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Taper and Volume Equations for Selected Appalachian Hardwood Species

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$$\sqrt{B^2 - 4AC}$$

$$(a_2 - Y)^3 J_2$$

$$V^3 - Y^3$$

$$I_1 = 1, X \leq a_2$$

The Author

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Abstract

Coefficients for five taper/volume models are developed for 18 Appalachian hardwood species. Each model can be used to estimate diameter at any point on the bole, height to any preselected diameter, and cubic-foot volume between any two points on the bole. The resulting equations were tested on six sets of independent data and an evaluation of these tests is included. A wide variety of volume tables can be constructed with the models; some examples are given.

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Errata Sheet

For Research Paper NE-490 the following corrections should be made:

Page 3--under the heading Analysis

H = height up the bole from groundline to lower should read

H = height up the bole from groundline (feet)

Pages 3, 5, & 11

Polynominal should read polynomials

Page 4--under table 3 (1st line)

The equation should read

$$D^2/DBH^2 = (10^{2b_0})(DBH^{2b_1-2})$$

Current interest in multiple-product timber harvesting has generated a need for improved volume prediction for individual trees and yield prediction for stands. Knowledge of total cubic-foot or board-foot volumes is no longer sufficient. We now need to know what portions of a tree can be used for specific products, and we need to identify the entire array of products that can be obtained from specific stands.

When available, empirical data will provide some of the answers; however, such information will generally be restricted to existing product specifications. Therefore, to interpolate and extrapolate empirical data, and to provide greater flexibility with changing specifications and with new products, a more generalized approach is needed. One of the most important elements in such a system is reliable taper/volume equations for the species in question. Such equations enable the user to estimate the diameter at any point on the bole, the height to any predetermined diameter, and the volume between any two points on the bole.

Although various methods of developing taper/volume equations have been proposed (Kozak and others 1969; Kozak and Smith 1966; Max and Burkhart 1976; Fries and Matern 1965; Bennett and Swindel 1972; Goulding and Murray 1976; Demaerschalk 1971, 1972, 1973a, 1973b; Bruce and others 1968; Ormerod 1973; Clutter 1980; Cao and others 1980; and Demaerschalk and Kozak 1977), the information is either theoretical or limited primarily to softwood species. Until recently taper functions for eastern hardwood species were not available. However, the work by Hilt (1980) for upland oaks in the Midwest, coupled with the results reported herein, should fill most of the voids.

Data Collection

The objective of this study was to develop taper and volume equations for selected commercial Appalachian hardwood species. Stem measurements were collected from 1,162 trees in West Virginia and southwestern Virginia. Eighteen species were sampled and data were obtained from 39 different stands on poor to excellent sites. Data for black tupelo and black oak came from only 5 stands, whereas sugar maple measurements came from 17 locations; the average was 10 stands per species.

Trees without forks below midheight were randomly selected, with stratification by species and dbh (diameter at breast height) class. Although a balanced distribution by dbh and total height was desired, dbh alone was used in selecting sample trees for practicality and economy. The resulting sample was well balanced by diameter class for nearly all species, whereas the total height distribution was somewhat uneven. With very few exceptions, three to five trees per 1-inch diameter class (between 5 and 22 inches) were selected for each species.

Table 1.—Simple statistics for trees included in the sample

Species	Variable	Range	Mean	SD	No. of sample trees
Red maple	dbh (inches)	5.1 - 22.0	12.4	5.0	76
	total height (feet)	48.1 - 103.6	76.2	12.7	
Sugar maple	dbh	5.3 - 22.3	12.9	4.9	63
	total height	46.9 - 109.1	80.1	12.9	
Sweet birch	dbh	4.6 - 18.9	11.2	4.0	64
	total height	54.7 - 87.7	70.3	9.4	
Yellow birch	dbh	5.3 - 22.4	12.8	4.7	66
	total height	53.3 - 90.4	73.8	7.8	
Hickory, sp.	dbh	4.9 - 22.5	13.0	5.0	60
	total height	47.0 - 113.2	80.2	17.7	
American beech	dbh	4.7 - 21.0	12.4	4.6	62
	total height	43.8 - 103.1	74.2	14.2	
White ash	dbh	5.8 - 21.5	13.7	4.6	70
	total height	61.0 - 105.1	85.6	11.0	
Yellow-poplar	dbh	4.7 - 22.4	13.4	5.2	75
	total height	44.6 - 118.5	84.0	17.4	
Cucumbertree	dbh	5.1 - 22.4	12.7	4.7	56
	total height	50.0 - 109.4	79.7	14.7	
Black tupelo	dbh	5.1 - 22.5	12.4	4.6	58
	total height	35.7 - 87.1	64.3	12.6	
Black cherry	dbh	5.7 - 21.6	12.7	4.8	78
	total height	45.0 - 107.2	77.6	14.0	
White oak	dbh	6.0 - 21.9	11.8	4.4	84
	total height	47.4 - 88.7	69.7	10.5	
Scarlet oak	dbh	6.0 - 20.2	12.4	3.9	49
	total height	53.4 - 93.3	74.9	10.5	
Chestnut oak	dbh	5.6 - 22.0	13.4	4.6	61
	total height	61.0 - 98.7	76.1	8.2	
Red oak	dbh	5.8 - 22.5	13.3	4.8	72
	total height	55.0 - 103.2	79.4	11.8	
Black oak	dbh	5.4 - 21.9	12.7	4.1	45
	total height	54.5 - 93.3	74.0	8.9	
Black locust	dbh	5.3 - 22.1	13.3	4.8	60
	total height	38.5 - 106.4	81.9	16.8	
American basswood	dbh	4.6 - 21.3	13.0	4.8	63
	total height	57.4 - 117.5	85.0	14.8	
All species	dbh	4.6 - 22.5	12.8	4.7	1162
	total height	35.7 - 118.5	77.1	14.0	

Dbh of the sample trees (Table 1) ranged from 4.6 to 22.5 inches (Mean = 12.8 inches) and total height ranged from 35.7 to 118.5 feet (Mean = 77.1 feet). Sample size ranged from 45 trees for black oak to 84 trees for white oak.

Dbh and bark thickness (at breast height) were measured and recorded for each sample tree. Stem profile data (diameters outside bark and height above ground) were obtained at eight points on the bole: stump (1 foot above ground) and at approximately 1/8, 1/4, 3/8, 1/2, 5/8, 3/4, and 7/8 of total height. Stem measurements for most of the sample trees were obtained during the fall, winter, and spring with a Barr and Stroud[1] optical dendrometer; however, the sample also includes direct measurements from 246 felled trees. Total height (in feet, from groundline to tip) was obtained either with the dendrometer or from direct measurement of felled trees.

Analysis

A variety of published taper equations was examined and tested on the stem measurements in the sample. The models ranged from simple to very complex; from models with only one coefficient to segmented polynominals having two join points and six coefficients. Some were discarded because they did not fit the data well and prediction was poor; others were discarded because they were too complex.

The five models that were retained (Bruce and others 1968; Demaerschalk 1972; Kozak and others 1969; Max and Burkhart 1976; and Ormerod 1973) still ran the gamut from simple to complex (Tables 2 - 6); however, they all fit the data reasonably well and yielded good predictions in subsequent tests. Variables and regression coefficients common to all five models are defined as follows:

DBH	=	diameter at breast height (inches)
TH	=	total tree height from groundline to tip (feet)
H	=	height up the bole from groundline to lower
D	=	diameter at height "H" (inches)
HL	=	height up the bole from groundline to lower limit of volume calculation (feet)
HU	=	height up the bole from groundline to upper limit of volume calculation (feet)
V	=	volume of bolewood section between "HL" and "HU" (cubic feet)
b_0 - b_6	=	regression coefficients estimated from the sample data.

