Optimal Stocking of Species by Diameter Class for Even-aged Mid-to-Late Rotation Appalachian Hardwoods

Joseph P. Roise, Joosang Chung, and Chris B. LeDoux[1]

Abstract.--Nonlinear programming (NP) is applied to the problem of finding optimal thinning and harvest regimes simultaneously with species mix and diameter class distribution. Optimal results for given cases are reported. Results of the NP optimization are compared with prescriptions developed by Appalachian hardwood silviculturists.

To achieve the desired objective from any thinning, it is necessary to regulate stand density, stand structure, stand quality, and species composition (Marquis et al. 1984). Consideration of all these variables simultaneously can represent a major computational challenge. However, several recent studies using nonlinear programming (NP) techniques appear to have overcome the computational limitations imposed by traditional optimization techniques. New perspectives on formulation and solution techniques have made simultaneous determination of optimal thinning plan, including timing, selection of species by diameter class, and rotation length, a tractable problem. Several recently published papers have explored these new perspectives. Roise (1986c) showed that the class of nonlinear programming algorithms referred to as nonlinear derivative free techniques can be used to optimize a wide variety of simulation problems, including optimal residual diameter class selections. Brooke et al. (1984) has reported significant progress in optimization of high level modeling systems using nonlinear programming. The optimization of simulation models is becoming more accessible.

The purpose of this paper is to present study results from an optimal species by diameter class stocking control problem. The optimal thinning plan and rotation length is solved for an Appalachian mixed hardwood stand using the MANAGE hardwood stand management simulator (LeDoux 1986a) incorporated into the Hook and Jeeves (1962) NP technique.

[1]*The authors are, respectively, Assistant Professor and Graduate Research Assistant, Department of Forestry, Box 8002, Raleigh, NC 27695, and Industrial Engineer and Project Leader, Northeastern Forest Experiment Station, USDA Forest Service, Box 4360, Morgantown, WV 26505.*

The following section briefly reviews the historical background on stand level decision makings. In the subsequent section, the general mathematical formulation for stand level decision makings is stated. Next, the model and the results of its application to a 1-acre Appalachian hardwood stand are described.

Background

As stated by Hann and Brodie (1980), stand level decision making involves such basic problems as finding the optimal species mix, planting density, thinning plan, fertilization plan, and rotation length. The literature indicates that most stand level optimization studies reduced the problem to simultaneously determining the optimal thinning regimes and rotation length. This reduction is probably because of the large number of variables necessary and the resulting computational burden when all variables are included.

Hann and Brodie (1980) further stated that nonlinear programming may have potential. The computational ability of several NP algorithms for even-aged stand management optimization have shown to be flexible optimization tools by Kao and Brodie (1980) and Roise (1984, 1986a). NP techniques have been used earlier to find optimal conversion strategies for uneven-aged stands (Adams and Ek 1974). Uneven-aged stand management has received a great deal of recent advance and clarification using nonlinear methods.

Haight (1987) has used the gradient projection method to solve uneven-aged stand management problems. Haight and Getz (1987) used a penalty function and gradient nonlinear programming method to compare optimal transition harvest regimes with either fixed end point or equilibrium end point constraints. They solve for the species composition and stand structure which optimize transition regimes and steady state condition. They conclude that solving for an optimal steady state and then solving for the optimal transition to that steady state will in general find suboptimal regimes when compared to solving for a more general equilibrium endpoint and transition regime simultaneously. Bare and Opalach (1987) used a direct search, derivative free, constrained nonlinear programming algorithm to solve for the optimal sustainable equilibrium diameter distribution and species mix in distance independent individual tree growth model.

For even-aged management, NP has not received as much attention. Kao and Brodie (1980) used a "modified flexible polyhedron method" to solve for the optimal thinning plan and rotation length. It is a modification of the simplex method of Nelder and Mead (1964) for constrained nonlinear programming. They also made a comparison between NP and DP by solving the same example problem with both techniques.

Roise (1986a) compared three NP techniques and discrete DP. The three NP techniques were the simplex method of Nelder and Mead, Hook and Jeeves' method, and Powell's method, all of which are derivative free multidimensional optimization techniques for unconstrained problems. He applied three different techniques to an example stocking control problem. He observed that the Hook and Jeeves' method performed better than others with regard to the relative computational efficiency and robustness of convergence for the given example.

