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Optimizing the Sequence of Diameter Distributions and Selection Harvests for Uneven-Aged Stand Management

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ABSTRACT. The determination of an optimal sequence of diameter distributions and selection harvests for uneven-aged stand management is formulated as a discrete-time optimal-control problem with bounded control variables and free-terminal point. An efficient programming technique utilizing gradients provides solutions that are stable and interpretable on the basis of economic principles. Methods and results are demonstrated using a whole-stand/diameter-class simulator developed for northern hardwoods stands in Wisconsin. Examples in which the objective is present net worth maximization over a 150-year planning horizon with a 5-year cutting cycle suggest two types of optimal equilibrium stand structures: a downward-sloping diameter distribution if large value premiums are assigned to the largest diameter classes and a truncated diameter distribution if premiums for larger trees are gradual or absent. Transition strategies vary in length and harvest pattern depending on the stumpage value function used. It is emphasized that equilibrium management regimes developed with static analysis are not optimal when used as starting conditions in dynamic formulations. FOREST SCI. 31:451–462.

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THE INCORPORATION of even-aged stand growth and yield simulators into dynamic programming frameworks has improved the economic analysis of silvicultural investment decisions in stand-level even-aged management (Brodie and Kao 1979, Martin and Ek 1981, Riitters and others 1982). Uneven-aged stand simulators cannot be readily incorporated into dynamic programming frameworks because the large number of decision variables involved would require long computation time and massive computer storage. We introduce a more efficient optimization technique based on optimal control theory and demonstrate its use in finding the optimal sequence of diameter distributions and selection harvests for an uneven-aged stand.

The decision variables facing a forest manager interested in applying uneven-aged management at the stand level are (1) the cutting cycle length, and (2) the diameter distribution and species composition after each selection harvest during a given planning horizon. We use the optimization technique to solve the diameter distribution problem, and it can be expanded to analyze both cutting cycle length and species composition problems.

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Hann and Bare (1979) have identified two separate but related issues in determining the optimal sequence of diameter distributions: (1) equilibrium diameter distribution, and (2) conversion strategy and conversion period length. Researchers have developed static-optimization techniques which focus on the first issue. Adams and Ek (1974) demonstrated a two-stage technique for determining the optimal equilibrium stand structure. In the first stage, they used a gradient projection method to determine the residual diameter distribution that maximized stand value growth for a given cutting cycle. In the second stage, having solved this nonlinear program for several alternative residual basal area levels, they chose the stand structure that satisfied a marginal value growth percent criterion (Duerr and Bond 1952). Adams (1976) subsequently distinguished between the use of value and basal area measures of growing stock as the appropriate constraints for determining investment-efficient diameter distributions. Martin (1982) derived investment-efficient diameter distributions using the Weibull distribution function. His problem formulation reduced the decision variables to the Weibull function parameters and the total number of trees. Solutions were obtained with a gradient projection method.

Adams and Ek (1974) addressed the second issue by formulating and solving a transition strategy problem with the constraint of achieving an investment-efficient diameter distribution after a specified number of periods. They used a gradient projection method, but the number of decision variables and constraints exceeded algorithm capacity for problems with more than three transition harvests.

The optimization technique presented here differs from these approaches because it considers the transition and equilibrium problems jointly. Interpretation of solutions generates new insights into the economics of uneven-age management.

This paper is presented in four sections. In the first we formulate the problem of determining the optimal sequence of diameter distributions and selection harvests for an existing stand without the constraints of period-to-period sustainability or specified equilibrium endpoint. We trace the development of this formulation through the forestry literature.

The second section shows how to solve this problem with a gradient-based nonlinear programming technique called the method of steepest descent.

In the third section we demonstrate the solution algorithm with the use of a whole-stand/diameter-class simulator for mixed-species northern hardwood stands developed by Ek (1974) and modified by Adams and Ek (1974). We use the algorithm to determine stand-specific management regimes for three different stumpage value functions. Finally, we examine the sensitivity of solutions to changes in the termination criterion and starting conditions for the algorithm.

We conclude with a discussion of optimal equilibrium stand structures and methods for expanding the problem statement and solution algorithm to consider cutting cycle and species composition problems.