In Tables 2 to 6, equation (1) estimates bole diameter at any height H above ground. This form of each model was used in all analyses to estimate the regression coefficients for the sample data. Equation (2) is the inverse of equation (1); it estimates the height above ground to any preselected bole diameter D. Equation (3) estimates cubic foot volume of the bole between HL and HU. Equation (3) was derived by integrating equation (1) between the limits HL and HU:

$$\int_{HL}^{HU} \frac{\pi}{4} DBH^2 \hat{f}(H)\, dH$$

The model by Bruce and others is a rather lengthy polynominal that the authors refer to as their "final equation" in

[1] The use of trade, firm, or corporation names in this publication is for the information and convenience of the reader. Such use does not constitute an official endorsement or approval by the U.S. Department of Agriculture or the Forest Service of any product or service to the exclusion of others that may be suitable.

Table 2.—Bruce and others' taper and volume functions

$$D^2/DBH^2 = b_1(X^{1.5})(10^{-1}) + b_2(X^{1.5} - X^3)(DBH)(10^{-2}) + b_3(X^{1.5} - X^3)(TH)(10^{-3}) + b_4(X^{1.5} - X^{32})(TH)(DBH)(10^{-5}) + b_5(X^{1.5} - X^{32})(TH^{0.5})(10^{-3}) + b_6(X^{1.5} - X^{40})(TH^2)(10^{-6}) \quad (1)$$

Where:

$X = (TH - H)/(TH - 4.5)$

H : Must use an iterative method; changing the value for H in equation (1) until the predicted value of D is satisfactorily close to the desired value for D (2)

$$V = (-.005454)(DBH^2)(TH - 4.5)\left[\frac{2A}{5}(XU^{2.5} - XL^{2.5}) + \frac{B}{4}(XU^4 - XL^4) + \frac{C}{33}(XU^{33} - XL^{33}) + \frac{E}{41}(XU^{41} - XL^{41})\right] \quad (3)$$

Where:

$XU = (TH - HU)/(TH - 4.5)$

$XL = (TH - HL)/(TH - 4.5)$

$A = (b_1)(10^{-1}) + (b_2)(DBH)(10^{-2}) + (b_3)(TH)(10^{-3}) + (b_4)(TH)(DBH)(10^{-5}) + (b_5)(TH^{0.5})(10^{-3}) + (b_6)(TH^2)(10^{-6})$

$B = -\left[(b_2)(DBH)(10^{-2}) + (b_3)(TH)(10^{-3})\right]$

$C = -\left[(b_4)(TH)(DBH)(10^{-5}) + (b_5)(TH^{0.5})(10^{-3})\right]$

$E = -\left[(b_6)(TH^2)(10^{-6})\right]$

Table 3.—Demaerschalk's taper and volume functions

$$D^2/DBH^2 = (10^{2b_0})(DBH^{2b_1 - 2}) \left[(TH - H)^{2b_2}\right] (TH^{2b_3}) \quad (1)$$

$$H = TH - \left[(10^{-b_0})(D)(DBH^{-b_1})(TH^{-b_3})\right]^{1/b_2} \quad (2)$$

$$V = \frac{(0.005454)(10^{2b_0})(DBH^{2b_1})(TH^{2b_3})(X_1^Z - X_2^Z)}{Z} \quad (3)$$

Where:

$$Z = (2 \times b_2) + 1$$

$$X_1 = TH - HL$$

$$X_2 = TH - HU$$

Table 4.—Kozak and others' taper and volume functions

$$D^2/DBH^2 = b_1(H/TH - 1) + b_2(H^2/TH^2 - 1) \quad (1)$$

$$H = \frac{(-b_1 TH) - \sqrt{(b_1 TH)^2 - 4b_2 \left(b_0 TH^2 - \frac{D^2 TH^2}{DBH^2}\right)}}{2 b_2} \quad (2)$$

Where:

$$b_0 = -b_1 - b_2$$

$$V = (0.005454\, DBH^2) \left[b_0 (HU - HL) + \frac{b_1 (HU^2 - HL^2)}{2TH} + \frac{b_2 (HU^3 - HL^3)}{3TH^2} \right] \quad (3)$$

Table 5.—Max and Burkhart's taper and volume functions

$$D^2/DBH^2 = b_1(H/TH - 1) + b_2(H^2/TH^2 - 1) + b_3(a_1 - H/TH)^2 I_1 + b_4(a_2 - H/TH)^2 I_2 \quad (1)$$

Where:

a_1, a_2 = join points

$I_1 = 1,\ H/TH \leqslant a_1$

$\quad = 0,\ H/TH > a_1$

$I_2 = 1,\ H/TH \leqslant a_2$

$\quad = 0,\ H/TH > a_2$

$$H = TH/2A \left[-B - \sqrt{B^2 - 4AC} \right] \quad (2)$$

Where:

$A = b_2 + I_1' b_3 + I_2' b_4$

$B = b_1 - 2 I_1' a_1 b_3 - 2 I_2' a_2 b_4$

$C = -b_1 - b_2 - D^2/DBH^2 + I_1' a_1^2 b_3 + I_2' a_2^2 b_4$

$I_1' = 1,\ D \geqslant d_1$

$\quad = 0,\ D < d_1$

$I_2' = 1,\ D \geqslant d_2$

$\quad = 0,\ D < d_2$

d_1 = estimated diameter at height $a_1 TH$

$\quad = DBH \sqrt{b_1(a_1 - 1) + b_2(a_1^2 - 1)}$

d_2 = estimated diameter at height $a_2 TH$

$\quad = DBH \sqrt{b_1(a_2 - 1) + b_2(a_2^2 - 1) + b_3 (a_1 - a_2)^2}$

$$V = 0.005454\, DBH^2 TH \left\{ \frac{b_2}{3}(X^3 - Y^3) + \frac{b_1}{2}(X^2 - Y^2) - (b_1 + b_2)(X - Y) - \frac{b_3}{3}\left[(a_1 - X)^3 I_1 - (a_1 - Y)^3 J_1\right] - \frac{b_4}{3}\left[(a_2 - X)^3 I_2 - (a_2 - Y)^3 J_2\right] \right\} \quad (3)$$

Where:

$X = HU/TH$

$Y = HL/TH$

$I_1 = 1,\ X \leqslant a_1$

$\quad = 0,\ X > a_1$

$I_2 = 1,\ X \leqslant a_2$

$\quad = 0,\ X > a_2$

$J_1 = 1,\ Y \leqslant a_1$

$\quad = 0,\ Y > a_1$

$J_2 = 1,\ Y \leqslant a_2$

$\quad = 0,\ Y > a_2$

Table 6.—Ormerod's taper and volume functions

$$D^2/DBH^2 = \left[\frac{TH - H}{TH - 4.5}\right]^{2b_1} \qquad (1)$$

$$H = TH - \left[\left(\frac{D}{DBH}\right)^{1/b_1}(TH - 4.5)\right] \qquad (2)$$

$$V = \frac{(0.005454\ DBH^2)\ (4.5 - TH)}{Y} \left[\left(\frac{HU - TH}{4.5 - TH}\right)^Y - \left(\frac{HL - TH}{4.5 - TH}\right)^Y\right] \qquad (3)$$

Where:

$Y = (2 \times b_1) + 1$

the development of red alder taper and volume systems. The model is conditioned so that D = 0 when H = TH. In addition to its size, particularly in the expression for volume, the model has another disadvantage for some users: It cannot be rewritten in terms of H; therefore, estimating height above ground to the 8-inch mark (for example) must be handled by an iterative procedure (e.g., interval halving, Newton-Raphson method, etc.)

Max and Burkhart's model is also a rather complicated approach to taper/volume equations. Their approach was to develop three separate submodels that describe the neiloid frustum of the lower bole, the paraboloid frustum of the middle bole, and the conical shape of the upper portion. The three submodels are then spliced together at two "join points" into an overall segmented polynominal tree model. The version selected for this study was their quadratic-quadratic-quadratic model. The equation is conditioned so that D = 0 when H = TH. To use this approach, one must first decide which of the three bole segments is appropriate; this determines the values for the two dummy variables. During the analyses, optimal join points were simultaneously estimated for each species along with the regression coefficients (Appendix Table 15). Join points are simply proportions of total height; the lower join point is usually close to 0.1 and the upper is usually between 0.6 and 0.7.

