# Forest Inventory and Analysis and Forest Health Monitoring: Piecing the Quilt

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**Abstract.**—Against the backdrop of a discussion about patchwork quilt assembly, the authors present background information on global grids. They show how to compose hexagons, an important task in systematically developing a subset of Forest Health Monitoring (FHM) Program plots from Forest Inventory and Analysis (FIA) plots. Finally, they outline the FHM and FIA grids, along with their current problems and future issues.

## Introduction

The curious title of this paper is meant to evoke an image of an old-fashioned patchwork quilt. One popular patchwork design, called *Grandmother's Flower Garden* (Brackman 1993), is featured in figure 1. One pattern represents the pistils of a flower, another pattern represents the petals, and a third pattern represents the actual garden. The flowers are reminiscent of the Steinman heptahex (Steinman 2001). A patchwork quilt with no fixed shape or size patch—for instance, one with triangles here and squares there—is called a crazy quilt.

Global grid developers have proposed lists of criteria for evaluating their grids. Clarke (2000) compared the proposed list of criteria for global grid development of Goodchild (1994) to that of Kimerling *et al.* (1999). Kimerling's list resembles Goodchild's, although with some obvious additions and an apparent subtraction. Carr (1998) cites the Goodchild criteria but reorders them as follows:

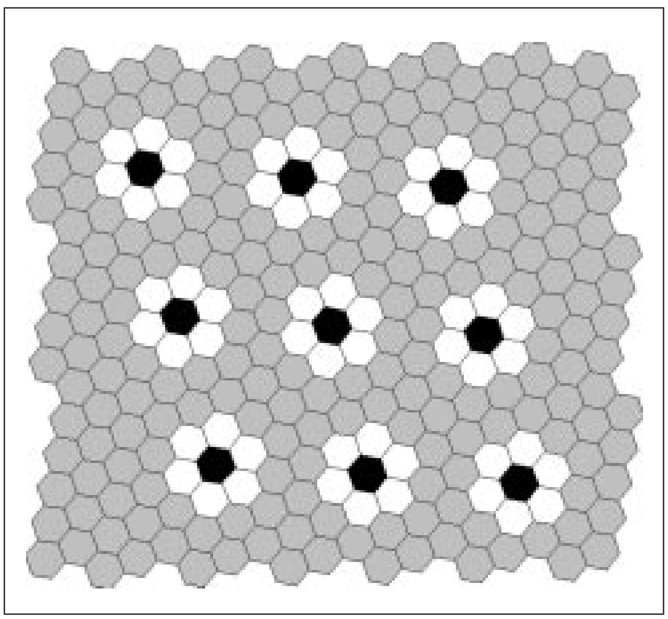
1. The domain is the globe.
2. Areas exhaustively cover the domain.
3. Areas are equal in size.
4. Areas are compact.
5. Areas are equal in shape.
6. Areas have the same number of edges.
7. Edges of areas are of equal length.
8. Edges of areas are straight on some projection.
9. Areas form a hierarchy preserving some properties for $m < n$ areas.
10. Each area is associated with only one point.
11. Points are maximally central within areas.
12. Points are equidistant.
13. Points form a hierarchy preserving some properties for $m < n$ points.
14. Addresses of points and areas are regular and reflect other properties.

The authors offer the following points of clarification:

First, areas are grid cells. For Forest Inventory and Analysis (FIA), a plot (a piece of sampled landscape, currently 672 $m^2$, while cells are $2.4 \times 10^7$ $m^2$) assigned to a particular cell must fall inside that cell.

Second, many measures of compactness exist, but one possible measure is maximum area per unit perimeter. Because the first eight criteria limit the choice of grid cell to an equilateral

Figure 1.—*Grandmother's flower garden.*



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triangle, rhombus, and regular hexagon, to show that the hexagon is the most compact cell is easy. Another possible measure of compactness is to minimize the maximum distance between any point and the grid point or any point and a potential plot. For FIA hexagonal cells of 2,400 hectares, no point is more than 3,040 meters from a grid point and no more than twice that distance from an actual plot. With the same size squares (which was the cell shape used in much of the United States before 1998), no point is more than 4,900 meters from a grid point and no more than twice that distance from an actual plot.