Problem Statement

The determination of the optimal sequence of diameter distributions and selection harvests for an uneven-aged stand can be stated in a discrete-time optimal-control formulation with free terminal point and bounded control variables:

$$\max_{\{Y_{ij}\}} z = \sum_{i=0}^{N-1} \sum_{j=1}^{M} P_{ij} X_{ij} Y_{ij} + \sum_{j=1}^{M} P_{Nj} X_{Nj} \tag{1}$$

subject to:

$$X_{i+1j} = X_{ij} - X_{ij}Y_{ij} + f_{ij}[X_{i1}(1 - Y_{i1}), \ldots, X_{iM}(1 - Y_{iM})] \qquad \begin{array}{l} i = 0, \ldots, N-1 \\ j = 1, \ldots, M \end{array} \tag{2}$$

$$0.0 \leq Y_{ij} \leq 1.0 \qquad \begin{array}{l} i = 0, \ldots, N-1 \\ j = 1, \ldots, M \end{array} \tag{3}$$

$$X_{0j} \qquad j = 1, \ldots, M \text{ (given)} \tag{4}$$

where

X_{ij} = a state variable representing number of trees per acre in diameter class j at the beginning of period i before cut,
Y_{ij} = a control variable representing the percentage of the number of trees per acre in diameter class j which are cut at the beginning of period i,
P_{ij} = the discounted net price per tree in diameter class j at the beginning of period i,
f_{ij} = a continuous nonlinear function with continuous partial derivatives representing the change in number of trees per acre in diameter class j during period i,
N = the number of periods in the planning horizon,
M = the number of diameter classes.

The objective function, equation (1), is formulated to seek control-variable values that maximize the present value of net returns (PNW), assuming that all remaining trees are harvested at the end of the planning horizon. No restrictions are placed on the form of the terminal diameter distribution. For a planning horizon greater than 150 years, the discounted value of the terminal diameter distribution is so small that it has no effect on the determination of the control variable values in earlier periods. In these cases, the value of the optimal management regime obtained from this formulation can be viewed as the present value of future income that could be obtained by managing the existing land and timber with an uneven-aged system indefinitely.

Constraints (2) represent the stand growth dynamics. The number of trees per acre in diameter class j at the beginning of period $i + 1$ equals the number of trees before cut in diameter class j period i less the amount cut plus the change in number of trees (f_{ij}) which is a function of the residual diameter distribution. The stand growth dynamics are used to compute the state variables for a given control variable sequence and a given initial stand diameter distribution. Constraints (3) ensure that control variables are fractions between 0.0 and 1.0. The initial diameter distribution is given by (4).

Optimal control theory has been applied to stand-level management problems, but results have either not been presented or have been unstable and difficult to interpret. Adams and Ek (1974) developed a control theoretic formulation of a conversion strategy problem in which the control variables were the number of trees cut per diameter class in each period. Because both state and control variables were constrained to be non-negative, the development and execution of a solution procedure was impractical. When the control variables were changed to the percentage of trees cut per diameter class in each period and bounded between 0.0 and 1.0 (Adams and Ek 1975), no constraints were needed on state variables. No solution procedure was presented, however.

Bullard (1983) adapted the control theoretic formulation of Adams and Ek (1974) to mixed-species even-aged hardwood stands. He demonstrated that a problem with eight decision variables and two periods could not be solved with a nonlinear programming technique.

Rapera (1980) formulated the optimal control problem defined by equations (1) to (4) and tested three solution procedures. He used the stand growth simulator given by Adams and Ek (1974) to model diameter class growth dynamics, using a stumpage value function that assigned premiums to large diameter trees. He found that the two procedures which used the gradient-based control-vector-iteration method (McDonough and Park 1975) converged to solutions within specified tolerances. However, values of optimal solutions varied by more than 25 percent depending on different initial values given to control variables. Harvest patterns ranged from periodic removal of all trees greater than 6 inches to the maintenance of a downward-sloping residual diameter distribution with a maximum tree size of 20 inches. Rapera concluded that many locally optimal solutions existed.

The solution procedure that we present is a multidimensional version of the method of steepest descent used by Dreyfus and Law (1977, p 102) to numerically solve single-variable discrete-time optimal-control problems. The method of steepest descent is a gradient-based method which finds solutions by setting first derivatives equal to zero. Since these conditions are necessary for any relative maximum or for any other types of stationary solutions, there is no way of distinguishing the absolute maximum. Thus, we never know if the global optimal solution has been found. Nevertheless, the values and harvest patterns for solutions obtained for different initial values of control variables do not vary widely. As a result, we can make stronger conclusions about optimal uneven-aged management regimes.

Gradient Method of Numerical Solution

The solution method seeks to improve the objective function value (z, equation (1)) by successive approximations of the control variables. Starting with an initial guess of the control variables $\{Y_{ij}^0\}$, $i = 0, \ldots, N-1$ and $j = 1, \ldots, M$, we seek formulas for finding a better sequence $\{Y_{ij}^1\}$, $i = 0, \ldots, N-1$ and $j = 1, \ldots, M$.