Both Demaerschalk's equation and the equation developed by Kozak and others are much easier to use. Both are conditioned so that D = 0 when H = TH, and both can be rewritten in terms of H. The model by Kozak and others can easily be solved with any electronic calculator possessing a square root function. Demaerschalk's model requires a somewhat more sophisticated machine since the exponents (regression coefficients) are not whole numbers. Demaerschalk's taper equation was derived from the basic logarithmic volume equation:

$$\text{Log } V = a + b \text{ Log } DBH + c \text{ Log } TH$$

Therefore, the volume equation obtained by integrating the taper function (Equation 3, Table 3) is compatible with this original logarithmic volume equation; that is, they both yield the same results. Actually, Demaerschalk's equation is compatible only if the model is not fitted separately for taper and volume.

Ormerod's equation has only one coefficient that needs to be estimated, hence the basic taper equation is very easy to use. The model is conditioned so that D = 0 when H = TH; and D = DBH when H = 4.5 feet.

All of the models were fitted to the sample data using a computer program for nonlinear regression. There were eight observations per sample tree and each observation consisted of DBH, TH, D, and H. The dependent variable in each case was D^2/DBH^2; where D = diameter outside bark and DBH = dbh outside bark.

Results of all the regression analyses are presented in the Appendix (Tables 12 - 16). Including all of the results allows the user to select whatever model he deems most appropriate. He can choose coefficients for individual species or use those for all species combined. Thus a user has great flexibility in customizing a taper/volume prediction system to suit his needs.

A summary of the regression results is presented in Table 7. One can readily see that there is little difference between models in how well they fit the sample data. Not too surprisingly, Ormerod's model with only one coefficient does not perform quite as well as do the more complicated equations. However,

Table 7.—Comparison of taper models for the overall equations (all species combined); $Y = D^2/DBH^2$ for all models[a]

Model	R^2	Standard error of the estimate
Bruce and others	.912	.132
Demaerschalk	.863	.165
Kozak and others	.864	.164
Max and Burkhart	.911	.133
Ormerod	.800	.200

[a] D = diameter outside bark
DBH = diameter at breast height outside bark

when one considers model complexity and limitations, as well as goodness of fit as evidenced by the statistics in Table 7, the decision as to which model is "best" is anything but clear cut.

Testing the Equations

The equations were evaluated to see how well a taper-based approach would work on independent sets of data, and also to see if any of the models were significantly better than the others. The tests compared diameter prediction, height prediction, and the prediction of volume for different-sized pieces (logs, bolts, and entire merchantable boles).

The data used in the testing were obtained from six different stands (three in northern West Virginia, two in the middle of the state, and one in the southern portion). Species composition varied from Allegheny hardwoods to upland hardwoods. Test area 1 was predominately black cherry, and test 6 was nearly half yellow-poplar. Mean stand age ranged from 55 to 70 years, and composite site index (for all species combined) varied from 60 to 80 feet at 50 years.

The test data consisted of 1,088 pieces from 593 trees (Table 8). Mean dbh of the test trees was 8.1 inches (range = 4.8 to 19.1 inches) and mean total height was 69 feet (range = 25 to 104 feet). Mean piece length was 13 feet; however, the size ranged from less than 1 foot for small cull sections to over 82 feet for entire merchantable portions of the bole.

Actual heights, to the lower and upper ends of each piece, were used in each equation to predict the large end and small end diameters outside bark. The predicted diameters were then compared to the actual values using the following three criteria (adapted from Cao and others 1980): (1) bias (the mean of the differences between the predicted and actual values), (2) mean absolute difference (the mean of the absolute differences), and (3) standard deviation of the differences. The same procedures were followed for comparing height predictions; actual large- and small-end diameters were used to estimate lower and upper heights respectively.

The comparison of volumes for each piece followed a similar pattern, except that no actual figures were available (i.e., the pieces were not immersed in water to determine true volumes). Hence, volume comparison was between two estimates: Smalian's formula versus the volume equations obtained by integrating the taper functions. Volume (including bark) had been previously computed by Smalian's formula for each log and bolt that was bucked from the test trees. Smalian's volume for longer test pieces was determined simply by summing the individual log and bolt volumes contained within the piece.

All comparisons were made by using the appropriate coefficients for individual species in each model. Biases, mean absolute differences, and standard deviations for these comparisons are shown in Table 9.

Table 8.—Summary statistics for the trees and pieces from the six test sites

	Test 1	Test 2	Test 3	Test 4	Test 5	Test 6
No. of trees	83	95	101	116	95	103
Dbh (in):						
Mean	8.3	8.7	6.8	8.4	8.8	7.9
Std. deviation	2.0	3.2	1.5	3.0	3.1	2.1
Minimum	5.0	5.0	4.8	5.0	5.0	5.0
Maximum	13.0	17.6	11.7	17.3	19.1	15.0
Total height (ft):						
Mean	61	70	67	69	73	72
Std. deviation	8	13	9	12	13	12
Minimum	42	25	44	46	47	48
Maximum	74	91	92	92	93	104
No. of pieces	123	149	182	228	200	206
Piece length (ft):						
Mean	12.3	13.1	11.7	13.6	11.8	14.9
Std. deviation	12.9	13.4	13.6	10.1	15.6	18.5
Minimum	1.0	1.2	1.2	1.0	0.6	1.1
Maximum	54.8	61.2	61.5	72.2	72.6	82.5
Dob small end (in):						
Mean	6.3	6.7	5.3	6.0	6.6	5.5
Std. deviation	1.8	2.8	1.4	2.3	2.6	1.6
Minimum	3.7	4.0	4.0	4.0	3.9	4.0
Maximum	10.6	15.5	11.0	15.8	16.0	12.7
Dob large end (in):						
Mean	7.8	8.3	6.4	7.8	8.0	7.4
Std. deviation	2.2	3.2	1.7	3.2	2.9	2.8
Minimum	4.1	4.2	4.2	4.1	4.1	4.0
Maximum	13.8	18.7	12.7	19.2	20.2	21.1

These measures of accuracy and precision indicate that all models do quite well in predicting diameter, height, and volume. Comparing models in Table 9 is easier if we combined the pieces from all six sites and recompute the three test criteria (i.e., a weighted average of the values in Table 9). If we then ignore any resulting negative signs, the absolute bias for predicting diameter ranged from 0.012 inches (0.2 percent of the mean) with the model by Kozak and others to 0.22 inches (3.2 percent of the mean) with Ormerod's model (Table 10). Mean absolute difference was about 1/2 inch for diameter with all five models. Absolute bias in height prediction ranged from 0.028 feet (0.1 percent of the mean) with Demaerschalk's model to 2.7 feet (12.5 percent of the mean) with Ormerod's model; mean absolute difference ranged from 3 to 5 feet. The coefficient of variation was about 100 percent for diameter and height for all five models. With volume prediction, absolute bias ranged from

Table 9.—Bias,[a] mean absolute difference (|D|), and standard deviation of the difference (SD) of the various models for six independent sets of data

