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ESTIMATING FOREST ECOSYSTEM EVAPOTRANSPIRATION AT MULTIPLE TEMPORAL SCALES WITH A DIMENSION ANALYSIS APPROACH[1]

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ABSTRACT: It is critical that evapotranspiration (ET) be quantified accurately so that scientists can evaluate the effects of land management and global change on water availability, streamflow, nutrient and sediment loading, and ecosystem productivity in watersheds. The objective of this study was to derive a new semi-empirical ET modeled using a dimension analysis method that could be used to estimate forest ET effectively at multiple temporal scales. The model developed describes ET as a function of water availability for evaporation and transpiration, potential ET demand, air humidity, and land surface characteristics. The model was tested with long-term hydrometeorological data from five research sites with distinct forest hydrology in the United States and China. Averaged simulation error for daily ET was within 0.5 mm/day. The annual ET at each of the five study sites were within 7% of measured values. Results suggest that the model can accurately capture the temporal dynamics of ET in forest ecosystems at daily, monthly, and annual scales. The model is climate-driven and is sensitive to topography and vegetation characteristics and thus has potential to be used to examine the compounding hydrologic responses to land cover and climate changes at multiple temporal scales.

(KEY TERMS: dimension analysis; evapotranspiration; empirical modeling; forest hydrology; water balance.)

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INTRODUCTION

Evapotranspiration (ET) is a major component of the hydrologic balances in terrestrial ecosystems. It represents 60-75% of precipitation inputs at the global (Vörösmarty *et al.*, 1998), continental (Sun *et al.*, 2002a), and regional scales (Lu *et al.*, 2005). ET is also the most important hydrologic component in influencing regional water availability and use (Zhang *et al.*, 2001, 2004; Sun *et al.*, 2006). In southern China, ET from well-forested eucalyptus plantations

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can be as high as 90% of annual precipitation (Zhou *et al.*, 2002a). In the last two decades, afforestation campaigns have increased both forest coverage area and productivity in China (Sun *et al.*, 2006). Concerns have been raised in China about the effects of large scale reforestation on water yield as plantations with exotic pioneer tree species such as eucalyptus transpire more water than do native plant communities (Zhou *et al.*, 2002a,b; Sun *et al.*, 2006). The most direct effect of climate and land use changes on hydrology is alteration of the magnitude and distribution of ET (Dow and DeWalle, 2000), with subsequent indirect effects on streamflow magnitude and distribution. Further, ET is also an indicator of ecosystem productivity and biodiversity (Currie, 1991; Law *et al.*, 2002), and in fact, it is the only variable that links hydrology and biological processes in most ecosystem models. Thus, improving our quantitative understanding of how environmental and biotic variables affect forest ET is essential in assessing impacts of land use and global environmental changes on water balance and ecosystem functioning.

It is well understood that the ET processes are mainly controlled by net radiation, atmospheric advection, and air turbulent transport, leaf area, and plant-available soil water (Milly, 1994). The literature describes numerous numerical models with different forms that are based either on physical processes or on empirical statistical approaches. One class of mathematical models is represented by formulas of the single-line type (e.g., Budyko, 1958) that relate annual ET to key driving variables such as annual precipitation, potential evapotranspiration (PET), and land topopgraphy (Zhang *et al.*, 2001, 2004; Donohue *et al.*, 2007). Zhang *et al.* (2004) suggest that Fu's formula (Fu, 1981) (Equation 1), which is based on dimensional analysis theories, is the best model for estimating annual ET:

$$E = E_0\left\{1 + \frac{r}{E_0} - \left[1 + \left(\frac{r}{E_0}\right)^m\right]^{\frac{1}{m}}\right\}, \tag{1}$$

where E is actual ET (mm/year), E_0 is PET rate (mm/year), or ET under unlimited soil water conditions, r is rainfall (mm/year), and m is a model parameter varying from 1 to infinity.

Fu's model (Equation 1) gives reasonable estimates of annual ET across large climatic gradients (Zhang *et al.*, 2004), but has several limitations: (1) the model is not applicable to smaller temporal (i.e., daily and monthly), (2) it does not explicitly consider soil and vegetation characteristics that affect water availability (Donohue *et al.*, 2007), and (3) it does not consider the effects of humidity or vapor pressure deficit (VPD) on ET directly when calculation of PET does not include the humidity variable. Studies have showed that relative humidity and VPD are important variables in controlling actual ET (Anthoni *et al.*, 2002).

The overall goal of this study was to use a dimension analysis approach similar to that adopted by Fu (1981) and described by Zhang *et al.* (2004) to construct a new ET model for moist coniferous forest ecosystems in China and the United States (U.S.). The new model can be applied to estimate daily and monthly ET in addition to annual ET. Specifically, our objectives were to (1) derive a new formula that relates ET to water availability in the soil and on vegetation, to atmospheric demand (air humidity), and to energy driving force (PET) and (2) examine the applicability of this model at multiple temporal (daily, monthly, and yearly) scales.

METHODS

Model Development

At any temporal and spatial scale, ET occurs as long as three conditions exist: a supply of water in soils or on vegetation surfaces is present; energy is available (often expressed as PET); and the atmosphere can still hold water vapor. Other factors such as leaf stomatal conductance, leaf area index, and atmospheric turbulence can also influence the actual ET by affecting these three major conditions. Based on these assumptions, we hypothesized that actual ET (E) can be described as a function of PET (E_0); actual availability of water in soil and on plant surfaces for evaporation (s), and air relative humidity (h) (0-1) representing atmospheric water demand through VPD or mass transfer:

$$E = F(E_0, s, h), \tag{2}$$

where F is a function to be determined.

The three variables on the right hand side of Equation 2 are independent. By definition, E_0 is the ecosystem ET from lands where water supply is unlimited, and thus is independent of s. We adopted a PET method (described later in the text) in which only air temperature and day length are needed to estimate E_0, thus E_0 is independent of atmospheric relative humidity, h. The variable, s, is affected mostly by precipitation and soil physical properties, and thus is independent of h. We imply that actual ET is always less than PET.

Mathematically, Equation 2 can be expressed in the following two partial differential equations. They can be used to examine how surplus of potential energy (E_0-E) and surplus of availability of water supply (s-E) drives the ET processes:

$$\begin{cases} \frac{\partial E}{\partial s} = f(E_0 - E, s, h) \\ \frac{\partial E}{\partial E_0} = \varphi(s - E, E_0, h) \end{cases}, \tag{3}$$

where f and φ are functions to be determined.

