of Natural Resources, Division of Forestry, 402 W. Washington St. Rm. W296, Indianapolis, IN 46204; and (JBM) Rural Technology Initiative, College of Forest Resources, University of Washington, Box 352100, Seattle, WA 98195-2100. JAK is corresponding author: to contact, call (506)453-4933 or email kershaw@unb.ca.

2006). This pair of studies provides a unique opportunity to look at both stand dynamics and production relationships in young upland hardwood stands over a relatively long time period (Morrissey and others, in review).

One limitation of both studies is a lack of individual stem height data. Height measurements, especially in the Central Hardwood Region, are not often made. Reasons for the lack of height measurement include: 1) time required to complete measurements; 2) chance of observer error; and 3) visual obstructions (Colbert and others 2002). These limitations are especially true in regenerating and self-thinning cutover stands, where stem densities are quite high, and other obstructions, such as briars and grapevines, are abundant.

On the other hand, accurate estimates of height are crucial for estimating individual tree volume and stand productivity (Husch and others 2003). Existing height-diameter equations are either local in nature or were developed for mature stands (e.g., Colbert and others 2002, Lootens and others 2007). Application of these equations to young stands may not reflect the dynamic height-diameter relationship in young stands, especially self-thinning stands. The objectives of this paper are to: 1) develop a set of flexible polymorphic height-diameter equations for upland hardwood species in southern Indiana; and 2) test the applicability of these equations for estimating heights in self-thinning upland hardwood stands.

STUDY SITES

This study was conducted on the HNF in south-central Indiana. The HNF is located within the Highland Rim and Shawnee Hills Sections of the Interior Low Plateau (Homoya and others 1985). The 80,000-ha forest is administratively divided between the Brownstown Ranger District, and the Tell City Ranger District, which is composed of the Patoka Lake and Tell City management units.

Seventy-four naturally regenerated clearcut stands on the HNF were selected for measurement in 1986. Initially, all clearcuts made prior to 1982 were considered. The initial list was reduced to those stands with complete management records. The list of available stands was sorted by ranger district, management unit, and stand age. A sorted list by stand age within each ranger district was used to select sample stands. Stands were systematically selected from each list (Brownstown and Tell City) and placed in a queue. Stands were sequentially selected from the queue until the end of the field season.

1986 Sample Design

Sampling was conducted between July 1986 and March 1987. The boundary of each stand was traversed prior to sampling to insure map accuracy and to determine orientation of sample grids. The sample grid in each stand was oriented to insure a distribution of sample points across the range of slope and aspects present. Grid azimuths were determined from stand maps using a compass. An initial sample point was selected and plots were spaced 63.5 m apart following the sample grid (2.5 plots/ha). Distances were determined using pacing and azimuths were followed using a hand compass; approximate locations were marked on stand maps by hand. Measurement plots were 0.004-ha circular plots with a radius of 3.6 m. Plots falling partially into adjacent stands were not sampled.

At each plot center, average canopy height and other plot-level factors were measured. All trees and woody shrubs greater than 1.37 m in height were tallied by species (see Table 1 for a list of common and scientific names), crown class, and origin (seedling, seedling sprout, stump sprout, or residual). Species of the dominant plot tree (tallest plot tree) and the largest oak tree (by diameter at breast height [d.b.h.])

Table 1.—Mean and range of individual tree measurements by species group, individual species and data source (1986 clearcut study or FIA plots) for the predominant species groups on the Hoosier National Forest, Indiana. Clearcut data are the dominant plot tree data. FIA data are from the Phase II subplots. Nomenclature is based on the USDA, Plants database (http://plants.usda.gov/).