We define $T_i(X_{i1}; \ldots, X_{iM}, Y_{i1}^0, \ldots, Y_{iM}^0)$ as the value of equation (1) from period i through period N, starting in period i with state $X_{i1}, \ldots, X_{iM}$ and using the guessed control sequence. The function T_i satisfies the recurrence relation:

$$\begin{aligned} T_i = \sum_{j=1}^{M} P_{ij}X_{ij}Y_{ij}^0 + T_{i+1}(X_{i1} - X_{i1}Y_{i1}^0 + f_{i1}, \ldots, X_{iM} \\ - X_{iM}Y_{iM}^0 + f_{iM}; Y_{i+1\,1}^0, \ldots, Y_{i+1\,M}^0) \qquad (5) \\ i = 0, \ldots, N-1 \end{aligned}$$

and the boundary condition:

$$T_N = \sum_{j=1}^{M} P_{Nj}X_{Nj}. \qquad (6)$$

We use the stand growth dynamics to compute the state sequence $\{X_{ij}^0\}$, $i = 1, \ldots, N$ and $j = 1, \ldots, M$, determined by $\{Y_{ij}^0\}$ and $\{X_{0j}\}$. To improve the value of (z) we need to know how it would behave if we changed Y_{ij}^0 but kept all other control variables fixed. Partial differentiation of (5) with respect to Y_{ij}^0 gives the answer:

$$\left.\frac{\partial T_i}{\partial Y_{ij}^0}\right|_0 = \left.P_{ij}X_{ij}\right|_0 - \left.X_{ij}\frac{\partial T_{i+1}}{\partial X_{i+1j}}\right|_0 + \sum_{k=1}^{M} \left.\frac{\partial T_{i+1}}{\partial X_{i+1k}}\right|_0 \left.\frac{\partial f_{ik}}{\partial Y_{ij}^0}\right|_0 \qquad (7)$$

where $|_0$ means expressions are evaluated in terms of the particular sequence $\{Y_{ij}^0, X_{ij}^0\}$. We can compute (7) if we know $\partial T_{i+1}/\partial X_{i+1k}$, $k = 1, \ldots, M$. Hence we take the partial derivative of (5) with respect to X_{ij}:

$$\left.\frac{\partial T_i}{\partial X_{ij}}\right|_0 = P_{ij}Y_{ij}^0 + (1 - Y_{ij}^0)\left.\frac{\partial T_{i+1}}{\partial X_{i+1j}}\right|_0 + \sum_{k=1}^{M} \left.\frac{\partial T_{i+1}}{\partial X_{i+1k}}\right|_0 \left.\frac{\partial f_{ik}}{\partial X_{ij}}\right|_0. \tag{8}$$

Partial differentiation of the boundary condition (6) yields:

$$\left.\frac{\partial T_N}{\partial X_{Nj}}\right|_0 = P_{Nj}. \tag{9}$$

This allows us to compute partial derivatives (8) for all periods i working backward from period N. Equations (7), (8), and (9) allow the computation of $\partial T_i/\partial Y_{ij}^0$, $j = 1, \ldots, M$, at each period of the guessed control sequence. We use this information to compute the change in each control variable δY_{ij}^0 so that

$$Y_{ij}^1 = Y_{ij}^0 + \delta Y_{ij}^0 \qquad i = 0, \ldots, N-1 \quad \text{and} \quad j = 1, \ldots, M. \tag{10}$$

Suppose we let the change in each control variable δY_{ij}^0 be proportional to the size of the partial derivative $\partial T_i/\partial Y_{ij}^0$:

$$\delta Y_{ij}^0 = p\left.\frac{\partial T_i}{\partial Y_{ij}^0}\right|_0 \qquad i = 0, \ldots, N-1 \quad \text{and} \quad j = 1, \ldots, M. \tag{11}$$

The resulting change in the objective function value can be approximated:

$$\delta z \cong \sum_{i=0}^{N-1} \sum_{j=1}^{M} \left.\frac{\partial T_i}{\partial Y_{ij}^0}\right|_0 \delta Y_{ij}^0 = p \sum_{i=0}^{N-1} \sum_{j=1}^{M} \left(\left.\frac{\partial T_i}{\partial Y_{ij}^0}\right|_0\right)^2. \tag{12}$$

For a specified improvement $\overline{\delta z}$ we can use (12) to compute p:

$$p = \frac{\overline{\delta z}}{\sum_{i=0}^{N-1} \sum_{j=1}^{M} \left(\left.\frac{\partial T_i}{\partial Y_{ij}^0}\right|_0\right)^2} \tag{13}$$

and use equations (11) and (10) to compute the new decision sequence $\{Y_{ij}^1\}$. Whenever the new values of the control variables are outside the bounds given by equations (3), we set their values equal to the nearest bound. The new decision sequence is used in dynamical equations (2) to compute the state sequence $\{X_{ij}^1\}$, and if $\{Y_{ij}^1\}$ and $\{X_{ij}^1\}$ improve the value of z by more than a predetermined limit, they are used as the starting point for the next iteration. If they do not improve the value of z, we replace $\overline{\delta z}$ by $\overline{\delta z}/2$ and compute a new solution, repeating the process until z improves by the desired amount. The process terminates when we seek an increase $\overline{\delta z}$ that is smaller than a predetermined limit and even this does not give the desired improvement in the value of z. No improvement in z is possible when all of the following conditions hold:

