

Optimizing Any-Aged Management of Mixed-Species Stands: II. Effects of Decision Criteria

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ABSTRACT. The effects of maximum present value and maximum volume objectives on the efficiencies of alternative silvicultural systems are determined by solving any-aged management problems for mixed-conifer stands in the Northern Rocky Mountains. Any-aged management problems are formulated with periodic planting and harvesting controls and without constraints on the stand age or size structure over time. Optimization is performed with the Stand Prognosis Model, a single-tree simulator that has been calibrated for many forest types in the western United States. With a maximum present value objective, 1987 stumpage prices, and no logging damage, steady-state, uneven-aged management using selection cuts is the target silvicultural system for a range of stand types. Optimal transition regimes for understocked stands include clearcutting and planting western white pine (*Pinus monticola* Dougl. ex D. Don), a high-value, fast-growing species. With a maximum merchantable volume objective, two types of management emerge as optimal: even-aged plantation management and an uneven-aged shelterwood system are both capable of producing the same high level of yield indefinitely. Both types of management rely on a series of thinnings to grow the most vigorous crop trees into the largest size classes. In addition, the shelterwood system ensures that adequate natural regeneration is established when the crop trees are harvested. Because the uneven-aged shelterwood system produces steady-state yields that are as high or higher than plantation management, these results call into question the widely held view that even-aged plantation management maximizes total merchantable volume production. *FOR. SCI.* 36(1):125-144.

ADDITIONAL KEYWORDS. Optimal harvesting, even-aged management, uneven-aged management, single-tree simulator, Stand Prognosis Model.

FOREST MANAGERS frequently choose between maximum present value and maximum yield as criteria for determining silvicultural policy. Comparisons of the effects of these criteria are usually made with simple, time-dependent models that predict yields in even-aged plantations. Assuming that all trees are cut at once, it is easy to show that the rotation age that maximizes present value is, in general, shorter than the rotation age that maximizes volume (e.g., Newman 1988), although there are special cases where the opposite is true (Binkley 1987).

Due to the obvious limitations of univariate models, forest biometricians have developed disaggregated models of stand growth that project the con-

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sequences of the full range of silvicultural options, including site preparation, planting, thinning, and the various regeneration harvest methods, in pure or mixed-species stands with even- or uneven-aged structures. These models, which include single-tree simulators and diameter-class, matrix models, are composed of difference equations for projecting the regeneration, growth, and survival of trees in different size classes. Tree size and species, stand density, and site characteristics are the determinants of the growth processes.

Unfortunately, general results about optimal harvesting are difficult to obtain with more complicated models. Nevertheless, optimal solutions to specific problems can be computed with numerical methods, and the examination of solutions to several problems can be used to gain insight. For example, Riitters et al. (1982) use dynamic programming to demonstrate that, when price premiums exist for large trees, thinned Douglas-fir (*Pseudotsuga menziesii* [Mirb.] Franco) plantations that maximize present value produce almost as much volume and have the same rotation ages as thinning regimes that maximize volume production. Adams and Ek (1974) and Bare and Opalach (1987) determine the effects of the decision criteria on steady-state size structures for uneven-aged stands. While each of these studies analyzes a particular timber management system, none rate different management systems under each objective.

Ranking management systems with economic and biological criteria is important for both private and public timberland managers. Private timberland managers are often interested in the stand-level tradeoffs between volume production and present value when choosing between even-aged and uneven-aged timber management (see, for example, Guldin and Baker 1988). Planners developing silvicultural policy for national forests in the United States are coming under increasing pressure to justify their selection of timber management systems on the basis of economic efficiency, timber production, and environmental consequences (see, for example, Benfield 1987). Although silvicultural policy decisions on national forests are based on results from forest-level optimization models, results from stand-level studies such as those presented here, when incorporated into forest-level analyses, may improve both the economic and biological performance of the forest.

To evaluate the effects of maximum present value and maximum cubic foot volume objectives on the relative efficiencies of different management systems, we develop an any-aged management formulation and apply it to mixed-conifer stands in the Northern Rocky Mountains. Any-aged management is a new concept resulting from a general optimization formulation that has been used to evaluate and compare the efficiencies of even-aged and uneven-aged management systems (Haight 1987). The any-aged management problem is to determine the best time sequence of harvests and plantings without constraints on the stand age or size structure (Haight and Monsrud 1990). With no constraints, optimal management may prescribe stand structures that vary from even-aged to irregular, and thus regimes are termed any-aged. Because the any-aged management problem includes even-aged and uneven-aged management problems as special cases, optimal any-aged regimes should be superior to regimes that fit these standard definitions of stand management.

Constructing any-aged management regimes requires a stand simulator that forecasts tree size distributions as a function of harvest intensity. We use the Stand Prognosis Model (Wyckoff et al. 1982), a single-tree simulator that has been calibrated for many forest types in the western United States.

Unlike previous attempts at solving optimization problems using single-tree simulators (e.g., Bare and Opalach 1987), we do not simplify the simulator structure to produce a tractable model. Thus, the methodology that we demonstrate can be applied to any forest type where single-tree simulators are used for management analysis. This is extremely important because single-tree simulators are among the most popular methods of forecasting stand growth and yield.

To solve any-aged management problems, we use a coordinate-search process called the "Method of Hooke and Jeeves" (Hooke and Jeeves 1961). The mechanics of the search process can be found in most operations research texts (e.g., Bazaraa and Shetty 1979), and its application to even-aged management problems is discussed in Roise (1986). A description of the coordinate-search process and its performance with the Prognosis Model are given in detail elsewhere (Haight and Monserud 1990).

In the first section, we discuss the any-aged stand management formulation and contrast it with the standard definitions of even-aged and uneven-aged management. Next, the Prognosis Model and its optimization program are briefly reviewed. In the third section we demonstrate the optimization methodology by determining management systems for a mixed-conifer stand in northern Idaho. Optimal any-aged and even-aged management regimes are compared to determine the effects of present value and cubic foot volume objectives on harvest intensity, species composition, and management system efficiency. The final section discusses the effects of initial stand structure on optimal management and some limitations of the problems formulated and solved here.

THE ANY-AGED STAND MANAGEMENT PROBLEM

The any-aged stand management problem is to determine the best sequence of species and diameter-class harvesting rates and planting intensities for an existing stand over an infinite time horizon. Harvesting and planting may take place simultaneously in any period, and there are no constraints on the stand size or age structure. Because the sequence of residual diameter distributions may take on any structure, the optimal management regime is any-aged (see Appendix A for a mathematical statement of the problem).

It is useful to compare the any-aged management problem with the standard definitions of even-aged and uneven-aged management. For even-aged management, the harvesting problem includes two components: the determination of the best thinning regime to employ in an existing stand before it is clearcut (or seedtree harvested) and the determination of the best sequence of actions to undertake in the subsequent plantation (or even-aged natural stand). These components are called the conversion problem and the plantation problem, respectively. The uneven-aged management problem is to determine the best sequence of selection harvests for a transition regime that terminates with a steady state. Both even-aged and uneven-aged problems are special cases of the any-aged management problem because they can be obtained by placing appropriate constraints on the harvesting and planting controls (Haight 1987). As a result, the unconstrained, any-aged management formulation provides superior solutions.

To determine the effects of each decision criterion, we first solve the any-aged management problem, and then we compare the any-aged management regime with harvesting and planting associated with optimal even-aged management. Because an infinite-time-horizon problem cannot be solved exactly, we must compute approximate solutions to the any-aged

management problem. Sensitivity analyses showed that horizons greater than 160 years are sufficient to approximate the infinite time horizon (Haight and Monserud 1990).