Species		1986 Clearcut Study							FIA Plots						
Common Name	Scientific Name	Number	d.b.h. (cm)			Height (m)			Number	d.b.h. (cm)			Height (m)		
			minimum	mean	maximum	minimum	mean	maximum		minimum	mean	maximum	minimum	mean	maximum
Oak	*Quercus* L.														
white oak	*Q. alba* L.	46	2.5	8.53	15.5	3.0	8.60	14.6	181	12.7	34.10	76.0	9.8	23.90	40.2
northern red oak	*Q. rubra* L.	56	1.8	7.54	14.5	3.7	8.42	13.4	33	13.5	39.08	72.4	13.1	26.79	38.7
black oak	*Q. velutina* Lam.	112	2.3	7.92	20.1	3.7	8.99	18.3	67	13.2	32.03	59.2	9.1	23.76	37.8
chestnut oak	*Q. prinus* L.	68	2.5	8.87	19.6	4.9	9.76	15.2	58	15.2	32.13	56.6	14.6	24.14	32.3
chinkapin oak	*Q. muehlenbergii* Engelm.	17	1.5	8.72	20.6	2.4	10.07	15.8	9	13.0	23.17	43.7	13.4	18.93	25.0
pin oak	*Q. palustris* Muenchh.	0		---			---		1		53.30			34.40	
post oak	*Q. stellata* Wangenh.	0		---			---		6	24.6	34.59	46.0	17.7	22.50	28.3
scarlet oak	*Q. coccinea* Muenchh.	5	9.7	12.44	16.0	7.3	10.36	14.6	28	15.2	34.44	55.6	13.1	25.68	32.9
shingle oak	*Q. imbricaria* Michx.	0		---			---		1		54.90			33.20	
Shumard oak	*Q. shumardii* Buckl.	3	1.8	3.50	6.9	2.4	3.63	5.5	2	39.4	42.42	45.5	26.8	27.13	27.4
Hickory	Carya Nutt.														
shagbark hickory	*C. ovata* (P. Mill.) K. Koch	3	5.3	5.83	6.1	4.9	6.10	7.9	21	13.5	24.66	49.0	12.5	21.55	28.6
pignut hickory	*C. glabra* (P. Mill.) Sweet	2	5.1	8.15	11.2	6.1	7.30	8.5	68	13.7	29.54	53.3	13.4	24.57	35.0
bitternut hickory	*C. cordiformis* (Wangenh.) K. Koch	4	4.6	7.82	12.7	4.9	7.78	10.4	12	19.3	30.42	45.0	18.3	26.34	39.3
mockernut hickory	*C. alba* (L.) Nutt. ex Ell.	1		3.30			4.30		0		---			---	
shellbark hickory	*C. laciniosa* (Michx. f.) G. Don	3	5.3	5.83	6.1	4.9	6.10	7.9	21	13.5	24.66	49.0	12.5	21.55	28.6
Yellow-Poplar	*Liriodendron tulipifera* L.	538	2.0	10.45	25.1	2.4	10.85	20.7	83	13.2	29.99	74.7	10.0	23.31	38.4
Black Cherry	*Prunus serotina* Ehrh.	269	1.0	10.24	24.4	2.4	9.71	18.9	23	3.0	19.28	36.6	4.0	16.82	22.9
Maple	*Acer* L.														
sugar maple	*Acer saccharum* Marsh.	63	1.0	6.80	15.2	3.0	7.92	12.8	181	12.7	22.40	70.9	8.8	19.24	38.1
red maple	*Acer rubrum* L.	66	3.0	8.43	16.0	3.7	9.34	15.2	96	12.7	22.31	51.0	11.9	19.76	32.6
boxelder	*Acer negundo* L.	0		---			---		2	19.0	20.07	21.1	15.8	16.61	17.4
Ash	*Fraxinus* L.														
white ash	*Fraxinus americana* L.	89	1.5	7.46	15.7	1.8	7.39	17.1	50	12.7	24.28	48.8	11.0	20.64	34.1
green ash	*Fraxinus pennsylvanica* Marsh.	0		---			---		1		29.20			14.63	
Elm	*Ulmus* L.														
American elm	*Ulmus americana* L.	7	5.1	10.60	17.8	4.9	7.74	11.6	19	12.7	19.94	36.8	11.9	16.14	24.1
red elm/slippery elm	*Ulmus rubra* Muhl.	108	1.5	9.40	16.8	3.0	9.41	16.5	7	14.0	20.83	31.2	14.6	19.16	26.2
winged elm	*Ulmus alata* Michx.	3	2.0	4.80	8.6	3.0	5.27	8.5	5	13.7	16.61	18.5	12.2	14.32	15.5
Siberian elm	*Ulmus pumila* L.	0		---			---		1		20.60			19.20	
ironwood/hophornbeam	*Ostrya virginiana* (P. Mill.) K. Koch	3	3.3	4.63	6.3	6.7	7.93	9.8	1		14.00			13.70	
bluebeech/hornbeam	*Carpinus caroliniana* Walt.	0		---			---		1		13.50			10.00	
Walnut	*Juglans* L.														
black walnut	*Juglans nigra* L.	7	5.1	10.13	17.0	3.7	7.84	11.6	11	13.5	24.59	40.1	8.5	21.58	29.9
butternut	*Juglans cinerea* L.	6	7.6	10.68	14.5	7.9	9.75	11.6	1		21.30			8.80	
Other Tree Species															
sassafras	*Sassafras albidum* (Nutt.) Nees	112	1.0	7.43	20.8	3.0	8.57	20.7	54	12.7	19.24	51.8	11.6	18.78	29.0
aspen	*Populus grandidentata* Michx.	96	2.5	11.22	23.9	4.3	12.92	20.7	11	26.2	31.86	43.9	25.0	29.98	32.0
American beech	*Fagus grandifolia*	4	4.3	7.55	11.4	3.7	6.88	12.2	40	13.2	25.74	66.8	10.4	19.51	33.8
blackgum	*Nyssa sylvatica* Marsh.	17	0.8	5.98	12.4	1.8	6.69	18.3	40	12.7	18.50	38.4	10.0	15.64	22.2
sycamore	*Platanus occidentalis* L.	16	2.0	9.21	18.3	3.7	9.74	14.6	13	14.5	34.35	53.8	12.8	23.56	36.0
black locust	*Robinia pseudoacacia* L.	15	5.6	8.79	12.7	5.5	7.87	12.8	9	14.2	17.41	24.4	11.9	15.78	25.9
American chestnut	*Castanea dentata* (Marsh.) Borkh.	1		11.20			8.50		0		---			---	
basswood	*Tilia americana* L.	8	2.5	9.85	12.7	6.1	10.44	15.2	0		---			---	
hackberry	*Celtis occidentalis* L.	1		9.10			7.30		1		31.80			25.90	
honeylocust	*Gleditsia triacanthos* L.	0		---			---		1		23.10			17.40	
persimmon	*Diospyros virginiana* L.	0		---			---		2	13.5	15.50	17.5	14.6	15.85	17.1
river birch	*Betula nigra* L.	0		---			---		2	26.4	27.18	27.9	15.2	17.37	19.5
sweetgum	*Liquidambar styraciflua* L.	0		---			---		1		27.40			28.00	
European alder	*Alnus glutinosa* (L.) Gaertn.	0		---			---		1		16.00			14.00	
Softwood Species															
eastern redcedar	*Juniperus virginiana* L.	0		---			---		53	12.7	19.98	47.0	5.8	12.53	22.2
eastern white pine	*Pinus strobus* L.	0		---			---		62	12.7	24.83	58.4	8.5	17.43	32.0
shortleaf pine	*Pinus echinata* P. Mill.	0		---			---		77	14.2	29.07	47.2	12.5	23.48	32.3
virginia pine	*Pinus virginiana* P. Mill.	0		---			---		17	12.7	18.59	25.4	13.1	16.55	22.9
Understory Woody Species															
dogwood	*Cornus florida* L.	0		---			---		7	4.8	10.56	18.5	4.3	7.92	11.9
red bud	*Cercis canadensis* L.	4	2.8	4.55	6.6	3.0	4.28	5.5	4	13.0	13.78	16.3	6.4	9.98	11.6
willow	*Salix* (L.) spp	2	9.4	9.80	10.2	9.8	10.10	10.4	0		---			---	
sumac	*Rhus typhina* L.	2	3.3	3.55	3.8	3.7	5.20	6.7	0		---			---	
buckthorn	*Rhamnus cathartica* L.	1		5.30			6.70		0		---			---	
hoptree	*Ptelea trifoliata* L.	1		3.60			3.00		0		---			---	

were recorded and d.b.h. and total height measured. Data were recorded on field tally sheets and later keypunched. A total of 1,801 plots were established and measured. A complete description of sample plot location and all measurements is found in Fischer (1987) and George and Fischer (1989).