In Equation 3, h is a dimensionless quantum, but E_0-E and s, s-E and E_0 are pairs of quanta with a dimension of length or time. According to the π law of dimensional analysis, the two pairs of quanta E_0-E and s, s-E and E_0 should have only one independent quantum in dimension in each pair. If s and E_0 are the quanta with independent dimension for E_0-E and s, s-E and E_0, respectively, there will be:

$$\pi_1 = x = \frac{E_0 - E}{s}$$

and

$$\pi_2 = y = \frac{s - E}{E_0}$$

and Equation 3 may be rewritten as

$$\begin{cases} \frac{\partial E}{\partial s} = f_1(x, h) \\ \frac{\partial E}{\partial E_0} = \varphi_1(y, h) \end{cases} \tag{4}$$

Equation 4 is one of the dimensionless forms of Equation 3. f_1 and φ_1 are functions to be determined. Equation 4 should satisfy the following boundary conditions:

$$\begin{cases} \frac{\partial E}{\partial s}|_{E=E_0} = 0 \\ \frac{\partial E}{\partial s}|_{h=1} = 0 \\ \frac{\partial E}{\partial E_0}|_{E=s} = 0 \\ \frac{\partial E}{\partial E_0}|_{h=1} = 0 \end{cases} \tag{5}$$

As discussed earlier, E_0, is independent of water availability, s, and relative humidity, h. So, for boundary Condition 1, when $E = E_0$, or actual ET (E) reaches the maximum, then E will become a constant (under a E_0), independent of soil moisture content (s), thus the derivative of E/s equals to zero. For boundary Condition 2, when relative humidity reaches the highest, 1.0, suggesting the water vapor holding capacity of the atmosphere is filled, actual ET (E) will stop and will not change with water availability, s. Similar verification analysis can be performed for the other three boundary conditions.

Solving Equation 5 subject to above boundary conditions, the following ET model is derived:

$$E = E_0\left\{1 + \frac{s}{E_0} - \left[1 + \left(\frac{s}{E_0}\right)^{N[k(h)+1]+1}\right]^{\frac{1}{N[k(h)+1]+1}}\right\}, \tag{6}$$

where N is a model parameter. Details of model development are provided in Appendix A. Equation 6 suggests that estimating ET at daily, monthly, or annual scale requires soil water availability, PET, and relative humidity, as driving variables, and an empirical model parameter, N. The form of this model is similar to Fu's model. However, this new model includes more climatic variables and can be applied for finer temporal scales when the input variables are available. Therefore, this new model is not limited to estimating annual or long-term ET as Fu's model does.

Analysis of Key Parameters and Variables in the ET Model

The Parameter N for Land Surface Characteristics. The parameter N in the ET model (Equation 6) is a dimensionless integration product that reflects the effects of land characteristics on runoff, soil water, and thus ET. The physical meaning of this parameter is similar to that of the m parameter in Fu's model (Equation 1). Land cover, soil properties, and landforms all have influences on this parameter. Its value may vary from 0 to infinity. When N equals zero, E will be reduced to zero, suggesting that the ET surfaces cannot hold any liquid water, as in the instance of pavement or roofs of houses. In this case, actual ET (E) will be zero even if the PET or precipitation is large. In another extreme scenario, $N = \infty$, E will be determined by the smaller value of s and E_0, suggesting that the ET surface can hold all liquid water, as in the instance of a flat pond with no flow outlets. The above analysis implies that land topographic gradients will have great impact on runoff, soil water storage, and finally ET. As this parameter is an empirical one that varies greatly among different landscapes and regions, the N values for different land surface conditions must be estimated by empirical data such as long-term historic watershed-scale hydrologic observations. We expect this parameter to vary greatly among different land uses (urban lands *vs.* forests). However, Fu (1996) suggests that ground topographic gradient (e.g., plain *vs.* mountains) is the major controlling factor among many other factors.

Relative Humidity Function $k(h)$. The form of the relative humidity function $k(h)$ can be determined by satisfying the following three conditions:

(1) $k(h)$ (0.0 to −1.0) decreases with increase in relative humidity, h (0.0-1.0).
(2) When h equals to zero, $k(h)$ should reach its maximum value of 0.0. In this case, the ET equation becomes the form proposed by Fu (1981) (Zhang *et al.*, 2004).
(3) When h equals 1.0, $k(h)$ should reach its minimum value of −1. In this situation, E equals zero.

Based on those requirements, relationship between $k(h)$ and h was determined as Equation 7, and was graphically presented in Figure 1.

$$k(h) = \frac{h}{h^2 - h - 1} \tag{7}$$

When N is bigger, the variable h has greater influence on the value of $N[k(h) + 1] + 1$ (Figure 1b), and

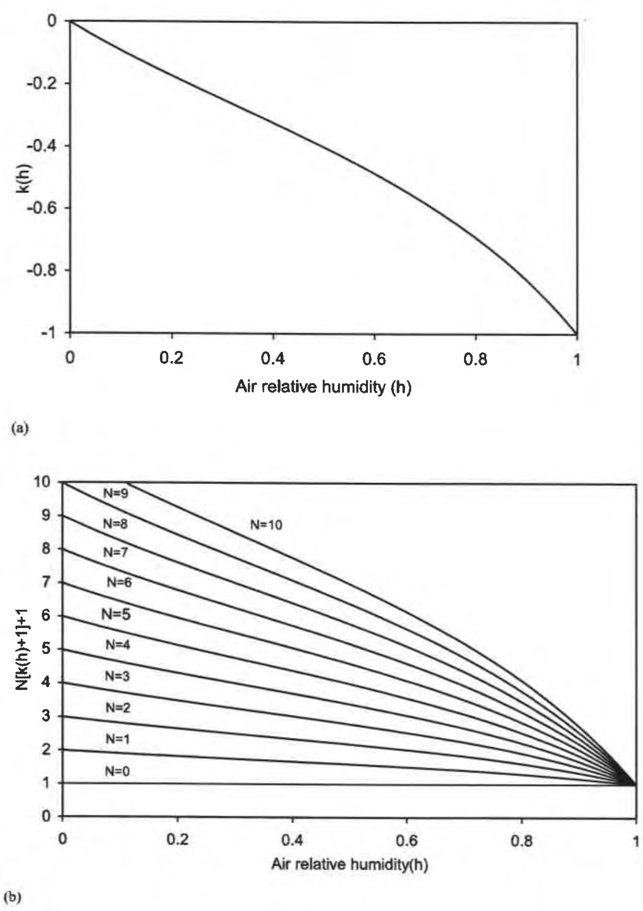

FIGURE 1. The Relationships for (a) Between h and $k(h)$ and (b) Between h and $N[k(h) + 1] + 1$.

thus E. Therefore, air humidity is an important factor in the calculation of actual ET, especially when the humidity is low and N is large (Figure 1b). Although selection of this relation is somewhat arbitrary (only one of the many forms that satisfy the three conditions above), it does provide a simple way to reflect the influence of relative humidity on $k(h)$, and eventually E.

Water Availability Variable, *s*. The term s in Equation 6 represents water in the plant rooting zone and on plant surfaces available to be transferred in a vapor form in the soil-plant-atmosphere continuum during the ET processes. Water sources may include intercepted water on plant surfaces and soil water subject to extraction by plant roots for transpiration and surface soil evaporation. This term is highly variable in space and time, and thus measurements are rarely available. On a long-term basis (such as yearly), total precipitation can represent the maximum water supply for ET. However, for finer temporal scales, such as daily and monthly ones, a simple water balance method is developed in this study to estimate water supply rates.