Third, what the hierarchy criteria mean is that the cell sizes should be somewhat flexible, as figure 2 shows. According to the criteria, the domain is divided into n cells, but the domain should be able to be divided into m cells, where m is some (but not necessarily any) number less than n, and m and n are integers. For instance, a network consisting of squares (which are rhombi with equal angles) is hierarchical because a square may be decomposed into $h^2$ smaller squares, where h is an integer. If h is odd, the centroid of the larger cell will coincide with a centroid of a smaller cell. An equilateral triangle, as figure 2(b) illustrates, may be decomposed into $h^2$ smaller equilateral triangles. A hexagon may be divided into $T = h^2 + hk + k^2$ smaller hexagons, where T stands for *triangulation*, and h and k are integers. The first several triangulation numbers are 1, 3, 4, 7, 9, 12, 13, 16, 19, 21, 25, and 27. Any product, rather than the multiple, of triangulation numbers is also a triangulation number. The decomposition of the hexagon is not quite as elegant as that of the square or the triangle because some of the smaller hexagons are cut in pieces around the perimeter of the larger hexagon. Reorientation of axes permits that $h \geq k \geq 0$. When h = k (e.g., when h = 3, k = 3, and T = 27), shown in figure 3(a), or when k = 0 (e.g., when h = 4, k = 0, and T = 16), shown in figure 3(b), the decompositions are a bit more elegant. In the former case, partial hexagons are either half-hexagons or third-hexagons; in the latter case, partial hexagons are half-hexagons. Further details on partial hexagons may be found in McCollum (2001).

Fourth, points are grid points, and by "the maximally central point," global grid developers mean the centroid of the cell. Such points are equidistant from each other. Many global grid developers believe the sample should be taken at the centroid, but FIA and Forest Health Monitoring (FHM) programs have avoided this rule. Details of how plots were assigned to hexagons are available in Brand *et al.* (2000). Basically, legacy FHM plots were favored first, then legacy FIA plots, then deleted plots could be reconstructed, and, finally, if a cell remained empty, a new plot was generated. Ties at any level of favor were broken by choosing the plot closest to hexagon center. Moreover, the Food Security Act (7 U.S.C. 2276) forbids release of plot locations or of any information that makes landowners' identity discernible. If plots did appear at grid point, the plot grid could