2003 Sample Design

Clearcut Follow-up Study

Only 70 of the original 74 stands selected in 1986 were available for sampling in 2003. Given time since the original study, technology used to initially establish field plots, and lack of permanent plot markers, no attempt was made to relocate the original 1986 plots.

Stands from the 1986 study were relocated and outlined using SOLO Field GPS software® (TDS, Corvallis, OR) to more accurately determine stand area. Sample plots were arranged on a 63.5 x 63.5 m grid (2.5 plots/ha) using SOLO Field GPS software, and permanent sample plot centers were established for future study. Each plot center was located using the SOLO Field GPS software, and plot centers were logged for future reference using latitude and longitude coordinates and elevation information. Each plot consisted of a 0.04-ha tree tally and a 0.004-ha reproduction plot.

Dominant height and other plot-level variables were recorded for each plot. One tree deemed to be representative of average dominant canopy height was selected and measured to the nearest 1.5 m using a clinometer. Within each 0.04-ha plot, all trees with a d.b.h. greater than 2.54 cm were tallied by species and measured using calipers. The d.b.h., crown class (dominant, intermediate, suppressed), and origin (sprout, seed) were recorded for each tree. Within each 0.004-ha plot, all trees less than 2.54 cm d.b.h. were tallied by species. A total of 1,330 plots were established and measured. A complete description of sample plot location and associated measurements is found in Kershaw and others (2006) and Morrissey (2006).

Thinning Trial

Five additional clearcut stands, similar in age to the ones described above, were selected for a thinning trial aimed at increasing the proportion of oak and hickory. In each stand, three 0.4-ha study plots were located and in each plot, 40 crop trees identified based on species (oak and hickory favored) and canopy class (dominant and codominant). Fifteen of the 40 crop trees in each plot were randomly selected and measured for total height, height to crown base, and crown width. The 225 measured crop trees were used to provide an independent test of the accuracy of the height-diameter equations developed in this study.

2005 FIA Plot Database

U.S. Forest Service Forest Inventory and Analysis (FIA) phase 2 plots located on the HNF were downloaded from the FIA online data server (http://fia.fs.fed.us/tools-data/tools/). Pertinent plot data and individual tree species, heights and d.b.h.s were extracted and imported into the R statistical software (R Development Core Team 2007). Plots from the full 2001-2005 measurement cycle were used.

METHODS

Individual species were grouped into 11 species groups (Table 1). Data from the 1986 clearcut sample were combined with FIA plot data located on the HNF to develop height-diameter equations on the basis of species groups and for more abundant individual species. These equations were validated using the 2003 clearcut studies. Table 1 contains a summary of the individual tree height and d.b.h. measurements by species and data source.

Height - Diameter Estimation

Height-Diameter Models

Several model forms are frequently used for estimating height-diameter relationships (Huang and others 2000, Trincado and Leal 2006). The basic 3-parameter form of the Chapman-Richards function (Clutter and others 1983) is utilized in this study:

[1] $$H = H_B + b_1 \bullet \left(1 - e^{-b_2 \bullet D}\right)^{b_3}$$

where H_B is breast height (1.37 m), H is total height, D is diameter at d.b.h., and b_i's are species-specific regression coefficients. This function was chosen because: 1) $H = H_B$ when $D = 0$; 2) the function is simple, yet quite flexible in form; and 3) the parameters have reasonable biological interpretation.

The b_1 coefficient is the asymptotic maximum height. For a given species, this coefficient represents the theoretic maximum height obtainable. For a given stand or plot, this parameter could be used to represent maximum tree height. By modifying equation [1] so that b_1 changes as a function of stand age, site index, dominant height, or stand density, equation [1] can be generalized to be more applicable to a wider range of sites and stand ages, or more regionally applied (e.g., Arabatzis and Burkhart 1992, Trincado and Leal 2006).

Since dominant canopy height was available, or easily calculated, in all datasets, and can be used to reflect differences in stand age and site index, equation [1] was modified to include dominant canopy height:

[2] $$H = H_B + \left(c_1 \bullet H_{dom}^{c_2}\right) \bullet \left(1 - e^{-b_2 \bullet D}\right)^{b_3}$$

where H_B, H, and D are as above, H_{dom} is dominant canopy height, and the b_i's and c_i's are species-specific regression coefficients.

The combined 2005 FIA data and dominant tree data from 1986 were used to derive height-diameter equations. Dominant height was calculated for each FIA subplot. Dominant height was a measured variable on the 1986 plots.

Dominant heights on FIA plots were calculated using Gingrich's (1967) stocking equations. For each FIA subplot, the tree list was sorted from tallest to shortest tree. The stocking contribution of each tree was computed using the equation for the "B-line", which represents canopy closure. Stocking contributions were accumulated until full B-line level stocking (1000 milacres) was achieved. Average height of the tallest trees required to reach B-line stocking was calculated and used to represent dominant height.

Equations [1] and [2] were fitted to individual species data and to data grouped by genus or growth type (Table 1). For all species, equations were fitted using a nonlinear regression package in R (R Development Core Team 2007) with non-weighted and weighted fits. Best fit was determined on the basis of the lowest Furnival's (1961) Index (FI), residual distributions, and parameter estimates and their associated standard errors.

Model Validation
Data from the 2003 plots were used to validate the equations developed above. No individual tree height data were measured; however, dominant plot height and crown class (dominant, intermediate, and suppressed) of each tree were available. Individual tree heights were predicted using the species-specific coefficients, d.b.h, and dominant plot height. Relative height, calculated as the ratio of predicted tree height to maximum predicted tree height within each plot, was calculated. Histograms of the distributions of relative heights by species and crown class were used to visually assess model performance. Dominant height for each plot was then estimated using predicted individual tree heights and Gingrich's (1967) "B-line" stocking equation as described for the FIA plots. Estimated dominant heights were compared to dominant heights measured in the field. For the thinning trial data, heights of measured crop trees were compared to predicted heights based on the equations developed in this study.