For even-aged management, the optimal plantation regime is found independently of the conversion regime, and it maximizes an infinite series of rents. The best infinite-series plantation is found by comparing the rents from plantations with different rotation ages. The optimal conversion regime maximizes the sum of the rents from conversion and plantation management. The optimal conversion period is found by comparing the rents from conversion and plantation regimes with different conversion ages.

Computing the rent for even-aged management depends on the management criterion. With a maximum present value objective, the value of an infinite series of plantations is obtained by multiplying the single-rotation present value by the well-known conversion factor. The sum of the present value of the conversion regime and the discounted value of an infinite series of plantations equals the overall rent. With a maximum volume production objective, the volume produced by an infinite series of plantations is infinite. This singularity is avoided with the use of average annual production (AAP), which is calculated in three steps. First, the total yield from thinnings and clearcut is computed for a management regime with a finite horizon; second, the initial stand volume is subtracted from the total yield; and third, the difference (net yield) is divided by the horizon length.¹ The AAP for a plantation is thus the total yield divided by rotation age (and is equivalent to mean annual increment); the optimal plantation maximizes AAP. The AAP for the conversion and plantation regime is found by choosing some large but finite horizon, computing the net yields from conversion and plantation management, and dividing net yields by the horizon length.

THE PROGNOSIS MODEL AND OPTIMIZATION

The Stand Prognosis Model is designed to simulate the development of forest stands in the Northern Rocky Mountains (Wykoff et al. 1982, Crookston 1985, Wykoff 1986, Hamilton 1986). The individual tree is the basic unit of projection, and stands with any combination of species and size classes can be accommodated. An important feature of the simulator is its regeneration establishment component (Ferguson et al. 1986, Ferguson and Crookston 1984). We used the Inland Empire version 5.2 of the Prognosis Model, which is calibrated for 11 conifer species occurring on 30 habitat types in northern Idaho and northwestern Montana.

When coupled with the coordinate-search process for optimization, the Prognosis Model is used to evaluate harvest regimes that are defined by a set of control variables representing the fractions of trees harvested by diameter class and species group in specified periods. In addition to harvesting, planting and site preparation may be scheduled in any period. These cultural activities are fixed rather than variable inputs, which improves the performance of the optimization program. The coordinate-search process determines the optimal harvest proportions for a given set of cultural activities. Repeated application of the optimization program allows comparison of the efficiencies of sets of cultural activities.

The coordinate-search algorithm does not guarantee convergence to a

¹ We subtract the initial stand volume from total yield to get an estimate of the production capability of a given stand.

globally optimal solution to a given harvesting problem. Performance tests showed that, for problems with more than about 10 active control variables (two thinnings), the algorithm converges to solutions that may vary considerably in present value and harvest pattern depending on the point at which the search starts and how broadly the control variables are defined (Haight and Monserud 1990). Convergence is improved by defining harvest controls for relatively wide diameter classes and broad species groupings. Therefore, for problems with a present value objective, we define three groups based on stumpage price, and with a cubic foot volume objective, we use one species group. Problems with both management objectives are formulated with three submerchantable diameter classes (0–2, 2–4, and 4–7 in.) and five merchantable diameter classes (7–10, 10–14, 14–18, 18–22, and 22–40 in.). These groupings proved very efficient and considerably reduced the variability resulting from different random starts.

We examine a range of local optima before selecting the best solution to a given problem. Each problem is solved at least three times (and usually six) using different starting points for the harvest controls. Starting points are determined either randomly or from solutions to problems with shorter time horizons. We are confident that additional trials will not change the overall conclusions reached below.

OPTIMAL MANAGEMENT OF A WELL-STOCKED, MIXED-CONIFER STAND

In this section we compare the efficiencies of alternative timber management systems that are computed with two maximization criteria: present value and merchantable cubic foot volume. Optimal regimes are computed for a well-stocked, mixed-conifer stand. The stand is located on the St. Joe National Forest in Idaho in the *Tsuga heterophylla/Clintonia uniflora* habitat type. The elevation is 3,400 ft, the slope is 25%, and the aspect is northwest. The stand has over 2,000 trees/ac varying in diameter between 0 and 22 in. (Table 1). The basal area is approximately 195 ft²/ac and the merchantable volume is 4,260 ft³/ac. The stand includes 5 species in the following percentages based on basal area: 41% western white pine (*Pinus monticola* Dougl. ex D. Don), 32% grand fir (*Abies grandis* [Dougl. ex D. Don] Lindl.), 10% western hemlock (*Tsuga heterophylla* [Raf.] Sarg.), 9% western redcedar (*Thuja plicata* Donn ex D. Don), and 8% Douglas-fir. Lodgepole pine (*Pinus contorta* Dougl. ex Loud.) and western larch (*Larix occidentalis* Nutt.) are added via natural regeneration even though parent trees are not included in the initial stand.

TABLE 1. Trees/ac by diameter class and species group for a mixed-conifer stand on the *Tsuga heterophylla/Clintonia uniflora* habitat type. Species group 1 includes western larch, grand fir, and Douglas-fir; group 2 includes western white pine and western redcedar; and group 3 includes western hemlock and lodgepole pine.

Diameter (in.)	Species group			Diameter (in.)	Species group		
	Group 1	Group 2	Group 3		Group 1	Group 2	Group 3
	----- trees/ac -----				----- trees/ac -----		
0–2	674	367	561	10–14	15	20	8
2–4	132	73	43	14–18	3	26	0
4–7	129	53	25	18–22	0	5	0
7–10	76	48	9	22+	0	0	0

The problem formulations are the same for both management objectives. Approximations of the any-aged management problem are formulated to find the best sequence of selection harvests on a 20-year cycle during a 160-year horizon.² The even-aged management problems include the determination of the best planting density, thinning regime, and rotation age for a white pine plantation and the determination of the best thinning regime for the mixed-conifer stand before it is clearcut and replaced with the white pine plantation. We investigate white pine plantations because, compared to managing other species, growing white pine is more efficient using either economic or physical production criteria. An example of a Prognosis keyword file for determining an optimal plantation regime is listed in Appendix B.

MAXIMIZE PRESENT VALUE WITH ANY-AGED MANAGEMENT

With a present value objective, approximations to the optimal any-aged management regime are computed using economic parameters that represent 1987 market conditions (Charles McKetta, Department of Forest Resources, University of Idaho, personal communication). The seven species are split into three groups, each with a different stumpage price. Group 1 includes larch, grand fir and Douglas-fir with a stumpage price of \$70 per thousand board feet (mbf) (Scribner), group 2 includes white pine and redcedar with a stumpage price of \$115/mbf, and group 3 includes western hemlock and lodgepole pine with a stumpage price of \$45/mbf. The minimum merchantable tree size is 7 in. in diameter at breast height, and merchantable tree volumes are measured with a 6-in. minimum top diameter. Precommercial thinning costs \$0.10/tree regardless of species. Planted white pine seedlings cost \$0.40/tree, and site preparation costs \$50/ac. The real discount rate is 4%, and the discount rate, prices, and costs, are assumed to be constant over time.

Figure 1 shows the first 100 years of the optimal any-aged management regime. During the first 40 years, the management strategy is to thin heavily from above removing almost all trees greater than 10 in. and a portion of the trees between 7 and 10 in. in diameter. Precommercial thinning takes a portion of the trees between 2 and 7 in. In years 60 and beyond, optimal harvesting approaches a steady state with commercial thinnings taking all merchantable trees and with precommercial thinnings controlling the number of younger trees. All seven species regenerate successfully so that planting is neither necessary nor economical. Selection harvests take trees from all three species groups; harvest volumes range between 9 and 18 mbf/ac. The present value of the 160-year regime is \$2,893/ac (the value of the initial stand is \$1,888/ac).