We developed the following procedure for estimating s using commonly available parameters and measurable climatic variables. As both plant canopy interception and soil water movement involve complex processes at a daily time-scale, simplifications were made for general applications at large spatial scales.

We assume:

(1) Plant canopy and litter interception losses can be represented as occurring at a fixed rate I_p when precipitation occurs. The I_p (dimensionless) values for different forest communities have been well reported for North America (Helvey and Patric, 1988) and China (Zhou *et al.*, 2002a).
(2) Water drains out of the rooting zone as subsurface flow when soil moisture content exceeds soil water holding capacity (WHC) (dimension in length). If this condition (soil moisture content greater than field capacity) occurs, soil water content is reset as the field capacity. This soil water routing scheme implicitly assumes that all precipitation will infiltrate to the soil zone and ground water recharge will occur when the soil water content exceeds soil field capacity. Soil WHC is a soil hydraulic parameter whose range is readily available from standard soil surveys. It was calculated as the product of field capacity in percent and mean rooting depth in the unit of length.

Therefore, s in Equation 6 can be estimated for shorter temporal scales by adding two key parameters, I_p and WHC.

$$\begin{aligned} s_i &= s_{i-1} + P_i - E_{i-1} & \text{if } s_i &< \text{WHC} + P_i \times I_p \\ s_i &= \text{WHC} + I_p \times P_i & \text{if } s_i &\geq \text{WHC} + P_i \times I_p, \end{aligned} \tag{8}$$

where s_i and s_{i-1} is water availability (mm) in day i or month i in the rooting zone and plant surface, while s_{i-1} is the water availability one day or one month earlier. The variable E_{i-1} is the actual ET in the day or month i−1. P_i is precipitation in day or month i. I_p and WHC are constants representing maximum plant canopy and litter interception rates regardless of rainfall characteristics and soil WHC (field capacity) (dimension in length) in the rooting zone, respectively. Canopy interception is an essential component of the forest hydrologic cycle. Forest canopy and litter interception rate (I_p) can be found in most forest hydrology textbooks and are well reported in the literature (Chang, 2003). It varies among both forest types and seasons but is generally less than 0.4. Similarly, forest rooting depth is also highly variable across biomes. Observed rooting depth values range from less than 1 m to as high as 15 m (Kleidon and Heimann, 1998). Most simulation models use 50-100 cm as a standard rooting depth for hydrologic simulations of water uptake (Kleidon and Heimann, 1998; Sun *et al.*, 1998; Amatya and Skaggs, 2001). In this study, we set the rooting depth as 80 cm as a conservative estimate for all five research sites. Over 80% of the root biomass is located within the top 80 cm soil layer for pine and hardwoods forests in the southern U.S. (Monk and Day, 1988). We consider this rooting zone constitutes "effective" rooting depth that plant roots can draw water from during the transpiration process. Field ground water table data from the forested sites suggest plant roots and associated unsaturated water movement in the ET processes could influence soil moisture content at least 80 cm in the soil profile (Sun *et al.*, 2000).

As any empirical modeling method, uncertainties do exist in the model we developed. For example, empiricisms exist in the internal relationships between h and $k(h)$, the parameter N, soil water routing schemes, and even in development of the overall functional relationships between E and E_0, s, and h. Our overall model development strategy was to build some kind of relation forms using rational methods first among the key climatic and physical variables and then use empirical data to verify the relations at multiple scales using different data sources such as watershed water balances and eddy fluxes data when they are available.

Estimating Potential ET (E_0)

Daily PET, E_0, was estimated using Hamon's method as described by Federer and Lash (1978) and Lu *et al.* (2005) (Equation 9). This method treats temperature as the main driving force for ET, but also includes other variables such as daytime length and saturated vapor pressure.

$$E_0 = 0.1651 \times D \times V_d \times k, \tag{9}$$

where E_0 is the PET (mm/day); D is the time from sunrise to sunset in multiples of 12 hours, computed as a function of date, latitude, slope, and aspect of the watershed; V_d is the saturated vapor density ($g \cdot m^{-3}$) at the daily mean temperature (T) (°C).

$$V_d = 216.7 \times V_s/(T + 273.3), \tag{10}$$

where V_s is the saturated vapor pressure (mb).

$$V_s = 6.108 \times \exp[17.26939 \times T/(T + 237.3)], \tag{11}$$

where k is the correction coefficient to adjust PET calculated using Hamon's method to measured values. Our previous studies indicated that it was appropriate to use k = 1.3 to estimate forest PET for the Coweeta (CW) site and k = 1.2 to estimate it for other sites (Lu *et al.*, 2005).

Previous regional ET model comparison studies suggest that Hamon's E_0 method gives comparable or slightly higher E_0 than the more data intensive methods, such as Priestly-Taylor equation (Priestley and Taylor, 1972), in the humid region (Vörösmarty *et al.*, 1998; Lu *et al.*, 2005). The Hamon's PET model has the advantages that it requires few input variables for a regional application.

Model Validation Procedures

Database Descriptions. Model evaluation was performed at individual sites. Five experimental forest sites in southern China and the southeastern U.S. were selected for detailed model evaluation at multiple temporal scales and deriving N parameters. Those five sites span a wide range of vegetation and hydrologic conditions (Table 1). The two research sites in southern China, Zhanjiang 1 (ZJ1) and Zhanjiang 2 (ZJ2), represented a stand-scale forest hydrologic study of tropical plantations (Zhou *et al.*, 2004). The main study objective of research at these two sites was to quantify total water use by eucalyptus plantations. For each plantation, canopy interception was determined as rainfall above the tree

TABLE 1. Characteristics of Five Research Sites for Model Development and Validation.

