STAND LEVEL HEIGHT-DIAMETER MIXED EFFECTS MODELS: PARAMETERS FITTED USING LOBLOLLY PINE BUT CALIBRATED FOR SWEETGUM

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Abstract.—Mixed effects models can be used to obtain site-specific parameters through the use of model calibration that often produces better predictions of independent data. This study examined whether parameters of a mixed effect height-diameter model estimated using loblolly pine plantation data but calibrated using sweetgum plantation data would produce reasonable predictions of sweetgum. Results showed model calibration resulted in sound predictions of arithmetic mean height for the sweetgum data used in model calibration. However, data at older ages did not occur along the same general linear trend as the younger data used in calibration resulting in poor predictions. It should not be concluded that a mixed models framework using different species in model fitting and calibration will produce poor results. Poor predictions probably were obtained at older ages because the data used in calibration were too young and not because the parameter estimates using loblolly pine could not be calibrated for sweetgum.

INTRODUCTION

Growth and yield models are an integral component of forest management. Models help managers identify the productive capability of a particular species and how different cultural treatments will likely affect economic returns. Mixed models are becoming a popular modeling tool to provide more site-specific predictions of stand development. Both linear and nonlinear mixed models have been used in the development of natural resource models (e.g., Lynch and others 2005, Trincado and Burkhart 2006, Trincado and others 2007, VanderSchaaf and Burkhart 2007). Mixed effects models provide an efficient means to obtain cluster-specific, or for this particular example, stand-specific, parameters through the prediction of cluster-specific random effects. For example, arithmetic mean height (H) can be predicted as a function of quadratic mean diameter (D_q):

$$\ln H = \beta_0 + \beta_1 \ln D_q + \varepsilon \qquad [1]$$

where ln - natural logarithm, H - arithmetic mean height (ft), D_q - quadratic mean diameter (in.), β_0,β_1 - parameters to be estimated, ε - random error, where it is assumed $\varepsilon \sim N(0, \sigma^2 I)$.

Equation [1] provides what is often termed a population average estimate of H for a given D_q. We assume that the parameters β_0 and β_1 are fixed, or that the parameter estimates apply to every experimental unit (e.g., stand) in a population. Whether a stand is located in Florida or Arkansas, the parameter estimates are assumed to be correct. However, stand-specific characteristics such as soil type, nutrient status, competition from herbaceous vegetation, elevation, aspect, and genetic stock may cause parameters to differ across stands. Thus, specific stands may have what are generally termed “random parameters” in mixed effects

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model terminology. Equation [1] can be altered by adding stand-specific random effects to the population average parameters to produce stand-specific parameters:

$$\ln H = (\beta_0+u_{0i})+(\beta_1+u_{1i})\ln D_q + \varepsilon \qquad [2]$$

where u_{0i},u_{1i}- stand-specific random effects, assumed to be $N(0,\sigma_0^2)$ and $N(0, \sigma_1^2)$, respectively;, (β_0+u_{0i}) - stand-specific intercept; (β_1+u_{1i}) - stand-specific slope; i - index for a specific stand; and all other variables as previously defined.

Additionally, a covariance, σ_{01}, can be assumed to exist between u_{0i} and u_{1i}, where the parameter is assumed to be N(0). In this particular case, linear mixed effects models produce an efficient estimate of stand-specific parameters because only five parameters are estimated using the model-fitting algorithm (β_0, β_1 σ_1^2, σ_1^2, σ_{01}). Based on the variance and covariance estimates, and Bayesian methodology, we can predict stand-specific random effects (u_{0i},u_{1i}) and then add them to the population average intercept and slope (β_0,β_1) estimates to obtain stand-specific parameters. The prediction of stand-specific random effects is conducted outside the model-fitting algorithm and thus degrees of freedom are not lost. A much less efficient means of obtaining stand-specific parameters would be to estimate parameters separately for each stand. Although the parameter estimation efficiency of mixed models is an advantage, often the greatest advantage is the ability to calibrate the model using data independent of those used in model fitting.

