Direct Sampling for Stand Density Index

Mark J. Ducey and Harry T. Valentine

ABSTRACT

A direct method of estimating stand density index in the field, without complex calculations, would be useful in a variety of silvicultural situations. We present just such a method. The approach uses an ordinary prism or other angle gauge, but it involves deliberately "pushing the point" or, in some cases, "pulling the point." This adjusts the usual limiting distances in prism sampling to match the limiting distances required for sampling with a specified stand density factor (SDF). The estimate of stand density associated with a point is just the number of trees tallied times the SDF; no diameter measurements are needed. We give guidelines on choosing an SDF on the basis of expected stand characteristics and equations for calculating the limiting distances and the amount of "push." Field implementation of the resulting method is extremely fast and simple.

Keywords: Reineke, relative density, density management diagram, horizontal point sampling

Foresters need a measure of stand density in a variety of silvicultural, forest health, and habitat management applications (Long 1985). Reineke's (1933) stand density index (SDI), along with related density measures such as those of Curtis (1971) and Long and Daniel (1990), has proven valuable and enjoys enduring popularity.

Zeide (2005) calls Reineke's SDI "the best measure we have" for evaluating stand density but bemoans its dismissal by such authorities as Spurr (1952, 282), Bickford et al. (1957), and Avery and Burkhart (2002, 324), in part on the grounds that basal area is simpler to measure than SDI. Basal area can, of course, be measured simply by multiplying the number of trees tallied at a prism point by the basal area factor of the prism used (Bitterlich 1948, Grosenbaugh 1952). By contrast, methods for obtaining SDI have relied on charts or on mathematical computation but have always required postprocessing of inventory data after data collection is complete. Stand prescriptions and density management recommendations written in terms of SDI must often be converted back to basal area for practical use (Zeide 2005). A direct method of estimating SDI in the field would be quite useful but has so far remained elusive.

Here, we propose just such a direct method. The approach involves modifying prism sampling by deliberately "pushing the point" (Oderwald and Gregoire 1995). Ordinarily, pushing the point is an error. We push the point (or, in some cases, do the opposite by "pulling the point") by a specified amount to change the basal area factor (BAF) of the prism to a stand density factor (SDF). The resulting field procedure is extremely fast and simple; stand density index at a point can be calculated by multiplying the number of tallied trees by the SDF, without any tree diameter measurements.

Stand Density Index

Reineke (1933) proposed a stand density index for even-aged stands of the form

$$\mathrm{SDI} = \mathrm{TPA} \times (\mathrm{QMD}/10)^R \tag{1}$$

where SDI is stand density index, TPA is trees per acre, QMD is stand quadratic mean diameter (in.), and R is an exponent that is constant for a particular species and independent of site and age. Reineke (1933) suggested $R = 1.605$ for several western conifer species. We will use that value of R for the specific recommendations and tables presented in this article, although our approach generalizes easily when R takes a slightly different value.

Long and Daniel (1990), building on earlier work by Stage (1968) and Curtis (1971), suggested that for uneven-aged or irregular stands a modified form of SDI might be more appropriate:

$$\mathrm{SDI} = \sum_i \mathrm{TPA}_i \times (D_i/10)^R \tag{2}$$

where the summation is over diameter classes in a stand, TPA_i is the number of trees in the ith diameter class, and D_i is the dbh of the ith class. Although the additive form of SDI in Equation 2 has some appealing properties (Shaw 2000), field evidence of an improvement over the original form remains weak (Ducey and Larson 2003, Woodall et al. 2003). In an even-aged context, however, where the stand diameter distribution is not too broad, Equations 1 and 2 give nearly identical values of SDI (Long and Daniel 1990, Ducey and Larson 2003, Long and Shaw 2005), and that result is critical for the sampling strategy developed here. Ducey and Larson (2003) develop exact relationships between the two density indices when the stand diameter distribution is a Weibull; the discrepancy is generally less than 10% until the distribution approaches a reverse-J shape. Their calculation also assumes that all diameters of trees (down to those with a minimal dbh) are included in the density estimate; the discrepancy would typically be less if small trees were excluded.