RESULTS

Height-Diameter Equations

Model Fits
Table 2 shows estimated parameters, their associated standard errors, nonlinear pseudo r^{2}'s (Kutner and others 2003), and FI by species group, individual species, and equation form (eqs. [1] and [2]). Non-weighted and weighted fits were compared on the basis of smallest FI. In general, optimal weight (smallest FI) was d.b.h.$^{-.5}$. For almost all species groups and individual species, equation [2], the equation incorporating dominant canopy height provided, a better fit to the height-diameter data than did the standard model (eq. [1]). Only shagbark hickory and blackgum had better associated fit statistics with equation [1] than equation [2].

For several species groups and individual species the shape (b_3) parameter was dropped from the equations (Table 2). Dropping the shape parameter reduces the three-parameter Chapman-Richards equation to the two-parameter formulation (Clutter and others 1983). For aspen, the exponential decay (b_2) parameter was fixed at a value of b_2 = .02. All attempts to obtain fits that met convergence criteria failed when b_2 was allowed to vary. The value of .02 was selected based on a grid search of values of b_2 (Kutner and others 2003).

Overall, both equations fitted the height-diameter data well (Table 2). Almost all pseudo r^{2}'s (Kutner and others 2003) were greater than .50 and most were greater than .70 (29 of 33 species for eq. [1] and 31 of 33 for equation [2]). For equation [1], the shape parameter, b_3, was generally greater than 1.0, indicating a sigmodial-shaped height-diameter relationship. For equation [2], the shape parameter was generally less than 1.0, indicating an asymptotically increasing height-diameter relationship within a given dominant height level. Comparisons of estimated heights based on the two equations are shown in Figure 1 along with the FIA and clearcut data for four selected species. Within a given dominant canopy height class, resulting height-diameter curves were generally flatter than curves resulting from equation [1] (Fig. 1).

Validation of Height-Diameter Relationships
Figure 2 shows relative height distributions by canopy class for a selected set of species. A complete summary by crown class and species is shown in Table 3. For all species groups, mean relative height decreases from the dominant canopy class, to the intermediate canopy class, to the suppressed canopy class. While there is overlap between classes, the distributions of relative heights reflect the three canopy classes

Table 2.—Regression statistics, estimated coefficents, and standard errors (in parentheses) by species and height-diameter equation form (Standard, Eq. 1 and Dominant Height, Eq. 2)

Species		Standard[b]					Dominant Height[b]					
	N	r^2	FI[a]	b_1	b_2	b_3	r^2	FI[1]	c_1	c_2	b_2	b_3
Oak	693	0.92	2.58	34.826	0.037	1.111	0.95	**2.09**	11.832	0.374	0.013	0.510
				(1.029)	(.003)	(.049)			(1.189)	(.021)	(.004)	(.041)
white oak	227	0.88	2.86	31.945	0.048	1.343	0.92	**2.34**	9.187	0.385	0.031	0.760
				(1.195)	(.006)	(.138)			(1.239)	(.038)	(.007)	(.106)
red oak	89	0.94	2.43	37.982	0.033	1.107	0.97	**1.70**	9.196	0.482	0.007	0.388
				(2.798)	(.007)	(.107)			(3.547)	(.052)	(.011)	(.087)
black oak	179	0.93	2.26	38.205	0.031	1.051	0.96	**1.82**	11.764	0.397	0.011	0.475
				(2.871)	(.006)	(.082)			(2.994)	(.04)	(.009)	(.075)
chestnut oak	126	0.92	2.24	33.017	0.040	1.113	0.94	**1.91**	16.042	0.323	0.008	0.482
				(2.726)	(.009)	(.12)			(8.572)	(.049)	(.015)	(.113)
other oak species	72	0.90	2.90	34.128	0.038	1.104	0.94	**2.14**	6.848	0.427	0.066	---
				(4.007)	(.013)	(.204)			(1.513)	(.06)	(.008)	(---)
Hickory	110	0.81	3.12	34.140	0.051	1.379	0.84	**2.88**	10.029	0.370	0.040	0.970
				(2.885)	(.014)	(.301)			(3.198)	(.087)	(.018)	(.289)