MAXIMIZE PRESENT VALUE WITH EVEN-AGED MANAGEMENT

The even-aged management problem includes the determination of the best conversion regime for the mixed conifer stand and the best plantation regime that follows the clearcut. Both problems are solved with the set of economic parameters used in the any-aged management analysis.

To analyze plantation management, we compare the present values of

² Application of the optimization model to several stands on this site showed that including planting as a fixed input did not improve objective function values. Thus, the any-aged management results presented here do not include planting in any time period.

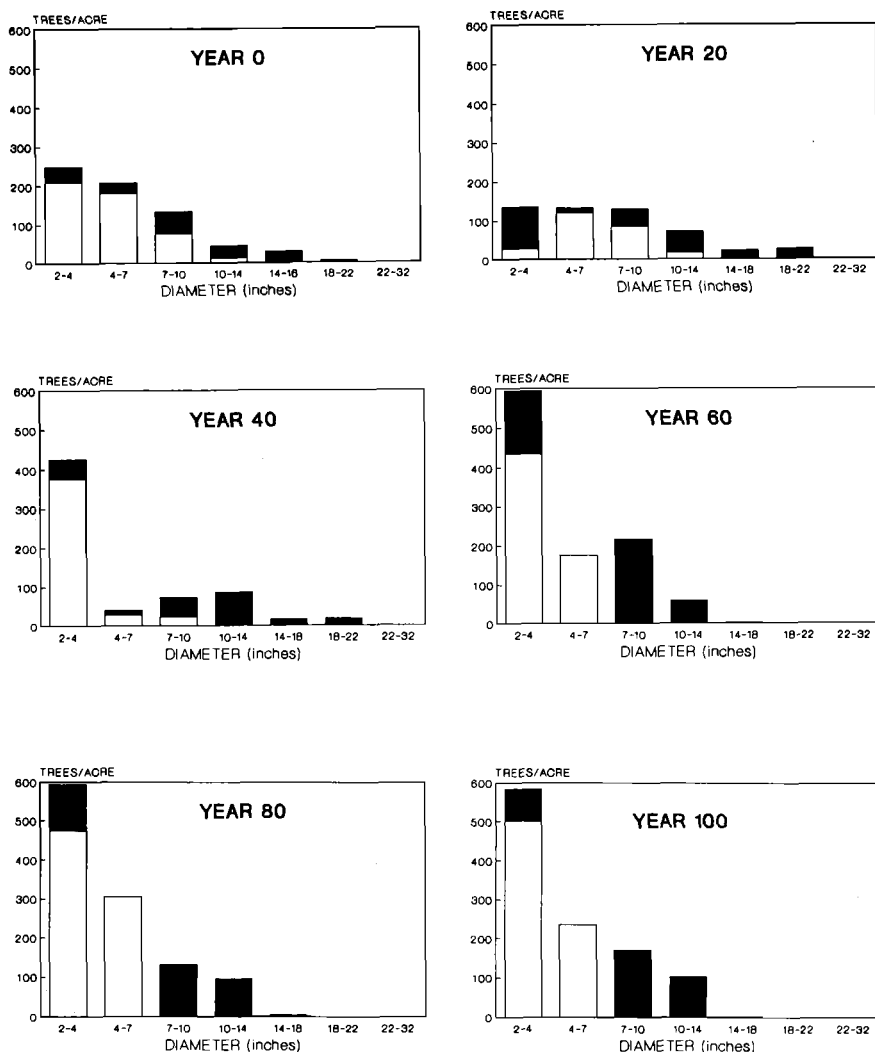


FIGURE 1. The first 100 years of the any-aged management regime that maximizes the present value of a mixed-conifer stand in the *Tsuga heterophylla/Clintonia uniflora* habitat type. Shaded areas show numbers of trees cut.

thinned and unthinned plantations for a wide range of white pine planting densities (0, 150, 300, 600, 1000, 1800, and 3000 trees/ac) and rotation ages (60, 80, 100, 120, 140, and 160 years). Thinnings (and their intensity) are determined by the optimizer and can occur on a 20-year cycle, beginning in year 20 after the planting; of course, thinnings are not scheduled if the optimizer determines it is suboptimal.

The infinite-series present values of the plantation alternatives are given in Table 2. For each planting density, present values of thinned plantations increase with rotation age until they level off at 120 years. Longer rotation ages increase present value by less than 1% of the 120-year regime's value. Optimal planting density is between 150 and 300 trees/ac, depending on rotation length. Planting 300 trees/ac is best for the 60 and 80-year rotations, whereas 150 is best (and has a higher present value) for the longer rotations. Note that present values change less than 10% across the planting density

TABLE 2. Projected infinite-series present values (\$/ac) of white pine plantations on the *Tsuga heterophylla*/*Clintonia uniflora* habitat type.

Planting density (trees/ac)	Rotation age (yr)					
	60	80	100	120	140	160
----- Thinning -----						
0	119	153	159	158	158	158
150	168	192	223	233	232	233
300	190	201	208	217	220	221
600	132	178	200	215	219	219
1,000	9	74	94	112	116	117
1,800	-281	-185	-167	-158	-158	-159
3,000	-818	-685	-642	-622	-620	-620
----- No thinning -----						
0	77	56	17	-12	-30	-40
150	126	72	0	-49	-78	-94
300	92	26	-54	-108	-139	-154
600	43	-48	-147	-214	-251	-270
1,000	-122	-214	-307	-371	-411	-431
1,800	-454	-531	-623	-691	-729	-749
3,000	-1004	-1041	-1119	-1179	-1213	-1231

range of 150 to 600 trees/ac (300 to 600 trees/ac is the common planting density range). Extremely high planting densities ($\geq 1,800$ trees/ac) all have negative present values. We use a 120-year rotation for the optimal plantation regime because of the small gain in present value for longer rotations.

Unthinned plantations are considerably less efficient than thinned plantations. The optimal plantation regime without thinning involves planting 150 seedlings/ac and clearcutting in year 60 (\$126/ac present value). For comparison, the optimal management with thinning (plant 150 seedlings/ac with a 120-year rotation) has a present value that is almost twice as large (\$233/ac). Generally, present values of management without thinning decrease with increasing rotation age and increasing planting density. All planting densities with rotation ages of 120 or more produce present values that are negative.

