New Method for Determining the Relative Stand Density of Forest Inventory Plots

Christopher W. Woodall and Patrick D. Miles[1]

Abstract.—Determining the relative density of Forest Inventory and Analysis plots is complicated by the various species and tree size combinations in the Nation's forested ecosystems. Stand density index (SDI), although developed for use in even-aged monocultures, has been used for stand density assessment in large-scale forest inventories. To improve application of SDI in uneven-aged, mixed species stands present in large-scale inventories, a model was developed whereby a stand's maximum SDI was a function of the stand's mean specific gravity (SG) of individual trees. A strong relationship was found between the mean SG of all trees in a stand and the 99th percentiles of the observed distribution of stand SDIs. A model is proposed whereby the mean SG of individual trees in a stand serves as a predictor of a stand's maximum stocking potential, regardless of the stand's diameter distribution and species composition.

Assessing the relative density of hundreds of thousands of forest inventory plots across the Nation is complicated by the diameter distributions, species compositions, and site conditions unique to every forest stand. Most techniques for assessing relative stand density were developed for application in individual stands consisting of monocultures or regionally common species mixtures (Reineke 1933, Krajicek *et al*. 1961, Gingrich 1967, Drew and Flewelling 1979). Although a substantial body of literature addresses the development of small-scale, stand-specific relative density measures, scant research has been conducted to develop effective relative density assessment techniques for use at strategic scales inclusive of all tree species and size combinations.

Stand density index (SDI) is a method for estimating relative stand density. SDI was first proposed by Reineke (1933) as a stand density assessment tool based on size-density relationships observed in fully stocked monocultures. SDI is defined as the equivalent trees per hectare at a quadratic mean diameter of 25 cm and is formulated as the following:

$$SDI = tph\ (d.b.h._q/25)^{1.6} \qquad (1)$$

where *SDI* is stand density index, *tph* is number of trees per hectare, and $d.b.h._q$ is quadratic mean diameter (cm) at breast height (1.4 m) (Long 1985). The only way to appropriately determined SDI in stands with Gaussian diameter distributions is to use the summation method (Long and Daniel 1990, Shaw 2000, Ducey and Larson 2003) by which the SDIs for individual diameter at breast height (d.b.h.) classes are added for the entire stand. The SDI summation method is formulated as follows:

$$SDI = \Sigma\ tph_i\ (d.b.h._i/25)^{1.6} \qquad (2)$$

where $d.b.h._i$ is the midpoint of the tph_i diameter class (cm), and *i*th is the number of trees per hectare in the *i*th diameter class (Long 1995, Shaw 2000).

The SDI of even-aged monocultures is typically compared to an empirically observed, species-specific maximum SDI for determining a stand's relative density. Maximum SDI (SDI_{max}) is defined as the maximum possible density for a given mean tree size in a self-thinning population (Long 1996). SDI_{max} has typically been determined strictly through empirical means, finding the heaviest stocked stand on the landscape. Percentages of species' SDI_{max} have been related to prominent stages of stand development (Long 1985), making their determination valuable for strategic-scale assessments of stocking. A relative density of 25 percent of SDI_{max} is associated with the onset of competition, 35 percent of SDI_{max} is associated with the lower limit of full-site occupancy, and 60 percent SDI_{max} is associated with the lower limit of self-thinning (Long and Daniel 1990).

[1] Research Foresters, U.S. Department of Agriculture, Forest Service, North Central Research Station, St. Paul, MN 55108. Phone: 651–649–5141; e-mail: cwoodall@fs.fed.us.

SDI has rarely been applied in mixed species stands (Binkley 1984, Puettman *et al.* 1993, Torres-Rojo and Martinez 2000, Williams 2003) because of a lack of empirical and theoretical information. In most studies, investigators were able to empirically determine SDI for specific forest types in local areas but were unable to state any broader conclusions (Binkley 1984, Puettman *et al.* 1993, Williams 2003). As an alternative to empirically determining SDI_{max} for mixed species stands, past research in monocultures suggests that SDI_{max} may be predicted using species' specific gravities (SGs; Dean and Baldwin 1996). Dean and Baldwin (1996) suggest that species-specific variation in the maximum mechanical leverage canopies exert on stems may help explain species variation in SDI_{max}. They found that species' SG was inversely related to SDI_{max}. The SDI_{max} versus SG relationship has not been further explored or applied in stand inventory/management activities and may serve as a novel methodology for estimating SDI_{max}. Therefore, the goal of this study is to develop and validate a technique for estimating SDI_{max} for stands containing diverse tree species and size combinations using the mean specific gravities (SG_m) for individual trees.