$$\partial T_i/\partial Y_{ij} = 0.0 \quad \text{for} \quad 0.0 < Y_{ij} < 1.0 \tag{14}$$

$$\partial T_i/\partial Y_{ij} \leq 0.0 \quad \text{for} \quad Y_{ij} = 0.0 \tag{15}$$

$$\partial T_i/\partial Y_{ij} \geq 0.0 \quad \text{for} \quad Y_{ij} = 1.0. \tag{16}$$

A control sequence $\{Y_{ij}\}$ which satisfies these conditions also satifies Kuhn-Tucker necessary conditions for constrained optimization, which are satisfied by any relative minima, maxima, or other stationary solution. Thus, we must compare

TABLE 1. Initial diameter distribution, volume, and stumpage value functions for a site index 60 northern hardwood stand.

Function	Diameter class midpoint (inches)							
	6	8	10	12	14	16	18	20
Initial diameter distribution (trees/acre)	69.9	49.3	37.5	29.7	24.1	10.7	2.1	0.3
Volume (ft^3/tree)	3.9	10.1	17.9	27.4	38.4	51.0	65.1	80.8
Stumpage value, regime 1 (\$/ft^3)	0.01	0.01	0.01	0.01	0.025	0.05	0.08	0.10
Stumpage value, regime 2 (\$/ft^3)	.03	.04	.05	.06	.07	.08	.09	.10
Stumpage value, regime 3 (\$/ft^3)	.05	.05	.05	.05	.05	.05	.05	.05

the values of solutions that are obtained from several different starting guesses for the control sequence to make an estimate of the globally optimal solution.

Demonstrating the Gradient Method

Stand Growth Projection.—We employ a whole-stand/diameter-class simulator for northern hardwoods reported by Ek (1974) and modified by Adams and Ek (1974). The simulator was incorporated into static-optimization techniques by Adams and Ek (1974) and Adams (1976). The simulator computes f_{ij}, the change in the number of trees per acre in a 2-inch diameter class j during a 5-year growth period i as a function of the residual stand structure at the start of the growth period. This change is equivalent to upgrowth from diameter class $j - 1$ less upgrowth into diameter class $j + 1$ less mortality from diameter class j. Models for diameter class upgrowth and mortality are a function of site index, diameter class midpoint, number of trees in the class, and total stand basal area and number of trees. A separate ingrowth model is used to predict upgrowth into the smallest diameter class (6 inches).

The largest diameter class is 20 inches. Due to limits on the growth data used to construct the simulator, Adams and Ek (1974) and Adams (1976) assumed that only trees 18 inches in diameter and smaller were retained in the residual stand. Trees were allowed to grow up to 20 inches during a 5-year growth period, but were cut at the beginning of the next period. To make our results comparable to the solutions given by Adams (1976), we introduce the same constraint by setting the value of the control variable for the 20-inch diameter class equal to 1.0 in each period. The solution procedure is otherwise unaffected.

Optimal Management Regimes.—To demonstrate the gradient method we develop three management regimes for a site index 60 mixed hardwood stand. These are developed to maximize present value of net returns (PNW) from harvests taken in 5-year intervals over a 150-year planning horizon with a real interest rate of 4 percent and with three different functional relationships between tree diameter and stumpage value: (1) a sawlog objective where price per cubic foot is constant to 12 inches and increases rapidly to 20 inches, (2) a cordwood and sawlog objective where price per cubic foot increases at a constant rate with increasing tree size, and (3) no market premium for larger trees (Table 1). The initial diameter distribution is the profit-maximizing pre-harvest equilibrium diameter distribution developed for a sawlog objective and corresponding stumpage value function with a 4 percent real interest rate presented by Adams (1976). Net cubic foot volume per tree is from the cordwood volume table given by Adams and Ek (1974).