Ordinary cruise data, such as prism sampling data, give estimates of the original form of SDI that have a bias that varies with sample size (Ducey and Larson 1997). That problem is not shared by the additive form (Ducey and Larson 1999), but both forms require that the raw data be processed to give estimates of stand QMD, or of TPA by diameter class, before SDI can be estimated. This has limited the usefulness of SDI in situations where an "instantaneous"

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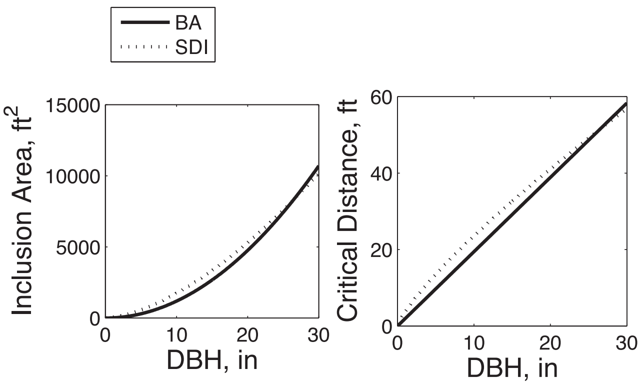

Figure 1. Area of the inclusion zone and critical distance (or radius of the inclusion zone) for ordinary prism cruising with a BAF 20 prism and sampling with probability proportional to stand density index with a stand density factor of 25. Critical distances are the same as those given in Table 1. BA, basal area.

density measure would be helpful, for example in controlling the marking of a uniform thinning.

Prism Sampling and SDI Sampling

Intuitively, each tree tallied in prism cruising represents the same amount of basal area per acre (the BAF) because prism cruising samples trees with probability proportional to the basal area of the tree. More specifically, recall that the basal area of a tree is $0.005454D^2$, where D is dbh (in.) and 0.005454 includes a factor of $\pi/4$ and unit conversions from in.^{2} to ft^2. Recall also that the prism or other angle gauge is used to judge a "limiting distance" $L = kD$, where L is the limiting distance in feet, and k is a constant determined by the diopter of the prism or the angle of the gauge. If the tree is closer than that limiting distance, it is tallied; otherwise, it is not. The inclusion zone is an area centered on the tree; if the sample point falls inside the inclusion zone of the tree, it is tallied, and otherwise, it is not. The inclusion zone is a circle with radius L, so its area (in ft^2) is πL^2 or $\pi k^2 D^2$. The expansion factor (EF), or number of trees/ac that a tallied tree represents, is as follows:

$$\mathrm{EF} = 43{,}560/(\pi k^2 D^2) \tag{3}$$

where 43,560 is a conversion factor with units of ft^2/ac. When we multiply EF by the basal area of one tree, to obtain the basal area one tallied tree represents, we obtain the equation

$$\mathrm{BAF} = \mathrm{EF} \times 0.005454D^2 = 43560 \times 0.005454D^2/(\pi k^2 D^2) \tag{4}$$

which is a constant because the factors of D^2 in the numerator and denominator cancel, leaving only unit conversion constants and the number k that determines the BAF of the prism. The original and still classic work tying this fundamental result to probability sampling, and firmly establishing the unbiasedness of prism sampling, is Grosenbaugh (1958).

Look again at Equation 2. The contribution of an individual tree on an acre to SDI is $(D/10)^R = (D^R/10^R)$. To obtain a sampling method in which each tallied tree represents a constant SDF, we need to cancel out this D^R in the numerator. To do so, we need the inclusion zone *area* in the denominator to be proportional to D^R, not D^2. That means the *radius* of the inclusion zone must be proportional to $D^{R/2}$ instead of D. When $R = 1.605$, these inclusion zone radii or critical distances are similar, but not identical; see Figure 1 for an example. What we would like is a method to adjust the usual prism sampling radius, so that it matches the SDI sampling radius. This would give us a sampling method that is proportional to the SDI contribution of individual trees.