Parameters	Carteret (CT)	Coweeta (CW)	Florida (FL)	Zhanjiang 1, China (ZJ1)	Zhanjiang 2, China (ZJ2)
Location	34° 48′N, 76° 42′W	35° 03′N, 82° 25′W	29° 54′N, 81° 30′W	21° 05′N, 109° 54′E	20° 54′N, 109° 52′E
Altitude	3 m	710-1,000 m	43-43-44 m	8-10 m	8-10 m
Watershed Size (ha)	25.0	12.0	140.0		
Dominant Climate	Subtropical, marine	Subtropical, marine	Subtropical, marine	Tropical, marine	Tropical, marine
Long-term Annual Precipitation	1,340 mm, convection and hurricane	1,245-2,314 mm, convection and orographic formation	1,400 mm, convection formation	1,300-2,500mm, convection and hurricane	1,300-2,500mm, convection and hurricane
Mean Annual Air Temperature (°C)	16.2	12.6	21.0	23.5	23.5
Slope (%)	<0.2	<42.0	<2.0	<2.0	<2.5
Soils	Fine sandy loam (<3m), field capacity: 0.22	Deep sandy loams on bedrock (0-6 m), field capacity: 0.33	Sandy soils on deep clay (<3 m), field capacity: 0.22	Sandy soil of sedimentary origin, field capacity: 0.25	Basalt-derived clay soil, field capacity: 0.40
Vegetation	Mature loblolly pine plantation I_p = 0.25 for all seasons	Mature mixed deciduous hardwoods (oak) I_p = 0.12 for growing season I_p = 0.079 for dormant season	Unmanaged mature cypress-slash pine plantations I_p = 0.15 for all seasons	Intensively managed eucalyptus plantation I_p = 0.16 for all seasons	Intensively managed eucalyptus plantation I_p = 0.20 for all seasons
Available Data [precipitation, runoff, relative humidity, and PET (E_0)]	Daily, monthly, and yearly: 13 years (1988-2000)	Daily, monthly, and yearly: 6 years (1985-1990)	9 years of daily data (1984-1992); Monthly and yearly: 15 years (1978-1992)	Daily: 1 year (1999-2000) no runoff data	Daily: 1 year (1999-2000) no runoff data
N value determination (model calibration)	Four years (1990-1991) of yearly data	Five years (1985-1988, 1990) of yearly data	Five years (1985-1987, 1989-1990) of yearly data	One year data (1999-2000)	One year data (1999-2000)
Model validation (monthly and annual scale only)	1992-2000	N/A	1978-1984, 1988	N/A	N/A

canopy, as recorded by a tipping bucket, minus stemflow, and throughfall collectors installed under the canopy. Soil evaporation was measured by several lysimeters in different locations on both sites. Tree transpiration at the stand level was determined by scaling up measured sap flux density of 18-20 trees using a heat-pulse system developed by Edwards Industries of New Zealand (Zhou *et al.*, 2002b, 2004). The sum of canopy interception, soil evaporation, and transpiration was compared with modeled total daily ET. Volumetric soil moisture at four depths (50, 150, 250, and 350 cm) was recorded by TDR-based soil moisture sensors (Theta Probes, Delta T Devices, UK). The CW, Carteret (CT), and FL sites represent three small forested watersheds in a warm and humid subtropical climatic environment of the southeastern U.S. The CW watershed represents a steep upland forest ecosystem of the southern Appalachian Mountains, while the FL and CT watersheds are forested wetlands on the flat coastal plains. The CT watershed was a mid-rotation loblolly pine plantation originally converted from forested wetlands by construction of parallel ditches, a common forestry practice in the Atlantic coastal region. All three sites had a long-term hydrologic monitoring history, and represent three of the best forest hydrologic research facilities in the southern U.S. (Sun *et al.*, 2002b). All three watersheds were gauged by either a flume or weir at the watershed outlet and continuous flow and associated climatic variables were requested for the study period. Details of watershed experimental designs and research findings are found in Swank and Crossley (1988) for CW, Riekerk (1989) and Sun *et al.* (1998) for FL, and Amatya and Skaggs (2001) for the CT site.

Procedures of Model Calibration and Validation

Model validations were performed at the daily, monthly, and annual temporal scales after the land surface parameter N had been determined by a model optimization procedure for each site by pooling all datasets over the study periods. Once the N values were derived by this model calibration process, they remained unchanged for model testing at the multiple temporal scales. For FL and CT sites for which we had long-term data, we used only part of the data during this model calibration (fitting) process and reserved another section of the dataset for model validation at multiple temporal scales. Unfortunately, we had daily ET data only from the ZJ1 and ZJ2 sites. So, model calibration and verification only at the daily step was conducted for these two sites. For each of the three U.S. sites, we compared the annual sum of predicted daily or monthly ET to the annual measured ET as determined by the watershed water balance method (ET = precipitation – streamflow). Modeled annual ET by the model developed in this study (Equation 6) and Fu's model (Equation 1) were compared with the annual ET used for model validation at the daily and monthly scales for the three U.S. sites.

Linear correlations as well as graphic comparisons were conducted to determine model performance. Daily scale validation was performed at the two Chinese sites. Monthly and annual prediction errors were calculated as annual sum of predicted ET minus estimated annual ET that was estimated as the differences of annual precipitation and stream runoff.

RESULTS AND DISCUSSION

Modeled Land Surface Characteristic Parameter N

As described earlier, the value of parameter N was determined by fitting Equation 6 and comparing modeled and calculated E with the watershed water balance method (Table 2). Long-term annual actual ET (E) for both the CW, CT, and FL watersheds was estimated as the difference between measured annual precipitation and streamflow (Table 2). The daily E values for the ZJ1 and ZJ2 sites were reported in

TABLE 2. Estimated Land Surface Parameter N From Long-Term Hydrometeorological Records.

Sites and Data Time Period	Total Precipitation or S (mm/year)	Total Runoff (mm/year)	E (Precipitation-Runoff) (mm/year)	E_0 (mm/year)	Average h	Derived N in Equation (6)	Derived m in Equation (1)
Carteret (CT) (1988-1991)	1,523	400	923	1,148	0.79	19.86	7.36
Coweeta (CW) (1985-1988, 1990)	1,492	549	943	986	0.68	8.32	4.64
Florida (FL) (1985-1987, 1989-1990)	1,402	212	1,190	1,417	0.76	10.63	4.79
Zhanjiang 1 (ZJ1) (1999-2000)[1]	1,555	No data	826[1]	1,379	0.80	2.97	1.92
Zhanjiang 2 (ZJ2) (1999-2000)[1]	1,525	No data	1,141[2]	1,290	0.80	10.55	4.28

[1]Data in this row represent incomplete accumulation for 1999 and 2000;

[2]Data represent as the sum of measured canopy interception and sapflow. No runoff measurements from ZJ1 and ZJ2.

Zhou *et al.* (2002b, 2004) as the daily sums of direct measurements of tree transpiration, soil evaporation, and canopy interception. Large variability of the N value was found. N was largest for the CT site and lowest for the ZJ1 site. This suggests that poorly drained wet flat lands with high water retention capacity such as those at CT and FL generally have higher N values. The high-gradient upland watershed (CW) with a deep soil ($\approx$6 m) has a surprisingly low N value, which suggests that soil depth also influences N. The lowest N value was found at the ZJ1 site, which has well drained sandy soils. These results suggest that land topographic gradient is not the only factor that determines the N value and that soil depth and hydraulic properties may also play an important role in determining this land characteristic parameter. Sites with well-drained sandy soils, such as ZJ2, may have lower N values than those sites with deep soils (CW) or poorly drained wetlands (CT and FL). Using the same datasets, values for the parameter m, in Fu's formula (Equation 1), were also obtained by optimization (Table 2). Fu's model was not designed for modeling daily and monthly ET, so we contrasted our model's performance to Fu's at the annual scale for the three U.S. sites only.