Management regime 1, developed with a sawlog objective, can be divided into

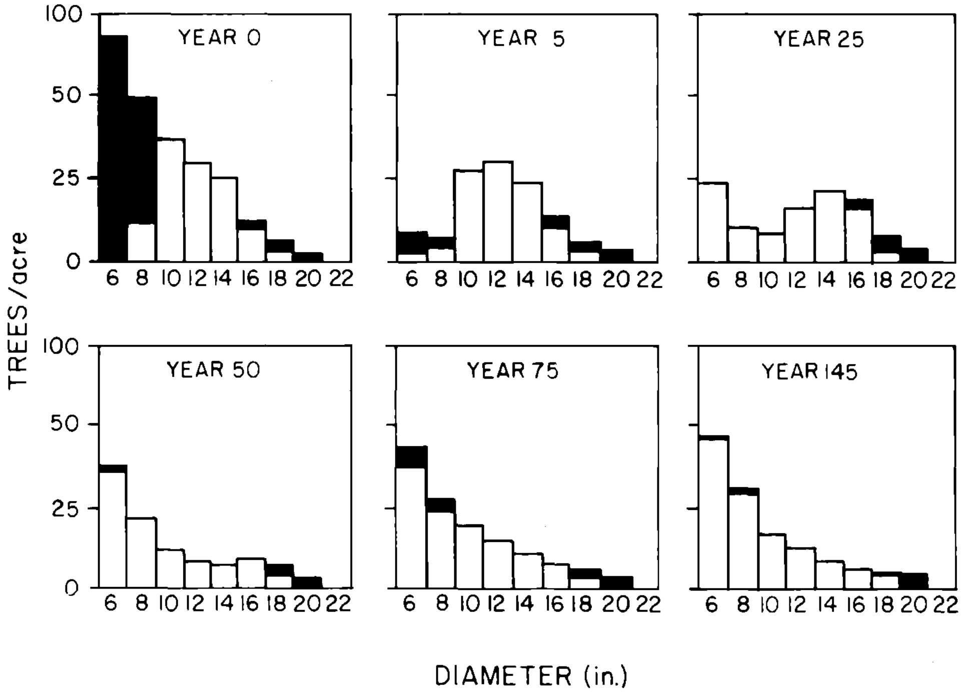

FIGURE 1. Residual diameter distributions at selected points of management regime 1. Shaded regions show number of trees cut.

two 75-year blocks on the basis of harvesting patterns and resulting sequences of residual diameter distributions (Table 2). All 6-inch and 77 percent of the 8-inch trees are cut at the start. These initial heavy cuts in the smallest diameter classes accelerate the growth of trees in the 10-inch and larger diameter classes. These trees are cut in future periods when they reach the three most valuable diameter classes. A downward-sloping diameter distribution is present at the end of 75 years due to natural regeneration (modeled by the ingrowth function) and low levels of cut in the smallest diameter classes. The sequence of residual diameter distributions shows the management of a pulse over the 75-year period (Fig. 1).

During the second 75 years a downward-sloping diameter distribution is maintained by natural ingrowth, mortality, and cuts taken from the 20-inch diameter class. Due to the high value of 20-inch trees, the values of cuts stay between $24 and $32/acre even though the total number of trees harvested in each period is low.

Management regime 2, where price per cubic foot increases at a constant rate with increasing tree size, removes at the first harvest all trees in diameter classes greater than 6 inches (Table 3). Subsequent harvests take all 10-inch trees and 65–95 percent of the 8-inch trees, leaving an increasing number of trees in the 6-inch diameter class and a small number of trees in the 8-inch diameter class.

A similar harvest pattern exists for management regime 3—no market premium for large diameter trees—which specifies that all trees in the 8-inch or greater diameter classes are cut at the start (Table 4). Upgrowth into the 8-inch diameter class is cut in each subsequent period. The residual diameter distributions include increasing numbers of trees in the 6-inch diameter class and no trees in the 8-inch or larger diameter classes.