The Hard Way and the Easy Way

There is precedent in the scientific literature for attempting an adjustment to the inclusion zone radius. Zöhrer (1978) pointed out that ordinary prism or relascope sampling was one possibility in a continuum of sampling schemes, in which the radius varied proportional to D raised to some power. This idea was expanded by Bitterlich (1978) and Kuusela (1979); Bitterlich (1984, 39–40) gives a short summary in English. However, the primary focus of this work (to the degree that it had an applied focus) was on sampling with probability proportional to a local volume table, which implies an exponent greater than 2. Furthermore, Zöhrer (1978, 1979) and Bitterlich (1978, 1984) envisioned this approach as a rather painstaking method, in which a highly accurate dendrometer (such as a tele-relascope) would be used in conjunction with a portable computer or programmable calculator, to select trees with an angle that would depend on D. Kuusela (1979) also envisioned sampling with a continuously varying angle. This idea may well have been viewed as impractical, and rightly so; to our knowledge nobody has taken up Bitterlich's (1984) challenge that the approach "still needs a lot of research."

Our approach is different, and it is motivated by the similarity between the curves in Figure 1. Instead of using a continuously changing angle, we will simply add to or take away from the limiting distance enforced by an unchanging prism or angle gauge. The procedure for doing this is relatively easy.

In ordinary prism cruising, remember that the prism must be held over the sample point or "plot center." (For a gauge such as a target on a stick, it is the cruiser's eye that should be kept over the plot center.) Failing to do so—e.g., standing over the plot center and moving the prism around the plot at arm's length—causes an error known as pushing the point (Oderwald and Gregoire 1995). If the prism is not over the plot center but is pushed toward a particular tree, then the distance between the plot center and the prism has been effectively added to the limiting distance for that tree. The tree's inclusion area becomes "too large," and the result is an overestimate. Conversely, if the prism is backed away from the point, so that the cruiser sights across the point when looking at the tree, we will call this pulling the point. Pulling the point reduces the limiting distance for a tree and leads to an underestimate. The ordinary procedure, pushing the point, and pulling the point are illustrated in Figure 2.

Now, look again at Figure 1. For a few trees (those that are very small, or those with a diameter very close to where the two curves cross), the limiting distance for ordinary prism sampling is very close to what would be needed for sampling with probability proportional to stand density index. For a broad range of diameters (those between zero and the crossing point), the limiting distance for prism sampling is slightly less than that for SDI sampling, but only a slight push would be needed to make up the difference. For large trees—those to the right of the crossing point—the limiting distance for prism sampling is greater than that for SDI sampling, so the prism would have to be pulled back from the plot center to obtain the correct distance.