Although several mixed effects models have been developed, no study has examined whether parameters fit using data from one species can be used to obtain cluster-specific random effects of another species. The advantage of using parameters from a species where more complete data in terms of stand ages, planting densities, site qualities, stand development, etc. exist is that more reasonable extrapolations of stand development for another species may be obtained. Cluster-specific random parameters depend on the amount of estimated variability in the random effects and the population average parameter estimates. Thus, differences in site requirements, structural constraints, growth habits, etc. among species may not allow for mixed effects models fit using one species to produce reasonable predictions of stand development for another species. This situation may occur when the population average parameter estimates of the model fitting species are not correct for the model calibration species, and/or the variability in the cluster-specific random effects of the model fitting species is not representative of the variability in the model calibration species.

Loblolly pine (*Pinus taeda* L.) is one of the most commercially important species in the Southeastern United States. Several long-term studies have been initiated and many growth and yield models have been developed for this species. Sweetgum (*Liquidambar styraciflua* L.) can generate revenue for a landowner but to a much smaller extent than loblolly pine. Hence, very few growth and yield models have been developed to predict stand development of plantations for sweetgum. Unlike many other hardwood species, sweetgum has a growth habit similar to loblolly pine. Sweetgum trees when young have a strong excurrent growth habit. They have conically shaped crowns and self-prune well (Kormanik 1990). Additionally, sweetgum is intolerant of shade and is therefore a relatively fast-growing species similar to loblolly pine. Thus, it may be possible to fit a mixed effects model using loblolly pine data and then to calibrate the model using sweetgum data to obtain reasonable predictions of future sweetgum stand development. It is the objective of this paper to determine the feasibility of using this approach to predict the height-diameter relationship of sweetgum plantations.

METHODS

Data Used in Model Fitting

Tree and plot measurements were obtained from long-term studies of planted loblolly pine and sweetgum in southeastern Arkansas. Observations of H and D_q were obtained from permanent research plots located in a loblolly pine plantation near Monticello, AR (http://www.afrc.uamont.edu/growthyield/montthinprun/index.html). The stand was planted in 1958 at a spacing of 8 ft square using 1-0 seedlings obtained from a state nursery located in Arkansas. Genetic stock was of a local seed source. Plots were originally established in 1970 and were measured at age 12 prior to conducting various thinning and pruning treatments. Thinning treatments from below at age 12 removed trees until residual basal areas per acre of 40, 60, 80, and 100 sq ft were obtained. At ages of 15, 24, 27, 30, 35, and 40, thinning treatments reduced basal areas to 30, 50, 70, and 90 sq ft per acre. Pruning treatments consisted of branch removal up to heights of 25, 40, and 50 percent of total tree height at ages 12 and 15. These thinned and pruned plots were also measured at ages 16, 19, 37, 43, 45, and 48. All observations used in model fitting were plot-level values obtained just prior to thinning treatments. Unthinned control plots were established in 1984, measured at age 27, and then remeasured at ages of 30, 35, 37, 40, 43, 45, and 48. Table 1 provides a summary of plot-level characteristics across all inventory ages and treatments. Site index was determined to be near 62 ft (base age 25).

Table 1.—Plot-level characteristics of the loblolly pine model fitting dataset (n = 559), and the sweetgum model calibration and fitting (equations 1 and 3) dataset (ages 5, 6, 7, and 8) and the sweetgum model validation dataset (ages 14 and 15). Where: Min—minimum, Max—maximum, Age—from seed, TPA—trees per acre, QMD—quadratic mean diameter, BAA—basal area per acre, H—arithmetic mean height.