Model	Test site	Diameter outside bark			Height			Volume including bark		
		Bias	|D|	SD	Bias	|D|	SD	Bias	|D|	SD
		inches			*feet*			*cubic feet*		
Bruce and others	1	.156	.342	.321	.794	2.156	1.994	-.059	.240	.408
	2	.087	.428	.448	- .283	2.649	2.511	-.119	.299	.550
	3	.134	.383	.446	1.046	3.329	3.911	.050	.253	.642
	4	.178	.434	.483	.642	2.920	3.225	.025	.376	.895
	5	.383	.605	.667	2.058	4.365	5.439	.157	.311	.440
	6	.050	.375	.448	.362	2.688	2.662	-.090	.305	.570
Demaerschalk	1	.063	.332	.298	.661	2.674	2.179	.033	.252	.420
	2	-.018	.489	.369	- .203	3.858	2.864	.137	.338	.570
	3	.039	.453	.419	.360	4.316	3.912	.119	.300	.642
	4	-.054	.608	.502	- .552	4.375	3.185	.234	.493	1.013
	5	.227	.669	.640	1.318	4.914	4.733	.313	.437	.745
	6	-.131	.524	.500	-1.086	4.329	3.746	.046	.315	.507
Kozak and others	1	.056	.322	.293	.475	2.553	2.124	.027	.247	.385
	2	-.051	.471	.374	- .566	3.735	2.848	.086	.326	.575
	3	-.019	.434	.433	- .307	4.226	4.197	.070	.289	.642
	4	-.025	.576	.480	- .436	4.294	3.226	.282	.506	1.088
	5	.261	.657	.653	1.507	4.868	4.751	.352	.459	.805
	6	-.144	.508	.499	-1.404	4.312	3.777	.011	.307	.513
Max and Burkhart	1	.044	.335	.346	- .108	2.024	1.979	-.160	.267	.448
	2	-.039	.448	.386	- .873	2.926	2.645	-.145	.335	.552
	3	.004	.392	.444	- .284	3.186	3.617	-.021	.260	.628
	4	.058	.473	.484	- .297	3.177	3.145	.056	.376	.836
	5	.308	.633	.691	1.551	4.151	4.175	.186	.332	.550
	6	-.054	.392	.454	- .407	2.774	2.575	-.100	.298	.540
Ormerod	1	-.101	.314	.322	-1.162	2.922	2.811	-.164	.261	.446
	2	-.313	.483	.449	-3.494	4.767	4.439	-.290	.402	.590
	3	-.204	.405	.481	-2.872	4.960	6.793	-.107	.279	.621
	4	-.310	.546	.510	-3.294	5.320	5.028	-.235	.421	.800
	5	.019	.537	.633	- .706	4.922	6.342	-.041	.323	.466
	6	-.355	.545	.685	-4.398	6.020	7.640	-.318	.398	.751

[a]Positive bias = overestimation; negative bias = underestimation.

0.003 cubic feet (0.1 percent of the mean) for the model by Bruce and others to 0.19 cubic feet (4.7 percent of the mean) with Ormerod's model. Mean absolute difference was between 0.3 and 0.37 cubic feet with a coefficient of variation approaching 200 percent.

It appears from the results in Table 9 that the effects of test location were minimal. In general, the models were less accurate and less precise on the trees from site 5. The trees from this area were larger, although mean dbh and mean total height were not significantly greater than on site 2.

In a further attempt to determine the best model, the accuracy and precision of each model were examined separately for different portions of the bole. The comparisons previously described, using the six independent sets of data, were repeated three times. First, only the butt pieces (of any length) from the six test sites were used in the comparisons. Second, only butt pieces ⩽ 12.3 feet in length were used. And, third, only upper pieces (of any length) were used to compare the models. To keep from burdening the reader with too many numbers, I have summarized the results from these additional tests in Table 10. Weighted mean values for the three test criteria, computed over

Table 10.—Bias,[a] mean absolute difference (|D|), and standard deviation of the difference (SD) of the various models averaged over all six test sites for different bole sections

Model	Test pieces used	Diameter outside bark			Height			Volume including bark		
		Bias	|D|	SD	Bias	|D|	SD	Bias	|D|	SD
		inches			*feet*			*cubic feet*		
Bruce and others	All[b]	.169	.434	.498	.807	3.086	3.644	.003	.304	.628
	Butts - any	.248	.500	.540	.604	2.099	2.690	-.045	.547	.890
	Butts ≤ 12.3 ft	.229	.426	.522	.791	1.841	2.378	-.139	.241	.400
	Uppers - any	.118	.391	.464	.941	3.734	4.023	.034	.145	.265
Demaerschalk	All	.017	.530	.494	.028	4.191	3.672	.158	.368	.710
	Butts - any	.019	.522	.486	.089	4.284	3.753	.337	.626	1.011
	Butts ≤ 12.3 ft	.266	.525	.476	2.160	4.425	3.634	.164	.258	.365
	Uppers - any	.015	.535	.499	- .012	4.132	3.617	.040	.199	.304
Kozak and others	All	.012	.511	.494	- .156	4.117	3.741	.152	.369	.745
	Butts - any	.003	.502	.490	- .161	4.148	3.874	.323	.635	1.071
	Butts ≤ 12.3 ft	.212	.502	.492	1.652	4.184	3.871	.141	.258	.386
	Uppers - any	.017	.517	.496	- .152	4.099	3.650	.040	.195	.302
Max and Burkhart	All	.059	.455	.501	- .033	3.116	3.234	-.014	.316	.623
	Butts - any	.116	.512	.520	- .121	2.246	2.555	-.063	.570	.887
	Butts ≤ 12.3 ft	.123	.417	.464	.200	1.804	2.077	-.100	.234	.398
	Uppers - any	.022	.417	.485	.024	3.687	3.496	.017	.149	.237
Ormerod	All	-.217	.486	.548	-2.742	4.971	5.974	-.193	.354	.646
	Butts - any	-.426	.582	.647	-5.087	6.473	7.518	-.434	.653	.889
	Butts ≤ 12.3 ft	-.363	.508	.624	-4.454	5.974	7.668	-.282	.313	.467
	Uppers - any	-.080	.422	.462	-1.205	3.987	4.429	-.035	.158	.278

[a]Positive bias = overestimation; negative bias = underestimation.
[b]All pieces = mean values from Table 9.

all six test sites, as well as weighted mean values from Table 9, are presented.

Looking at the values in Table 10, it is evident that for the most part the models performed best in the upper part of the bole and poorest in the lower butt section. In the upper bole, the actual difference among the models was very small. Except for bias in height prediction, there was no significant difference among the five models for any of the remaining criteria in the upper part of the bole. This may indicate that taper is relatively uniform in the upper bole of hardwood trees, even though a wide variety of species and sizes are considered. On the other hand, predictions for butt sections of the bole resulted in the greatest difference among the models.

Next, the mean values in Table 10 were ranked from 1 (lowest = best) to 5 (highest = poorest). The results of this ranking process are shown in Table 11. We see that for overall prediction (all sections of the bole) the "best" model depends on your objective. The model by Kozak and others was the best predictor of diameter. This model was the most accurate (lowest bias) and most precise (lowest standard deviation). For predicting height, however, Max and Burkhart's model did the best job by all three criteria, whereas the model by Bruce and others was better for predicting cubic foot volume. When the ranks were summed for diameter, height, and volume, the least bias (most accurate) models for overall prediction were Demaerschalk's, Max and Burkhart's, and the one by Kozak and others. The model by Max and Burkhart was somewhat more precise overall. When the three criteria are considered together, the models by Bruce and others and Max and Burkhart finished in first place. The models by Demaerschalk and Kozak and others ranked next, followed by Ormerod's.