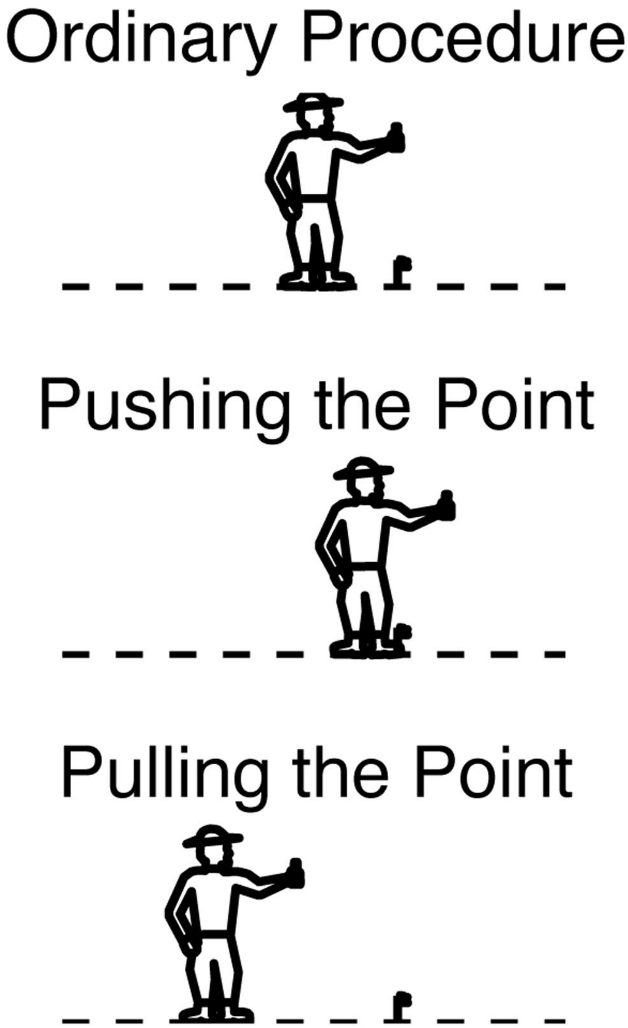

Figure 2. The correct procedure for ordinary prism sampling (top panel), pushing the point (middle panel), and pulling the point (bottom panel). In all three cases, the flag is the sample point. The forester is looking through the prism at a tree somewhere to the right of the sample point. Ordinarily, the prism is held over the sample point, as in the top panel; pushing or pulling the point would be an error. In the procedure developed here, pushing and pulling the point are used deliberately to adjust the critical distance.

An Example

Before proceeding farther with equations and other mathematical details, let us consider a concrete example to show how the method would be used in the field. Table 1 gives the required information for sampling with an SDF of 25, using a prism that would ordinarily have a BAF of 20. When we follow the procedure, every tallied tree will represent 25 units of SDI, no matter what its diameter.

The first column of Table 1 gives diameter classes. The second column gives the ordinary limiting distances for a BAF 20 prism. But these are not the limiting distances we want to use, if we want to sample with an SDF of 25. The limiting distances for SDF 25 are given in the third column. The difference between the third column and the second column is the amount of "push" that is needed for a tree of a particular diameter; these values are given in the fourth column.

Now, suppose our forester establishes a sample point in a stand. She looks at the first tree in the sweep, holding the prism normally.

Table 1. Critical distances and required push for sampling with an SDF of 25, using a BAF of 20 ft^2/ac prism. When the push is shown as negative, the prism must be pulled back across the point.

dbh (in.)	BAF = 20 critical distance	SDF = 25 critical distance	Required push (ft)
2	3.9	6.5	2.6
4	7.8	11.3	3.5
6	11.7	15.7	4
8	15.6	19.7	4.1
10	19.5	23.6	4.1
12	23.3	27.2	3.9
14	27.2	30.8	3.6
16	31.1	34.3	3.2
18	35	37.7	2.7
20	38.9	41	2.1
22	42.8	44.3	1.5
24	46.7	47.4	0.8
26	50.6	50.6	0
28	54.5	53.7	−0.8
30	58.3	56.7	−1.6

The tree appears to be just barely "out." But she can plainly see that the tree has a dbh of about 10 in., so she checks the table and then takes a couple of steps toward the tree, pushing the point by 4.1 ft. The tree is now clearly in, so it is tallied.

The second tree our forester checks is a very large tree for this stand—about 30 in. dbh. It seems borderline, holding the prism normally. But checking the table, this size tree should have a push of −1.6 ft, or in other words, a pull of 1.6 ft. When our forester pulls the point, she realizes this tree is now out. The tree is not tallied.

The next four trees she checks are mid-sized trees—not even 24 in. dbh. They are all clearly in, with the BAF 20. Since any tree less than 26 in. dbh with this table has a positive push, and pushing cannot cause a tree to drop out of the tally, all four trees are tally trees.

The final tree the cruiser checks looks to be about 16 in., but after pushing the point by 3.2 ft—which, after all, is a guess based on an eyeballed diameter—the tree appears borderline. If she were doing extremely crude work, she could just guess again, or flip a coin, or count the tree as half a tally. But our cruiser is trying to do a professional job, so she treats this tree just like one should for a borderline tree in prism cruising: she measures the diameter of the tree and measures the distance from the sample point to the tree. Then, she uses the diameter to get the limiting distance from the third column of Table 1, where they are calculated for SDF 25. (She does not use the ordinary BAF 20 limiting distance.) If she were truly fastidious, and recorded the diameter to 0.1 in., she could calculate an exact limiting distance back in the office, using a formula that we will give below. Whether she uses the table or a formula, if the actual distance is less than the limiting distance, she tallies the tree. In this case, we suppose that she does tally it. No other trees are close enough to be "in."