The substantial gain in present value for plantations with thinning results from regular and fairly heavy commercial thinnings and precommercial thinnings beginning in year 40. The optimal 120-year regime in Figure 2 is typical of plantation management found by the optimizer. Commercial thinnings are from above and in years 40 and 60 concentrate on removing the rapidly growing white pine crop trees. During this time the stand is regenerating naturally with Douglas-fir, grand fir, western redcedar, and western hemlock. By year 80 the white pine crop trees have been harvested and the stand takes on a mixed-species and mixed-age composition that resembles the any-aged regime in Figure 1. It is important to note that at year 120 the stand is well stocked with a majority of trees in the submerchantable diameter classes. Clearcutting at this point is suboptimal because the advance regeneration is lost. However, in the context of plantation management, the yields from additional thinnings, when discounted 100 years or more to the beginning of the rotation, increase present value by less than 1%. Nevertheless, these regimes suggest that converting the initial plantation to mixed-species, uneven-aged management is economical and can be sustained indefinitely. This conjecture is supported by the present values of the optimal 140- and

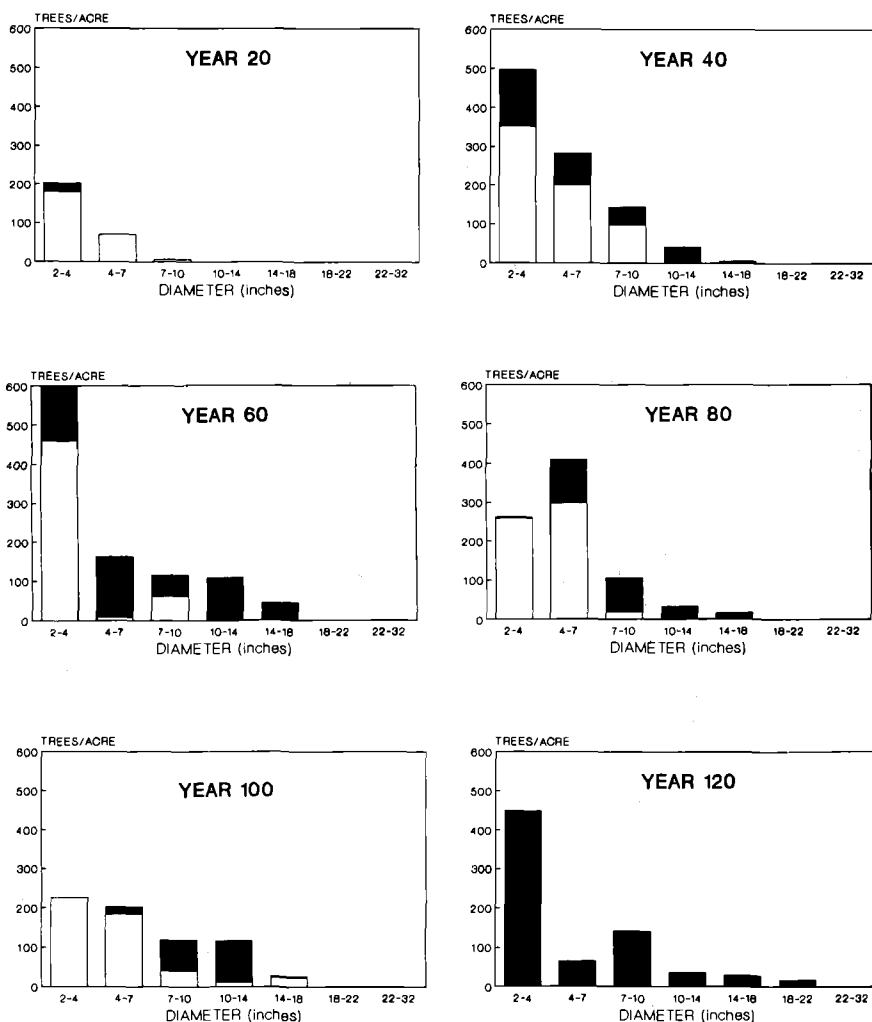


FIGURE 2. Optimal thinning regime for a white pine plantation that maximizes present value in the *Tsuga heterophylla*/*Clintonia uniflora* habitat type. Shaded areas show numbers of trees cut. Planting density is 150 seedlings/ac.

160-year regimes (Table 2), which are essentially identical to the optimal regime illustrated in Figure 2.

The thin-from-above rule employed by the optimizer removes trees based on their value growth rate rather than their volume growth rate. The largest crop trees were removed because their value growth rate over the next 20-year period is smaller than the discount rate. Smaller trees respond to the increased growing space with higher value growth rates. Note that no damage is assumed to befall the residual stand, a shortcoming that results from the lack of a good thinning damage model. The harvest strategy for maximizing present value contrasts strongly with management regimes that maximize volume yield discussed in the next subsection.

To determine the best conversion regime for converting the existing mixed conifer stand to a plantation, we compare optimal thinning regimes with 20-year cutting cycles that begin in year 0 and terminate with clearcuts in years 20, 40, . . . , or 160. The present values of the conversion regimes

increase with the conversion year (Table 3). When the discounted value of the optimal plantation is added to the present value of each conversion regime, the total value of management also increases with the age of clearcut. As the clearcut age increases, the optimal conversion regime approaches the any-aged management regime described in Figure 1. Thus, the best strategy for the mixed-conifer stand is the indefinite postponement of plantation management and the establishment of a sequence of selection harvests.

MAXIMIZE MERCHANTABLE CUBIC FOOT VOLUME WITH ANY-AGED MANAGEMENT

With the cubic foot volume objective, there are no production costs, the stumpage price is effectively \$1/ft³, and the discount rate is zero. The seven tree species are grouped together for harvesting. The minimum merchantable tree size is 7 in. in diameter at breast height, and cubic foot volume is measured with a minimum 6-in. top diameter.

The optimal 160-year any-aged management regime involves growing the best trees from the initial stand into the largest diameter class during the first 80 years and using a shelterwood regeneration method to create a second cohort of trees that grows into the largest diameter class during the second 80 years (Figure 3). Harvesting begins with a heavy precommercial thinning in year 0 and includes commercial thinnings every 20 years. By year 80 the 60 crop trees/ac (all with diameter >18 in.) are removed, along with most of the trees smaller than 2-in. in diameter (not shown). What remains is a well-stocked, naturally regenerated stand with approximately 500 trees/ac between 2 and 7 in. in diameter. This cohort of trees is then managed for the next 80 years in a manner similar to the preceding 80 years on the original well-stocked stand. A precommercial thinning in year 120 removes slow-growing natural regeneration that came in after the overstory removal in year 80. By year 160 there are 110 crop trees/ac, all larger than 14 in. in diameter. There is also another cohort of small trees (not shown) ready to be released by overstory removal. After subtracting the initial volume of 4,260 ft³/ac, the optimal any-aged management regime produces 183 ft³/ac/yr over the 160-year horizon.

Because the initial stand is well stocked, there is a possibility that the high level of volume production is partly due to a transient response induced by

TABLE 3. Projected present values (\$/ac) of conversion and plantation regimes starting with a mixed-conifer stand on the *Tsuga heterophylla*/*Clintonia uniflora* habitat type.

Management component	Conversion year								
	0	20	40	60	80	100	120	140	160
Conversion regime ^a	1,888	2,522	2,671	2,805	2,864	2,878	2,888	2,891	2,893
Conversion and plantation regime ^b	2,121	2,628	2,720	2,827	2,874	2,883	2,890	2,892	2,893

^a Present values include yields from optimal thinning regimes that terminate at a given conversion year.

^b Present values include yields from optimal thinning regimes that terminate at a given conversion year plus the discounted value of the white pine plantation described in Figure 2.