If we look at how well the various models predicted values for butt pieces of any length, we find that the rankings lead

Table 11.—Rankings of the five models for diameter, height, and volume prediction

Model	All pieces				Butt pieces				Butts ⩽ 12.3′				Upper pieces			
	Rankings for			Sum	Rankings for			Sum	Rankings for			Sum	Rankings for			Sum
	Bias	|D|	SD		Bias	|D|	SD		Bias	|D|	SD		Bias	|D|	SD	
DIAMETER OUTSIDE BARK																
Bruce and others	4	1	3	8	4	1	4	9	3	2	4	9	5	1	2	8
Demaerschalk	2	5	2	9	2	4	1	7	4	5	2	11	1	5	5	11
Kozak and others	1	4	1	6	1	2	2	5	2	3	3	8	2	4	4	10
Max and Burkhart	3	2	4	9	3	3	3	9	1	1	1	3	3	2	3	8
Ormerod	5	3	5	13	5	5	5	15	5	4	5	14	4	3	1	8
HEIGHT																
Bruce and others	4	1	2	7	4	1	2	7	2	2	2	6	4	2	4	10
Demaerschalk	1	4	3	8	1	4	3	8	4	4	3	11	1	5	2	8
Kozak and others	3	3	4	10	3	3	4	10	3	3	4	10	3	4	3	10
Max and Burkhart	2	2	1	5	2	2	1	5	1	1	1	3	2	1	1	4
Ormerod	5	5	5	15	5	5	5	15	5	5	5	15	5	3	5	13
VOLUME INCLUDING BARK																
Bruce and others	1	1	2	4	1	1	3	5	2	2	4	8	2	1	2	5
Demaerschalk	4	4	4	12	4	3	4	11	4	4	1	9	4	5	5	14
Kozak and others	3	5	5	13	3	4	5	12	3	3	2	8	5	4	4	13
Max and Burkhart	2	2	1	5	2	2	1	5	1	1	3	5	1	2	1	4
Ormerod	5	3	3	11	5	5	2	12	5	5	5	15	3	3	3	9
SUM OF RANKINGS																
Bruce and others	9	3	7	19	9	3	9	21	7	6	10	23	11	4	8	23
Demaerschalk	7	13	9	29	7	11	8	26	12	13	6	31	6	15	12	33
Kozak and others	7	12	10	29	7	9	11	27	8	9	9	26	10	12	11	33
Max and Burkhart	7	6	6	19	7	7	5	19	3	3	5	11	6	5	5	16
Ormerod	15	11	13	39	15	15	12	42	15	14	15	44	12	9	9	30

to nearly identical conclusions. About the only change was that the difference between the top four models was less pronounced. Thus, so far, there seems to be little rationale for proclaiming one model superior.

However, if we look just at the more extreme portions of the bole, upper pieces of any length, and butt pieces ⩽ 12.3 feet, it is evident that Max and Burkhart's model ranked highest in most categories. Their model was more accurate (less biased) for all predictions in the lower 12.3 feet of the bole, and the precision of the model ranked high as well. However, even though Max and Burkhart's model ranked at the top for upper-bole predictions, this was mainly due to its consistent performance; because, as noted earlier, there was no significant difference among the five models in this portion of the tree.

Discussion and Application

Although the study results do not establish an indisputable "best" taper/volume model for Appalachian hardwoods, they do show some general trends. Based on the regression analyses using sample data (Table 7), the independent tests with data from six different stands, and our knowledge of the models' complexities and limitations, we can make some qualified recommendations:

(1) If the computations will not present a problem, use Max and Burkhart's model. Although this model did not rank highest in all cases, it was the most consistent performer. And it was particularly good, compared to the other four models, for predictions in the lower bole. However, it is not a simple model to use; therefore, a computer (and the neces-

sary software) or at least a programmable calculator with sufficient memory is required for efficient calculations.

(2) If, on the other hand, you can sacrifice some accuracy in the lower bole to gain simplicity in use, then the model by Kozak and others would be a good choice. Overall, their model, while not too precise, was as accurate as Max and Burkhart's (Table 11), and it is very easy to use.

Of course, as a user, the choice is yours; the coefficients presented in the Appendix tables provide considerable flexibility in designing a taper/volume estimating system to fit your needs. In selecting a model, you should consider the relative magnitude of the values in Tables 9 and 10 as well as the various rankings. Sometimes the rankings imply greater differences than were actually observed.

Although the models were fitted to outside bark data, the user can easily convert dob (diameter outside bark) estimates to diameter inside bark by using one of the bark options from Grosenbaugh (1974). Three bark options were presented by Grosenbaugh (1974), in his STX 3-3-73 timber cruising package, for estimating diameters inside bark at any point on the bole using the ratio of dbh inside bark to dbh outside bark. The three options are briefly described below (from Colaninno and others 1977):

Option 1 $D_{ib} = D_{ob}\ (DBH_{ib}/DBH_{ob})$
Bark thickness is assumed to be a constant proportion of dob throughout the height of the tree.

Option 2 $D_{ib} = D_{ob}\ (1.0 - (1.0 - DBH_{ib}/DBH_{ob})(1.0/(2.0 - D_{ob}/DBH_{ob})))$
Implies the proportion of bark decreases hyperbolically up the tree.

Option 3 $D_{ib} = D_{ob}\ (DBH_{ib}/DBH_{ob})(9.0/(10.0 - D_{ob}/DBH_{ob}))$
Implies the proportion of bark increases hyperbolically up the tree.

DBH_{ib}/DBH_{ob} ratios can be determined by measuring bark thickness in the field or by using average values determined from the data used in this study (Appendix, Table 17). To estimate height above ground (H) to a specified diameter inside bark (dib), the dib value must first be converted to diameter outside bark using one of the bark options.

However, if bark option 1 is satisfactory, the DBH_{ib} values can simply be used whenever inside bark predictions of diameter, height, or volume are desired. This eliminates the need to convert any of the final estimates.

All three options are presented because, so far, no one seems sure which option is most appropriate for a particular hardwood species. Although Option 1 is probably adequate for most hardwoods, studies by Wiant and Koch (1974), Boehmer and Rennie (1976), and Colaninno and others (1977) indicate that a single option is not adequate for all hardwood species. And a single option may not be satisfactory for trees of the same species from different geographical areas. A summary of recommendations for several species is presented below:

	Wiant and Koch (1974)	Boehmer and Rennie (1976)	Colaninno and others (1977)
	(STX bark option recommended)		
Yellow-poplar	1	1	2
Red maple	1	—	—
Hickory, sp.	—	—	2
Red oaks	1	1	1
White oaks	—	1	3

Even though a substantial amount of testing was done, there are still a couple of unanswered questions. Since the independent test data contained few big trees (mean dbh was 8.1 inches), additional testing in the large diameters (dbh 18+ inches) is desirable. Butt swell in larger trees might alter the results, although at least some of the taper models should be more accurate than Smalian's formula for volume estimation. Smalian's formula normally overestimates the volume of butt logs from large trees.

This leads to the second question: How close are taper-based volume estimates to the true volume? Since all of the tests (including those in most other studies) have simply compared one estimate against another, we really do not know. However, current research at the Forestry Sciences Laboratory in Princeton, West Virginia, using water displacement techniques, should soon provide the answer.

One problem is that total height is one of the required variables. Often this is not measured when timber is cruised; but it should be. It is the least ambiguous height measurement on a tree (no guessing about the 4- or 8-inch mark, etc.); and if a taper-based system is used, it is the only height measurement needed for most trees (height to where the bole breaks up would also be required on some trees). Actually, measuring total height is a small price to pay for having a complete, accurate, and consistent estimating system for bolewood. For some uses, an acceptable alternative would be to measure total height on a subsample of the cruised trees. These data could be used to construct a total-height/dbh curve for estimating the heights of the remaining trees.

Any of the taper models can easily be used to prepare a variety of different volume tables: total volume or merchantable volume, inside or outside bark, for an individual species or all species combined, etc. These are just a few of the options available to the user. Four example volume tables are included in this report (Appendix Tables 18 - 21) to show what can be obtained just by changing the upper and lower limits (HU and HL) of volume calculation. Short computer programs that will generate volume tables such as presented here have been developed for each of the five models. Source listings and input instructions for any or all programs are available upon request from the author. Note that volumes in Table 20 plus those in Table 21 equal the values in Table 19. This demonstrates the consistency of taper-based volume calculations. Note also that certain dbh-height combinations in Tables 18-21 are obviously unrealistic; however, these are permitted to simplify the programs and avoid arguments over arbitrary cutoff points.

As we have seen, a taper-based system, regardless of the model used or the goal in mind, provides accurate and consistent estimates of diameter, height, and volume for Appalachian hardwoods. Some models perform slightly better than others, but at the expense of simplicity. However, with the information provided (Tables 12 - 16), the user has considerable latitude in choosing the model and the coefficients that best suit his needs.