Now, our forester's dot tally (this is all the data that is strictly required) for this point shows six tallied trees. She immediately calculates that the SDI around this point is six trees, times an SDF of 25, giving a SDI of 150.

Later, she wants to calculate an average SDI for the stand. At the end of the day, she has put in 20 sample points, and her dot tally shows 140 trees. So the average SDI of the stand is 140 trees, divided by 20 sample points, times an SDF of 25, giving an SDI of 175. If our forester had kept the dot tally separate by sample point, she could also calculate confidence limits for this average using standard procedures.

It is worth mentioning here that our forester could have recorded other data, in addition to how many trees were tallied at each point. For example, if the dot tally were broken out by diameter class, then she could easily calculate the SDI contribution of each diameter class. The same could be said of crown classes, acceptable or unacceptable growing stock, trees infected or not infected with mistletoe, and so on. Anything one might do with a regular prism tally to break out basal area into different categories can be done with the stand density tally to break out stand density into different categories.

Finally, just as in ordinary prism cruising, if our forester had recorded the diameter of each tallied tree, she could calculate an expansion factor for each tree and "blow up" any attributes of those trees (such as crown area or board foot volume) to a per-acre basis. We will return to that subject below.

Design Guidelines

In prism cruising, it is important to select an appropriate BAF. In stand density cruising, it is important to select an appropriate SDF. In fact, the first step in choosing an SDF is to decide what BAF prism you will use.

Most foresters have a well-established preference for a certain BAF in particular kinds of stands. This preference is formed on the basis of experience and on a few basic principles. Of these principles, two are worth mentioning. The first is that a good BAF leads to tallying several trees at a sample point: not too few, not too many. A variety of rules-of-thumb specify what "too few" and "too many" mean, but something in the range of 4 to 12 often feels right. What is important here is that the limiting distances for ordinary prism cruising and stand density cruising will be similar if the BAF and SDF are well-matched, so the number of trees tallied in each method should also be similar. If one chooses an appropriate BAF, and then a well-matched SDF, the number of tallied trees should be about the same.

The second key principle in choosing an appropriate BAF is that the largest trees that will be tallied cannot be included from so far away that they may not actually be seen. In the sampling literature, this is called "nondetection bias." Again, if an appropriate BAF is used, and the SDF is well-matched, the limiting distances for large trees in the two methods will be very close. If nondetection bias is not a problem in prism cruising, it will not be a problem in stand density cruising.

Once an appropriate BAF has been chosen, then an appropriate SDF can be found that meets two goals. First, it should produce limiting distances that are close to that for the chosen BAF, so the push or pull can be done easily, quickly, and with reasonable accuracy. Second, the SDF should be a round number for easy calculations in the field. We have calculated and compared limiting distances for prism cruising and stand density cruising using a range of commercially available BAFs and for a range of maximum diameters. Recommendations for appropriate SDFs are given in Table 2. These recommendations should be taken as suggestions, not commandments. However, they all give SDFs that are round numbers, and for which the largest push or pull is no more than a few feet.

Detailed Calculations

Once a BAF and SDF have been chosen, a table similar to Table 1 is needed to give the limiting distances and the push for stand density cruising. It could also be useful to calculate the expansion factor for any size tree. The simplest way to construct a table like Table 1 starts with the expansion factor, so we begin there.

Table 2. Recommended stand density factors as a function of original BAF and maximum diameter of trees expected in the stand.