Model Calibration and Validation

Model Performance at the Daily Scale. Daily E for each research site was computed by Equation 6 as a function of modeled daily s determined by Equation 8 or direct field measurements, daily E_0 calculated from Equation 9, measured daily h, and the N values derived in Table 2. Of the five sites, only ZJ1 and ZJ2 had measured daily ET data for direct day to day model verification. For the other three sites, annual actual ET in a calendar year was estimated as the difference between measured precipitation and runoff at the watershed scale. This estimation may have inherent errors when the change of water storage is large. This would occur in extremely wet or dry years, and was especially common for the two wetland sites (CT and FL), where the inter-annual ground water table level variability was large (Sun *et al.*, 1998; Amatya and Skaggs, 2001). Another potential factor was runoff measurement errors during extreme flow events that resulted in submergence of weirs, such as events caused by hurricanes at the poorly drained CT site (Amatya and Skaggs, 2001). As a result, the years 1988-1989, 1998-1999 at the CT site, year 1984 for the FL site, and year 1989 (a record wet year) at the CW site were eliminated from the databases, primarily because the watershed water balance equation was invalid when applied at

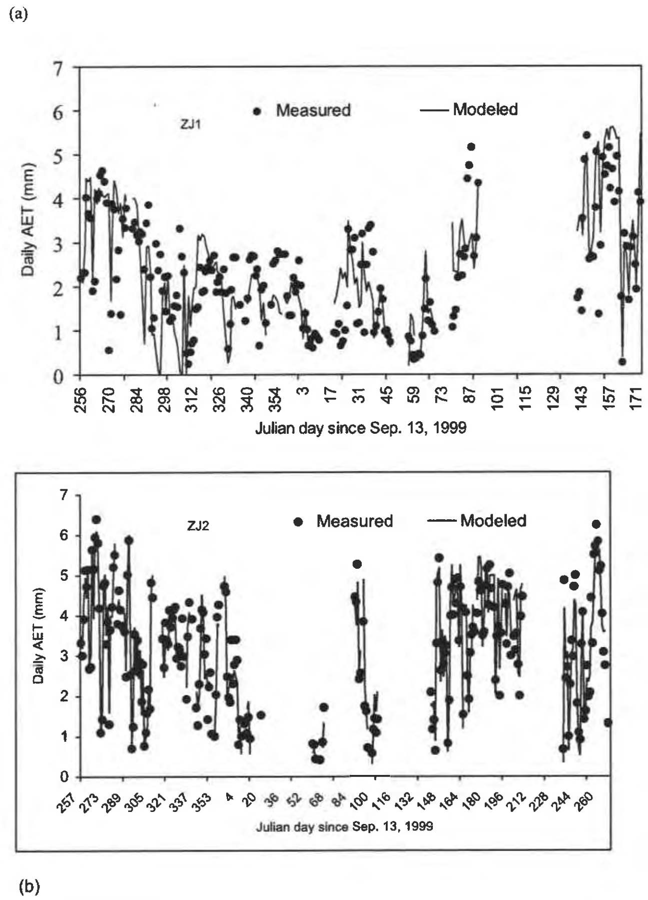

FIGURE 2. Modeled and Measured Daily Actual Evapotranspiration, E, in Zhanjiang 1 (ZJ1) and Zhanjiang 2 (ZJ2) Sites During 13-Sept-1999 to 23-Sept-2000. Some data were missing as a result of lighting-induced equipment failures.

the calendar year scale. For example, the CW watershed received a 500 mm surplus of rainfall during the second half of 1989 and this resulted in a large ground water recharge. Consideration only of the watershed water balance would lead to overestimation of actual ET in such a case. For the three U.S. sites, we evaluated model performance at the three temporal scales (daily, monthly, and yearly) by comparing annual measured ET (precipitation-runoff) to accumulated annual ET calculated at the daily, monthly, and annual scale by the same Equation 6.

It appears that the ET model performed well in capturing the dynamics of daily ET for sites of ZJ1 and ZJ2 (Figure 2). Linear regression analysis suggests that measured and modeled ET are highly correlated for ZJ1 and ZJ2 with an adjusted R^2 values of 0.509 ($p < 0.001$) for ZJ1 and 0.901 ($p < 0.001$) for ZJ2 (Figure 3). The model performed better for site ZJ2 than for site ZJ1, possibly because the plantations at ZJ1 were not fully stocked (which implies

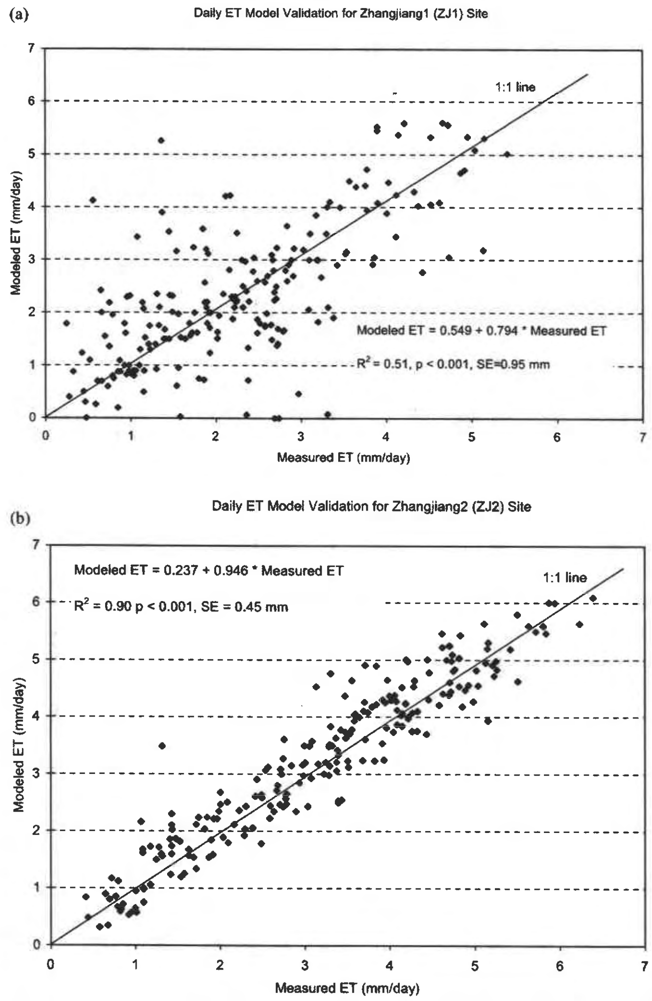

FIGURE 3. Modeled Daily Actual Evapotranspiration Is Closely Correlated to Field Measurements at the Zhangjiang1 (ZJ1) (a) and Zhangjiang2 (ZJ2) (b) Sites.

lower leaf area index and ET) and possibly because measurement errors for some of the ET components may have been larger for ZJ1 (Zhou *et al.*, 2004).

Model Performance at the Monthly and Annual Scales. Because there was only one year of data for ZJ1 and ZJ2 sites, we excluded these two sites from model comparison analysis and focused on the three U.S. sites that had multiple years of data. Unfortunately, as mentioned earlier, these three U.S. sites did not have daily or monthly actual ET data, so we have to use annual totals of simulated and measured for evaluating model performance.