Figure 2.—*Hierarchy of (a) squares and (b) triangles.*

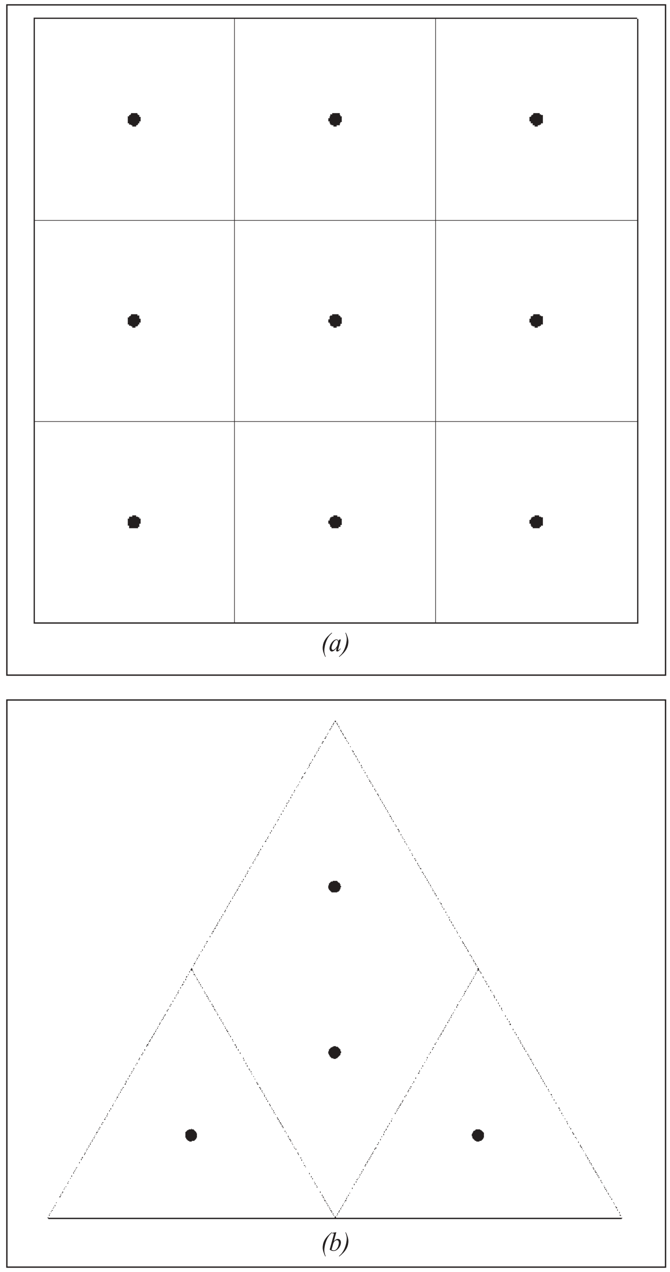
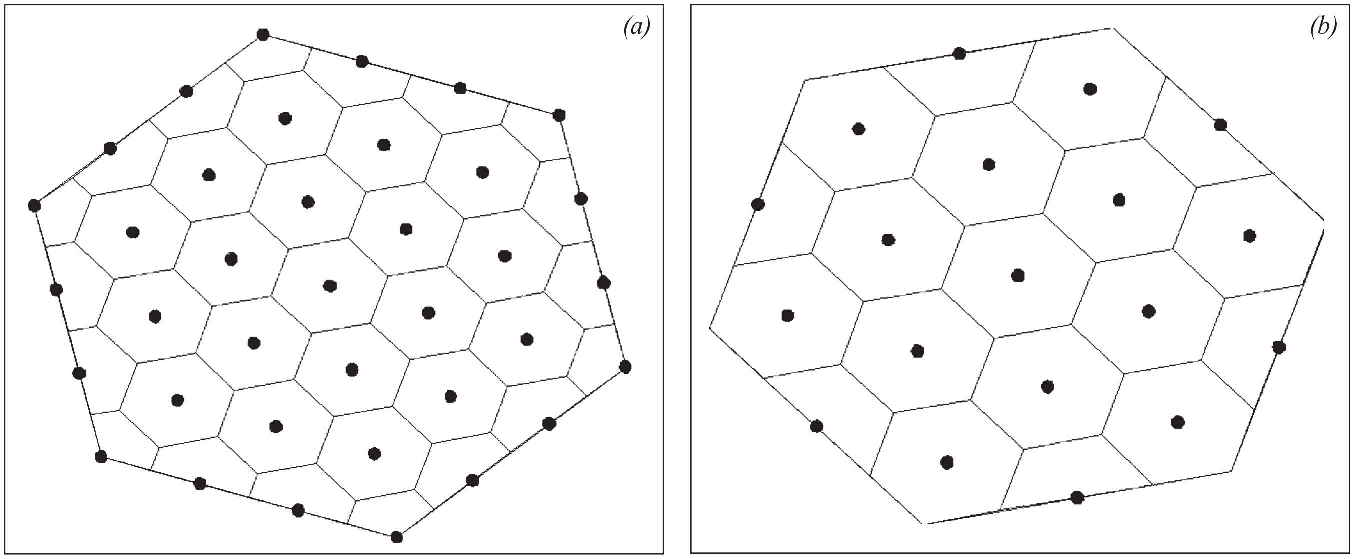


Figure 3.—*Hierarchy of hexagons.*




be reconstructed, and landowners' privacy, if not the integrity of the data, would be at risk.