Roise (1986b) applied Hook and Jeeves' method to solving for optimal thinning regimes including optimal diameter class distribution. The following section briefly describes his unconstrained objective function derivation procedures. Two advantages of this formulation in increasing the computational efficiency of Hook and Jeeves' method were observed: (1) the separable functional form of the dynamic stocking control was well suited for coordinate optimization and (2) empty diameter classes reduced the number of active variables to be evaluated.

Mathematical Programming Formulation

The mixed species forest stand management problem can be formulated in terms of timing and intensity of thinnings and final harvest.

$$\text{Max } f(t_j, X_{ijn}, t_{T+1}) \qquad [1]$$

subject to

$$X_{ijn} \varepsilon \Omega \qquad i = 1,2,\ldots,N$$
$$H_{ijn} >= X_{ijn} \qquad j = 1,2,\ldots,T$$
$$t_j, X_{ijn}, H_{ijn} \geq 0 \qquad n = 1,2,\ldots,S$$

where N is the number of diameter classes per species; S is the number of species; T is the number of thinnings; f is the objective function; X_{ijn} is the amount of timber to remove from diameter class i and species n at thinning j; H_{ijn} is the amount of species n in diameter class i at thinning j, $H_{ijn} = g[(H_{k(j-p)n} - X_{k(j-p)n}, t_j]$, p = 1,2,...,j-1, k = 1,2,...N, where g is a growth and yield model with decision variables of size class thinning intensity by species and interval between thinnings; Ω represents the operational constraints of the production system; t_j is the time between thinning j-1 to j; and t_{T+1} is the elapsed time between the last thinning and final harvest.

Assuming that there are no external production constraints, the number of decision variables required is TNS+T+1. The number of decision variables represents the maximum possible problem size. This will be discussed in more detail later when discussing problem reduction.

In order to solve problem [1] using Hook and Jeeves' method, the formulation is transformed into an unconstrained objective functional form as outlined by Roise (1986a). The transformed formulation is

$$\text{MAX } f(t'_j, r'_{ijn}, t'_{T+1}) \qquad [2]$$

where t'_j, r'_{ijn}, and t'_{T+1} are transformed decision variables t_j, X_{ijn}, and t_{T+1}, respectively. This unconstrained function can be solved by any unconstrained optimization technique including Hook and Jeeves' method. Formulation [2] can be interpreted as a barrier function. This formulation enables simultaneous optimization of the time of thinnings, amount of trees to remove for each diameter class by species, and rotation length.

Optimization and Simulation Model Description

All optimization procedures have two parts: the optimization algorithm and the system model. Hook and Jeeves' algorithm and the MANAGE stand management simulator are the two parts used here. Hook and Jeeves' algorithm controls the iteration processes to reach a local optimum point by adjusting decision variables. It is not within the scope of this article to discuss how to determine the global optimum, which is known as an unsolvable problem using NP when the system model is nonconvex. The system model, MANAGE, evaluates the decision variables passed from the Hook and Jeeves' algorithm and passes back an evaluation of present net worth.

MANAGE is a complete management systems simulator for mixed hardwood stands. It consists of a stand growth and yield simulator, GROW (Brand 1981), a thinning and harvest production estimator, ECOST (LeDoux 1985), a tree conversion model, BUCKING (Ledoux 1986b), and economic analysis model, PNW (LeDoux 1983). It includes a number of options which reflect various management activities in the real world. Only the PNW objective function option is used. The decision variables from the Hook and Jeeves' algorithm, the removal ratio of all species by diameter classes and thinning times, are passed to MANAGE. MANAGE removes the specified number of trees for each class, bucks the stems, and estimates present net value according to log grade values and net conversion costs. The estimated PNW for each entry is summed and passed to the Hook and Jeeves' algorithm where it is considered an objective function valuation, given decision variables.

The Hook and Jeeves' algorithm is illustrated in figure 1. The algorithm searches for the local optimum with a two step method: exploratory and pattern searches. Exploratory searches are directional steps along all coordinate axes from a fixed point, called the "base point." When exploratory searches are completed for all directions, the function gradient is estimated and the search continues in that direction. This movement along the gradient is called a "Pattern Search." The two step searching method is repeated until all the exploratory searches fail to improve the objective value. After successful searches for time variables or pattern searches, the stand conditions are stored for the next iteration to take advantage of dynamic stand structure. If an exploratory search fails, the step size is reduced by some percent of previous step size. It iterates until the step size decreases below a set tolerance. At this time, the last base point is reported as the local optimal solution.