The model developed by this study performed well for all scales when comparing against measured annual totals (Figure 4). Across sites, the FL site had the highest variability for both measured and simulated annual ET, which suggests that the model captured the inter-annual variability reasonably well. This can be seen more clearly in Figure 5, which shows statistically significant correlations ($p < 0.01$) between annual measured ET and annual totals simulated by the same formula with inputs at three different temporal scales. When the sites were pooled together, the adjusted R^2 values were 0.40, 0.44, 0.63, and 0.48 for the daily, monthly, and annual models of Equation 6, and for Fu's model, respectively. When calibration data were excluded for the CT and FL sites, the adjusted R^2 values lowered slightly to 0.49 and 0.58 for the monthly and annual models (Equation 6), but were reduced to 0.16 (not significant at the 0.05 level) for Fu's model. Daily scale model validation-only analysis was not performed due to lack of sufficient data. Averaged cross-site annual prediction errors were found to be within 7% across methods and models. Equation 6 predicted slightly better with an increase in the temporal scale as showed by the averaged absolute prediction errors of 7% (−11.8 to 0.0%), 6% (−15.8 to 1%), and 5% (−13.8 to 0.0%) for daily, monthly, and annual models, respectively. At the individual site level, among the three sites it appears that our model performed best at the FL site, which had a wider ET range (768-1,158 mm/year) than had the other two (Figure 6). The R^2 values for this site were 0.45, 0.56, 0.71, and 0.74 for the daily, monthly, and annual models of Equation 6, and for the Fu model (annual model only), respectively. When calibration data were excluded, R^2 was reduced somewhat to 0.58 and 0.60 for the monthly and annual models, and 0.57 for Fu's model (annual model only). In contrast, the data for CW and CT were much more scattered and the range of ET values was much narrower, and consequently both Fu's model and Equation 6 had relatively higher simulation errors (Figures 6a and b) and the correlations between simulated and modeled were not significant at the 0.05 level, not as good those for the FL site.

Validation of ET models at daily to monthly temporal scales was rarely conducted in most hydrologic studies due to lack of measurements at such scales. Our study certainly suffered from lack of long-term high resolution ET data to verify the model performance at multiple scales (daily and monthly) as well. In addition, we found that the annual ET calculated by the watershed balance equation as the residual of precipitation and streamflow could have large errors for certain years when the change of soil water storage was large. However, the long-term hydrologic databases derived from an array of forested watersheds across a climatic and topographic gradient in this study provided a unique opportunity to calibrate and test the behaviors of the new ET model. In fact,

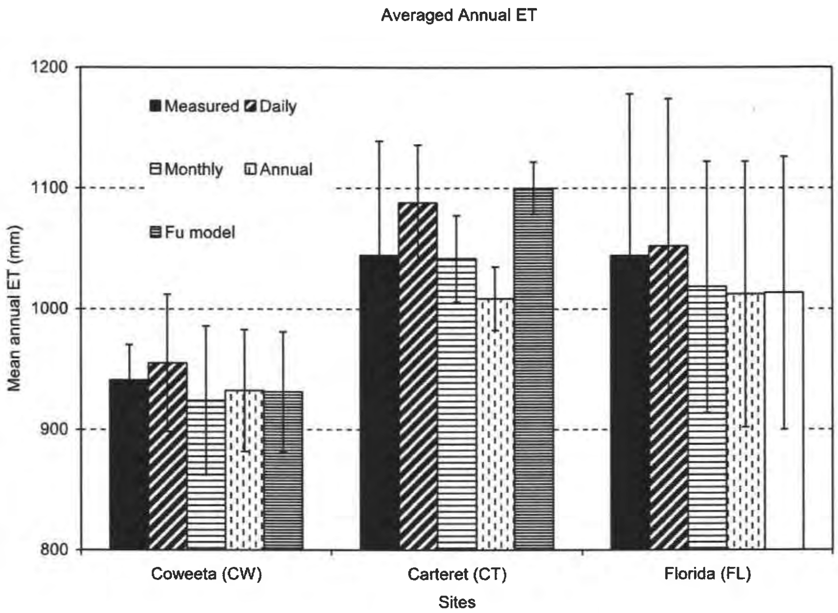

FIGURE 4. Summary of Across-Site Comparison of Annual ET Measurements and Simulated Annual ET Using Daily, Monthly, and Annual Time Step Methods at Three Forested Watersheds in Southern U.S. Error bars indicate variability (1 SD) of annual total ET.

during the model validation phase of the present study, we have identified several years in which streamflow may have measurement errors may have occurred, and these possible errors require further investigation. The relatively low simulation errors (<15.8%) across multiple scales suggest that the model developed in this study has promise to capture the multiple temporal dynamics of ET across a wide range of watersheds.

Performance Comparison to Fu's Model. As mentioned earlier, model comparisons between our model and Fu's model can be carried out only at the annual scale as Fu's model was not designed for finer temporal scales. In general, the ET model developed in this study performed better than Fu's model at the annual temporal scale, notably for the CT site where ET was overestimated by the later model. Modeling errors of Fu's model were most pronounced at the CT site (Figures 4 and 6b). Fu's model tended to overestimate annual ET when it was less than 1,100 mm/year but overestimated it when it is above 1,100 mm/year. (Figure 5c). Across the three sites, the averaged absolute prediction error for annual ET by the Fu (1981) model was 7.6%, ranging from −12.7% to 0.0%.

At the annual temporal scale, the soil moisture term in Equation 6 would be annual precipitation, the same as the term, *r*, in Fu's model. So, the differences between our model and Fu's model lie on two aspects: (1) Fu's model uses a fixed empirical parameter, *m*, that is mostly related to land surface characteristics Fu (1996) (Equation 1) and (2) Our model uses averaged relative air humidity and a calibrated empirical parameter, *N*, that reflects soil and topographic controls on water balances. Better match between observed ET and predictions by our model (Figures 4 and 5c) suggests that adding relative humidity might be essential for predicting annual ET. The relative humidity for watersheds examined in this study located in the humid regions differs little among sites and time. The effect of humidity on ET predictions might be more important for other regions that have low humidity with high temporal and spatial variability.

