Mapped Plot Patch Size Estimates

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Abstract.—This paper demonstrates that the mapped plot design is relatively easy to analyze and describes existing formulas for mean and variance estimators. New methods are developed for using mapped plots to estimate average patch size of condition classes. The patch size estimators require assumptions about the shape of the condition class, limiting their utility. They may have some value as landscape metrics.

The Forest Inventory and Analysis (FIA) program of the U.S. Department of Agriculture Forest Service demarcates various forest conditions that occur on field plots. This relatively new concept results in a map for each plot showing where the conditions occur. A new condition is defined by changes in characteristics such as land use, tree species, tree size, and tree density. In the past, some FIA regions forced plots to contain a single condition by rotating them into a pure condition class. Plot rotation led to a small bias in the estimates (Birdsey 1995), and was replaced with plot mapping as recommended by Hahn *et al*. (1995).

A recent paper (Van Deusen 2004) provides a simple derivation of mapped plot estimators for the mean and variance for particular conditions. This paper briefly reviews those results and develops a new method for estimating condition patch size from mapped plots. The new method requires assumptions about the shape of the condition, i.e., circular or square. Such shape assumptions are crude approximations to the actual shape. Regardless, the resulting patch size estimates may have value as a landscape metric.

Review of Theory

The simple forest inventory model used in the original derivation (Van Deusen 2004) is used here. Assume two conditions, C and B, where C is a circular condition surrounded by condition B (fig. 1). We want estimates of the mean and variance of type C, and the other types that surround it are denoted as B. The spatial shape of type C could be anything in practice. Sampling is systematic or simple random and uses fixed area circular plots with radius d. The edge of type C is shown by a dash line and a perimeter band overlaps the outer edge of area C shown by solid lines. The plot contains both conditions when the plot center falls within the perimeter band. The condition boundary is mapped when it crosses a plot, but there is no need to map the perimeter band.

The following notation is used (Van Deusen 2004):

a_i = the proportion of the area of plot i that is within condition C.

A = area of condition C.

$\tilde{A}$ = area in the perimeter band that is outside of condition C plus the area of C.

r = $\frac{A}{\tilde{A}}$, the ratio of area C to the area of C plus the outer perimeter band.

y_i = a variable that can be measured on each randomly located plot that completely or partially overlaps condition C. For plots that do not overlap C, y_i=0.

μ = the per unit area mean of variable y for condition C, e.g., cubic meter per hectare pine volume.

$\tilde{\mu}$ = the per unit area mean of variable y in condition C inclusive of the outer perimeter.

area; the outer perimeter area contains no variable y by definition, and $\mu = \frac{\tilde{\mu}}{r}$.

n = the number of plots that contain some condition C.

The FIA sampling design involves randomly locating plot centers in a forest area. The FIA plot consists of a fixed configuration of four circular 1/24-acre subplots. The development here is simplified by using a single circular plot, but the results apply directly to the FIA plot design. After a plot is located, the amount of variable y is recorded and expanded to a per acre value. With the FIA subplot configuration, a total of 1/6 acre is sampled, so y is multiplied by 6 to expand it to a per acre value.

Consider randomly locating plot centers within the model forest area (fig. 1) and recording the amount of variable y in the plot. When plot i contains no condition C, y_i=0 and a_i=0 It follows immediately that an estimator for the ratio, r, is

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$$\hat{r} = \sum_{i=1}^{n} \frac{a_i}{n} \qquad (1)$$

Likewise, an estimator of $\tilde{\mu}$ is

$$\hat{\tilde{\mu}} = \sum_{i=1}^{n} \frac{y_i}{n} \qquad (2)$$

Putting equations (1) and (2) together provides an estimator for the average amount of y within condition C,

$$\hat{\mu} = \frac{\sum_{i=1}^{n} y_i}{\sum_{i=1}^{n} a_i} . \qquad (3)$$

Equation (3) is not unbiased because it involves the ratio of two random variables (Thompson 2002), but it is consistent.