Application to Example Stand Problem

Data from an example 1-acre stand was provided from the USDA Forest Service (Morgantown, WV). The example stand is 70 years old with site index 70. Average stand d.b.h. is 13.3 inches. The stand has 135 trees composed of multiple species such as basswood, hardmaple, white oak, red oak, beech, yellow-poplar, cucumbertree, and other minor species. Among the species, the first four species are considered economically valuable and others less so. The economically less important species were grouped as "Other Hardwoods" for the analysis. A total of 31 species, including "Other hardwoods," are appeas-

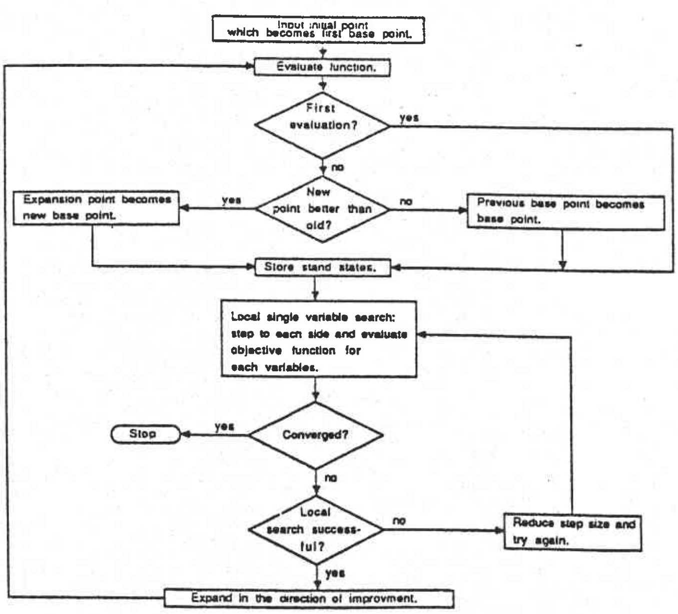


Figure 1.--Flowchart of the Hook and Jeeves' method.

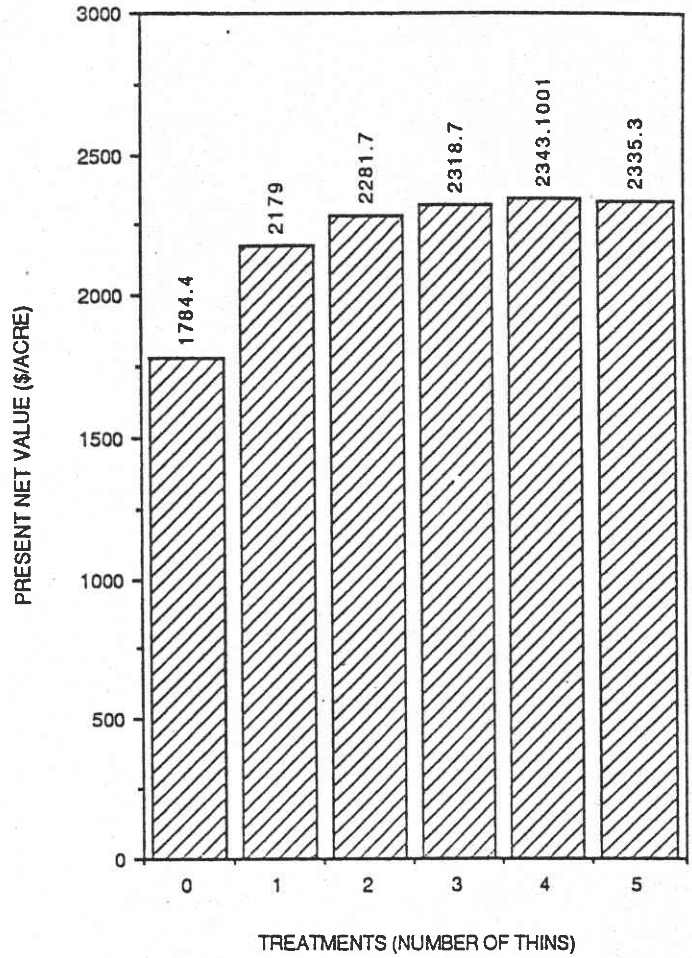


Figure 2.--Comparison of best local optima found with 0 to 5 thinnings.

able. The initial stand structure at age 70 is shown in figure 4 as diameter distributions by species. Real annual interest is assumed to be 4%. The Ecologger I skyline yarding machine is the logging option used.