shagbark hickory	24	0.88	**2.46**	27.451	0.090	2.077	0.88	2.52	28.262	-0.009	0.091	2.101
				(1.974)	(.025)	(.728)			(19.533)	(.209)	(.028)	(.946)
pignut hickory	70	0.73	3.22	34.735	0.046	1.229	0.77	**2.97**	8.070	0.412	0.055	1.230
				(4.973)	(.022)	(.448)			(3.658)	(.122)	(.026)	(.536)
bitternut hickory	16	0.93	2.93	196.808	0.004	0.922	0.95	**2.42**	14.992	0.364	0.023	---
				(1172.678)	(.029)	(.365)			(10.375)	(.152)	(.009)	(---)
Yellow Poplar	621	0.84	2.40	66.915	0.010	0.829	0.89	**2.00**	8.909	0.354	0.070	---
				(13.909)	(.004)	(.039)			(.664)	(.019)	(.004)	(---)
Black Cherry	292	0.68	2.21	127.857	0.003	0.774	0.80	**1.72**	6.003	0.401	0.083	0.714
				(522.805)	(.018)	(.104)			(.758)	(.029)	(.026)	(.13)
Maple	408	0.83	2.72	28.060	0.058	1.262	0.87	**2.41**	14.739	0.283	0.017	0.593
				(1.118)	(.007)	(.104)			(2.53)	(.028)	(.01)	(.08)
sugar maple	244	0.82	2.88	27.455	0.062	1.350	0.86	**2.54**	12.899	0.333	0.015	0.580
				(1.36)	(.01)	(.156)			(3.406)	(.041)	(.013)	(.109)
red maple	164	0.86	2.46	29.007	0.053	1.175	0.89	**2.14**	16.357	0.259	0.016	0.581
				(1.905)	(.01)	(.137)			(4.432)	(.037)	(.014)	(.111)
Ash	140	0.90	2.53	38.760	0.036	1.256	0.94	**2.02**	11.029	0.411	0.012	0.524
				(5.235)	(.01)	(.135)			(4.807)	(.049)	(.016)	(.119)
white ash	139	0.91	2.40	38.452	0.038	1.292	0.94	**1.98**	11.212	0.382	0.017	0.588
				(4.612)	(.009)	(.131)			(3.296)	(.05)	(.014)	(.123)
Elm[c]	157	0.78	2.12	37.886	0.025	0.988	0.82	**1.89**	17.816	0.265	0.010	0.546
				(14.795)	(.018)	(.153)			(18.403)	(.042)	(.026)	(.135)
American elm	26	0.78	2.47	26.226	0.053	1.375	0.83	**2.14**	11.595	0.235	0.050	---
				(8.695)	(.043)	(.685)			(4.502)	(.087)	(.016)	(---)
red elm/slippery elm	115	0.76	2.03	71.603	0.011	0.936	0.82	**1.77**	30.154	0.300	0.003	0.528
				(2.204)	(.048)	(.015)			(147.415)	(.048)	(.031)	(.14)
Walnut	25	0.80	3.62	56.173	0.021	1.200	0.84	**3.27**	12.509	0.309	0.038	---
				(71.35)	(.042)	(.594)			(8.565)	(.138)	(.02)	(---)
black walnut	18	0.86	3.41	32.934	0.062	1.938	0.92	**2.50**	6.882	0.452	0.055	---
				(8.66)	(.04)	(1.005)			(3.429)	(.123)	(.018)	(---)
Other Tree Species	433	0.81	2.85	33.174	0.038	1.061	0.83	**2.70**	19.996	0.186	0.021	0.680
				(2.16)	(.006)	(.076)			(2.802)	(.028)	(.008)	(.079)
sassafras	166	0.84	2.34	27.942	0.061	1.298	0.88	**1.98**	10.033	0.316	0.036	0.610
				(2.676)	(.013)	(.147)			(1.874)	(.041)	(.018)	(.121)
aspen	107	0.76	2.91	53.867	0.022	---	0.85	2.04	3.759	0.628	0.020	0.146
				(9.231)	(.005)	(---)			(.927)	(.058)	(---[d])	(.075)
American beech	44	0.78	3.17	27.261	0.070	1.700	0.81	**2.96**	9.587	0.324	0.050	1.111
				(1.974)	(.022)	(.608)			(4.177)	(.131)	(.026)	(.498)
blackgum	57	0.86	**2.11**	22.921	0.074	1.464	0.86	2.12	20.405	0.044	0.065	1.296
				(2.675)	(.023)	(.325)			(5.418)	(.093)	(.031)	(.47)
other misc. species	59	0.88	2.64	77.888	0.008	0.882	0.90	**2.51**	21.168	0.163	0.032	---
				(83.4)	(.015)	(.166)			(6.62)	(.064)	(.007)	(---)
Softwood Species	209	0.62	3.95	28.029	0.069	2.161	0.80	**2.84**	1.829	0.815	0.122	3.968
				(2.387)	(.019)	(.657)			(.369)	(.06)	(.019)	(1.18)
eastern redcedar	53	0.63	2.93	17.580	0.117	3.716	0.59	**2.42**	16.143	0.146	0.031	---
				(1.652)	(.042)	(2.245)			(10.589)	(.213)	(.009)	(---)
eastern white pine	62	0.74	3.01	49.071	0.016	---	0.80	**2.59**	10.142	0.462	0.019	0.959
				(11.981)	(.005)	(---)			(6.704)	(.095)	(.022)	(.321)
shortleaf pine	77	0.49	3.35	26.003	0.117	3.223	0.82	**1.98**	1.300	0.922	0.080	---
				(1.299)	(.042)	(2.331)			(.34)	(.077)	(.008)	(---)
Virginia pine	17	0.43	2.49	20.534	0.074	---	0.53	**1.96**	3.972	0.534	0.081	---
				(4.895)	(.038)	(---)			(2.23)	(.174)	(.03)	(---)
Understory Woody Species	15	0.89	1.15	24.398	0.035	1.084	0.89	**1.15**	24.507	0.045	0.026	---
				(47.004)	(.101)	(.579)			(31.487)	(.131)	(.033)	(---)