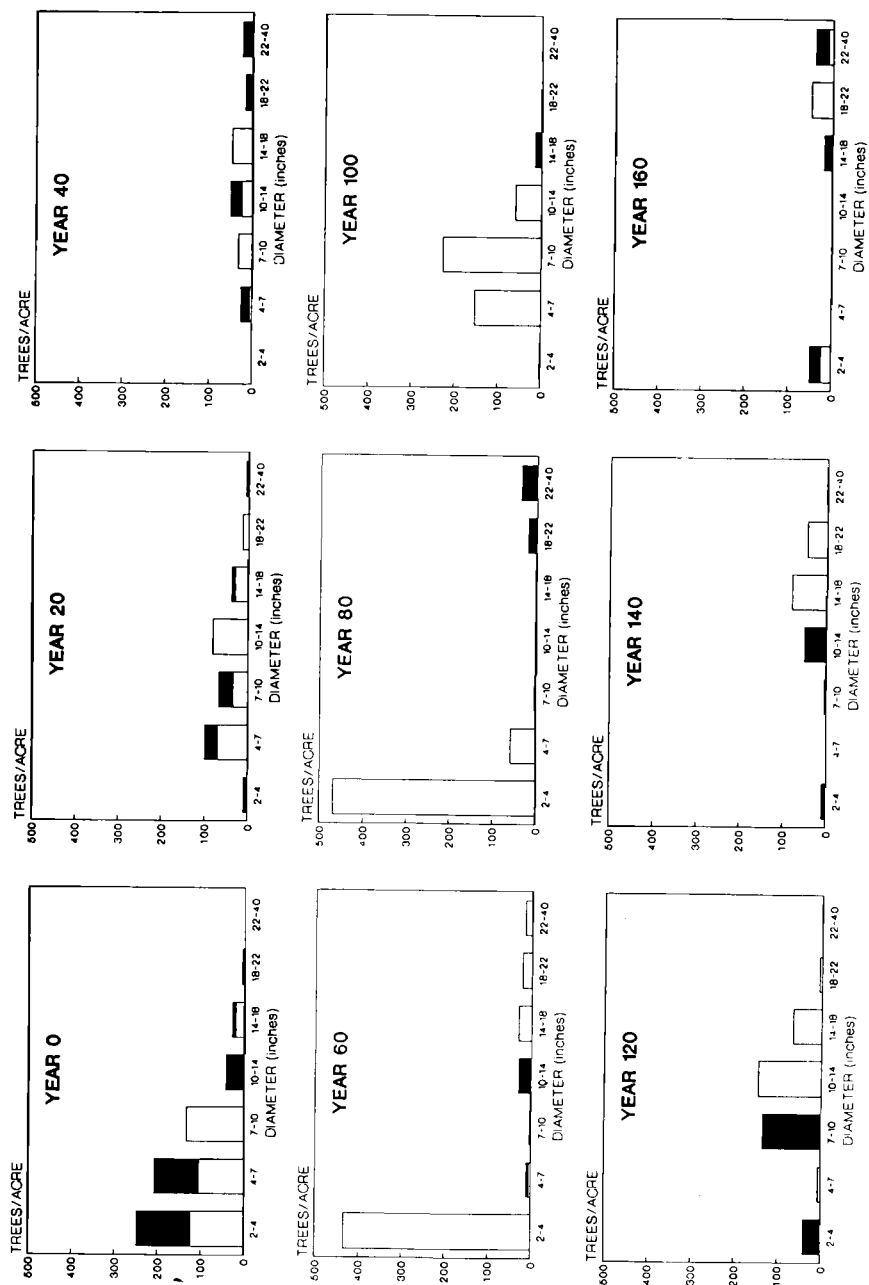


FIGURE 3. The 160-year, any-aged management regime that maximizes the cubic foot volume production of a mixed-conifer stand in the *Tsuga heterophylla*/*Clintonia uniflora* habitat type. Shaded areas show numbers of trees cut.

the initial conditions. To examine this, we used the tree list at the end of the optimal any-aged regime (year 160) as the initial stand for another 160-year, any-aged optimization.³ After subtracting the initial volume of 11,480 ft³/ac, the optimal any-aged regime produces 148 ft³/ac/yr over the second 160-year period. We repeated this procedure starting with the tree list in year 320. After subtracting the initial volume of 12,710 ft³/ac, the optimal any-aged regime produces 158 ft³/ac/yr for years 320 through 480. The average annual production for the entire 480 years is 162 ft³/ac/yr. Optimal harvesting for years 160 to 480 is very similar to the series of shelterwoods obtained for the first 160 years. Species composition is stable; merchantable volume comes primarily from grand fir, with lesser yields from Douglas-fir and western hemlock. Although western redcedar and hemlock (the climax species for this habitat type) are usually in the understory, the steady sequence of thinnings and overstory removals keeps the stand from reaching climax. White pine is not an important stand component after the first overstory removal in year 80. Because of the abundance of vigorous natural regeneration, it is clear that a sequence of shelterwood regeneration harvests can be practiced indefinitely without planting.

MAXIMIZE MERCHANTABLE CUBIC FOOT VOLUME WITH EVEN-AGED MANAGEMENT

The first stage of the even-aged management analysis is to determine the best plantation regime to establish after the mixed-conifer stand is clearcut. As in the preceding analysis that maximized present value, we compute optimal thinning regimes for white pine plantations with planting densities that range from 0 to 3,000 seedlings/ac and rotation ages between 60 and 160 years. The optimizer has the option of scheduling a thinning during any 20-year cycle, beginning in year 20. The stand is clearcut at the end of the rotation.

The average annual productions of selected plantation regimes are given in Table 4. Note that an extremely wide range of planting densities is examined, even though common management practice calls for between 300 and

TABLE 4. *Projected average annual merchantable cubic foot volumes (ft³/ac/yr) produced by white pine plantations on the Tsuga heterophylla/Clintonia uniflora habitat type.*

Planting density (trees/ac)	Rotation age (yr)						Rotation age (yr)					
	60	80	100	120	140	160	60	80	100	120	140	160
	----- Thinning -----						----- No thinning -----					
0	86	106	119	129	135	139	82	100	106	105	103	100
150	106	128	141	145	147	147	95	114	119	120	117	113
300	118	135	144	151	155	155	114	122	127	124	119	113
600	133	151	159	163	162	161	120	135	138	134	125	119
1,000	141	158	163	167	169	167	125	137	137	133	127	120
1,800	149	165	170	172	172	168	128	140	141	138	131	123
3,000	148	163	168	174	174	172	124	136	137	132	125	118

³ Storage limitations necessitated this concatenation of 160-year optimizations, which results in a constrained optimum for the entire horizon.

600 seedlings/ac for plantations. Our optimization results show that increasing the planting density improves yields, until a plateau is reached (i.e., within 5% of the maximum) when densities exceed 1,000 seedlings/ac. Yields also improve as rotation age increases, up to 120 years. Negligible improvement (<1%) results from longer rotations (140 and 160 years). The maximum yield found by the optimizer is a plantation with 3,000 seedlings/ac managed for either a 120- or 140-year rotation (174 ft³/ac/yr).

It should come as no surprise that yields increase with planting density up to some plateau. As planting density increases so too does the opportunity for the optimizer to select the most rapidly growing trees for management, while removing the rest with free precommercial thinning. Of course, such high planting densities are extremely uneconomical when the costs of planting, site preparation, and precommercial thinning are considered (see Table 2).

Compared to the best thinned plantation, the projected yield for the best unthinned plantation is 15% less (138 ft³/ac/yr for a 100-year rotation with 600 seedlings/ac). Generally, the shape of the optimal yield surfaces (volume yield vs. planting density vs. rotation age) for regimes with and without thinning are very similar; the unthinned yields are approximately 15 to 20% less than the yields with thinning. Very long rotations (>120 years) without thinning have considerably less yield than corresponding rotations with thinning. Unthinned plantations have lower yields due to lack of density control, which results in inefficient use of growing space and mortality that cannot be captured in the thinning yields. These losses in volume yield are consistent with results from optimization studies for Douglas-fir in the Pacific Northwest (Riitters et al. 1982).