The objective is to maximize PNW with decision variables of timing and intensity of thinning and final harvest. The number of species, the number of diameter classes, and the number of thinnings determine the total number of decision variables required. In this example, the initial stand has 5 species, including "Other hardwoods." Twenty five 2-inch diameter classes per species are used. This allows for all possible growth projections, as it turns out only 15 diameter classes were needed. At this age, precommercial thinning is not an option. Starting from zero thinning, we increased the number of thinnings by one in each subsequent run until the peak of the local optimal point is recognized (fig. 2). Results are summarized in table 1.

Each number of thinning type was solved three times with different randomly selected initial points (matrix of decision variables). All other non-control parameters remain fixed. When one or more thinnings are involved, the three runs converge on separate local optimum. This is an expected result

Table 1.--Summary of results from three different initial points for six different numbers of thinnings.

No. of thins	Initial point $/ac	PNW $/ha	NFE	Local optimal $/ac	PNW $/ha
0	1780.6	4398.1	6	1784.4	4407.5
0	751.9	1857.2	15	1784.4	4407.5
0	321.9	795.1	18	1784.4	4407.5
1	1092.6	2698.7	821	2179.0	5382.1
1	1004.2	2480.4	1321	2166.9	5352.2
1	1384.7	3420.2	1550	2124.0	5246.3
2	1294.6	3197.7	1035	2281.7	5635.8
2	830.2	2050.6	1347	2234.6	5519.5
2	1448.9	3578.8	1421	2261.8	5586.6
3	1407.9	3477.5	1319	2318.7	5727.2
3	1510.9	3731.9	1584	2318.6	5726.9
3	1543.0	3811.2	1124	2309.5	5704.5
4	1390.4	3434.3	1992	2343.1	5787.5
4	1231.9	3042.8	2598	2315.1	5718.3
4	1081.0	2670.1	2267	2232.0	5513.0
5	1217.9	3008.2	2095	2335.3	5768.2
5	1380.3	3409.3	1517	2307.4	5699.3
5	1047.3	2586.8	2167	2309.8	5705.2

if the objective function is nonconvex (Roise 1986a). Nonconvexity results among other reasons from sigmoidal individual tree growth functions, discrete thinning events, and nonlinear tree value functions (Roise 1986b). NP techniques may or may not provide a global optimum for nonconvex problems. By comparing several runs starting from different points, we may find the best optimal solution. However, the global optimum can never be guaranteed in solving nonconvex problems.

The best optimal solution from zero to five thinnings are compared in figure 2. Objective value reaches a peak at four thinnings and then decreases at five thinnings. When there are five thinnings, one thinning time interval goes to zero in two of the three runs from different initial points. This indicates that for those two, five thinning initial points, the local optima found has only four thinnings.

The number of functional evaluations (NFE) is used as a measure for the computational efficiency. The total NFE required generally increases as the number of active variables increase because more exploratory searches are required. Moreover, different initial matrices of decision variables can results in wide fluctuation of NFE, since some may start much closer to the optimum than others. Figure 3 illustrates the convergence rates of all 15 runs, when the number of thinning is greater than or equal to one. NFE ranges from approximately 800 to 2,598 according to the number of thinnings and the initial matrix of decision variables. In general, 90% of a local optimum PNW is reached within 25% to 30% of total NFE.

The number of decision variables represents the problem size. The problem size was reduced greatly by taking advantage of empty diameter classes as specified by Roise (1986b). The NP algorithm searches only in the vector space of active