Methods

Plot data from the national Resources Planning Act (RPA) database were used as observations in this study (Smith *et al.* 2004). The RPA database contains plot and tree data collected by the Forest Inventory and Analysis (FIA) program of the U.S. Department of Agriculture (USDA) Forest Service. Briefly, the plot design for FIA inventory plots consists of four 7.2-m, fixed-radius subplots spaced 36.6 m apart in a triangular arrangement with one subplot in the center of the triangle. All trees located on forested subplots with a d.b.h. of at least 12.7 cm are inventoried. (For further information on the RPA database and FIA sample design, refer to Smith *et al.* (2004) and Bechtold and Patterson [in press].) The study data set consisted of data from all fully forested plots (n = 119,235) from the RPA database that had at least one tree of the selected eight species representing diverse growth conditions and forest ecosystems across the United States: loblolly pine (*Pinus taeda*), ponderosa pine (*Pinus ponderosa*), Douglas fir (*Pseudotsuga menziesii*), paper birch (*Betula payrifera*), trembling aspen (*Populus tremuloides*), white oak (*Quercus alba*), lodgepole pine (*Pinus contorta)*, and red maple (*Acer rubrum*) (n = 119,235). A validation data set was created using all fully forested inventory plots (n = 29,307) from the RPA database that did not contain any of the study tree species.

For all study plots, the *tph* and *SDI* (equation [2]) for 10-cm d.b.h. classes were determined for study species and other species in each plot. The SG for all study trees was based on data available from the USDA Forest Service Forest Products Lab (U.S. Department of Agriculture, Forest Service 1999). The relationship between the 99th percentile SDI (SDI_{99}) for classes of SG_m (0.015 SG_m class width, 26 classes) for the study data set was modeled as follows:

$$E(SDI_{99}) = b_o + b_1(SG_m) \qquad (3)$$

where E(.) is statistical expectation, SG_m is the mean SG for all trees per plot, and b_0 and b_1 are parameters to be estimated. SDI_{99} was used instead of SDI_{max} as the response variable because the process of modeling SDI_{max} relationships can be highly affected by outliers. Therefore, for predicting SDI_{max} based on mean stand SGs, SDI_{99} serves as a surrogate for SDI_{max}. The ability of the regression model (equation [3]) to estimate SDI_{max} was evaluated using the validation data set by predicting SDI_{99} for SG_m classes (0.025 SG_m class width, 13 classes) and computing relative residuals [(observed – predicted)/observed)].

Results/Discussion

The ability of SG_m to predict SDI_{max} was evaluated for the SDI_{99} within classes of SG_m. For predictions of SDI_{99}, SG_m explained 92 percent of the variation ($\hat{b}_0$=2057.3, $\hat{b}_1$ = – 2098.6) (fig. 1, table 1). The model's ability to predict SDI_{99} was evaluated using the validation data set. Analysis of the relative residuals for the 13 classes indicates a slight bias of the estimated linear relationship so that the SDI_{99} may be overpredicted (table 2, fig. 2). The mean of the relative residuals was 0.05 (table 2). The absolute mean of relative residuals for the 13 validation data set classes of SG_m was 0.08 (table 2).

Figure 1.—*99th percentile SDIs by mean stand SG for over 119,000 RPA plots.*

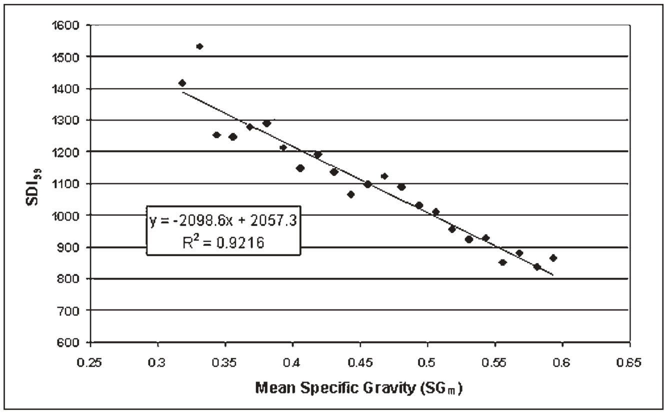

Figure 2.—*Relative residuals for predictions of SDI_{99} for validation data set SG_m classes.*