Fifth, cells have regular addresses if deducing the addresses of a cell's neighbors is easy. This characteristic is important in developing grids of varying resolution.

The additional criterion in Kimerling *et al.* (1999) was that "the grid system have a simple relationship to the traditional latitude-longitude graticule" (Kimerling *et al.* 1999, 273).This particular criterion seems in conflict with an earlier criterion in the same paper, namely that "areal cells have the same shape, ideally a spherical polygon with edges that are great circles" (Kimerling *et al.* 1999, 272). FHM did its best to adhere to both criteria. It used a variation of a grid based on great circles, but it numbered its plots according to the latitude and longitude of the grid point. Plot numbers had seven digits, where the first two digits were degrees of latitude, the next three digits were degrees of longitude, the sixth digit was the number of eighths of latitude, and the seventh digit was the number of eighths of longitude. For instance, a grid point at 30 degrees 1 minute north, 89 degrees 59 minutes west would be assigned an identification number of 3008918. With this numbering system, no obvious way existed to deduce this cell's neighbors.

FHM has offered its own reasons for accepting hexagons as the ideal cell. Among the benefits of a hexagonal network that D.L. Cassell cited in the FHM 1992 Activities Plan (Alexander and Barnard 1992) were the following:

1. It is spatially compact.
2. It provides uniform spatial coverage.
3. It is very flexible for altering the grid density.
4. It is less likely than a square grid to coincide with anthropogenic features.
5. It generally leads to smaller variance estimates than a random selection.

Moreover, the hexagon patch is superior to the square in computing contagion index (Parresol and McCollum 1997) because no quarrel need exist about whether to use rook's rule or king's rule, and the patch is superior to the random shape because no quarrel need exist about how to weight edge lengths.

## Composing Hexagons

In the interest of brevity, the authors will call a FIA Phase 2 (P2) hexagon a $C_1$ and an historic FHM Phase 3 (P3) hexagon a $C_{27}$; in general, a *T*-fold composition of FIA hexagons will be called a $C_T$. The variables H and K provide a convenient coordinate system, as figure 4 shows. The bold numbers in the upper left of each cell represent H, and the row and the plain text numbers in the lower right of each cell represent K, the path. This addressing scheme facilitates the hierarchy criteria, because to generate $C_{16}$ hexagons, one must choose the conjunctions of every fourth row and every fourth path of hexagon centers, and then perform Thiessen polygon expansions. In general, for a *T*-fold composition, *T* different possible starting points ($a_{0,0}$,$b_{0,0}$) exist. Additional grid points are selected by the following equation:

$$\{(a_{i,j}, b_{i,j})\} \quad = \quad \{a_{0,0} + i \cdot h + j \cdot (h+k), b_{0,0} + i \cdot k - j \cdot h)\} \qquad (1)$$

where *i* and *j* are index variables belonging to the set of integers.

Thus, when trying to generate $C_{16}$ hexagons, solve $T = 16 = h^2 + hk + k^2$. From above, ($h$,$k$) = (4,0). If the starting point ($a_{0,0}$,$b_{0,0}$) is (0,0), the generated points include (4,0) if ($i$,$j$) = (1,0), (8,0) if ($i$,$j$) = (2,0), and (8,-4) if ($i$,$j$) = (1,1). The index variables may become as negative or positive as necessary to cover the domain.

Equation (1) may also be used to assign hexagons to panels according to the interpenetrating design, as described in such papers as Reams and Van Deusen (1999) or Roesch and Reams (1999). For the seven-panel P2 scheme, this equation was followed. Because 5 and 10 are not triangulation numbers, the equations were not followed in the 5- and 10-panel P2 schemes. Rather, P2 hexagons were numbered in panels 1 to 10 in interpenetrating fashion across the country, and then 10 panels were collapsed into 5 panels.