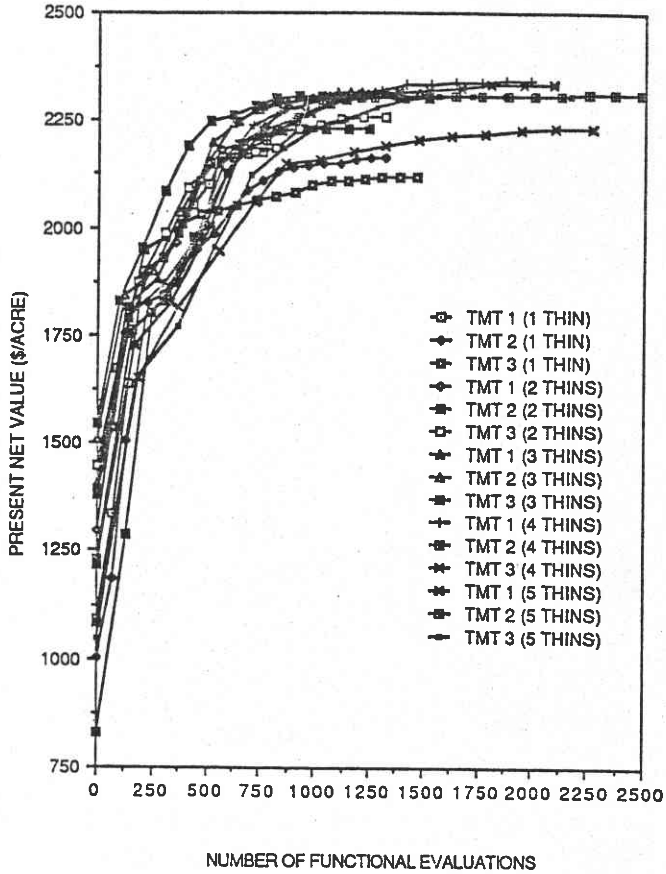


Figure 3.--Convergence rates for treatments from different initial points.

variables. As shown in table 2, the number of active variables was only 18% to 28% of the total variables for treatments with more than two thinnings. This reduction is related to the number of occupied diameter classes in the initial stand structure, the growth rate of species on the stand, and harvesting practices. The ratio of active to total variables for a Douglas-fir stand was observed to be approximately 0.3 by Roise (1986b).

For illustration, the results of the example problem, stand structure by species over time, are shown in figure 4. This was the best optimal solution for all 19 runs. Thinning entries occur at age 70, 90, 120, and 150 and final harvest at age 160. In general, trees are removed from the upper diameter classes. The discrepancy between the number of residual trees of a graph and

Table 2.--Reduction of the number of variables.

No. of thins	Total no. of variables	Aver. no. of active var.	Ratio
0	1	1	1.00
1	77	46	0.60
2	253	70	0.28
3	379	86	0.23
4	505	99	0.20
5	631	109	0.17

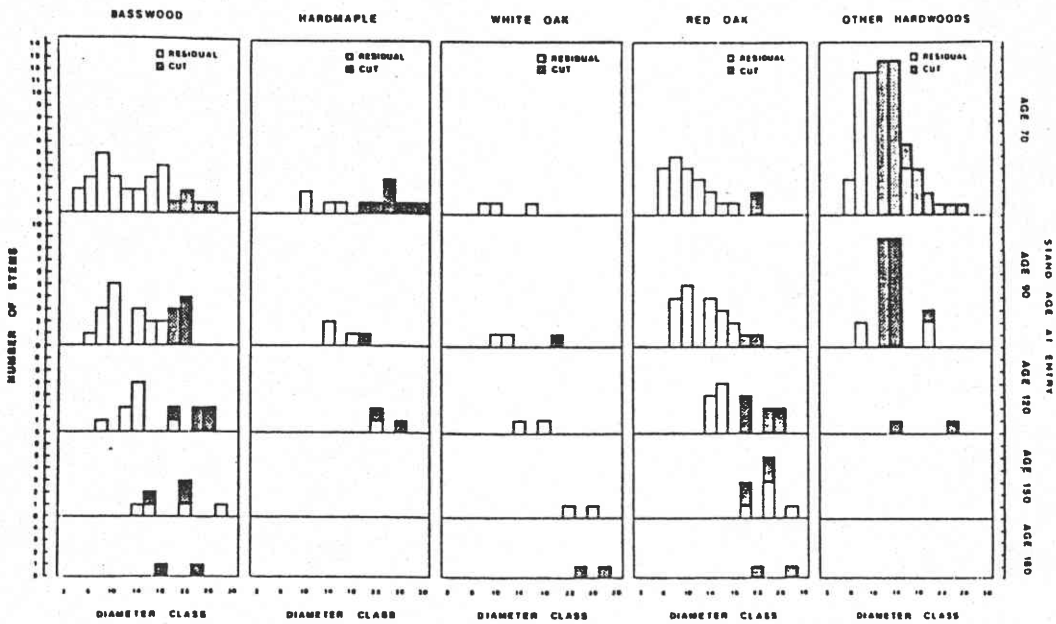


Figure 4.--Diameter class distribution before and after each thinning and final harvest.

the total number of stems in next graph is caused by mortality during that growth period.

How do these results compare to what a silviculturist would prescribe for the stand? To examine this question, we used the method of the prescribing silvicultural treatments in hardwood stands of the Alleghenies which Marquis et al. (1984) developed. Comparison is made on results from their method with results from our method for the first thinning. It should be noted that there is a major difference in objective between the two sets of results. Our objective is to maximize present net value of the stand explicitly considering expensive cable logging technology and detailed stump-to-mill costs. The silvicultural objective is to maximize the net undiscounted value of the stand.