[a]Bolded FI indicates best fit

[b]italic parameter estimates indicate parameter estimates that are not significantly different from 0 at a=.05

[c]Elm species include all Ulmacea spp (*Ulmus, Ostrya*, and *Carpinus*)

[d]value of .02 fixed after grid search for "best" value, could not achieve convergence of full model

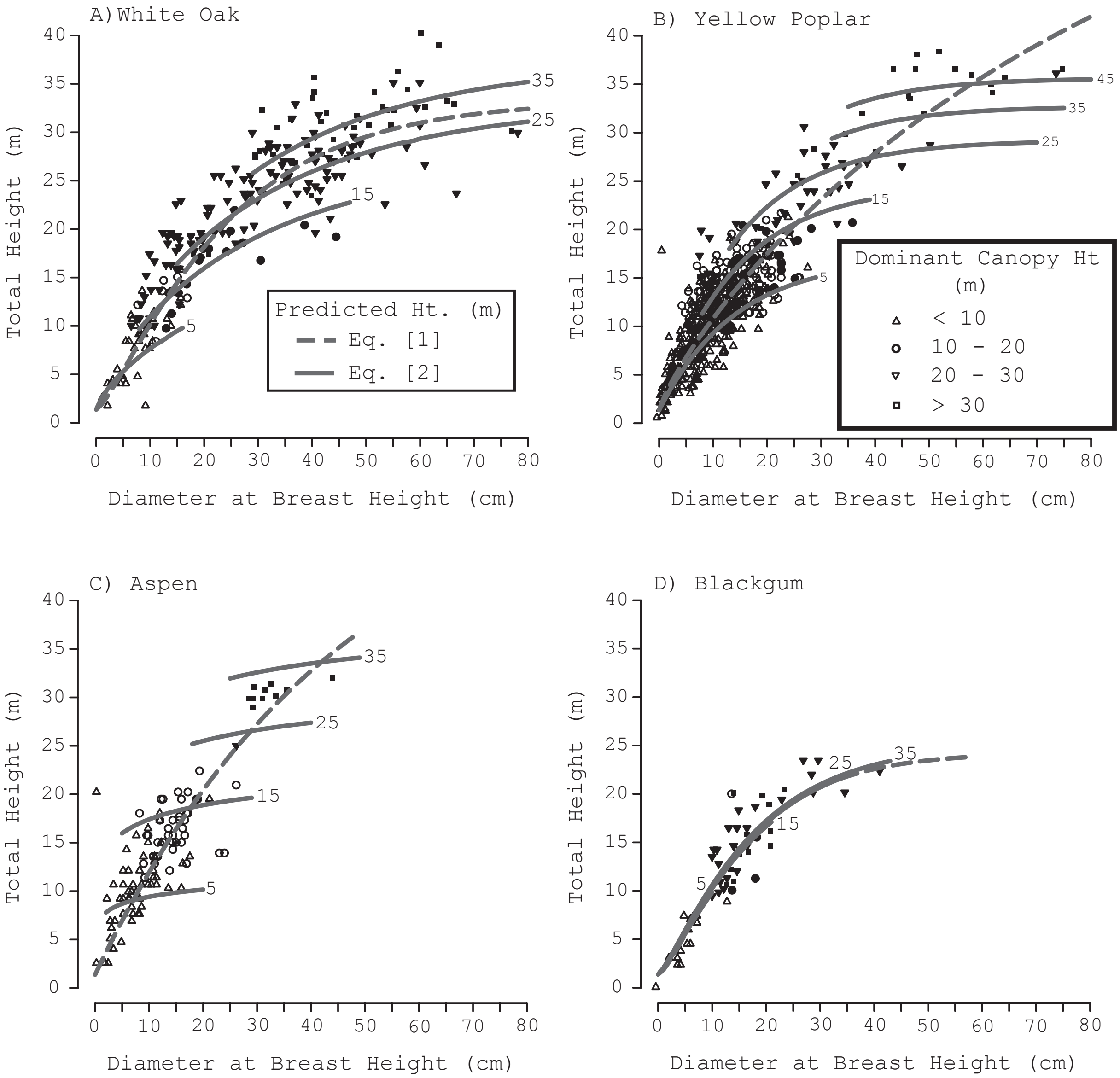

Figure 1.—Observed total height versus d.b.h. and predicted total height for equations [1] and [2] for selected species on the Hoosier National Forest: A) white oak, B) yellow-poplar, C) aspen, and D) blackgum.

very well (Table 3 and Fig. 2). Figure 3 shows the relationship between predicted dominant height, based on predicted heights and Gingrich's (1967) "B-line" stocking equation (as described above). Predicted dominant height was generally greater than dominant heights measured in the field. For the measured crop trees, heights were, on average, under-predicted (Table 4 and Fig. 4). For some species, especially white oak and red oak, heights were substantially under-predicted (> 1.5 m), while for most of the other species errors in height prediction were less than 1.0 m.

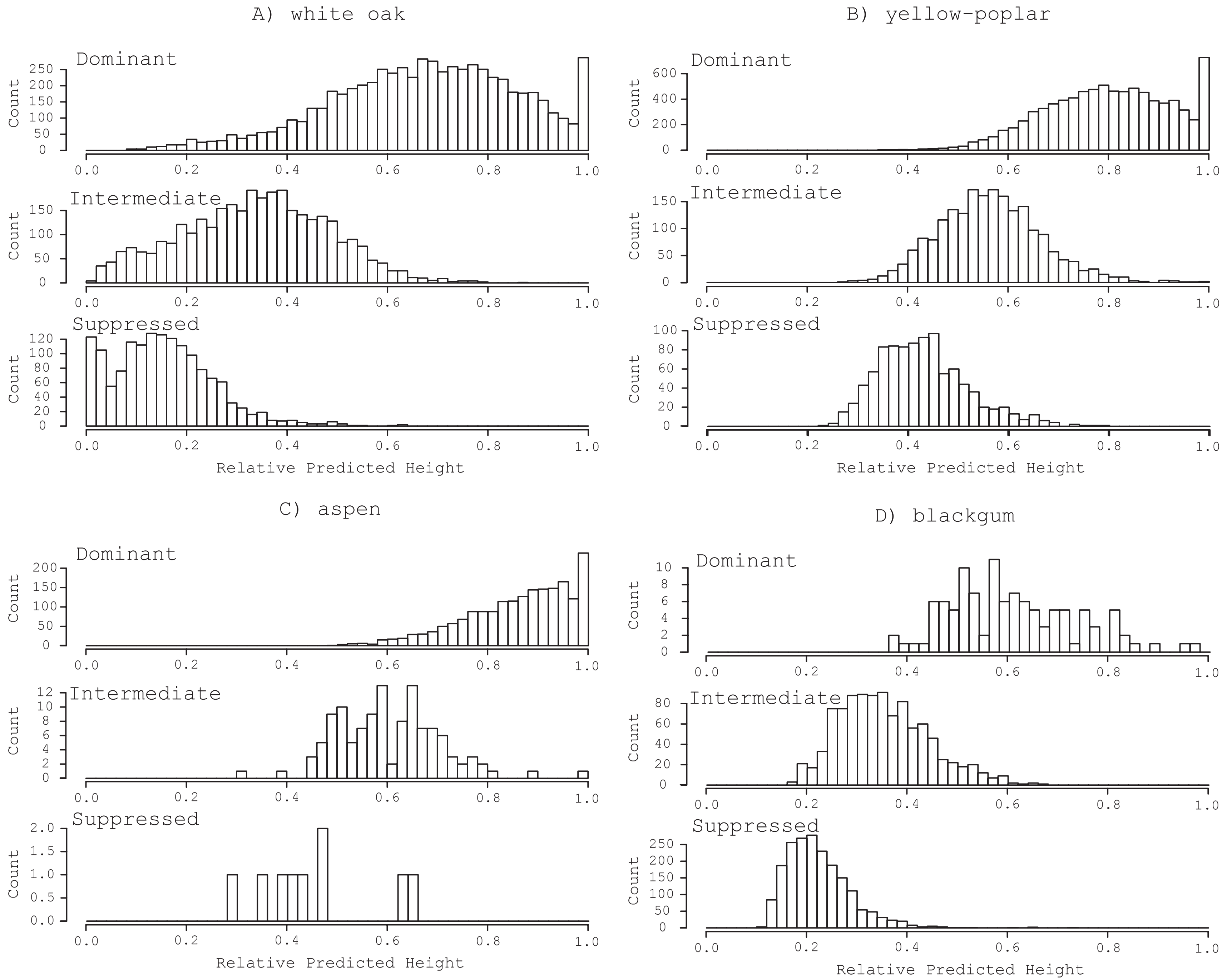

Figure 2.—Distribution of relative predict total heights by canopy class for selected species on the Hoosier National Forest: A) white oak, B) yellow-poplar, C) aspen, and D) blackgum.