Age		TPA	QMD (in.)	BAA (sq ft)	H (ft)
		Loblolly pine			
	Min	10	5.5	22	34
	Mean	127	14.8	80	69
All ages	Max	728	27.7	188	99
		Sweetgum			
	Min	312	1.7	7	10
	Mean	427	2.0	10	11
5	Max	513	2.3	14	13
	Min	312	1.9	8	13
	Mean	409	2.3	12	14
6	Max	491	2.6	17	17
	Min	290	2.5	11	15
	Mean	391	2.7	16	17
7	Max	491	3.2	26	21
	Min	268	3.1	17	19
	Mean	382	3.5	25	21
8	Max	468	3.9	37	25
	Min	245	4.6	34	31
	Mean	363	5.0	49	38
14	Max	446	5.5	69	44
	Min	245	5.0	39	37
	Mean	366	5.4	57	41
15	Max	446	6.1	76	48

For sweetgum, observations of H and D_q were obtained from permanent research plots established in 1979 in an unthinned plantation located near Monticello, AR, using 1-0 seedlings of local seed source planted at a 9 ft square spacing (Ku and others 1981, Guo and others 1998). Fertilizer treatments (N only [205 lb N/ac], P only [123 lb P/ac], and the combination of the two nutrients) and controls were established when the plantation was 4 yrs old. Trees were measured beginning at age 5 and remeasured at ages of 6, 7, 8, 14, and 15. After fertilization at age 4 no additional treatments were applied. Table 1 provides a summary of plot-level characteristics by plantation age. The natural fertility is moderate and the site index was determined to be 80 ft at base age 50 (Guo and others 1998).

Table 2.—Population average (β_0 and β_1) parameter estimates and associated standard errors and random effects variance (σ^2_0, σ^2_1) and covariance (σ_{01}) parameter estimates for the final mixed-effects model. There were a total of 559 observations used in model fitting and the total number of clusters was 45. Parameter estimates for equations [1] and [3] using the sweetgum model fitting dataset (ages 5, 6, 7, and 8, n = 96) are also given along with their associated standard errors. Where: -2LL—twice the negative log-likelihood, AIC—Akaike's Information Criterion, MSE—mean square error, and Adj. R^2 is the adjusted R^2.

Validation results when predicting arithmetic mean height as a function of quadratic mean diameter for the sweetgum ages of 14 and 15 are also provided (n = 48). Eqn. [1]—sweetgum refers to using OLS to estimate parameters when pooling data from all plots using ages 5, 6, 7, and 8, while Eqn. [1]—individual sweetgum plots refers to using OLS to estimate parameters separately for each individual plot using ages 5, 6, 7, and 8.

	Model fitting results							
	Mixed—loblolly pine		Eqn. [1]—sweetgum		Eqn. [3]—sweetgum		Eqn. [1]—individual sweetgum plots	
Parameter	Estimate	SE	Estimate	SE	Estimate	SE		
β0	2.2336	0.0293	1.6877	0.0392	0.8215	0.1144	-	-
β1	0.7563	0.0147	1.1080	0.0405	0.5350	0.0795	-	-
β2	-	-	-	-	0.7586	0.0966	-	-
σ^2_0								
σ^2_1	0.0297	-	-	-	-	-	-	-
σ_{01}	0.0084	-	-	-	-	-	-	-
	-0.0149	-	-	-	-	-	-	-
-2LL	-1672.0000		-191.8000		-236.9000		-	
AIC	-1664.0000		-		-		-	
MSE	0.0019		0.0071		0.0043		-	
Adj. R^2	-		0.8886		0.9330		-	
							Model validation results	
Bias	8.8400		5.7700		-2.4500		4.8800	
Variance	4.4500		5.6100		5.1200		10.3900	
MSE	82.5200		38.8700		11.1400		34.2500	

Model Development and Parameter Estimation

Parameters of equation [2] were estimated for the loblolly pine plantation dataset using SAS Proc MIXED (Littell and others 1996), which assumes random errors are normally distributed and subsequently estimates parameters using maximum likelihood. Rather than simply assuming β_0 and β_1 were random across plots, likelihood ratio tests were conducted to determine if assuming β_0 was random, β_1 was random, and if assuming a covariance term existed between u_{0i} and u_{1i} (σ_{01}), produced better model fit statistics. For the sake of brevity, complete likelihood ratio test results are not presented. Based on likelihood ratio tests and Akaike's Information Criterion, a model where both β_0 and β_1 were considered to vary across plots and which includes a covariance term, σ_{01}, was found to be best (Table 2).