Figure 4 shows the optimal thinning regime for the white pine plantation with 600 seedlings/ac and a 120-year rotation (163 ft³/ac/yr). This regime has the highest yield of those that are within the bounds of common planting densities and therefore also within the range of data used to construct the regeneration model (see Ferguson et al. 1986). Heavy precommercial thinning removes over 3,400 trees/ac in year 20, most of which are volunteers below 2 in. in diameter (not shown). In year 60 another precommercial thinning removes 2,000 trees/ac less than 2 in. in diameter (not shown), and a light commercial thinning from below removes 66 trees/ac between 7 and 10 in. in diameter. This leaves 234 crop trees/ac. In year 80 an additional 74 trees/ac with diameters between 10 and 14 in. are thinned from below, leaving 105 crop trees/ac and a basal area of 193 ft²/ac. The plantation is clearcut in year 120, removing the remaining 86 white pine crop trees/ac; these trees have a basal area of 350 ft²/ac and a yield of 16,000 ft³/ac.

The best conversion regime for the mixed conifer stand is determined by comparing optimal regimes with thinnings on a 20-year cycle beginning in year 0 and terminating in clearcuts in years 20, 40, . . . , or 160. For each conversion regime, the initial stand volume of 4,260 ft³/ac is subtracted from the total yield from thinnings and clearcut; this allows for a comparison in terms of average annual production (AAP). The AAP of optimal conversion regimes decrease from a maximum of 227 ft³/ac/yr for conversion year 20 to 183 ft³/ac/yr for conversion year 160 (Table 5). The AAP of conversion followed by plantation management is computed over a 160-year time horizon by calculating a weighted average of the conversion period AAP and the AAP from the best plantation (163 ft³/ac/yr produced by the 120-year rotation with 600 seedlings/ac). The weights are the proportion of the 160-year regime used by each of the two types of management. The resulting distribution of AAP for conversion and subsequent plantation management climbs

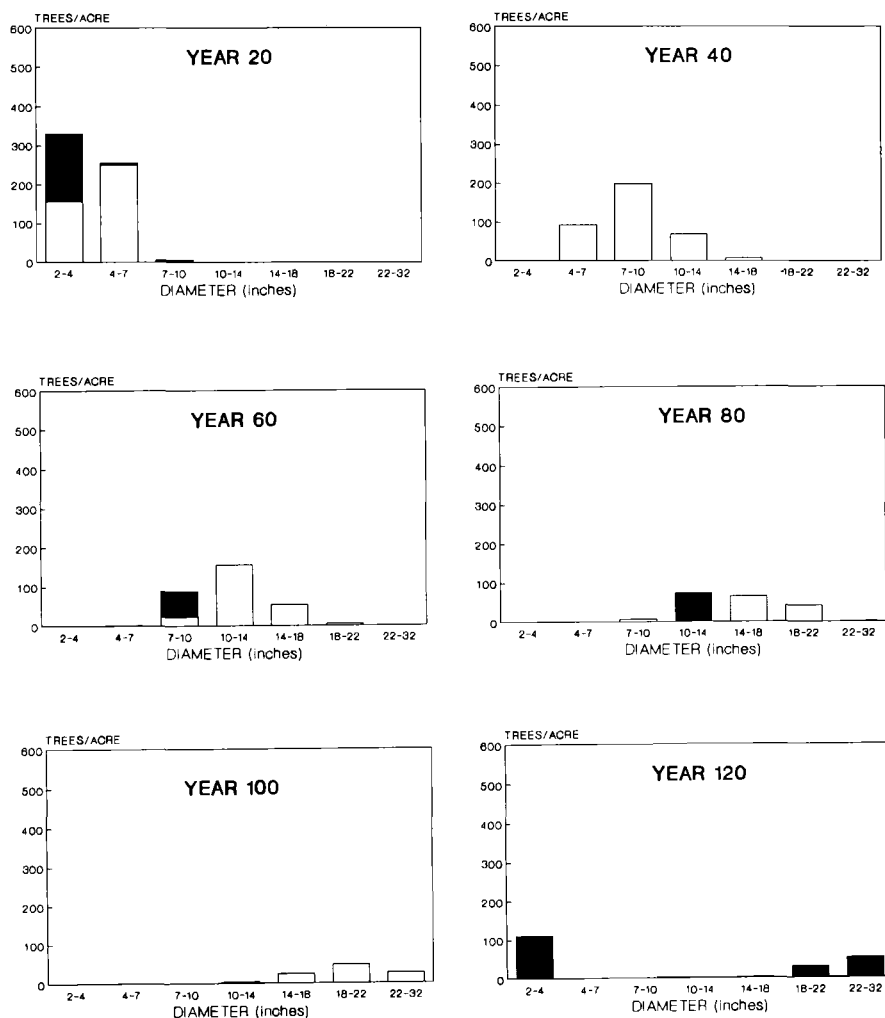


FIGURE 4. Optimal thinning regime for a white pine plantation that maximizes merchantable cubic foot volume in the *Tsuga heterophylla*/*Clintonia uniflora* habitat type. Shaded areas show numbers of trees cut. Planting density is 600 seedlings/ac.

quickly to a plateau by conversion year 60 (Table 5). There is a minor peak at conversion year 80 (187 ft³/ac/yr), but generally the differences in AAP among conversion years are negligible after year 60.

The optimal solutions to the any-aged and even-aged management problems with 160-year horizons have almost the same volume yield (183 and 187 ft³/ac/yr, respectively). The yields are even closer when we extend the horizon. The solution to the 480-year, any-aged optimization suggests that a steady-state yield of approximately 162 ft³/ac/yr can be produced indefinitely with a sequence of shelterwood harvests. This yield is essentially identical to the yield from an infinite sequence of optimal white pine plantations (163 ft³/ac/yr). Thus, we conclude that there are two management strategies that maximize merchantable volume yield. The first is to clearcut the stand in year 80 and begin the optimal white pine plantation regime; thinnings occur both before and after the clearcut. The second is to postpone conversion to plantation management indefinitely and instead use the sequence of shelter-

TABLE 5. Projected average annual cubic foot volumes (ft³/ac/yr) produced by conversion and plantation regimes starting with a mixed-conifer stand on the Tsuga heterophylla/Clintonia uniflora habitat type.

Management component	Conversion year								
	0	20	40	60	80	100	120	140	160
Conversion regime ^a	—	227	224	220	211	199	194	185	183
Conversion and plantation regime ^b	163	171	178	184	187	186	186	182	183

^a The average annual production of a conversion regime equals the total yield from thinnings and clearcut, minus the volume in the initial stand, divided by the conversion year. The AAP cannot be computed when the stand is clearcut immediately.

^b Average annual increments are computed from the total yields over a 160-year horizon. Total yields include optimal thinning regimes that terminate at a given conversion year plus the yields from the optimal white pine plantation described in Figure 4.

wood harvests suggested by the optimal any-aged management regime. Scrutiny of these seemingly disparate management strategies reveals that the actual differences are minor, even though the plantations are planted to white pine and the shelterwood regime is dominated by grand fir that has regenerated naturally.

It is important to note that at rotation age (year 120) the optimal plantation includes nearly 600 trees/ac with diameters less than 4 in. Because this advanced regeneration is lost, clearcutting at this point may be suboptimal. We therefore solve a sequence of two 160-year, any-aged management problems starting with the tree list obtained from the plantation in year 120. Optimal management involves a sequence of shelterwood harvests similar to the any-aged regime shown in Figure 3. The optimal regime produces 172 ft³/ac/yr over the 440-year horizon, which is 6% greater than the yield from a sequence of plantations (163 ft³/ac/yr). Thus, we conclude that converting the white pine plantation to a naturally regenerated, mixed-conifer stand using shelterwood harvests produces a slightly higher yield than a series of plantations.