A model-based derivation of estimator (3) is provided in Van Deusen (2004), which yields the following variance estimator:

$$\mathrm{Var}(\hat{\mu}) = \frac{\sigma^2}{\sum_{i=1}^{n} a_i} , \qquad (4)$$

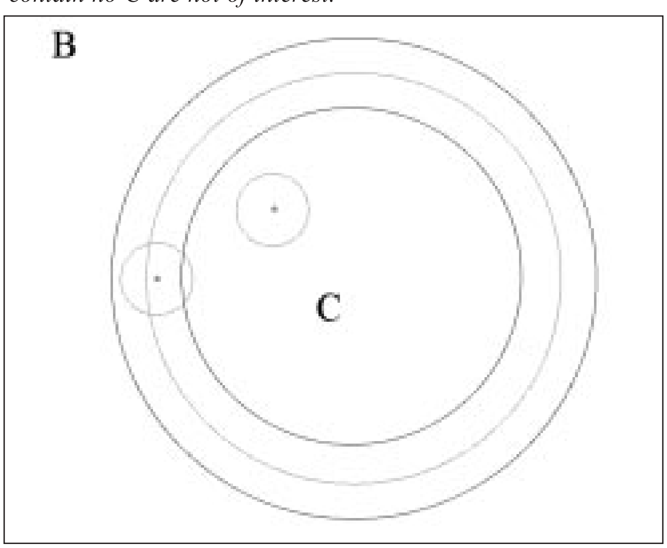
where

$$\hat{\sigma}^2 = \frac{\sum_{i=1}^{n} \hat{e}_i^2}{\left(\sum_{i=1}^{n} a_i\right) - 1} , \qquad (5)$$

and

$$\hat{e}_i = y_i - a_i \hat{\mu} .$$

Figure 1.—*An area of condition C surrounded by condition B. Condition C is bounded by the dashed line that is contained in a perimeter band of width equal to the diameter of the fixed area circular plots. Plots with centers that fall within the band will contain some of both conditions. All other plots contain only one condition. One plot that is fully in condition C is shown along with a plot that overlaps the boundary. Plots that contain no C are not of interest.*



Note that $n = \sum_{i=1}^{n} a_i$ when all plots are completely in the condition of interest. Therefore, equations (3), (4), and (5) reduce to the usual equations for simple random sampling when all plots are fully in the condition of interest or plot mapping is not implemented.

Area Estimation

Within the FIA design, three layers are referred to as P1, P2, and P3. The basic FIA plots are called the P2 layer, whereas forest health plots comprise the P3 layer. The P1 layer traditionally consisted of photo points that were classified as being forest or nonforest. Thus, the P1 layer is a higher resolution sample used to estimate forest area. The P2 layer has traditionally been a subset of the P1 layer and could potentially be used to produce a double sampling variance estimate of area. FIA is currently in the process of moving toward a P1 layer that provides wall-to-wall forest/nonforest coverage using remotely sensed data such as TM. Some uncertainty exists in forest area estimates with either approach, although FIA does not typically show the variance in forest area estimates. Confidence intervals for total volume estimates for a county or State include only the between plot sampling variance.

FIA data users have typically relied on plot expansion factors to determine the number of acres in their area of interest. These expansion factors are included in the standard FIA database (FIADB) and indicate the number of acres that a plot represents. It is difficult to justify the concept of a plot expansion factor because it implies that plots are selected with variable probability when they are actually established on a grid. Recognizing this, FIA is phasing out the plot expansion factor concept. Area expansion in this paper is based on the assumption that each plot represents 6,000 acres, adjusted depending on how many panels are being used.

Suppose we want to estimate the area or volume in condition C from mapped plots in a State. For any approach, one first needs to obtain a list of all the plots in the State that contain the desired condition. The approach used here is to compute the average volume in condition C with equation (3) and then multiply by the summed and adjusted expansion factors as follows:

$$\hat{V}_C = \hat{\mu} \sum_{i \in C} a_i E_i \qquad (6)$$

where the expansion factor is reduced in proportion to the amount of C on the plot. For the example application, E_i = 6000 * 5/3 = 10000 for all plots, because three of five panels are used.

Equation (6) provides a way to estimate total volume in a condition class from mapped plots. The total area part of the estimate, however, could be improved with a remote sensing derived P1 layer. In any event, the focus of this paper is the patch area estimator, which does not require knowledge of the total forest area.

Patch Area

The average patch area of condition classes can be estimated from mapped plot data, if one is willing to make an assumption about the shape of a condition. A feasible shape assumption would be that the condition of interest is approximately circular. If the average radius of circular condition C (fig. 1) is R, the average area of condition C is Area(C)=πR^2. The radius of the plots used (fig. 4) is d, so the average area of condition C plus the perimeter band is Area(C+band)=$\pi (R+d)^2$. The ratio of this area is

$$r = \frac{R^2}{(R+d)^2} \tag{7}$$

the same ratio estimated by equation (1). Given the known value for d and an estimate of r, the unknown condition average-radius is estimated from

$$\hat{R} = \frac{d}{\frac{1}{\sqrt{\hat{r}}} - 1} \tag{8}$$

A circular patch area estimator follows immediately from equation (9):

$$\hat{P}_C = k\pi\hat{R}^2 \tag{9}$$

where k is a constant to convert the units used for R and d into appropriate units for area. This estimator requires only the mapped plot information that is already available in the FIA database.