In many cases random effects account for nearly all autocorrelation among observations when using longitudinal datasets (Trincado and Burkhart 2006, VanderSchaaf and Burkhart 2007, Vonesh and Chinchilli 1997); however, a modeler can also directly model the random error structure. When parameters are estimated in a mixed-effects model framework using data from one species and calibrating for another, the measurement intervals may not be the same among the datasets. This difference can cause problems when trying to estimate covariances of the random errors because a covariance structure that is appropriate for the model fitting dataset may not be appropriate for the model calibration dataset. For this particular study, since measurement intervals differed among the datasets, the random error covariance-variance matrix was assumed to be $\sigma^2 I$.

To determine whether a mixed effects model fit using loblolly pine and calibrated using sweetgum would produce better predictions of H than a conventionally estimated height-diameter equation using sweetgum, parameters of equation [1] were estimated. The conventional approach is to use data of the species of interest, which in this case is sweetgum, and ordinary least squares. To account for the impacts of fertilization and stand density on the height-diameter relationship, a second conventional equation was developed that included age as an additional regressor:

$$\ln H = \beta_0 + \beta_1 \ln D_q + \beta_2 \ln Age + \varepsilon \qquad [3]$$

where Age—years from seed (plantation age plus 1 year), and all other variables as previously defined.

Data used in estimating parameters of equations [1] and [3] were the same data used in calibrating equation [2], ages 5, 6, 7, and 8 from the sweetgum plantation. Proc Mixed of SAS (SAS Inc, Cary, NC) was used to estimate parameters. Initial attempts were made to include basal area per acre rather than age in equation [3], but the basal area parameter was highly non-significant (alpha level = 0.4674).

Prediction errors were compared between the three approaches using the sweetgum data at ages of 14 and 15 and the validation process proposed by Arabatzis and Burkhart (1992). The difference between the observed and predicted arithmetic mean height($e_{ij} = H_{ij} - \hat{H}_{ij}$)for each individual plot (i) and age (j) was calculated for all three parameter estimation approaches. The mean residual ($\bar{e}$) and the sample variance (v) of residuals were computed and considered to be estimates of bias and precision, respectively. An estimate of mean square error (MSE) was obtained combining the bias and precision measures using the following formula:

$$MSE = \bar{e}^2 + v$$

Values of MSE were compared between the three approaches (equation [1] fit using sweetgum data, equation [3] fit using sweetgum data, equation [2] fit using loblolly pine data but calibrated using sweetgum data) to determine which one was most appropriate for this particular sweetgum plantation dataset. To account for logarithmic transformation bias, the procedure recommended by Baskerville (1972) and also described by Trincado and others (2007) was used. All validation statistics presented in this paper are based on untransformed errors.

RESULTS AND DISCUSSION

All three modeling (equations [1]—[3]) approaches produced bias predictions of H when extrapolating beyond the range of data used in model calibration or fitting (Table 2). The sweetgum-calibrated mixed effects model produced the most bias, resulting in the greatest MSE. However, using a mixed effects model actually produced the lowest variance in predictions.

Figure 1 demonstrates that model calibration of equation [2] resulted in sound predictions of H for the sweetgum data used in model calibration. However, Hs at ages of 14 and 15 were not on the same linear