CONCLUSIONS

Quantifying regional ecosystem ET at the daily to monthly temporal scales is often expensive and remains challenging in hydrology. Directly measuring ET at high temporal and spatial resolutions are rare, so models are indispensable in constructing ecosystem water balances and testing new hypotheses. Although measuring ET at the shorter temporal scales by using the eddy covariance and energy balance methods (Wilson *et al.*, 2001) across a global network has become possible, modeling is still the practical tool for estimating this variable and scaling up the point to watershed scale measurements to the regional scale. We used a dimensional analysis approach to develop a

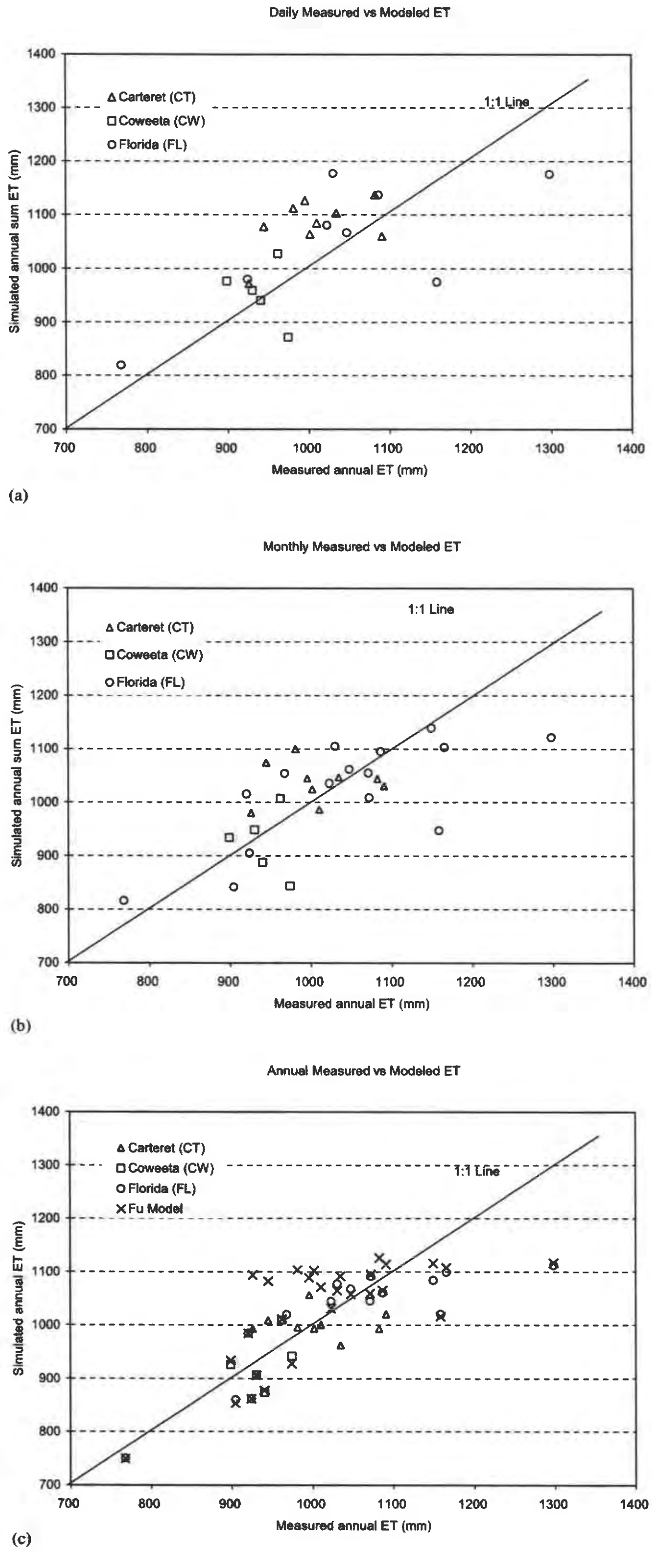

FIGURE 5. A Comparison of Measured Annual ET to (a) Annual Sum of Modeled Daily ET, (b) Annual Sum of Modeled Monthly ET, and (c) Modeled Annual ET, During Multiple Years at Three Forested Watersheds in the Southern U.S.

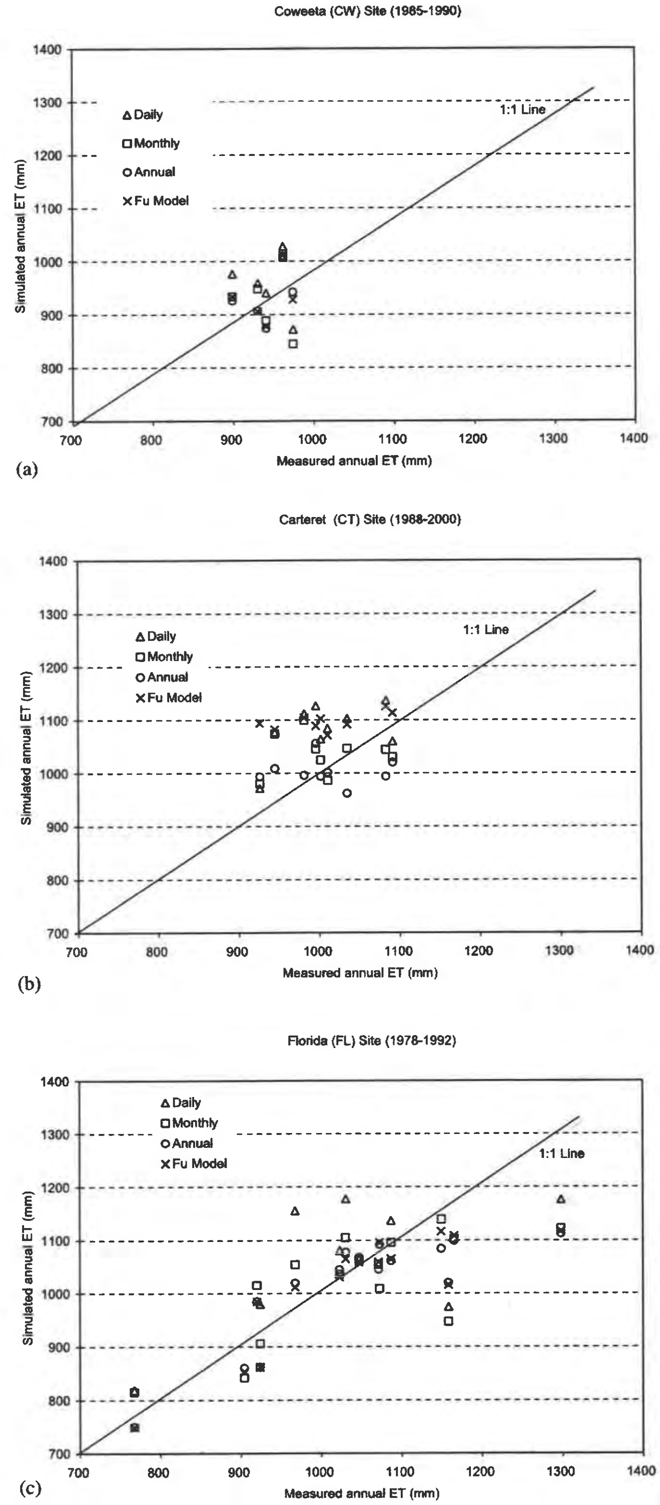

FIGURE 6. A Comparison of Measured Annual ET and Simulated Total Annual ET by Three Methods at (a) Coweeta (CW), (b) Carteret (CT), and (c) Florida (FL) Sites in the Southern U.S.