Table 3.—Mean, standard deviation, and range (in parentheses) of relative tree heights by species group and canopy class for the 2003 clearcut samples from the Hoosier National Forest, Indiana

Species Group	Canopy Class					
	Dominant		Intermediate		Suppressed	
	Mean	Std. Dev.	Mean	Std. Dev.	Mean	Std. Dev.
Oak	0.67	0.18	0.34	0.14	0.15	0.10
	(0.08, 1.00)		(0.01, 0.86)		(0.00, 0.64)	
Hickory	0.71	0.19	0.35	0.13	0.21	0.08
	(0.19, 1.00)		(0.03, 0.99)		(0.02, 0.58)	
Yellow-Poplar	0.80	0.12	0.56	0.10	0.43	0.09
	(0.35, 1.00)		(0.26, 1.00)		(0.23, 0.79)	
Black Cherry	0.76	0.14	0.51	0.12	0.36	0.12
	(0.33, 1.00)		(0.24, 1.00)		(0.16, 0.75)	
Maple	0.70	0.13	0.45	0.10	0.30	0.06
	(0.18, 1.00)		(0.07, 0.97)		(0.07, 0.73)	
Ash	0.69	0.13	0.45	0.10	0.30	0.08
	(0.32, 1.00)		(0.14, 1.00)		(0.15, 0.69)	
Elm	0.65	0.13	0.37	0.10	0.24	0.07
	(0.27, 1.00)		(0.14, 1.00)		(0.12, 0.71)	
Walnut	0.80	0.14	0.54	0.16	0.46	0.15
	(0.41, 1.00)		(0.29, 0.81)		(0.25, 0.79)	
Other Tree Species	0.74	0.15	0.46	0.12	0.24	0.09
	(0.27, 1.00)		(0.10, 1.00)		(0.08, 0.84)	
Softwood Species	0.68	0.18	0.36	0.14	0.21	0.10
	(0.26, 1.00)		(0.13, 0.70)		(0.03, 0.62)	
Understory Woody Species	0.46	0.10	0.33	0.09	0.21	0.06
	(0.25, 0.89)		(0.03, 0.75)		(0.08, 0.58)	

Table 4.—Prediction errors from the best fit equations applied to the crop tree measurements from the oak thinning study on the Hoosier National Forest, Indiana.

Species	Sample	Error		
	Size	Ave(Error)	Ave|Error|	rMSE
white oak	65	1.68	2.04	2.56
red oak	17	2.85	2.85	3.04
chestnut oak	24	-0.07	1.63	1.93
black oak	17	-1.15	1.95	2.71
scarlet oak	34	-0.03	1.55	2.31
shagbark hickory	3	0.43	2.32	2.76
pignut hickory	2	-1.25	1.25	1.27
bitternut hickory	1	6.43	6.43	6.43
yellow poplar	25	-0.82	1.45	1.79
black cherry	31	0.53	2.19	2.86
sugar maple	1	-0.47	0.47	0.47
black walnut	3	-0.26	1.62	1.72
white ash	2	0.24	0.24	0.24
Average	225	0.60	1.92	2.48

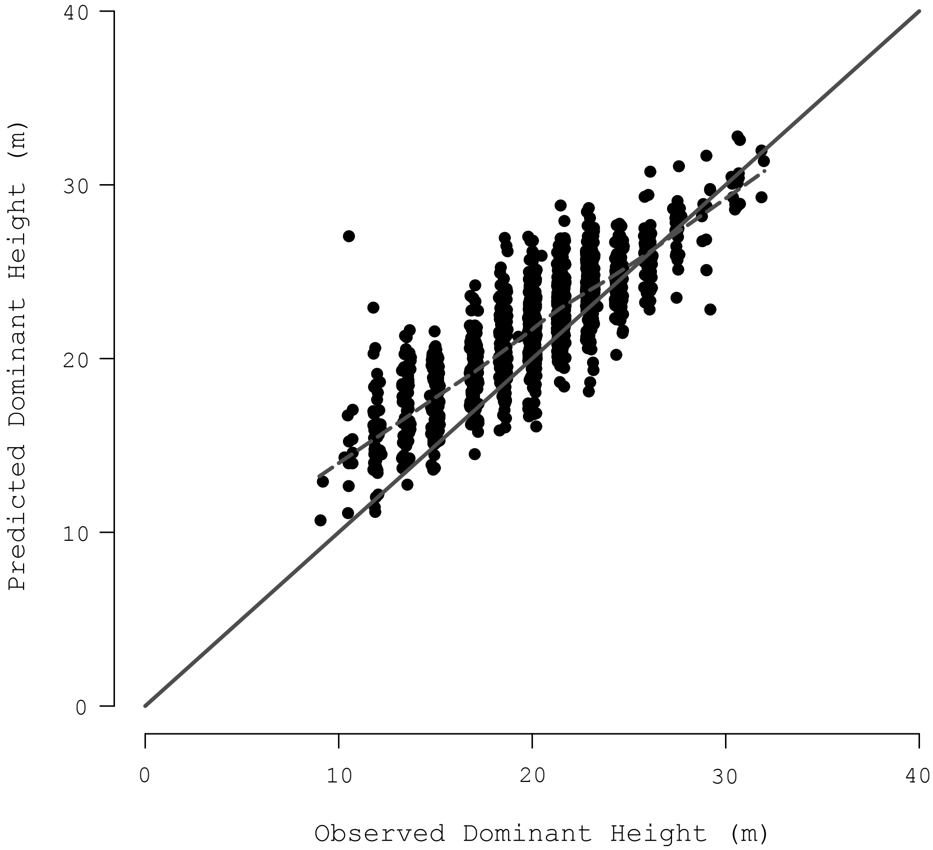

Figure 3.—Predicted versus observed dominant height for the 2003 clearcut plots on the Hoosier National Forest. Solid line is the 1:1 line and the dashed line is a lowess smoothed trend line.