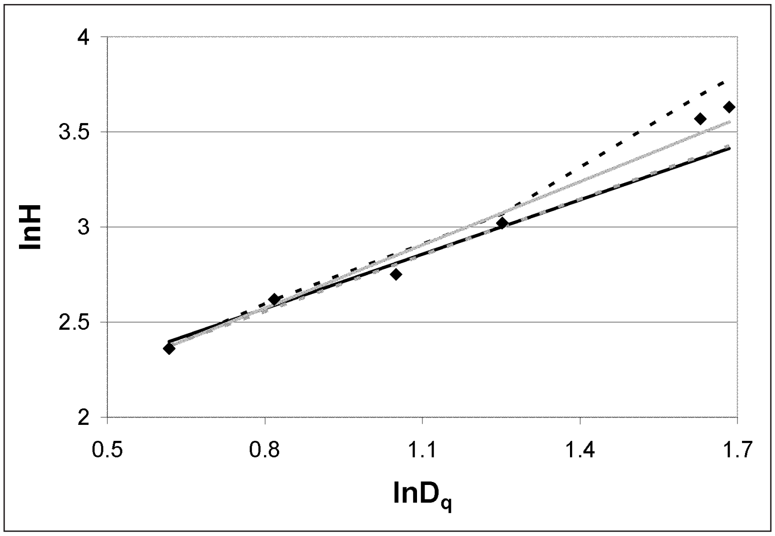

Figure 1.—Observed values of logarithmically transformed arithmetic mean height and quadratic mean diameter at ages 5, 6, 7, 8, 14, and 15 for the phosphorus-treated plot in block 1 (diamonds). The solid black line are predictions using a mixed effects calibrated model (equation 2), the dotted black line are predictions using equation [3], the solid gray line are predictions using equation [1] when pooling data from all plots, and the dotted gray line are predictions using equation [1] fit specifically to observations at ages 5, 6, 7, and 8 of this plot.

trend as Hs at ages of 5, 6, 7, and 8 and therefore predictions at ages of 14 and 15 were generally poor. Calibrated predictions were always less than the observed heights (Figs. 1 and 2). Equations [1] and [3] were most likely superior in predicting H at ages 14 and 15 because they ignored the trends of individual plots at ages of 5, 6, 7, and 8 and fit a model that better captured the general trend across all plots. If the Hs at ages 14 and 15 were on the same linear trend for individual plots as the Hs at ages 5, 6, 7, and 8, the mixed effects model would probably have been superior.

In an attempt to check the validity of the previous sentence, parameters using equation [1] were estimated for each individual sweetgum plot, and then validation statistics were conducted as described previously. Predictions using ordinary least squares (OLS) for each individual sweetgum plot produced less bias relative to the calibrated mixed effects model; however, the variance was much greater (Table 2). Overall, the MSE was less when estimating parameters using OLS for each individual plot compared to a mixed effects model framework. However, similar to the calibrated mixed effects model, the two conventional approaches often resulted in better predictions at ages 14 and 15 than OLS fits of each individual plot using younger data. Figure 1 shows that indeed parameter estimates using OLS based on data from this particular plot at ages 5, 6, 7, and 8 failed to capture the general trend at ages 14 and 15 (in fact the OLS and calibrated mixed effects model essentially follow the same trend). Thus, for this particular sweetgum dataset, the use of younger ages both to calibrate a mixed effects model and to estimate parameters by means of OLS produced poor predictions at older ages since all data were not on the same linear trend. Perhaps a non-linear model would have produced better results; however, the logarithmic transformation was used to help linearize the data. For either of the four approaches, prediction errors do not appear to be related to D_q (Fig. 2).

These results demonstrate that a mixed effects model calibrated for a species other than the one used in model fitting will produce very good predictions of the data used in model calibration. However, this study shows that when extrapolations beyond the range of data used in model calibration may yield very poor predictions. These results probably were observed not because loblolly pine was used in the mixed effects model fitting, but rather because the sweetgum data at older ages were not on the same linear trend as the younger sweetgum data used in model calibration (perhaps resulting from decreases in diameter increment at older ages due to excessive stand density). This observation is demonstrated by the poor MSE