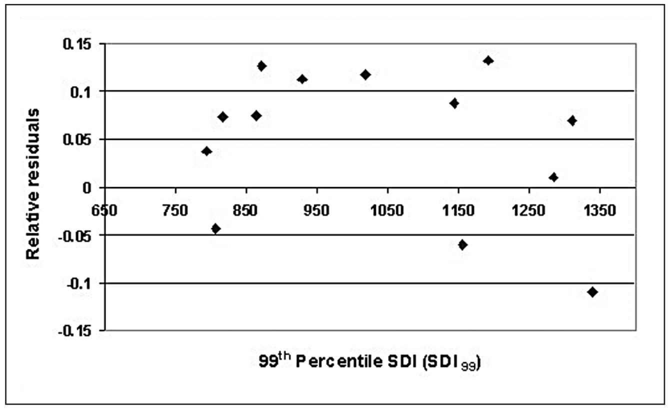

Table 1.—*Maximum observed and 99th percentile stand SDs for 119,235 RPA plots by classes of mean stand SG.*

Mean SG classes	Number of sample plots	Maximum observed stand SDI	99th percentile observed stand SDI
0.3126–0.3250	855	2,819	1,413
0.3251–0.3375	1,697	1,908	1,529
0.3376–0.3500	3,546	1,814	1,252
0.3501–0.3625	4,894	2,285	1,242
0.3626–0.3750	5,884	1,775	1,275
0.3751–0.3875	11,056	2,640	1,288
0.3876–0.4000	6,084	1,883	1,210
0.4001–0.4125	5,470	1,951	1,145
0.4126–0.4250	5,290	2,162	1,190
0.4251–0.4375	5,149	1,718	1,134
0.4376–0.4500	5,750	2,075	1,062
0.4501–0.4625	4,678	1,811	1,095
0.4626–0.4750	8,478	1,704	1,120
0.4751–0.4875	7,030	1,396	1,087
0.4876–0.5000	6,491	1,309	1,026
0.5001–0.5125	6,150	1,347	1,009
0.5126–0.5250	5,928	1,266	951
0.5251–0.5375	5,592	1,299	921
0.5376–0.5500	4,891	1,507	923
0.5501–0.5625	3,961	1,403	848
0.5626–0.5750	3,133	1,417	876
0.5751–0.5875	2,514	1,439	834
0.5876–0.6000	1,546	1,404	865

Table 2.—*Observed and predicted 99th percentile SDIs for 29,307 RPA validation plots for 13 classes of mean stand SG.*

Weighted mean SG classes	Number of sample plots	Observed 99th percentile SDI	Predicted 99th percentile SDI	Relative residuals[a]
0.3001–0.3250	1,637	1,310	1,401	0.07
0.3251–0.3500	1,214	1,191	1,349	0.13
0.3501–0.3750	1,987	1,284	1,297	0.01
0.3751–0.4000	1,714	1,144	1,244	0.09
0.4001–0.4250	2,210	1,339	1,192	– 0.11
0.4251–0.4500	1,367	1,019	1,139	0.12
0.4501–0.4750	2,780	1,156	1,087	– 0.06
0.4751–0.5000	2,606	929	1,034	0.11
0.5001–0.5250	2,994	872	982	0.13
0.5251–0.5500	5,445	864	929	0.08
0.5501–0.5750	3,245	817	877	0.07
0.5751–0.6000	1,602	794	824	0.04
0.6001–0.6250	506	807	772	– 0.04

[a] Relative residuals = (observed – predicted)/observed.

Because the majority of past SDI research focused solely on pure species stands (Reineke 1933, Stage 1968, Long 1985, Sterba and Monserud 1993, Woodall *et al.* 2003), self-thinning relationships underlying SDI has been assumed to be affected by mixed species compositions. Values of SDI_{max} that guide SDI application in stand-stocking assessments are always listed by single species (Long 1985). Unfortunately, vast acreages of forests of the United States are covered by mixed species stands. A finding from Dean and Baldwin (1996) forms the basis of our attempt to develop a method for estimating more stand-specific SDI_{max}. Dean and Baldwin (1996) found that a species' SG was inversely related to its SDI_{max}. The same result was found in our study. We attempted to take this premise a step farther and determine the mean SG for all trees in a stand, regardless of species. Results indicated a relationship between SDI_{99} and SG_m for classes of SG_m. Validation of our model to predict a stand's SDI_{99} based on its SG_m indicated a slight bias toward overpredicting SDI_{99} (0.08). The nearly 29,000 plots in the validation data set, however, represent unique combinations of uncommon tree species across the United States (e.g., Osage-orange [*Maclura pomifera*] and Ohio buckeye [*Aesculus glabra*]) in which trying to determine a SDI_{max} would be nearly impossible using other methodologies.

Methods for assessing relative stand density in strategic-scale assessments may be augmented by the results of this study. By using the summation method to determine current stand SDI and SG_m to predict SDI_{99} as a surrogate for SDI_{max}, we may quantify relative stand density across the Nation regardless of a stand's species and tree size combinations. SDI methods presented in this study warrant future refinement and application in strategic-scale density assessment situations such as found in national fire hazard reduction efforts.