## Forest Health Monitoring

Before 1999, the FHM program measured one plot per 64,800 hectares. Cells were hexagons numbered in four panels. The plan was to measure one panel per year plus a one-third overlap from the rest of the cycle. A State might have 240 FHM plots, so that 60 plots existed in each panel, and 20 plots existed in each overlap. Thus, in 12 years, a plot would be measured three times in its own panel and once in its own overlap.

When FIA and FHM were combined, P2 plots were to be put on a hexagonal grid of approximately 2,400 hectares per cell, while P3 plots were to be put on a grid approximately 1/16th as dense. The overlap would no longer be measured, and a fifth panel would be added. The hypothetical State with 240 FHM plots would have 400 P3 plots. Ideally, one-fifth of the plots would be measured each year.

Figure 4.—*Hexagons numbered in rows and paths.*

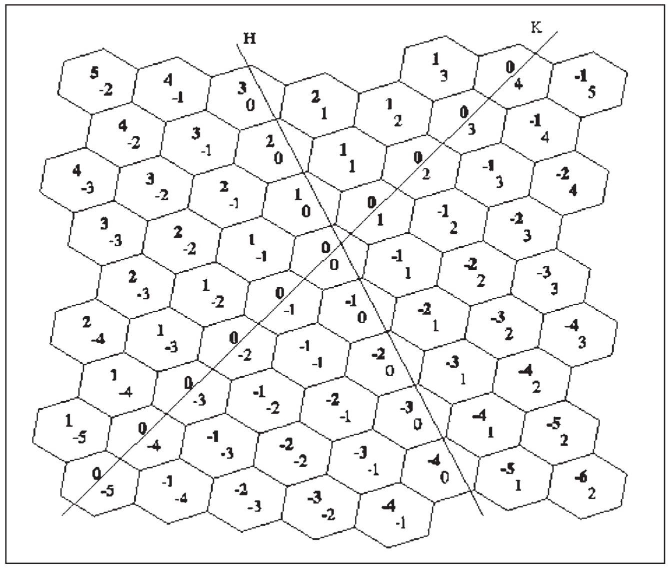
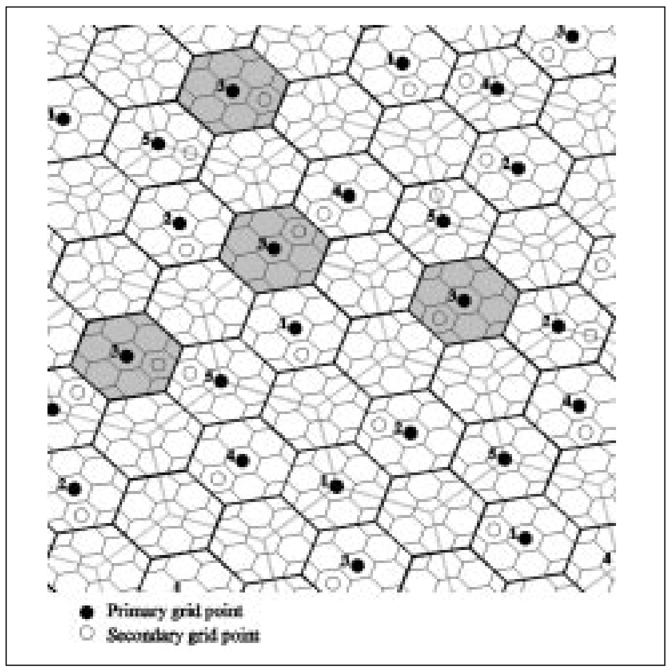



Figure 5.—*Intensification method, panels 1–4, old FHM States.*




Many States already had P3 plots on a grid of 64,800 hectares per cell. For the first four panels in those States, primary grid points (illustrated in this paper's figures by filled circles) were located at the centers of $C_{27}$ hexagons, as figure 5 illustrates. Secondary grid points (illustrated in this paper's figures by open circles) were generated near the primary grid points. The secondary grid point would usually belong to the same P2 panel as