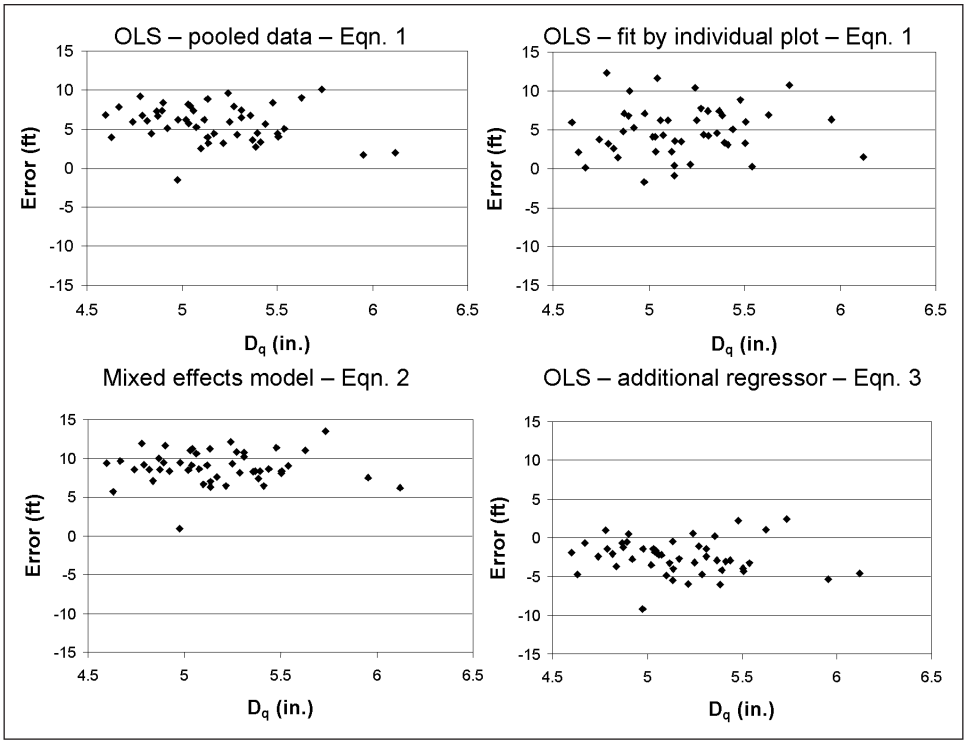

Figure 2.—Untransformed errors when predicting arithmetic mean height at ages of 14 and 15 for equation [1] when estimating parameters for each individual plot, equation [1] when pooling data from all individual plots, and for equations [2] and [3]. n = 48 for each predictive alternative.

results even when estimating parameters for each sweetgum plot using OLS (Table 2 and Fig. 1). Results from this study, as well as similar unpublished calibration studies conducted by the author, demonstrate that calibrating a mixed effects model using young data and extrapolating to older ages may produce poor results. Perhaps if the 14- and 15-year-old data were used in model calibration along with the ages of 5, 6, 7, and 8, predictions at ages of 20 and 25 would be better. Although height measurements were obtained at older ages for this sweetgum study, an ice storm reduced the tops of many trees at age 19. Thus, damage from the ice storm would be a confounding factor if data from these older ages were used.

It is not exactly clear why the data at ages of 14 and 15 were not on the same linear trend for individual plots. Perhaps a log transformation is not sufficient to produce linearity. The stands were fertilized a second time when they were 24 years old; thus fertilization was not a factor at ages 14 and 15.

In conclusion, results from this study demonstrate a mixed effects model fit using one species but calibrated for another did not produce adequate predictions when extrapolating beyond the range of data used in model calibration. This result was observed because data at ages 14 and 15 were not on the same general linear trend as the data used in model calibration, ages 5, 6, 7, and 8. It should not be concluded that mixed effect models do not produce adequate predictions and that a mixed effects model fit using one species cannot be calibrated for another species. Results from this study are additional evidence that users of models need to be careful when extrapolating beyond the range of data used in model fitting or model calibration. Further research needs to be conducted to determine if calibrating mixed effects models using data from young ages as well as during mid-rotation will produce adequate predictions for economic rotation ages.

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