> "Our guidelines on financial maturity are based on studies of individual tree financial maturity in New York, Pennsylvania, and West Virginia (Grisez and Mendel 1972, Mendel et al. 1973), plus unpublished results of computer simulator runs in Allegheny hardwood stands of varying species composition. Financial maturity of the stands in these computer runs was defined as the culmination of mean annual increment in dollar value. No interest rates or present net worth calculations were involved." (Marquis et al. 1984)

The method prescribes the distribution of cuts summarized in table 3 and illustrated in figure 5.

As can be seen, our method cuts trees exclusively from large diameter classes, whereas the silvicultural method prescribes cutting over all diameter classes, except in the sapling classes where both methods prescribe no cutting. In describing a commercial thinning, Marquis et al. (1984) state that trees should be removed primarily from the smallest and largest sizes, retaining those that are inbetween since these are the ones increasing most rapidly in value as they grow into size class they qualify for grade 1 sawtimber or veneer. Our method ignores the smaller sizes and increases cutting in the large to medium sizes. However, it should be noted that our method concentrates the first thinning on removal of low value species from the overstory and only cuts high value species with diameters greater than 20 inches. This is in agreement with the prescription developed using Marquis et al. (1984).

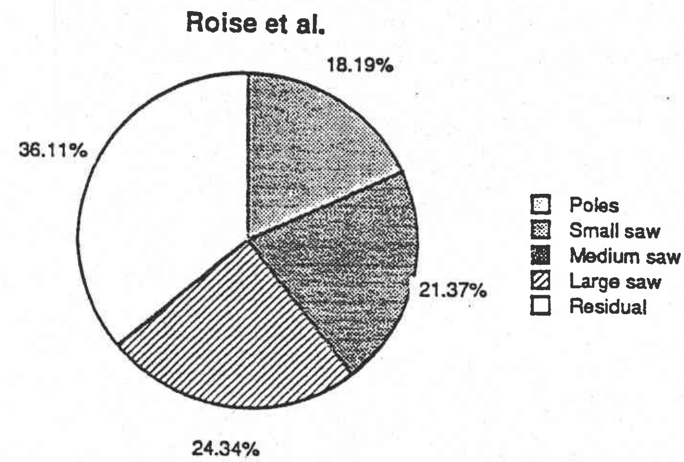
Our method also prescribes a much more intensive thinning than the silviculturists': residual basal area of 53 vs. 92 square feet per acre. There are several reasons for this difference. Our method cuts a much higher amount than what the silvicultural method prescribes because of inventory holding costs and expensive skyline cable stump-to-mill costs that must be offset

Table 3.--Comparison of results between methods of Marquis et al. and Roise et al. (basal area, ft²/acre).

Size class	Original stand	Cut	Residual stand	Cut	Residual stand
		Marquis et al.		*Roise et al.*	
Saplings	0.2	0.0	0.2	0.0	0.2
Poles	21.2	21.2	0.0	0.0	21.2
Small saw	50.1	5.0	45.1	26.9	23.2
Med. saw	40.5	12.2	28.3	31.6	8.8
Large saw	36.0	18.0	18.0	36.0	0.0

at each thinning. However, silviculturists are quick to point out that reducing a hardwood stand to such a low density will lead to problems later: lower tree quality, increased understory of interfering plants, and increased wind damage. Epicormic branching, forking, and other detrimental effects or costs may become serious at relative densities below 60% or 70%.

In general, thinning tends to increase competing understory and may lead to regeneration difficulties later in the rotation.



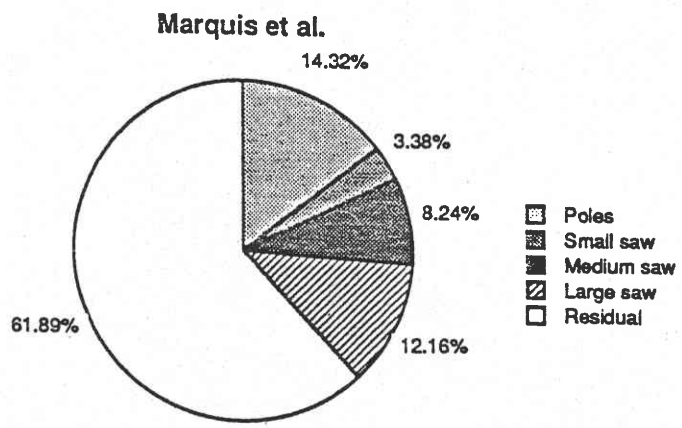


Figure 5.--Relative log size class distributions by Marquis et al. and Roise et al.