Imposition of the maximum-tree-size constraint does not affect the solutions

TABLE 2. *A portion of management regime 1 (PNW = $171.42/acre).*

	Diameter class midpoint (inches)								Totals		
Year	6	8	10	12	14	16	18	20	trees/acre	ft²/acre	$/acre
A. Cut trees/acre											
0	69.9	37.8	0.0	0.0	0.0	0.4	0.6	0.3	109.0	29.3	13.6
5	6.4	1.1	.0	.0	.0	2.9	3.5	.8	14.7	13.6	33.4
10	3.5	.0	.0	.0	.0	2.5	3.9	.9	10.8	13.2	34.4
15	.4	.0	.0	.0	.0	2.0	4.3	1.0	7.7	12.7	35.3
20	.0	.0	.0	.0	.0	.5	4.8	1.1	6.4	11.6	34.4
25	.0	.0	.0	.0	.0	.4	5.4	1.1	6.9	12.2	36.7
30	.0	.0	.0	.0	.0	.0	5.7	1.2	6.9	12.9	39.0
35	.0	.0	.0	.0	.0	.0	5.4	1.4	6.8	12.8	39.1
50	2.1	.0	.0	.0	.0	.0	2.9	2.1	7.1	10.3	31.8
75	8.0	.6	.0	.0	.0	.0	.7	2.6	11.9	8.6	24.9
100	4.8	1.0	.0	.0	.0	.0	.7	3.0	9.5	9.3	28.2
145	.3	.8	.0	.0	.0	.0	.1	3.5	4.7	8.3	29.2
B. Residual trees/acre											
0	0.0	11.5	37.5	29.7	24.1	10.3	1.5	0.0	114.6	90.5	75.2
5	3.4	7.3	28.1	29.7	24.1	11.4	1.5	.0	105.5	86.4	76.8
10	8.0	5.6	20.5	27.2	24.2	12.4	1.6	.0	99.5	82.4	78.6
15	14.1	5.8	14.7	23.3	23.1	13.7	1.8	.0	96.5	78.7	80.4
20	19.2	7.4	11.0	19.1	21.8	15.6	1.9	.0	96.0	76.0	83.4
25	23.1	9.7	9.1	15.2	19.4	16.8	2.1	.0	95.4	72.8	84.2
30	26.4	12.1	8.5	12.1	16.6	16.5	2.4	.0	94.6	68.8	81.9
35	29.4	14.5	8.9	10.0	13.8	15.4	2.8	.0	94.8	65.0	77.8
50	37.4	21.6	12.6	8.7	8.5	10.1	4.1	.0	103.0	59.3	65.7
75	25.7	22.8	18.1	13.1	9.4	7.3	4.7	.0	101.1	62.0	63.7
100	23.5	15.8	14.8	13.5	11.5	9.3	5.9	.0	94.3	64.7	75.9
145	39.5	21.4	13.2	10.0	8.6	7.7	6.6	.0	107.0	62.1	72.4

obtained for management regimes 2 and 3 because the maximum residual tree sizes are 8 and 6 inches, respectively. The impact of the constraint on regime 1 depends on the value per cubic foot assigned to trees greater than 20 inches. If this is the same as the value of a 20-inch tree, the constraint would have no effect, but if it increases at the same rate for trees with increasing diameter, a pulse-transition harvest pattern with a maximum tree size greater than 20 inches would be obtained.

Sensitivity Analysis.—There are two problems connected with the use of the method of steepest descent to solve these problems: (1) the gradient method converges at a very slow rate as a stationary point is approached, and (2) there is no assurance that a stationary point is a global optimum, because the production surface defined by the stand simulator is not convex (see Bullard (1983) for proof of nonconvexity). We therefore performed sensitivity analysis to evaluate these drawbacks.

Solutions obtained with the gradient method depend on the tolerance used to terminate the algorithm. For a smaller tolerance, the method terminates closer to a stationary point and execution times are longer.

We terminated the algorithm when an additional iteration improved the objective function value by less than $0.001. The range in values of $\overline{\delta z}$ examined was bounded by $128.0 and $0.1. Reducing the termination criterion to $0.0001 caused differences of less than one tree per acre per diameter class in residual diameter distributions for the first eight periods of all three management regimes,

TABLE 3. A portion of management regime 2 (PNW = $299.13/acre).

Year	Diameter class midpoint (inches) 6	8	10	12	14	16	18	20	Total trees/acre	ft²/acre	$/acre
A. Cut trees/acre											
0	0.0	49.3	37.5	29.7	24.1	10.7	2.1	0.3	153.7	106.0	225.6
5	.0	14.9	.0	.0	.0	.0	.0	.0	14.9	5.3	6.3
10	.0	21.7	1.7	.0	.0	.0	.0	.0	23.4	8.5	10.3
15	.0	27.6	3.6	.0	.0	.0	.0	.0	31.2	11.6	14.3
20	.0	32.9	4.7	.0	.0	.0	.0	.0	37.6	14.1	17.5
25	.0	37.2	5.1	.0	.0	.0	.0	.0	42.3	15.8	19.6
30	.0	40.7	4.9	.0	.0	.0	.0	.0	45.6	16.9	20.8
35	.0	43.4	4.5	.0	.0	.0	.0	.0	47.9	17.6	21.5
50	.0	48.9	3.0	.0	.0	.0	.0	.0	51.9	18.7	22.3
75	.0	53.7	1.4	.0	.0	.0	.0	.0	55.1	19.5	22.8
100	.0	55.8	.7	.0	.0	.0	.0	.0	56.5	19.9	23.0
145	.0	57.0	.3	.0	.0	.0	.0	.0	57.3	20.1	23.1
B. Residual trees/acre											
0	69.9	0.0	0.0	0.0	0.0	0.0	0.0	0.0	69.9	13.8	8.6
5	116.5	4.2	.0	.0	.0	.0	.0	.0	120.7	24.3	15.7
10	149.1	9.8	.0	.0	.0	.0	.0	.0	158.9	32.7	21.8
15	172.3	13.5	.0	.0	.0	.0	.0	.0	185.8	38.6	26.1
20	189.6	14.9	.0	.0	.0	.0	.0	.0	204.5	42.4	28.8
25	203.2	14.4	.0	.0	.0	.0	.0	.0	217.6	45.0	30.2
30	214.2	13.1	.0	.0	.0	.0	.0	.0	227.3	46.7	31.0
35	223.5	11.5	.0	.0	.0	.0	.0	.0	235.0	47.9	31.5
50	244.0	7.1	.0	.0	.0	.0	.0	.0	251.1	50.4	32.1
75	262.8	3.1	.0	.0	.0	.0	.0	.0	265.9	52.7	32.8
100	271.3	1.5	.0	.0	.0	.0	.0	.0	272.8	53.8	33.1
145	276.3	.7	.0	.0	.0	.0	.0	.0	277.0	54.5	33.4