It is also important to note that this 440-year regime that begins with a white pine plantation produces 6% greater yield than the optimal any-aged regime (480 years) for the well-stocked, mixed-conifer stand (Figure 1). Clearly, optimal management and resulting yield are sensitive to initial stand conditions. Species composition is important in this case, for the shelterwood regime undertaken after plantation establishment manages to maintain a vigorous white pine component (something that the optimal any-aged regime is unable to do in the mixed-conifer stand). A better solution to the any-aged management problem for the mixed-conifer stand may be found if additional harvesting and planting control variables are added for white pine. We do not pursue this problem because, without price incentives, the coordinate-search optimization method often produces unstable solutions to large problems with both planting and species-specific harvest controls (Haight and Monserud 1990).

DISCUSSION

In the discussion of the results it is important to choose commonly accepted silvicultural terms to describe the any-aged regimes. With a present value objective, optimal any-aged management of the mixed-conifer stand (see

Figure 1) approaches a steady-state, uneven-aged selection regime that is consistent with the traditional definition of sustainability: approximately the same diameter distribution is achieved after each cutting cycle (Adams and Ek 1974). With a volume maximization objective, the optimal any-aged regime is best described as an uneven-aged shelterwood system (see Figure 3). The system is a shelterwood because the future stand is regenerated naturally by moderately heavy thinnings from below that open up the stand just before the overstory is removed (Smith 1962). Note that the cohort of sub-merchantable trees that is released by the overstory removal may include trees in 3 or more 10-year age classes, technically making the stand uneven-aged. The management of the resulting cohort of trees resembles intensive plantation management, however, with frequent thinnings (both precommercial and commercial) to favor the crop trees. Examination of the 480-year regime indicates that the yields from this form of management can be sustained indefinitely, even though a given diameter distribution may never recur.

The analysis of the mixed-conifer stand provides evidence that, with a maximum present value objective, uneven-aged management using the selection system is the target silvicultural system for stands on this site. When the decision criterion is changed to a maximum merchantable volume objective, two types of management emerge as optimal: plantation management and an uneven-aged shelterwood system are both capable of producing the same high level of yield indefinitely. Both types of management rely on a series of thinnings to grow the most vigorous crop trees into the largest size classes. In addition, the shelterwood system ensures that adequate regeneration is established when the crop trees are harvested. The important conclusion is that this uneven-aged shelterwood system can be just as efficient as plantation management as long as the stand is initially well stocked and adequate natural regeneration is available.

To investigate the influence of initial stocking on these results, we compute optimal any-aged and even-aged management regimes for other stands with similar site characteristics but with different species compositions and stocking levels. With a maximum present value objective, optimal any-aged management regimes all converge to a sequence of selection harvests that maintains a multispecies, uneven-aged stand structure. Optimal transition regimes, however, depend on the initial stand structure. Both young stands with less than approximately 50 ft² basal area and few white pine trees and old stands with inadequate natural regeneration are immediately clearcut and replaced with white pine plantations. In contrast to the fully stocked, mixed-conifer stand described above, these stands do not contain enough vigorous, small-diameter trees to sustain selection harvests. In these cases, planting white pine is chosen because it immediately establishes a vigorous stand where one does not exist and is not chosen because plantations are inherently superior. With natural regeneration, precommercial thinning, and diameter-limit cutting, white pine plantations evolve into fully stocked, uneven-aged, mixed-conifer stands that sustain a sequence of selection harvests. In the event that old stands do contain sufficient natural regeneration after the overstory removal, there is no need to plant white pine. A series of selection cuts every 20 years favors the most vigorous pole-sized trees by removing merchantable trees from above and excessive regeneration from below.

With a maximum volume yield objective, optimal management also depends on the initial stand structure. For understocked stands, the optimal management is to clearcut immediately and establish a white pine plantation.

This plantation can either be repeated when rotation age is reached or instead be converted to an uneven-aged, mixed-conifer stand using a sequence of shelterwood harvests. For overstocked or overmature stands, both even-aged plantation management and a regime obtained by solving the any-aged management problem are optimal. The apparent disparity in this result disappears on close examination: the any-aged regime calls for complete over-story removal immediately. The new stand consists of the remaining sub-merchantable trees and the subsequent natural regeneration. Thus, the only difference between the management regimes is in the type of regeneration obtained: natural or planted. The average annual production is virtually identical in either case.

When the objective is to maximize total merchantable volume production, the traditional view is that the clearcutting system using artificial regeneration is the method of choice (Assmann 1970, Guldin and Baker 1988). Our results show that this maxim is not generally true. Hasse and Ek (1981) came to a similar conclusion after comparing plantation management to three uneven-aged selection systems for northern hardwoods. In the current study, solutions to the any-aged management problem repeatedly show that optimal uneven-aged management systems are at least as productive as plantation management. When the stand is fully stocked (or even over-stocked) and adequate natural regeneration is available, a sequence of shelterwood harvests can produce volume yields that match those from plantation management. In addition, plantations can also be converted, without any loss in productivity, to uneven-aged, mixed-conifer stands that are regenerated indefinitely with a sequence of shelterwood harvests.

The reader should note that both the growth and mortality models used in these simulations include the effects of white pine blister rust (*Cronartium ribicola*). Future management practices will likely call for planting genetically improved seedlings that are more resistant to infection. With lower seedling and sapling mortality rates, fewer seedlings will need to be planted to produce a given number of mature trees.

Our study results are logical outcomes of the stand dynamics that are represented by the Prognosis Model, and these results should be used to evaluate model structure: if the optimal management strategies are not realistic or sensible, they may point to shortcomings in Prognosis.⁴ For example, any-aged management strategies obtained with an economic objective involve heavy thinning from above and assume that residual trees respond positively to increases in growing space. In practice, thinning from above may damage residual trees and advanced regeneration as a result of logging damage or sun scald. Because a thinning damage model is not available to take these effects into account, our results should be viewed with caution. The large increase in yield due to thinning indicates that the development of a thinning damage model would be a fruitful area of research. Unrealistic solutions may also point to deficiencies in our management experience. For example, planting densities for white pine plantations in practice rarely reach 1,800 seedlings/ac, yet optimization results show that high planting densities maximize average annual production. This result and others can be used as a basis for designing silvicultural trials to expand our experience.

Finally, our results demonstrate the importance of site-specific conditions

⁴ In fact, this happened several times in the course of this study. The use of the optimizer as a model validation tool will be addressed in a subsequent paper.

on the determination of management strategies. We recommend that this optimization program be used along with other decision-making tools by forest managers who are responsible for making stand-level, silvicultural prescriptions.