Conclusions

The SDI_{max} that may be attained by any individual stand is affected by the stand's species composition and size distribution. Because SDI_{max} may be unique for individual stands, a stand-specific model is suggested to predict SDI_{max}. The SG of individual species may be used to define the stem mechanics driving self-thinning dynamics resulting in a stand-specific SDI_{max}. This study found a relationship between the SDI_{99} by classes of mean stand SG and SG_m of all trees in a stand. If SG_m may be considered a predictor of SDI_{99}, as a surrogate for SDI_{max}, relative densities of individual stands may be estimated across large scales, regardless of diameter distributions and species compositions.

Literature Cited

Bechtold, W.A.; Patterson, P.L., eds. [In press]. Forest Inventory and Analysis national sample design and estimation procedures. U.S. Department of Agriculture, Forest Service. 106 p.

Binkley, D. 1984. Importance of size-density relationships in mixed stands of Douglas-fir and red alder. Forest Ecology and Management. 9: 81–85.

Dean, T.J.; Baldwin, V.C., Jr. 1996. The relationship between Reineke's stand-density index and physical stem mechanics. Forest Ecology and Management. 81: 25–34.

Drew, T.J.; Flewelling, J.W. 1979. Stand density management: an alternative approach and its application to Douglas-fir plantations. Forest Science. 25: 518–532.

Ducey, M.J.; Larson, B.C. 2003. Is there a correct stand density index? An alternate interpretation. Western Journal of Applied Forestry. 18: 179–184.

Gingrich, S.F. 1967. Measuring and evaluating stocking and stand density in upland hardwood forests in central states. Forest Science. 13: 39–53.

Krajicek, J.E.; Brinkman, K.A.; Gingrich, S.F. 1961. Crown competition—a measure of density. Forest Science. 7: 35–42.

Long, J.N. 1985. A practical approach to density management. Forestry Chronicle. 61: 23–27.

Long, J.N. 1995. Using stand density index to regulate stocking in uneven-aged stands. In: Uneven-aged management—opportunities, constraints, and methodologies. Misc. Pub. No. 56. Missoula, MT: U.S. Department of Agriculture, Forest Service, Montana Forest and Conservation Experiment Station: 110–122.

Long, J.N. 1996. A technique for the control of stocking in two-storied stands. Western Journal of Applied Forestry. 11(2): 59–61.

Long, J.N.; Daniel, T.W. 1990. Assessment of growing stock in uneven-aged stands. Western Journal of Applied Forestry. 5: 93–96.

Puettman, K.J.; Hann, D.W.; Hibbs, D.E. 1993. Evaluation of the size-density relationships for pure red alder and Douglas-fir stands. Forest Science. 39: 7–27.

Reineke, L.H. 1933. Perfecting a stand-density index for even-aged stands. Journal of Agricultural Research. 46: 627–638.

Shaw, J.D. 2000. Application of stand density index to irregularly structured stands. Western Journal of Applied Forestry. 15: 40–42.

Smith, W.B.; Miles, P.D.; Vissage, J.S.; Pugh, S.A. 2004. Forest resources of the United States, 2002. Gen. Tech. Rep. NC-241. St. Paul, MN: U.S. Department of Agriculture, North Central Research Station. 137 p.

Stage, A.R. 1968. A tree-by-tree measure of site utilization for grand fir related to stand density index. Resour. Note INT-77. Ogden, UT: U.S. Department of Agriculture, Forest Service, Intermountain Forest and Range Experiment Station.

Sterba, H.; Monserud, R.A. 1993. The maximum density concept applied to uneven-aged mixed-species stands. Forest Science. 39: 432–452.

Torres-Rojo, J.M.; Martinez, A. 2000. Relative stand density index for mixed even-aged stands. Agrociencia. 4: 497–507.

U.S. Department of Agriculture, Forest Service. 1999. Wood handbook—wood as an engineering material. Gen. Tech. Rep. FPL-113. Madison, WI: U.S. Department of Agriculture, Forest Service, Forest Products Laboratory.

Williams, R.A. 2003. Use of stand density index as an alternative to stocking percent in upland hardwoods. Northern Journal of Applied Forestry. 20: 137–142.

Woodall, C.W.; Fiedler, C.E.; Milner, K.S. 2003. Stand density index in uneven-aged ponderosa pine stands. Canadian Journal of Forest Research. 33: 96–100.

Additional Readings

Long, J.N.; Smith, F.W. 1984. Relation between size and density in developing stands: a description and possible mechanism. Forest Ecology and Management. 7: 191–206.