Figure 6.—*Intensification method, panel 5, old FHM States.*

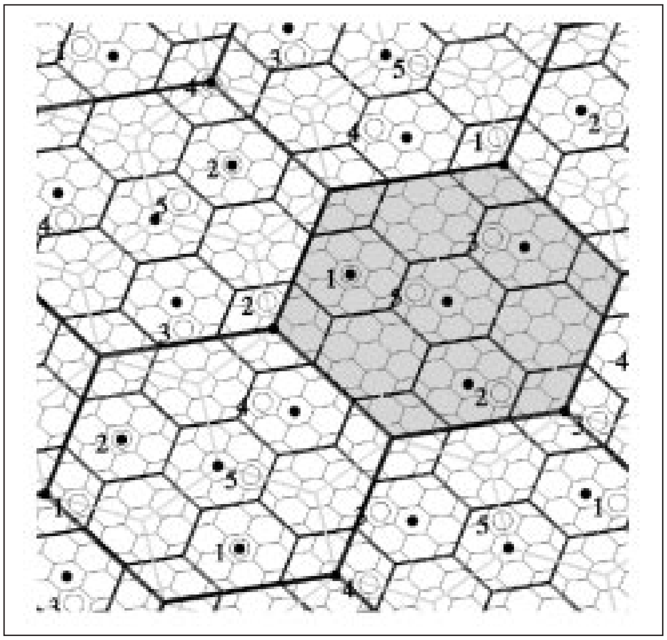
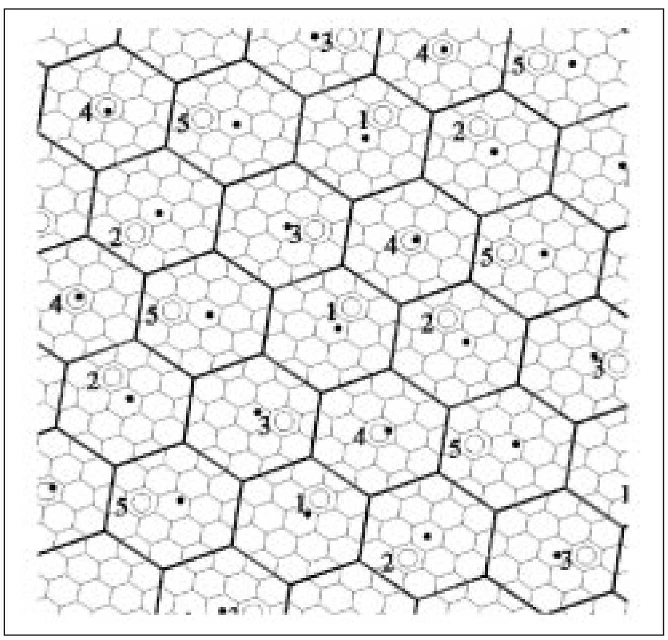


Figure 7.—*Panel layout, new FHM States.*



the intended panel of the P3 plot. Details on panel assignment may be found in Brand *et al.* (2000). The grid was intensified by locating three existing grid points of the same panel and then generating a fourth primary grid point of that panel. Again, a secondary grid point was generated. The result is a Y-shaped pattern, known to electrical engineers and linguists as a "wye," somewhat reminiscent of the mapped plot design (Alexander and Barnard 1992).

For the fifth panel in these States, the conjunctions at every ninth row and every ninth path of P2 hexagons were chosen as primary grid points, thereby unwittingly establishing $C_{81}$ hexagons, as figure 6 shows. The reader may realize that the intensification method used in the old FHM States leads to an expected ratio of P2 grid points to P3 grid points of 16.2 to 1. Five out of every nine $C_9$ cells will be filled. In the $C_{81}$ hexagon highlighted in figure 6, four $C_9$ cells that are filled form wye pattern. Around the perimeter of the $C_{81}$ hexagon, three other $C_9$ cells that are filled exist, but each one is shared with two other hexagons. The four empty $C_9$ cells also form a wye.