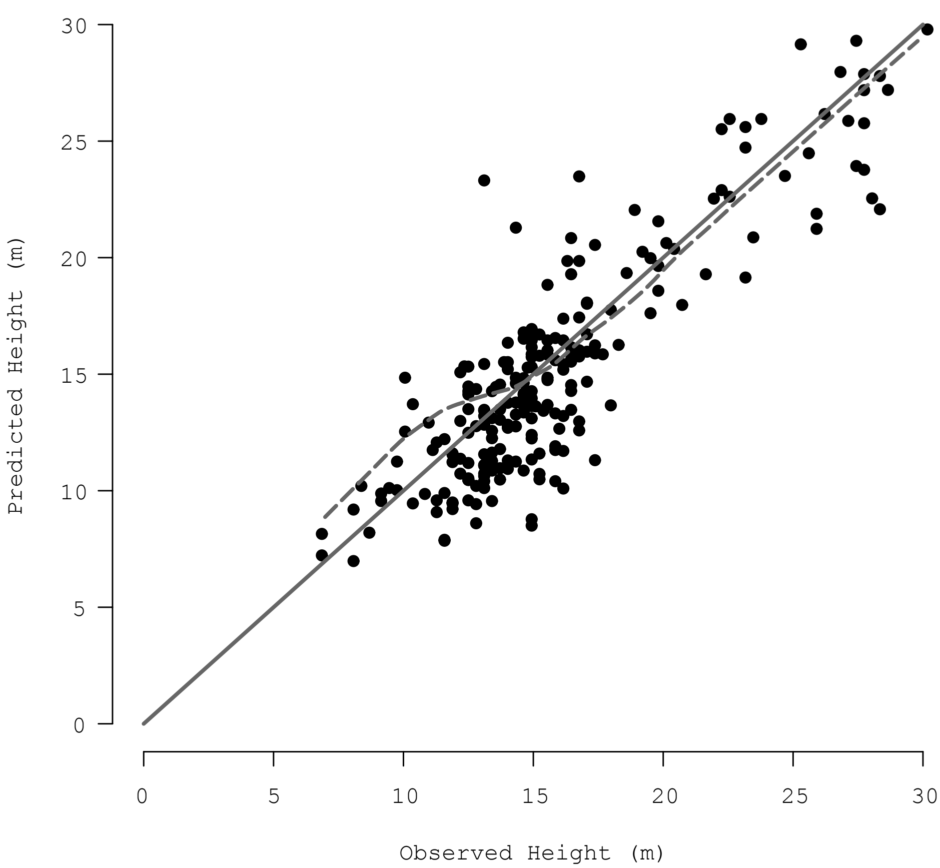

Figure 4.—Predicted versus observed heights for crop trees in the thnning trials on the Hoosier National Forest. Solid line is the 1:1 line and the dashed line is a lowess smoothed trend line.

DISCUSSION

Accurate height-diameter equations are a valuable tool for forest managers. While d.b.h. is easily and accurately measured, height is time consuming and prone to error (Arabatzis and Burkhart 1992, Colbert and others 2002, Trincado and Leal 2006). While several equation forms are frequently used (Huang and others 2000), many of these equations are applicable only to a limited geographic area or limited range of stand ages (Trincado and Leal 2006). As a result, height-diameter equations are often classified as local or general (Arabatzis and Burkhart 1992, Trincado and Leal 2006). General height-diameter equations typically modify a local equation by incorporating additional variables such as stand age, site index, dominant height, maximum height, or other stand structure variables (Huang and others 2000).

The equations developed here started with a local height-diameter equation (Eq. 1) and incorporated dominant height to produce a general height diameter equation (Eq. 2). The b_1 parameter in the local equation, which represents the asymptotic maximum height, was replaced by a power function of dominant height. With the exception of two species, shagbark hickory and blackgum, all species and species groups had better fits with equation 2 than equation 1 (Table 2). Several other researchers (e.g., Bi and others 2000, Diéguez-Aranda and others 200, Wang and Tang 1997) also found that incorporating dominant height into local height-diameter equations resulted in improved fits to data for several tree species; however, Arabatzis and Burkhart (1992) did not find that dominant height was as good as other stand parameters for modeling regional height-diameter relationships in loblolly pine. Leite and Andrade (2003) considered dominant height to be one of the fundamental structural variables in development of height-diameter equations.

Not only were fits improved in this study, resulting equation forms were altered (Fig. 1) by incorporating dominant height into the height-diameter relationship. For all species, the shape parameter (b_3) for equation 1 was greater than 1.0, producing a sigmodial shape height-diameter relationship. The shape parameter for equation 2 was generally smaller than 1.0 producing a monotonically increasing height-diameter relationship (Fig. 1). This change in shape is important to understand since the equations were developed across a wide range of tree and stand ages. For example, predicted heights for a 20-cm d.b.h. yellow-poplar (Fig. 1B) would be about 15 m and would increase to about 25 m for a 40-cm d.b.h. tree based on equation 1; however, using equation 2, the increase is much smaller within a given dominant height class (approximately 17 m to 20 m for a 15-m dominant height stand and 21 m to 25 m for a 25-m dominant height stand).

Dominant height, as incorporated into the height-diameter equations developed here, enables the asymptotic maximum height to change as stands develop. Biologically, it makes sense that asymptotic maximum height should be related to dominant canopy height, since dominant canopy height is the average of the dominant (i.e., tallest) trees in a stand. Others have used stand density measures (e.g., Sharma and Zhang 2004, Staudhammer and LeMay 2000) to represent this change. While these equations may work for relatively pure even-aged stands, it is unlikely that they would adequately represent the relationships observed in the mixed species stands that characterize the Central Hardwoods Region.

The equations developed here cover a broad range of tree sizes and stand ages. Even though the data came only from very young stands and older mature stands, they appear to adequately predict height-diameter relationships in self-thinning stands (Figs. 2, 3, and 4). These equations are an important first step in assessing changes in productivity following clearcutting in south central Indiana.

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