Intensive thinning opens a stand up to wind pressures which the root system cannot immediately absorb. This increases risk of windthrow in the stand. In the paper by Roise (1988), found in this same volume, the importance of stand quality is shown. Three stand parameters--volume production, tree size, and quality--are shown to control optimal stand management. All of the above factors are reasons cited by silviculturists not to have low residual densities. The results from the two methods are different because they have different objectives, they use different growth estimation routines and they have different assumptions on logging technology.

Conclusion

The typical Central Appalachian hardwood stand is a complex mixture of species and sizes. The problem of finding the optimal species mix by diameter class over time has a large number of variables. The use of the empty diameter class concept reduces number of variables and enables more efficient solution of the problem. Knowledge of desired species and diameter class structure is indispensable for foresters to make sound stand management decisions. This technique can be used to gain such a knowledge.

In this study, we expand the application of the Hook and Jeeves' method to multiple species problems. The solution to the stand management problem for multiple species stands are obtainable. As the numbers of active variables increase, the NFEs required to converge increase rapidly as well. Use of empty diameter classes and dynamic structures will reduce NFE and computer expense.

Literature Cited

Adams, D. M.; Ek, A. R. 1974. Optimizing the management of uneven-aged forest stands. Canadian Journal of Forest Research. 4: 274-287.

Bare, B. Bruce, Opalach, Daniel. 1987. Optimizing species composition in uneven-aged forest stands. Forest Science. 33: 958-970.

Brand, G. J. 1981. GROW--a computer subroutine that projects the growth of trees in Lake States forests. Res. Pap. NC-207. St. Paul, MN: U.S. Department of Agriculture, Forest Service, North Central Forest Experiment Station. 11 p.

Brooke, Anthony; Drud, Arne; Meeraus, Alexander. 1984. High level modeling system and nonlinear programming. In: Broggs; Byrd; Schnabel, eds. Nonlinear optimization. Philadelphia, PA: SIAM: 178-198.

Hann, D. W.; Brodie, J. D. 1980. Even-aged management: basic managerial questions and available or potential techniques for answering them. Gen. Tech. Rep. INT-83. Ogden, UT: U.S. Department of Agriculture, Forest Service, Intermountain Forest and Range Experiment Station. 27 p.

Haight, R. G. 1987. Evaluating the efficiency of even-aged and uneven-aged stand management. Forest Science. 33: 116-134.

Haight, R. G.; Getz, W. M. 1987. Fixed and equilibrium endpoint problem in uneven-aged stand management. Forest Science. 33: 908-931.

Hook, R.; Jeeves, T. A. 1961. Direct search solution of numerical and statistical problems. J.A.C.M. 8: 212-221.

Kao, C.; Brodie, J. D. 1980. Simultaneous optimization of thinnings and rotation with continuous stocking and entry intervals. Forest Science. 26: 338-346.

LeDoux, Chris B. 1983. A simulation of the comparative costs and benefits of skyline strip thinning. Corvallis, OR: Oregon State University. 103 p. Ph.D. dissertation.

LeDoux, Chris B. 1985. Stump-to-mill timber production cost equations for cable logging eastern hardwoods. Res. Pap. NE-566. Broomall, PA: U.S. Department of Agriculture, Forest Service, Northeastern Forest Experiment Station. 6 p.

LeDoux, Chris B. 1986a. MANAGE: a computer program to estimate costs and benefits associated with eastern hardwood management. Gen. Tech. Rep. NE-112. Broomall, PA. U.S. Department of Agriculture, Forest Service, Northeastern Forest Experiment Station. 7 p.

LeDoux, Chris B. 1986b. Bucking logs to cable yarder capacity can decrease yarding costs and minimize wood wastage. Southern Journal of Applied Forestry. 10: 180-183.

Marquis, David A.; Ernst, Richard L.; Stout, Susan L. 1984. Prescribing silvicultural treatments in hardwood stands of the Alleghenies. Gen. Tech. Rep. NE-96. Broomall, PA: U.S. Department of Agriculture, Forest Service, Northeastern Forest Experiment Station.