For States new to FHM, for the most part, conjunctions of every fourth row and every fourth path of hexagons were chosen as primary grid points. Unwittingly, this selection process established $C_{16}$ hexagons, which figure 7 illustrates. Panel assignment was based on the interpenetrating design (Roesch and Reams 1999, Reams and Van Deusen 1999). For States with little or no coastline, the expected ratio of P2 grid points to P3 grid points is 16.0 to 1.

## Why Rip the Seams?

In the quilting hobby, "ripping the seams" means undoing stitches and sewing new patches rather than actually ripping. The reason the authors are ripping the seams is that a number of current difficulties exists. First, the actual ratio of P2 grid points to P3 grid points in many States is quite different from the expected ratio. Secondary grid points were overlaid on TIGER shapefiles (U.S. Census Bureau 2000) to see which States they were in and whether they landed on land or water. Table 1 shows the results.

The results show that States with proportionally large coastlines per unit area—Florida, Louisiana, and South Carolina, for instance—are undersampled. When the grid was first constructed, coastal water plots were eliminated with some method other than a TIGER land coverage. This method inadvertently eliminated some land plots; for security reasons, a map has been withheld. Some $C_{16}$ cells are unsampled—from $C_{16}$ center no P3 plot in any direction for at least 10,500 meters. Some of

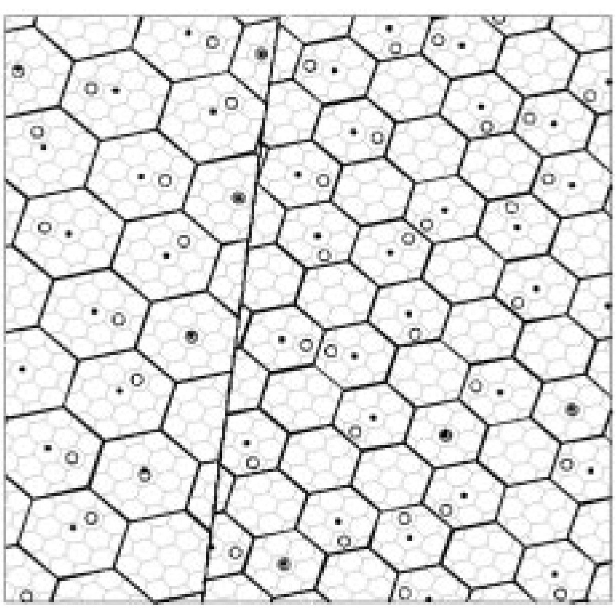
Table 1.—*Ratio of P2 to P3 hexagons in Southern States.*

| | Old P3 States | | | | | | New P3 States | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | Land only | | | All hexagons | | | | Land only | | | All hexagons | | |
| State | P2 | P3 | Ratio | P2 | P3 | Ratio | State | P2 | P3 | Ratio | P2 | P3 | Ratio |
| SC | 3,252 | 193 | 16.85 | 3,453 | 204 | 16.93 | LA | 4,770 | 290 | 16.45 | 5,592 | 303 | 18.46 |
| VA | 4,295 | 261 | 16.46 | 4,613 | 271 | 17.02 | FL | 5,823 | 359 | 16.22 | 7,069 | 377 | 18.75 |
| GA | 6,291 | 383 | 16.43 | 6,413 | 388 | 16.53 | TX | 28,224 | 1,746 | 16.16 | 28,955 | 1,774 | 16.32 |
| AL | 5,464 | 337 | 16.21 | 5,655 | 348 | 16.25 | AR | 5,621 | 349 | 16.11 | 5,732 | 358 | 16.01 |
| NC | 5,266 | 325 | 16.20 | 5,800 | 338 | 17.16 | KY | 4,293 | 267 | 16.08 | 4,359 | 275 | 15.85 |
| TN | 4,462 | 276 | 16.17 | 4,538 | 282 | 16.09 | OK | 7,423 | 465 | 15.96 | 7,535 | 472 | 15.96 |

these unsampled $C_{16}$ centers actually are land, and other $C_{16}$ centers are water, the secondary grid point, if not the plot, easily could be land. One way to address this issue would be to extend the P3 grid to the exterior boundary of the United States, as indicated in TIGER files (U.S. Census Bureau 2000).