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Appendix A

Parameters for Appalachian Hardwoods Taper and Volume Functions

TABLE 12. PARAMETERS FOR EASTERN HARDWOOD TAPER AND VOLUME FUNCTIONS. MODEL: D**2/DBH**2 = F(DBH,TH,H) --- A POLYNOMIAL BY BRUCE, ET AL. (1968). ESTIMATED VALUES OF D ARE OUTSIDE BARK.

SPECIES	NO. OF OBSERV.	REGRESSION COEFFICIENTS B1	B2	B3	B4	B5	B6	STANDARD ERROR	R SQUARE
ALL	9296	10.2828	-2.1059	6.9869	-0.9731	0.9282	-11.7092	0.1319	0.9125
RED MAPLE	608	10.4294	-3.2706	6.9694	-2.4575	0.3857	-3.6133	0.1000	0.9362
SUGAR MAPLE	504	10.6713	0.1364	0.7001	-3.1851	-1.3048	-3.6224	0.1160	0.9284
SWEET BIRCH	512	10.0763	-3.4468	7.6647	8.0964	3.1484	-25.8929	0.1522	0.8959
YELLOW BIRCH	528	10.0781	-3.9630	6.9500	8.9909	16.3795	-41.7064	0.1443	0.9155
HICKORY, SP.	480	10.0801	4.0467	-2.5104	-2.1674	0.6558	-14.7264	0.1170	0.9425
AM. BEECH	496	10.0164	-1.1153	2.5557	11.9980	11.0472	-34.0218	0.1274	0.9188
WHITE ASH	560	10.1506	-3.2644	7.8975	-0.3633	16.2545	-24.6575	0.1121	0.9318
YELLOW-POPLAR	600	10.1220	-3.1400	8.7678	4.9025	6.0196	-17.1372	0.0845	0.9535
CUCUMBERTREE	448	10.1864	-1.3422	6.3870	-4.2010	5.9840	-9.0812	0.0839	0.9560
BLACK TUPELO	464	10.0640	0.9529	3.1858	-2.3718	2.6607	-14.2303	0.1265	0.9309
BLACK CHERRY	624	9.7644	-5.2088	13.5189	3.0840	9.4764	-19.6663	0.0904	0.9398
WHITE OAK	672	9.7215	-3.4985	11.1399	3.7322	7.7467	-32.9728	0.1167	0.9465
SCARLET OAK	392	10.6262	-1.0242	6.8596	-10.9711	-6.3697	-2.8698	0.1396	0.9335
CHESTNUT OAK	488	10.2809	-0.6879	3.7916	-2.4675	3.4765	-10.9332	0.1082	0.9325
RED OAK	576	10.2266	-3.0538	8.1870	-0.0747	4.8519	-18.6862	0.1071	0.9425
BLACK OAK	360	9.6401	0.5290	2.5419	3.0013	5.1115	-30.2225	0.1116	0.9552
BLACK LOCUST	480	9.9436	-2.7018	8.0709	-0.6156	7.1518	-14.2425	0.0977	0.9427
AM. BASSWOOD	504	10.4019	-2.0130	8.8898	-0.3135	10.1272	-18.9616	0.1186	0.9245

TABLE 13. PARAMETERS FOR EASTERN HARDWOOD TAPER AND VOLUME FUNCTIONS. MODEL: D**2/DBH**2 = 10.0**(2.0*B0) * DBH**(2.0*B1-2.0) * (TH-H)**(2.0*B2) * TH**(2.0*B3) --- DEMAERSCHALK (1972). ESTIMATED VALUES OF D ARE OUTSIDE BARK.

SPECIES	NO. OF OBSERV.	REGRESSION COEFFICIENTS B0	B1	B2	B3	STANDARD ERROR	R SQUARE
ALL	9296	0.1714	0.9999	0.9183	-0.9743	0.1649	0.8632
RED MAPLE	608	0.1162	0.9882	0.8345	-0.8639	0.1134	0.9176
SUGAR MAPLE	504	0.1380	1.0219	0.9421	-0.9956	0.1346	0.9033
SWEET BIRCH	512	0.0431	0.9728	1.0314	-0.9972	0.1945	0.8293
YELLOW BIRCH	528	0.0220	0.9423	1.1125	-1.0456	0.1875	0.8568
HICKORY, SP.	480	0.2187	1.0436	1.0795	-1.1775	0.1786	0.8655
AM. BEECH	496	0.0326	0.9450	0.9520	-0.9027	0.1525	0.8831
WHITE ASH	560	0.0383	1.0009	0.8870	-0.8769	0.1471	0.8823
YELLOW-POPLAR	600	0.0406	0.9665	0.7508	-0.7292	0.0987	0.9363
CUCUMBERTREE	448	0.1023	1.0075	0.7689	-0.8029	0.1074	0.9277
BLACK TUPELO	464	0.2912	1.0062	0.9676	-1.0887	0.1743	0.8681
BLACK CHERRY	624	-0.0513	0.9426	0.7167	-0.6353	0.1067	0.9158
WHITE OAK	672	0.2291	0.9826	1.0833	-1.1469	0.2033	0.8371
SCARLET OAK	392	0.3219	1.0526	1.1728	-1.3219	0.2246	0.8269
CHESTNUT OAK	488	0.0764	0.9974	0.9002	-0.9073	0.1367	0.8918
RED OAK	576	0.2583	1.0102	0.9610	-1.0657	0.1598	0.8716
BLACK OAK	360	0.1571	0.9997	1.2836	-1.3162	0.2069	0.8451
BLACK LOCUST	480	0.0119	0.9715	0.8091	-0.7747	0.1181	0.9158
AM. BASSWOOD	504	0.1768	1.0206	0.7515	-0.8251	0.1452	0.8865

TABLE 14. PARAMETERS FOR EASTERN HARDWOOD TAPER AND VOLUME FUNCTIONS. MODEL: D**2/DBH**2 = B1*(H/TH-1.)+B2*(H**2/TH**2-1.) -- KOZAK, ET AL (1969). ESTIMATED VALUES OF D ARE OUTSIDE BARK.

SPECIES	NO. OF OBSERV.	REGRESSION COEFFICIENTS B1	B2	STANDARD ERROR	R SQUARE
ALL	9296	-2.5116	1.1587	0.1645	0.8638
RED MAPLE	608	-2.2122	0.9514	0.1139	0.9166
SUGAR MAPLE	504	-2.5086	1.1911	0.1340	0.9037
SWEET BIRCH	512	-2.8191	1.3999	0.1948	0.8281
YELLOW BIRCH	528	-2.9832	1.5358	0.1914	0.8503
HICKORY, SP.	480	-2.8371	1.4219	0.1799	0.8629
AM. BEECH	496	-2.5942	1.2382	0.1532	0.8815
WHITE ASH	560	-2.3971	1.0821	0.1457	0.8840
YELLOW-POPLAR	600	-2.0085	0.7622	0.0992	0.9354
CUCUMBERTREE	448	-2.0518	0.8045	0.1063	0.9288
BLACK TUPELO	464	-2.7082	1.2785	0.1763	0.8645
BLACK CHERRY	624	-1.8830	0.6637	0.1078	0.9138
WHITE OAK	672	-3.0260	1.5193	0.2055	0.8331
SCARLET OAK	392	-3.1363	1.6148	0.2286	0.8198
CHESTNUT OAK	488	-2.4336	1.1115	0.1353	0.8935
RED OAK	576	-2.6281	1.2477	0.1598	0.8712
BLACK OAK	360	-3.1388	1.6648	0.2129	0.8351
BLACK LOCUST	480	-2.1358	0.8883	0.1167	0.9175
AM. BASSWOOD	504	-2.1236	0.8074	0.1437	0.8884

TABLE 15. PARAMETERS FOR EASTERN HARDWOOD TAPER AND VOLUME FUNCTIONS. MODEL: D**2/DBH**2=F(DBH,TH,H) - SEGMENTED POLYNOMIAL, MAX & BURKHART (1976) ESTIMATED VALUES OF D ARE OUTSIDE BARK.