but greater differences did occur in later periods. Objective function values improved by less than 1 percent.

Execution times for regimes 1, 2, and 3 were 72, 6, and 9 seconds, respectively, on a Control Data Corp. CYBER 73/16. Reducing the tolerance increased execution times by more than 100 percent.

We also analyzed a subset of solutions to each problem using different sets of starting values for the control variables. Two sets of starting values were the boundary conditions (0.0 and 0.99) and each remaining set utilized starting values randomly chosen between 0.0 and 0.99.

Management regime 1 was obtained with starting values set at 0.0. Solutions with different sets of starting values had differences of less than one tree per acre per diameter class in residual diameter distributions for the first eight periods. Markedly different diameter distributions were established and maintained in subsequent periods. The maximum difference between objective function values obtained with different starting values was less than 2 percent. Execution times varied from 200 to 400 seconds. Similar sensitivity results were obtained for management regimes 2 and 3.

Because of the stability of the solutions, we conclude that each of the three management regimes is very close to the globally optimal solution for the first eight periods. The long-term management regimes are dependent on starting values given to control variables because large discount factors make the marginal values of distant-period control variables smaller than the termination criterion,

TABLE 4. A portion of management regime 3 (PNW = $274.39/acre).

Year	Diameter class midpoint (inches) 6	8	10	12	14	16	18	20	Totals trees/acre	ft²/acre	$/acre
A. Cut trees/acre											
0	0.0	49.3	37.5	29.7	24.1	10.7	2.1	0.3	153.7	106.0	181.2
5	.0	19.1	.0	.0	.0	.0	.0	.0	19.1	6.8	10.0
10	.0	29.3	.0	.0	.0	.0	.0	.0	29.3	10.2	14.9
15	.0	36.2	.0	.0	.0	.0	.0	.0	36.2	12.6	18.5
20	.0	41.2	.0	.0	.0	.0	.0	.0	41.2	14.4	21.0
25	2.4	44.8	.0	.0	.0	.0	.0	.0	96.5	16.1	23.4
30	5.3	47.2	.0	.0	.0	.0	.0	.0	52.5	17.5	25.2
35	6.8	48.5	.0	.0	.0	.0	.0	.0	55.3	18.3	26.1
50	6.8	50.2	.0	.0	.0	.0	.0	.0	57.0	18.9	27.0
75	3.5	52.4	.0	.0	.0	.0	.0	.0	55.9	19.0	27.5
100	1.5	54.4	.0	.0	.0	.0	.0	.0	55.9	19.4	28.2
145	.1	56.5	.0	.0	.0	.0	.0	.0	56.6	19.9	29.1
B. Residual trees/acre											
0	69.9	0.0	0.0	0.0	0.0	0.0	0.0	0.0	69.9	13.8	14.1
5	116.5	.0	.0	.0	.0	.0	.0	.0	116.5	22.8	23.3
10	151.6	.0	.0	.0	.0	.0	.0	.0	151.6	29.7	30.3
15	178.2	.0	.0	.0	.0	.0	.0	.0	178.2	35.0	35.6
20	198.8	.0	.0	.0	.0	.0	.0	.0	198.8	39.0	39.8
25	212.5	.0	.0	.0	.0	.0	.0	.0	212.5	41.7	42.5
30	220.3	.0	.0	.0	.0	.0	.0	.0	220.3	43.3	44.1
35	225.0	.0	.0	.0	.0	.0	.0	.0	225.0	44.2	45.0
50	233.4	.0	.0	.0	.0	.0	.0	.0	233.4	45.8	46.7
75	247.7	.0	.0	.0	.0	.0	.0	.0	247.7	48.7	49.6
100	260.7	.0	.0	.0	.0	.0	.0	.0	260.7	51.3	52.3
145	273.4	.0	.0	.0	.0	.0	.0	.0	273.4	53.8	54.9

and the algorithm terminates before the values of these control variables reach a stationary point. The slow convergence rate for distant-period control variables shows that many near-optimal long-term management regimes exist but, due to the limits on execution time, prevented us from analyzing long-term stationary solutions.