A patch-area estimator can also be derived under the alternative assumption that the condition shape is square. Let the length of one side of the condition be S. The analogous procedure as used with circular shapes gives the following estimate:

$$\hat{S} = \frac{d}{\frac{1}{\sqrt{\hat{r}}} - 1} \tag{10}$$

The square patch-area estimator is

$$\hat{P}_S = k\hat{S}^2 \tag{11}$$

Therefore, the square patch assumption results in smaller patches by a factor of π. This shows that patch area estimates are sensitive to the shape assumption.

Patch-Area Variance Estimates

The patch-area estimators involve the denominator, $1/\sqrt{\hat{r}} - 1$, which presents the possibility of dividing by zero. This occurs when the estimate of r is 0 or 1. A suggested solution to this potential problem is to use an ad hoc, but more robust, estimator for r:

$$\tilde{r} = \frac{\sum_{i \varepsilon C} a_i}{n(n-1)}. \tag{12}$$

Estimator (11) prevents the estimate of r from reaching 1.0. An r estimate of 0.0 implies that no plots were in the condition, so this contingency is not a problem.

The only random component of the patch-size estimators is $\tilde{r}$, and a suggested standard error estimator for it is

$$SE(\tilde{r}) = \frac{\sum_{i \varepsilon C} (a_i - \hat{r})^2}{n(n+1)}, \tag{13}$$

which is the usual standard error estimator for $\hat{r}$. Since $\tilde{r}$ is divided by n+1 instead of n, this should provide a conservative estimate. Now treat the patch-size estimator as a function of $\tilde{r}$, i.e., $P_C(\tilde{r})$. Establish approximate 95 percent upper and lower bounds on the patch-size estimates from the bounds on $\tilde{r}$ as $P_C\left(\tilde{r} \pm 2\sqrt{SE(\tilde{r})}\right)$. In practice, this involves computing the upper and lower bounds on $\tilde{r}$, and then using these values to compute upper and lower patch-size estimates.

Example Application

Data from the first three annual panels in Maine were used to demonstrate the patch size estimators. The analysis consists of computing the results broken down by forest type for the entire State. The estimated number of acres, number of sample plots, and mean condition volume estimates are displayed (table 1) for forest types that have at least 10 sample plots. A sample plot can be counted in more than one row, because it can contain more than one forest type. The per acre estimate of net cubic foot volume is computed with equation (3). These figures may not correspond to official FIA reports, because different methods are being used and this is an out-of-date data set.

Patch size means and confidence bounds (table 2) are computed for each forest type assuming circular patches. The plot radius, d, is set to the radius of a circular 1/6-acre plot. The mean patch size varies from less than an acre for nonstocked areas to 107 acres for the sugar maple/beech/yellow birch type.

It is also possible to estimate the number of patches by type (table 3) by dividing the patch sizes (table 2) into the total area in the forest type (table 1). It isn't clear how these statistics would be used, but they might be useful in conjunction with the average patch size estimates (table 2) as another landscape metric.

Table 1.—*Number of acres, sample size, and net cubic feet per acre by forest type based on the first three panels for Maine. A constant expansion factor of 10,000 acres per plot is assumed. Results are shown for sample sizes of 10 or more.*

Forest type	Total acres	Sample size	Cubic feet/acre
Non-stocked	51,471.00	10.00	67.57
Tamarack	80,283.00	12.00	1,122.99
Balsam poplar	121,199.00	18.00	947.07
Red maple/lowland	200,563.00	29.00	943.21
White spruce	149,275.00	21.00	1,449.91
Northern red oak	178,840.00	24.00	1,923.19
Black ash/American elm/red maple	144,677.00	19.00	967.62
Eastern white pine	535,345.00	73.00	2,736.53
Red maple/upland	640,588.00	85.00	1,204.09
White oak/red oak/hickory	127,683.00	16.00	1,246.48
Cherry/ash/yellow-poplar	168,696.00	21.00	1,090.47
Red spruce	1,021,258.00	126.00	2,166.10
White pine/hemlock	122,117.00	14.00	2,801.73
White pine/red oak/white ash	377,068.00	45.00	1,864.22
Aspen	926,498.00	112.00	1,252.95
Northern white-cedar	1,020,298.00	123	1,907.68
Black spruce	436,904.00	52.00	911.06
Eastern hemlock	700,215.00	83.00	2,049.81
Paper birch	1,386,688.00	162.00	937.76
Balsam fir	1,780,338.00	206.00	949.10
Red spruce/balsam fir	1,044,594.00	118.00	1,117.62
Sugar maple/beech/yellow birch	6,626,599.00	715.00	1,427.02