SPECIES	NO. OF OBSERV.	REGRESSION COEFFICIENTS B1	B2	B3	B4	A1	A2	STANDARD ERROR	R SQUARE
ALL	9296	-3.9964	2.0087	-2.4624	41.9337	0.6381	0.1241	0.1328	0.9113
RED MAPLE	608	-3.9701	2.0384	-2.4816	25.1129	0.6034	0.1208	0.1015	0.9342
SUGAR MAPLE	504	-3.2884	1.6583	-2.3854	35.6192	0.5133	0.1209	0.1130	0.9321
SWEET BIRCH	512	-4.7790	2.5105	-2.7567	67.2975	0.6738	0.1082	0.1544	0.8928
YELLOW BIRCH	528	-3.5897	1.8331	-2.1859	55.3868	0.5857	0.1204	0.1480	0.9112
HICKORY, SP.	480	-3.6673	1.8308	-2.6598	43.4116	0.6060	0.1463	0.1150	0.9444
AM. BEECH	496	-3.4340	1.7394	-2.6702	34.6121	0.5172	0.1321	0.1307	0.9145
WHITE ASH	560	-3.8568	1.9413	-2.4527	887.9036	0.6130	0.0345	0.1128	0.9311
YELLOW-POPLAR	600	-4.8424	2.4509	-2.7919	22.0258	0.6978	0.1241	0.0849	0.9530
CUCUMBERTREE	448	-4.8937	2.4684	-2.6771	212.2433	0.7215	0.0507	0.0856	0.9542
BLACK TUPELO	464	-4.5210	2.2799	-3.0741	29.6952	0.6600	0.1694	0.1313	0.9255
BLACK CHERRY	624	-4.9738	2.5251	-3.0382	167.3393	0.6904	0.0544	0.0921	0.9375
WHITE OAK	672	-4.2643	2.1658	-3.0514	77.1744	0.6281	0.1207	0.1200	0.9435
SCARLET OAK	392	-3.9750	1.9950	-2.7309	70.5026	0.6274	0.1264	0.1471	0.9261
CHESTNUT OAK	488	-3.2804	1.6103	-2.3807	42.5301	0.5429	0.1188	0.1067	0.9343
RED OAK	576	-4.4635	2.2812	-2.6462	58.2976	0.6752	0.1131	0.1105	0.9389
BLACK OAK	360	-3.7816	1.9023	-2.3461	70.8956	0.6522	0.1258	0.1122	0.9547
BLACK LOCUST	480	-3.7918	1.8454	-1.9766	65.2427	0.6859	0.0830	0.0982	0.9420
AM. BASSWOOD	504	-5.2178	2.5939	-3.0258	193.6099	0.7251	0.0576	0.1187	0.9244

TABLE 16. PARAMETERS FOR EASTERN HARDWOOD TAPER AND VOLUME FUNCTIONS. MODEL: D**2/DBH**2 = ((TH-H)/(TH-4.5))**(2.0*B1) --- ORMEROD (1973). ESTIMATED VALUES OF D ARE OUTSIDE BARK.

SPECIES	NO. OF OBSERV.	REGRESSION COEFFICIENT B1	STANDARD ERROR	R SQUARE
ALL	9296	0.7247	0.1995	0.7998
RED MAPLE	608	0.7215	0.1328	0.8865
SUGAR MAPLE	504	0.7697	0.1657	0.8524
SWEET BIRCH	512	0.7820	0.2386	0.7418
YELLOW BIRCH	528	0.8223	0.2429	0.7583
HICKORY, SP.	480	0.8003	0.2277	0.7800
AM. BEECH	496	0.7686	0.1876	0.8220
WHITE ASH	560	0.7092	0.1789	0.8250
YELLOW-POPLAR	600	0.6423	0.1215	0.9029
CUCUMBERTREE	448	0.6619	0.1276	0.8972
BLACK TUPELO	464	0.7382	0.2216	0.7854
BLACK CHERRY	624	0.6303	0.1219	0.8895
WHITE OAK	672	0.7595	0.2693	0.7130
SCARLET OAK	392	0.7865	0.2951	0.6989
CHESTNUT OAK	488	0.7333	0.1681	0.8354
RED OAK	576	0.7352	0.2036	0.7905
BLACK OAK	360	0.8934	0.2672	0.7396
BLACK LOCUST	480	0.6965	0.1363	0.8872
AM. BASSWOOD	504	0.5907	0.1801	0.8242

Appendix B

Average Dbh_{ib}/Dbh_{ob} Ratios
for the Sample Data

Table 17.—Average DBH_{ib} DBH_{ob} ratios for the sample data by species

Species	Average ratio
Red maple	.942
Sugar maple	.942
Sweet birch	.939
Yellow birch	.948
Hickory, sp.	.915
American beech	.968
White ash	.913
Yellow-poplar	.896
Cucumbertree	.912
Black tupelo	.866
Black cherry	.923
White oak	.929
Scarlet oak	.926
Chestnut oak	.887
Red oak	.921
Black oak	.906
Black locust	.861
American basswood	.907
All species	.918

Appendix C

Example Volume Tables Prepared with the Taper Model by Kozak and Others*

- Total volume - Table 18
- Merchantable volume to 4-inch top - Table 19
- Merchantable volume to 8-inch top - Table 20
- Volume between 8- and 4-inch points - Table 21

*All volumes are inside bark. Bark option 1 was used to make the conversion.

Table 18. --GROSS PEELED VOLUME IN CUBIC FEET (EXCLUDING BARK) BY TOTAL HEIGHT.
MODEL BY KOZAK, ET AL. (Y = D**2/DBH**2) USING COEFFICIENTS FOR ALL SPECIES
VOLUME BETWEEN LOWER LIMIT OF 0.0 FEET
AND UPPER LIMIT OF 0.0 INCHES

	TOTAL HEIGHT FROM GROUND-LINE TO TIP :									
DBHOB (IN.)	30 FEET	40 FEET	50 FEET	60 FEET	70 FEET	80 FEET	90 FEET	100 FEET	110 FEET	120 FEET
4	1.1	1.4	1.8	2.1	2.5	2.8	3.2	3.6	3.9	4.3
5	1.7	2.2	2.8	3.3	3.9	4.4	5.0	5.6	6.1	6.7
6	2.4	3.2	4.0	4.8	5.6	6.4	7.2	8.0	8.8	9.6
7	3.3	4.4	5.4	6.5	7.6	8.7	9.8	10.9	12.0	13.1
8	4.3	5.7	7.1	8.5	10.0	11.4	12.8	14.2	15.6	17.1
9	5.4	7.2	9.0	10.8	12.6	14.4	16.2	18.0	19.8	21.6
10	6.7	8.9	11.1	13.3	15.6	17.8	20.0	22.2	24.4	26.7
11	8.1	10.8	13.4	16.1	18.8	21.5	24.2	26.9	29.6	32.3
12	9.6	12.8	16.0	19.2	22.4	25.6	28.8	32.0	35.2	38.4
13	11.3	15.0	18.8	22.5	26.3	30.0	33.8	37.5	41.3	45.1
14	13.1	17.4	21.8	26.1	30.5	34.8	39.2	43.5	47.9	52.2
15	15.0	20.0	25.0	30.0	35.0	40.0	45.0	50.0	55.0	60.0
16	17.1	22.7	28.4	34.1	39.8	45.5	51.2	56.9	62.6	68.2
17	19.3	25.7	32.1	38.5	44.9	51.4	57.8	64.2	70.6	77.0
18	21.6	28.8	36.0	43.2	50.4	57.6	64.8	72.0	79.2	86.4
19	24.1	32.1	40.1	48.1	56.1	64.2	72.2	80.2	88.2	96.2
20	26.7	35.5	44.4	53.3	62.2	71.1	80.0	88.9	97.7	106.6
21	29.4	39.2	49.0	58.8	68.6	78.4	88.2	98.0	107.8	117.6
22	32.3	43.0	53.8	64.5	75.3	86.0	96.8	107.5	118.3	129.0
23	35.3	47.0	58.8	70.5	82.3	94.0	105.8	117.5	129.3	141.0
24	38.4	51.2	64.0	76.8	89.6	102.4	115.2	128.0	140.8	153.6
25	41.7	55.5	69.4	83.3	97.2	111.1	125.0	138.8	152.7	166.6
26	45.1	60.1	75.1	90.1	105.1	120.1	135.2	150.2	165.2	180.2
27	48.6	64.8	81.0	97.2	113.4	129.6	145.8	161.9	178.1	194.3
28	52.2	69.7	87.1	104.5	121.9	139.3	156.7	174.2	191.6	209.0
29	56.0	74.7	93.4	112.1	130.8	149.5	168.1	186.8	205.5	224.2
30	60.0	80.0	100.0	120.0	140.0	159.9	179.9	199.9	219.9	239.9