LITERATURE CITED

- ADAMS, D. M., and A. R. EK. 1974. Optimizing the management of uneven-aged forest stands. *Can. J. For. Res.* 4:274-287.
- ASSMANN, E. 1970. *The principles of forest yield study*. Pergamon Press, Oxford. 506 p.
- BARE, B. B., and D. OPALACH. 1987. Optimizing species composition in uneven-aged forest stands. *For. Sci.* 33:958-970.
- BAZARAA, M. S., and C. M. SHETTY. 1979. *Nonlinear programming: theory and algorithms*. Wiley, New York. 560 p.
- BENFIELD, F. K. 1987. The administrative record and the range of alternatives in national forest planning: Applicable standards and inconsistent approaches. *Environ. L.* 17:371-392.
- BINKLEY, C. S. 1987. When is the optimal economic rotation longer than the rotation of maximum sustained yield? *J. Environ. Econ. Manage.* 14:152-158.
- CROOKSTON, N. L. 1985. User's guide to the event monitor: An addition to the Prognosis Model. USDA For. Serv. Gen. Tech. Rep. INT-196.
- FERGUSON, D. E., and N. L. CROOKSTON. 1984. User's guide to the Regeneration Establishment Model-a Prognosis Model extension. USDA For. Serv. Gen. Tech. Rep. INT-161.
- FERGUSON, D. E., A. R. STAGE, and R. J. BOYD. 1986. Predicting regeneration in the grand fir-cedar-hemlock ecosystem of the northern Rocky Mountains. *For. Sci. Monogr.* 26.
- GULDIN, J. M., and J. B. BAKER. 1988. Yield comparisons from even-aged and uneven-aged loblolly-shortleaf pine stands. *South. J. Appl. For.* 12:107-114.
- HAIGHT, R. G. 1987. Evaluating the efficiency of even-aged and uneven-aged stand management. *For. Sci.* 33:116-134.
- HAIGHT, R. G., and R. A. MONSERUD. 1990. Optimizing any-aged management of mixed-species stands: I. Performance of a coordinate-search process. *Can. J. For. Res.* (In press.)
- HAMILTON, D. A., JR. 1986. A logistic model of mortality in thinned and unthinned mixed conifer stands of northern Idaho. *For. Sci.* 32:989-1000.
- HASSE, W. D., and A. R. EK. 1981. A simulated comparison of yields for even- versus uneven-aged management of northern hardwood stands. *J. Environ. Manage.* 12:235-246.
- HOOKE, R., and T. A. JEEVES. 1961. Direct search solution of numerical and statistical problems. *J. Assoc. Comp. Mach.* 8:212-221.
- NEWMAN, D. W. 1988. The optimal forest rotation: A discussion and annotated bibliography. USDA For. Serv. Gen. Tech. Rep. SE-48.
- RIITTERS, K., J. D. BRODIE, and C. KAO. 1982. Volume versus value maximization illustrated for Douglas-fir with thinning. *J. For.* 80:86-89,107.
- ROISE, J. P. 1986. An approach for optimizing residual diameter class distributions when thinning even-aged stands. *For. Sci.* 32:871-881.
- SMITH, D. M. 1962. *The practice of silviculture*. Ed. 7. Wiley, New York. 578 p.
- WYKOFF, W. R., N. L. CROOKSTON, and A. R. STAGE. 1982. User's guide to the Stand Prognosis Model. USDA For. Serv. Gen. Tech. Rep. INT-133.
- WYKOFF, W. R. 1986. Supplement to the user's guide for the Stand Prognosis Model-Version 5.0. USDA For. Serv. Gen. Tech. Rep. INT-208.

APPENDIX A

The any-aged stand management problem is to determine the best sequence of species and diameter-class harvesting rates and planting intensities for an existing stand over an infinite time horizon. Define the n -dimensional vector $x(t)$ as the stand state at the beginning of time t , and let $u(t)$ be an m -

dimensional vector of harvest controls where n and m are the number of state and control variables, respectively. Typically $\mathbf{x}(t)$ represents the frequency distribution of trees by diameter and species class and $\mathbf{u}(t)$ represents the harvest levels for various classes. Each element $u_j(t)$, $j = 2, \dots, m$, of $\mathbf{u}(t)$ is defined as the fraction of trees harvested from diameter and species class j , and for biological realism, each element is constrained to be between 0.0 and 1.0. Because planting is required either to establish plantations or to augment natural regeneration in uneven-aged stands, the control variable for the smallest size class $u_1(t)$ is unconstrained. When $u_1(t) < 0$, $u_1(t)$ is the number of trees planted; when $u_1(t) > 0$, $u_1(t)$ is the number of trees harvested from the seedling size class.

In addition to the state and control variables, we define a practical unit of time and a resource valuation function. In forest growth and yield models the period is usually some integer multiple of 5 years, depending on the remeasurement period used in the inventory system. Let $R[\mathbf{x}(t), \mathbf{u}(t)]$ denote the revenue obtained in period t , where the resource is in state $\mathbf{x}(t)$ at the beginning of this period before harvesting, and the harvest and planting control $\mathbf{u}(t)$ takes place at the beginning of the period. Let δ denote a discount factor related to the real discount (interest) rate r by $\delta = 1/(1 + r)$.

Using the above notation and letting T represent the planning horizon length, we define the T -horizon stand optimization problem as

$$\max_{\{\mathbf{u}(t), t=0,1,\dots,T-1\}} J_T[\mathbf{x}(0)] = \sum_{t=0}^{T-1} \delta^t R[\mathbf{x}(t), \mathbf{u}(t)] + \delta^T G[\mathbf{x}(T)] \quad (1)$$

where $\mathbf{x}(0)$ is the initial stand state and $G[\mathbf{x}(T)]$ is the terminal payoff function for the stand in state $\mathbf{x}(T)$. The maximization equation is solved subject to an n -dimensional difference equation for the stand dynamics:

$$\mathbf{x}(t + 1) = \mathbf{f}[\mathbf{x}(t), \mathbf{u}(t)], \quad t = 0, 1, \dots, T - 1. \quad (2)$$

Equation (2) is an implicit representation of the stand growth processes, which include natural regeneration, tree growth, and survival.

The solution to equations (1) and (2) is the control variable set $\{\mathbf{u}(t), t = 0, 1, \dots, T - 1\}$ that maximized $J_T[\mathbf{x}(0)]$, the present value of the existing stand over a T -period planning horizon. When $T \rightarrow \infty$, equations (1) and (2) define the *any-aged* management problem. In this case, harvesting and planting may take place simultaneously in any period, and there are no constraints on the stand size or age structure. Because the sequence of residual diameter distributions may take on any structure, the optimal management regime is any-aged.

APPENDIX B

This appendix lists the keyword file used by the Stand Prognosis Model (see Wykoff 1986 and Crookston 1985) in the development of the optimal plantation management regimes. This run begins with bare ground and plants 600 white pine/ac after site preparation is completed. During the 120-year projection (24 cycles of 5 years), the regeneration establishment model (Ferguson et al. 1986) will be called if thinning removes at least 20 ft²/ac. If the thinning leaves less than 80 ft²/ac then a regeneration cut is assumed to have occurred; site preparation is performed and the establishment and development of both advanced and subsequent regeneration are simulated. If the thinning leaves at least 80 ft²/ac then site preparation is not done, and only

advanced regeneration is simulated. Thinning keywords (THINDBH) do not appear in this file because they are written dynamically during execution by the optimizer.

```
STDIDENT
THE BOBS          HOOKE & JEEVES; PLANT 600 TREES & GROW 120 YEARS
NOTREES
STDINFO          18.0      570.0      0.00      8.0      3.0      34.0
NUMTRIP          5.
TIMEINT          5.
NUMCYCLE          24.
ESTAB            0.
PLANT            2.      1.      600.
MECHPREP          1.      50.
BURNPREP          1.      30.
MINREP           10.
OUTPUT           0.
END
COMMENT
Schedule a normal call to ESTAB whenever a regeneration cut occurs:
END
IF
  (FRAC((CYCLE-1)/4)EQ 0) AND (CYCLE NE 1) &
  AND (((BBA-ABA)GT 20) AND (ABA LT 80))
THEN
ESTAB
MECHPREP          0.      20.
END
ENDIF
COMMENT
Otherwise pump in advanced regeneration every 20 years:
END
IF
  (FRAC((CYCLE-1)/4)EQ 0) AND (CYCLE NE 1) &
  AND (((BBA-ABA)LE 20) OR (ABA GE 80))
THEN
ESTAB
MECHPREP          0.      0.
TALLYONE          3.
END
ENDIF
PROCESS
STOP
```