Table 2.—*Mean patch size estimates (acres) for Maine by forest type, with approximate 95 percent upper and lower confidence bounds (assuming circular patches and sorted by mean patch size).*

Forest type	Lower bound	Mean	Upper bound
Non-stocked	0.47	0.78	1.30
Tamarack	1.64	2.24	3.12
Balsam poplar	1.93	2.62	3.63
Red maple/lowland	2.82	3.35	4.01
White spruce	2.79	3.64	4.84
Northern red oak	3.99	5.01	6.39
Black ash/American elm/red maple	4.14	5.40	7.18
Eastern white pine	4.99	5.40	5.85
Red maple/upland	6.17	6.62	7.11
White oak/red oak/hickory	4.99	7.04	10.34
Cherry/ash/yellow-poplar	5.97	8.27	11.89
Red spruce	11.89	12.57	13.30
White pine/hemlock	9.28	14.21	23.48
White pine/red oak/white ash	13.53	15.26	17.30
Aspen	14.36	15.30	16.32
Northern white-cedar	15.00	15.89	16.85
Black spruce	14.06	16.21	18.84
Eastern hemlock	16.89	18.36	20.02
Paper birch	22.42	23.52	24.69
Balsam fir	26.15	27.19	28.29
Red spruce/balsam fir	34.41	36.76	39.35
Sugar maple/beech/yellow birch	105.45	106.99	108.57
Means	**14.70**	**16.03**	**17.85**

Conclusions

FIA is installing mapped plots nationwide. Some estimators for basic statistics, such as per acre volume and variance, have been reviewed. New estimators for average patch size are also presented. The possibility of making patch size estimates is unique to the mapped plot design. Patch size estimates depend on assumptions about patch shape and are nonrobust to the shape assumption. This limits their application and interpretation. They may be useful, however, as a basic landscape or ecological metric.

All the estimators presented in this study depend only on standard FIA measurements. They can therefore be implemented with little cost when deemed appropriate. They are a fortuitous byproduct of the mapped plot design that may prove useful.

Table 3.—*Number of patches in Maine by forest type, with approximate 95 percent upper and lower confidence bounds (assuming circular patches).*

Forest type	Lower bound	Mean	Upper bound
Non-stocked	39,545.96	65,886.00	109,046.95
Tamarack	25,693.23	35,770.69	48,887.27
Balsam poplar	33,367.62	46,203.98	62,727.81
Red maple/lowland	50,054.23	59,856.03	71,080.45
White spruce	30,839.63	41,016.24	53,579.08
Northern red oak	27,989.92	35,671.28	44,790.60
Black ash/American elm/red maple	20,148.13	26,812.97	34,944.24
Eastern white pine	91,525.70	99,165.71	107,257.23
Red maple/upland	90,042.63	96,765.52	103,834.91
White oak/red oak/hickory	12,350.25	18,138.77	25,600.04
Cherry/ash/yellow-poplar	14,187.58	20,404.11	28,271.76
Red spruce	76,762.60	81,251.39	85,908.70
White pine/hemlock	5,201.45	8,593.88	13,160.27
White pine/red oak/white ash	21,790.56	24,710.10	27,869.38
Aspen	56,775.40	60,561.61	64,506.11
Northern white-cedar	60,554.92	64,217.45	68,018.59
Black spruce	23,190.67	26,953.18	31,083.90
Eastern hemlock	34,981.36	38,138.02	41,468.53
Paper birch	56,173.46	58,969.30	61,849.32
Balsam fir	62,926.01	65,466.82	68,069.13
Red spruce/balsam fir	26,545.79	28,415.00	30,360.33
Sugar maple/beech/yellow birch	61,032.87	61,934.75	62,844.00
Means	**41,894.54**	**48,404.67**	**56,598.12**

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