Maximum dbh (in.)	BAF (ft^2/ac)				
	10	15	20	30	40
Less than 18	15	25	30	50	60
18–24	15	20	30	40	60
24–30	12	20	25	40	50
30–36			25	40	50
36–48					40

In ordinary prism cruising, the easiest way to calculate the expansion factor of a tree is as follows:

$$\mathrm{EF} = \mathrm{BAF}/(0.005454D^2) \tag{5}$$

If a table has been set up with D in the first column, and the expansion factors have been calculated, then the limiting distance, or radius of the inclusion zone, can be calculated as follows:

$$L = \sqrt{43560/(\pi \mathrm{EF})} \tag{6}$$

Calculating these limiting distances completes the second column of Table 1.

Next, we need the expansion factors under stand density sampling. These can be obtained easily by using the equation

$$\mathrm{EF}_{\mathrm{SDI}} = \mathrm{SDF}/[(D/10)^R] \tag{7}$$

Then, the limiting distances (L_{SDI}) can be calculated just as in Equation 6, only using the expansion factors from Equation 7. This completes the third column of Table 1.

Finally, one needs to know the amount of push that is needed for a particular size tree. This is simply $L_{\mathrm{SDI}} - L$. If $L_{\mathrm{SDI}} - L$ is positive, the cruiser pushes the point toward each tree; if $L_{\mathrm{SDI}} - L$ is negative, the cruiser pulls the point away. That completes the fourth column of Table 1.

When an exact check of the limiting distance for a borderline tree is needed, the exact diameter can be inserted in Equation 7, and Equation 6 will give the limiting distance. If the recorded distance from the sample point to the tree is less than the limiting distance, the tree should be kept in the tally.

Finally, if one were to use this method to estimate things other than SDI for a stand, recording the diameters of the tallied trees will allow calculation of an expansion factor using Equation 7. Once that is done, data analysis would proceed exactly as for ordinary prism sampling, except that the expansion factors will be different and each tree will still count as a constant amount of SDI, but not basal area.

Conclusions

A simple modification of prism cruising gives a method of sampling with probability proportional to stand density index. SDI can be estimated on the basis of the number of trees tallied and a stand density factor that is analogous to the basal area factor in ordinary prism cruising.

Stand density sampling is a design-unbiased form of probability proportional to size sampling. Stand density sampling should also have low variance when SDI or closely related variables are of interest. Strictly speaking, this approach estimates the additive form of SDI (Equation 2). However, the value of Reineke's (1933) original SDI (Equation 1) is nearly identical to the additive form in the even-aged stands for which it was originally developed (Long and Daniel 1990, Ducey and Larson 2003, Long and Shaw 2005). Ordinary prism cruising provides biased estimates of Reineke's original density index (as do fixed-area sample plots), so even as stands become slightly irregular, the difference may not be appreciable.

In theory, stand density sampling could be used just like prism cruising as a general sampling approach, if tree diameters are measured and used to calculate expansion factors. However, we are reluctant to advocate replacing prism cruising with stand density sampling for everyday use. The need to push the point in stand density sampling does create an opportunity for error by the cruiser; therefore, we speculate that the results from ordinary prism cruising would be more precise and repeatable under typical field conditions. The chief advantage of stand density sampling is its ability to provide rapid (nearly instantaneous) estimates of SDI in the field, with only trivial calculation (multiplication of number of trees by the stand density factor). This suggests two primary situations where stand density sampling would be of use:

1. Rapid diagnosis and description of stand conditions. Immediate calculation of SDI would allow quick assessment of stand dynamics, and the potential for timber operations, forest health concerns, and wildlife habitat values while the forester is still on site. This would allow more effective coupling and synthesis with quantitative density description on the one hand and silvicultural insight on the other, which neither density indices nor their graphical counterparts (such as density management diagrams) can replace (Long and Shaw 2005).
2. Control of marking intensity in thinnings and other partial harvests. Stand density sampling could be used to assess residual density, both in total and as it is partitioned among stand components, during the marking process itself and in any followup quality control. For example, the timber marker might periodically spin a stand density point, to verify that the residual stand meets the silvicultural prescription. The need to translate target structures to and from basal area would be eliminated.