semi-empirical ET model that can estimate actual ET at multiple temporal scales. This work represents an extension of the ET model proposed originally by Fu (1981) and examined by Zhang *et al.* (2004). We found that Fu's model had larger prediction error for one wetland site (CT) at the annual temporal scale. This suggests adding the relative humidity variable is essential to improve model performance. Soil moisture and relative humidity are important variables for modeling ET at finer temporal scales. Depending on the scale of interest, our model requires different input variables that are readily available for most landscapes. This modeling exercise suggests that daily to monthly ET can be quantified with reasonable confidence by using commonly available information about meteorology (precipitation, relative humidity, and air temperature), soils (field capacity), and plants (rooting depth and canopy rainfall interception rate). This process-based approach allows improved understanding of the processes and factors that control the water loss from ecosystems. However, uncertainty remains regarding factors that control the empirical parameters (i.e., N) in the derived ET model. Because our study covered a limited variety of sites and surface conditions (e.g., all forested), more research is needed to extrapolate and generalize the mechanisms by which land surface characteristics control ET at different temporal scales and under different climatic regimes. More field watershed-scale or regional scale hydrologic data are needed so that factors affecting the key empirical parameters (i.e., N) in the ET model can be explored further. Models are needed to link surface features such as watershed forest leaf area to the ET model parameters for wide-range applications. In addition, model calibration is necessary when the model is applied to an area for which the N parameter has not been published. Prediction in such areas will involve derivation of N values using historical hydrologic data. Once the model is calibrated, it will be very useful for a wide range of model application purposes. Because the model we derived in this study is sensitive to land surface characteristics through canopy interception, water extraction from the plant rooting zone, and also PET, it may be useful for predicting hydrologic responses to changes in land use, land cover, and climate at multiple temporal scales. Linking this model with physiologically meaningful vegetation parameters that are readily derived remote sensing data can be powerful for understanding the hydrologic cycles at a larger scale (Donohue *et al.*, 2007).

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APPENDIX A: ANALYTICAL SOLUTIONS OF EVAPOTRANSPIRATION AT MULTIPLE TEMPORAL SCALES

Based on differential mathematical theorems, a necessary condition for Equation 4 in the main text to have a general solution is:

$$\frac{\partial^2 E}{\partial s \partial E_0} = \frac{\partial^2 E}{\partial E_0 \partial s}, \tag{A1}$$

where

$$\frac{\partial^2 E}{\partial s \partial E_0} = \frac{\partial f_1(x,h)}{\partial E_0} = \frac{\partial f_1}{\partial x}\cdot\frac{\partial x}{\partial E_0} = \frac{1}{s}(1-\varphi_1)\frac{\partial f_1}{\partial x}$$

and

$$\frac{\partial^2 E}{\partial E_0 \partial s} = \frac{\partial \varphi_1(y,h)}{\partial s} = \frac{\partial \varphi_1}{\partial y}\cdot\frac{\partial y}{\partial s} = \frac{1}{E_0}(1-f_1)\frac{\partial \varphi_1}{\partial y}$$

Therefore,

$$\frac{1}{s}(1-\varphi_1)\frac{\partial f_1}{\partial x} = \frac{1}{E_0}(1-f_1)\frac{\partial \varphi_1}{\partial y} \tag{A2}$$

As

$$\frac{s}{E_0} = \frac{E_0 + s - E}{E_0}\cdot\frac{s}{E_0 + s - E} = \frac{1+y}{1+x} \tag{A3}$$

Equation A2 becomes

$$\frac{(1+x)\frac{\partial f_1}{\partial x}}{1-f_1} = \frac{(1+y)\frac{\partial \varphi_1}{\partial y}}{1-\varphi_1} \tag{A4}$$

The left side of Equation A4 is a function of only x and h, and the right is a function of only y and h. A necessary condition for both sides to be independent of x and y is that they must equal to a term including only the variable h:

$$\frac{(1+x)\frac{\partial f_1}{\partial x}}{1-f_1} = \frac{(1+y)\frac{\partial \varphi_1}{\partial y}}{1-\varphi_1} = N[k(h)+1] \tag{A5}$$

The right side of Equation A5 includes a constant N that is dependent of the three variables x, y, h, and a term $k(h)$ which is a function of h only. The integral constant on the right hand side of Equation A5 took an arbitrary form, but it reflects the influences of both land surface characteristics (N) and atmospheric humidity (h). The form of k(h) is described later in this paper. Thus, Equation A5 can be written as

$$\frac{(1+x)\frac{\partial f_1}{\partial x}}{1-f_1} = N[k(h)+1] \tag{A6}$$

$$\frac{(1+y)\frac{\partial \varphi_1}{\partial y}}{1-\varphi_1} = N[k(h)+1] \tag{A7}$$

Integrating Equation A6 and A7 results in

$$f_1(x,h) = 1-(1+x)^{-N[k(h)+1]} \tag{A8}$$

and

$$\varphi_1(y,h) = 1-(1+y)^{-N[k(h)+1]} \tag{A9}$$

A common solution for E can be obtained from any of the above two equations. Here, solve Equation A8 as one example. The variable E_0 can be seen as an integral constant. Thus, Equation A8 can be written as an ordinary differential equation:

$$\frac{dE}{ds} = 1-\left(\frac{s}{E_0+s-E}\right)^{N[k(h)+1]} \tag{A10}$$

Assuming $U = \frac{E_0+s-E}{s}$, then Equation A10 can be written as

$$\frac{d(U^{N[k(h)+1]+1}-1)}{U^{N[k(h)+1]+1}-1} = -\{N[k(h)+1]+1\}\frac{ds}{s} \tag{A11}$$

Because $E \le E_0$, therefore, $U \ge 1$. Integrating Equation A11 gives

$$U = \frac{1}{s}(K+s^{N[k(h)+1]+1})^{\frac{1}{N[k(h)+1]+1}} \tag{A12}$$

K is an integral constant, a function of E. Therefore,

$$E = E_0+s-(K+s^{N[k(h)+1]+1})^{\frac{1}{N[k(h)+1]+1}} \tag{A13}$$

K can be obtained by calculating $\frac{\partial E}{\partial E_0}$ with Equation A13 and combining $\frac{\partial E}{\partial E_0}$ with Equation A8:

$$K = E_0^{N[k(h)+1]+1}+b \tag{A14}$$

where b is an integral constant.

Therefore, Equation A14 becomes

$$E = E_0+s-(E_0^{N[k(h)+1]+1}+s^{N[k(h)+1]+1}+b)^{\frac{1}{N[k(h)+1]+1}} \tag{A15}$$

Based on the boundary condition that when $s=0$, $E=0$, the integral constant b in Equation A15 should be zero. Finally, the following ET model is derived.

$$E = E_0\left\{1+\frac{s}{E_0}-\left[1+\left(\frac{s}{E_0}\right)^{N[k(h)+1]+1}\right]^{\frac{1}{N[k(h)+1]+1}}\right\} \tag{A16}$$