Nelder, J. A.; Mead, R. Mead. 1964. A simplex method for function minimization. Computer Journal. 7: 308-313.

Powell, M. J. D. 1964. An efficient method for finding the minimum of a function of several variables without calculating derivatives. Computer Journal. 7: 155-162.

Roise, J. P. 1986a. A nonlinear programming approach to stand optimization. Forest Science. 32: 735-748.

Roise, J. P. 1986b. An approach for optimizing residual diameter class distributions when thinning even-aged stands. Forest Science. 32: 871-881.

Roise, J. P. 1986c. Optimization of simulation models using nonlinear programming. Paper presented at ORSA/TIMS National meeting; 1986 May; Los Angles, CA.

Roise, J. P. 1988. Stand level sensitivity analysis on the effect of markets on optimal management regimes. In: Proceedings, Systems analysis in forest resource management, the 1988 symposium; 1988 March 29-April 1; Asilomar, CA.

United States Department of Agriculture

Forest Service

Rocky Mountain Forest and Range Experiment Station

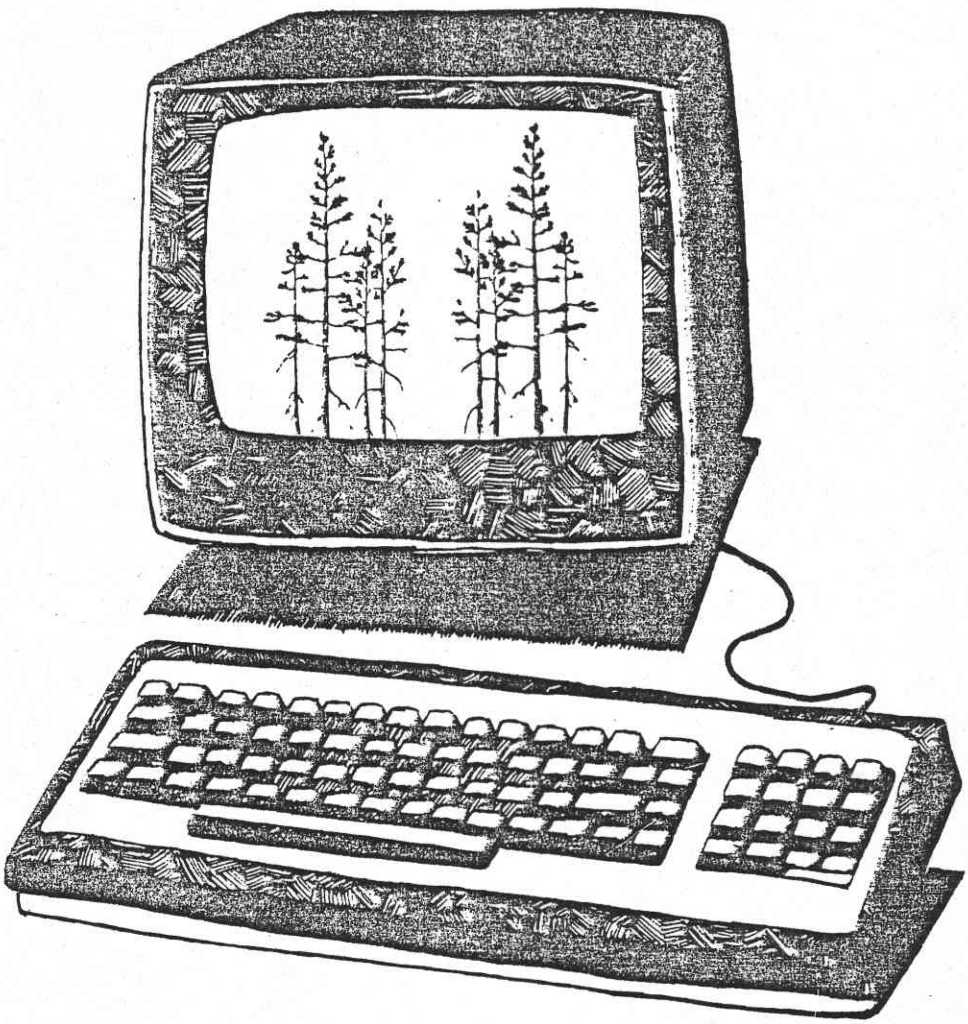
Fort Collins, Colorado 80526

General Technical Report RM-161

The 1988 Symposium on Systems Analysis in Forest Resources

March 29 to April 1, 1988

Asilomar Conference Center
Pacific Grove, California



278 pages