Finally, stand density sampling could provide a simple point density estimate for use in ecological studies (such as evaluations of regeneration) that is directly tied to stand-level density estimation.

Spurr (1952) and a number of authors writing later have objected that whatever the advantage of SDI might be, it is more than offset by the difficulty of measurement. The availability of a relatively simple technique for measuring SDI should enhance its usefulness.

Literature Cited

AVERY, T.E., AND H.E. BURKHART. 2002. *Forest measurements,* 5th ed. Wiley, New York. 488 p.

BICKFORD, C.A., F.S. BAKER, AND F.G. WILSON. 1957. Stocking, normality, and measurement of stand density. *J. For.* 55(2):99–104.

BITTERLICH, W. 1948. Die Winkelzählprobe. *Allgemeine Forst- und Holzwirtschaftliche Zeitung* 59(1/2):4–5.

BITTERLICH, W. 1978. Klassische und Praktische Relaskopstichprobe. *Allgemeine Forstzeitung* 89:271–274.

BITTERLICH, W. 1984. *The relascope idea: Relative measurements in forestry.* Commonwealth Agricultural Bureaux, Farnham Royal, United Kingdom.

CURTIS, R.O. 1971. A tree area power function and related density measures for Douglas-fir. *For. Sci.* 17(2):146–159.

DUCEY, M.J., AND B.C. LARSON. 1997. Thinning decisions using stand density indices: The influence of uncertainty. *West. J. Appl. For.* 12(3):89–92.

DUCEY, M.J., AND B.C. LARSON. 1999. Accounting for bias and variance in nonlinear stand density indices. *For. Sci.* 45(3):452–457.

DUCEY, M.J., AND B.C. LARSON. 2003. Is there a correct stand density index? An alternate interpretation. *West. J. Appl. For.* 18(3):179–184.

GROSENBAUGH, L.R. 1952. Plotless timber estimates—New, fast, easy. *J. For.* 50(1):32–37.

GROSENBAUGH, L.R. 1958. *Point sampling and line sampling: Probability theory, geometric implications, synthesis.* USDA Forest Service, Southern Forestry Experiment Station, Misc. Paper 160.

KUUSELA, K. 1979. Sampling of tree stock by angle gauge in proportion to tree characteristics. *Comm. Inst. For. Fenn.* 95(7):16.

LONG, J.N. 1985. A practical approach to density management. *For. Chron.* 61:23–27.

LONG, J.N., AND T.W. DANIEL. 1990. Assessment of growing stock in uneven-aged stands. *West. J. Appl. For.* 5(3):93–96.

LONG, J.N., AND J.D. SHAW. 2005. A density management diagram for even-aged ponderosa pine stands. *West. J. Appl. For.* 20(4):205–215.

ODERWALD, R.G., AND T.G. GREGOIRE. 1995. Overstated volumes from "pushing the point." *South. J. Appl. For.* 19(4):162–165.

REINEKE, L.H. 1933. Perfecting a stand-density index for even-aged stands. *J. Agric. Res.* 46(7):627–638.

SHAW, J.D. 2000. Application of stand density index to irregularly structured stands. *West. J. Appl. For.* 15(1):40–42.

SPURR, S.H. 1952. *Forest inventory.* Ronald Press, New York.

STAGE, A.R. 1968. A tree-by-tree measure of site utilization related to Stand Density Index. USDA For. Serv., Intermountain For. Range Exp. Sta., Res. Note INT-77.

WOODALL, C.W., C.E. FIEDLER, AND K.S. MILNER. 2003. Stand density index in uneven-aged ponderosa pine stands. *Can. J. For. Res.* 33(1):96–100.

ZEIDE, B. 2005. How to measure stand density. *Trees* 19(1):1–14.

ZÖHRER, F. 1978. Fundamentale Stichprobenkonzepte der Forstinventur. I. *Allgemeine Forstzeitung* 89(5):180–185.

ZÖHRER, F. 1979. Fundamentale Stichprobenkonzepte der Forstinventur. II. *Allgemeine Forstzeitung* 90(5):112–115.