Conclusions

Optimal Equilibrium Stand Structures.—Our solutions suggest two types of optimal equilibrium stand structures: (1) a downward-sloping diameter distribution if large value premiums are assigned to trees in larger diameter classes (management regime 1), and (2) a truncated diameter distribution if premiums for larger trees are gradual or absent (management regimes 2 and 3).

A comparison of management regime 1 with an investment-efficient management regime developed by Adams (1976) for the same stumpage value function and interest rate demonstrates that dynamically determined management regimes differ from equilibrium regimes determined with static-optimization techniques. Furthermore, the PNW of management regime 1 ($171.42/acre) is 5 percent greater than that of equilibrium harvests taken over the same planning horizon plus the liquidation value of the terminal stand. Thus, a stand-specific management regime developed with a dynamic optimization technique will not converge to the equilibrium determined using a static optimization technique.

The difference in the values of dynamically and statically determined management regimes calls into question the use of static analysis for developing longrun equilibrium management regimes (Adams and Ek 1974, Adams 1976, Martin 1982) and for determining the value of an uneven-aged stand (Chang 1981, Hall 1983). A detailed comparison of the problems solved by dynamic and static optimization techniques is beyond the scope of this paper. A comparison of these problems is necessary, however, so that managers and analysts can select the proper decision-making tools.

The truncated equilibrium diameter distributions suggested by management regimes 2 and 3 result from the shapes of the ingrowth and price functions. The number of trees growing into the 6-inch diameter class is greatest for truncated residual diameter distributions. When the stumpage value function assigns equal or gradually increasing values per cubic foot for trees of increasing diameter, an incentive is given to liquidating the growing stock greater than 6 inches and capturing high levels of periodic ingrowth.

Management regimes 1, 2, and 3 are sensitive to the forms of the growth equations and parameters used in the equations. In particular, the maintenance of a truncated diameter distribution is dependent on the predicted high level of periodic ingrowth. Thus, these solutions should be viewed with caution until the accuracy of the growth model projections is validated.

Expanding the Solution Algorithm and Problem Statement.—The gradient method converges to the globally optimal harvest in the first period and beyond, though it does not produce stable solutions for the long-term management regime. A procedure has been developed[1] which takes advantage of the stability of the first-period solution to develop long-term management regimes that are stable and independent of starting values given to control variables.

The problem statement, which we used to determine the optimal sequence of diameter distributions for a 5-year cutting cycle, can be expanded to consider both cutting cycle length and species composition problems. Sequences of diameter distributions with cutting cycles which are multiples of the simulator projection interval can be analyzed by setting control variables equal to zero in the periods between harvests. Sequences of diameter distributions for species classes can be developed by defining state and control variables and growth equations for species and diameter classes.

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Biological Processes and Soil Fertility

Edited by J. Tinsley and J. F. Darbyshire. 1984. Martinus Nijhoff/Dr. W. Junk, Publishers.

Reviewed by A. G. Wollum II, Department of Soil Science, North Carolina State University, Raleigh, NC 27695-7619

This book was first published as the proceedings of the meeting of Commissions III and IV of the International Society of Soil Science as volume 76 of Plant and Soil, except for the preface and introductory chapter. Because of space limitations, not all of the papers presented at the conference were included in this book. Some not included were reserved by the authors for publication by some other venue. However, abstracts of all papers presented by traditional oral means or those presented by posters are included in the Conference Transactions.

The book is divided into seven sections, each of which contains at least a keynote address as well as contributed papers. Some sections contain one or more introductory lectures. There are a total of 22 contributed papers included in the book. Subjects include topics on nutrient cycling (in several sections); measurements of microbial populations; interactions of organisms, organic matter, and soil management; and effects of noxious materials on biological processes in soil.

In my opinion the strength of the book is in the keynote and introductory chapters. Most of these are well written, informative, and bring together information from a wide range of sources. In this respect, these chapters provide a good review for those individuals studying the soil-plant-microbe interaction. They are "timeless," and much of the information will be valid for years to come. I was somewhat annoyed that several of these chapters ended with allusions of advertisement for the next conference.

Some sections appear to be incomplete. I assume this is a result of the fact that not all contributions were published. Not having access to the Conference Transactions, one wonders what might be overlooked.

The book is printed on high quality paper and in an easy-to-read print. Generally the book is error free, although some typographic errors were noted. I do not believe the binding will wear well under heavy use.

Despite some minor shortcomings, I was excited about this book. Perhaps I found support for some of my biases. I will use it for supplementary reading material for my soil microbiology class. I also recommend it for those seeking to upgrade their backgrounds. It is an important book not only for soil microbiologists, but for those involved in soil fertility and plant nutrition work. Principles presented are equally applicable to wild-land as well as agronomic management schemes.