The second current issue has to do with borders—some $C_{16}$ centers exist in new States that were not selected as primary grid points. Other $C_9$ centers exist in new States that were selected, but this resulted from the rule of the old States. Figure 8 illustrates examples of these anomalies. The authors' opinion is that either the grid or the documentation should be amended. Because the exact border rule is difficult to determine, the authors recommend that secondary grid points should be in the same State as primary grid points, and primary grid points should be selected only under the rules of the State they are in rather than under the selection rules of neighboring States. This solution has another side effect. Currently, the primary grid point can be in one State, and the secondary grid point be in another, and both States are old or both States are new. The proposed solution could affect a number of plots along such State boundaries.

Figure 8.—*Border issue, new FHM State vs. old FHM State.*



A third current issue is plot registration. Current Global Positioning System (GPS) readings do not necessarily agree with previously digitized plot locations, some to the point that many plots are outside their original assigned P2 hexagon. Several reasons exist for this problem. First, when plots were digitized, and no intention of one day laying them on a national grid existed. Quality assurance in digitizing maps may not have been what it should have been. Plots marked on county maps were only rough approximations of plot locations marked on aerial photographs. Second, when plot coordinates were collected with GPS units, the coordinates were transcribed to field sheets and then keypunched in the office, thereby providing at least two sources of transcription error. Third, in dense forest, GPS receivers can have difficulty maintaining a fix on satellites. Fourth, some miscommunication occurred regarding plot lists and plot selection rules.

A side effect of poor plot registration has been dropping plots because they are outside their assigned hexagons. On one hand, FHM does not want to needlessly drop past data, nor does FIA. On the other hand, FHM realized the importance of spatial dispersion among plots before FIA did. Keeping legacy plots and maintaining spatial dispersion, however, are conflicting goals. The authors believe that the best compromise would be to replace P3 plots if the field location is determined to be outside the assigned P3 hexagon. The P3 hexagon is the intended $C_{16}$

hexagon in States new to P3 and is the intended $C_9$ hexagon in historical P3 States.

To ensure that P2 and P3 plots are in the correct hexagon, a new quality assurance effort will be instituted. First, plot coordinates should be digitized by "heads-up" screen digitizing. The authors have already implemented a program to do this in several Southern States. Second, county maps with latitude, longitude, and county boundaries clearly marked should be issued to field crews. Third, field crews need to make sure that they can download coordinates from GPS to their data recorders through supplied GPS cables.

## Future Issues

The authors numbered all P2 hexagons into rows and paths, but might have renumbered the plots themselves. Field crews do not like it when plot 201 is in one side of the county, 202 is on the other side, 203 does not exist at all, and 204 is in the center. The lowest numbered row and path could be the first plot in the county, then plots could be incremented columnwise within row and then rowwise until all plots were numbered in logical fashion. It could be easy for the field crew to know the panel of a plot based on its plot number. The authors recognize the disadvantages of renumbering plots as well: database maintenance may become more difficult, and such a numbering scheme might be too easy for intruders (Office of Management and Budget 1994) to reconstruct.

Another application would be the construction of larger hexagons. Although hexagon maps are elegant, they may be legally risky. A $C_1$ map would reveal the P2 hexagon centers. A $C_T$ map would not be risky as T grows large.

## Conclusions

First, if the plot intensity for P2 or P3 changes yet again, global grid axioms should be followed. The axioms for intensifying the P3 grid have not been followed in the old States. With the current grid, either cells of unequal size exist or empty cells exist. According to the global grid axioms, there should be one plot in every cell, and cells should be equal area, regular hexagons. Second, a favorite expression in the quilting hobby is "measure twice, cut once." Failure to do so will turn a rationally planned patchwork quilt into a crazy quilt.

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