Table 19. --GROSS PEELED VOLUME IN CUBIC FEET (EXCLUDING BARK) BY TOTAL HEIGHT.
MODEL BY KOZAK, ET AL. (Y = D**2/DBH**2) USING COEFFICIENTS FOR ALL SPECIES
VOLUME BETWEEN LOWER LIMIT OF 1.0 FEET
AND UPPER LIMIT OF 4.0 INCHES

DBHOB (IN.)	TOTAL HEIGHT FROM GROUND-LINE TO TIP : 30 FEET	40 FEET	50 FEET	60 FEET	70 FEET	80 FEET	90 FEET	100 FEET	110 FEET	120 FEET
4	0.1	0.2	0.2	0.3	0.3	0.4	0.5	0.5	0.6	0.7
5	0.8	1.1	1.5	1.8	2.1	2.4	2.8	3.1	3.4	3.7
6	1.6	2.2	2.8	3.4	4.0	4.6	5.2	5.9	6.5	7.1
7	2.5	3.4	4.3	5.2	6.2	7.1	8.0	8.9	9.9	10.8
8	3.5	4.7	6.0	7.3	8.6	9.8	11.1	12.4	13.7	15.0
9	4.5	6.2	7.9	9.5	11.2	12.9	14.6	16.2	17.9	19.6
10	5.7	7.8	9.9	12.0	14.2	16.3	18.4	20.5	22.6	24.7
11	7.0	9.6	12.2	14.8	17.4	20.0	22.5	25.1	27.7	30.3
12	8.5	11.6	14.7	17.8	20.9	24.0	27.1	30.2	33.3	36.4
13	10.0	13.7	17.3	21.0	24.7	28.3	32.0	35.7	39.3	43.0
14	11.7	15.9	20.2	24.5	28.7	33.0	37.3	41.6	45.8	50.1
15	13.4	18.3	23.3	28.2	33.1	38.0	43.0	47.9	52.8	57.7
16	15.3	20.9	26.5	32.2	37.8	43.4	49.0	54.6	60.3	65.9
17	17.3	23.7	30.0	36.4	42.7	49.1	55.5	61.8	68.2	74.5
18	19.5	26.6	33.7	40.9	48.0	55.1	62.3	69.4	76.5	83.7
19	21.7	29.7	37.6	45.6	53.5	61.5	69.5	77.4	85.4	93.4
20	24.1	32.9	41.7	50.6	59.4	68.2	77.1	85.9	94.7	103.6
21	26.6	36.3	46.1	55.8	65.5	75.3	85.0	94.8	104.5	114.3
22	29.2	39.9	50.6	61.3	72.0	82.7	93.4	104.1	114.8	125.5
23	31.9	43.6	55.3	67.0	78.7	90.4	102.1	113.8	125.5	137.3
24	34.8	47.5	60.3	73.0	85.8	98.5	111.3	124.0	136.8	149.5
25	37.8	51.6	65.4	79.3	93.1	106.9	120.8	134.6	148.5	162.3
26	40.9	55.8	70.8	85.8	100.7	115.7	130.7	145.7	160.6	175.6
27	44.1	60.2	76.4	92.5	108.7	124.8	141.0	157.1	173.3	189.4
28	47.4	64.8	82.1	99.5	116.9	134.3	151.6	169.0	186.4	203.8
29	50.9	69.5	88.1	106.8	125.4	144.0	162.7	181.3	200.0	218.6
30	54.5	74.4	94.3	114.3	134.2	154.2	174.1	194.1	214.1	234.0

Table 20.--GROSS PEELED VOLUME IN CUBIC FEET (EXCLUDING BARK) BY TOTAL HEIGHT.
MODEL BY KOZAK, ET AL. (Y = D**2/DBH**2) USING COEFFICIENTS FOR ALL SPECIES
VOLUME BETWEEN LOWER LIMIT OF 1.0 FEET
AND UPPER LIMIT OF 8.0 INCHES

	TOTAL HEIGHT FROM GROUND-LINE TO TIP :									
DBHOB (IN.)	30 FEET	40 FEET	50 FEET	60 FEET	70 FEET	80 FEET	90 FEET	100 FEET	110 FEET	120 FEET
4	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
5	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
6	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
7	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
8	0.4	0.6	0.9	1.1	1.4	1.6	1.9	2.2	2.4	2.7
9	1.8	2.6	3.3	4.1	4.9	5.6	6.4	7.1	7.9	8.7
10	3.3	4.6	5.9	7.1	8.4	9.7	11.0	12.3	13.6	14.9
11	4.8	6.6	8.5	10.3	12.2	14.0	15.9	17.7	19.6	21.4
12	6.4	8.8	11.3	13.7	16.1	18.5	21.0	23.4	25.8	28.3
13	8.1	11.2	14.2	17.2	20.3	23.3	26.4	29.4	32.5	35.5
14	9.9	13.6	17.3	21.0	24.7	28.4	32.1	35.8	39.5	43.2
15	11.8	16.2	20.6	25.0	29.3	33.7	38.1	42.5	46.9	51.3
16	13.8	18.9	24.0	29.1	34.3	39.4	44.5	49.6	54.7	59.8
17	15.9	21.8	27.7	33.5	39.4	45.3	51.2	57.1	63.0	68.9
18	18.1	24.8	31.5	38.2	44.9	51.6	58.3	65.0	71.7	78.3
19	20.5	28.0	35.5	43.1	50.6	58.1	65.7	73.2	80.8	88.3
20	22.9	31.3	39.7	48.2	56.6	65.0	73.5	81.9	90.4	98.8
21	25.5	34.8	44.2	53.5	62.9	72.3	81.6	91.0	100.4	109.7
22	28.1	38.5	48.8	59.1	69.5	79.8	90.2	100.5	110.8	121.2
23	30.9	42.3	53.6	65.0	76.3	87.7	99.1	110.4	121.8	133.1
24	33.8	46.2	58.6	71.0	83.5	95.9	108.3	120.7	133.2	145.6
25	36.8	50.3	63.9	77.4	90.9	104.4	118.0	131.5	145.0	158.6
26	40.0	54.6	69.3	84.0	98.6	113.3	128.0	142.7	157.3	172.0
27	43.2	59.1	74.9	90.8	106.6	122.5	138.4	154.2	170.1	186.0
28	46.6	63.7	80.8	97.9	115.0	132.1	149.2	166.3	183.4	200.5
29	50.1	68.4	86.8	105.2	123.6	141.9	160.3	178.7	197.1	215.5
30	53.7	73.4	93.1	112.7	132.4	152.1	171.8	191.5	211.3	231.0

Table 21.--GROSS PEELED VOLUME IN CUBIC FEET (EXCLUDING BARK) BY TOTAL HEIGHT.
MODEL BY KOZAK, ET AL. (Y = D**2/DBH**2) USING COEFFICIENTS FOR ALL SPECIES
VOLUME BETWEEN LOWER LIMIT OF 8.0 INCHES
AND UPPER LIMIT OF 4.0 INCHES

	TOTAL HEIGHT FROM GROUND-LINE TO TIP :									
DBHOB (IN.)	30 FEET	40 FEET	50 FEET	60 FEET	70 FEET	80 FEET	90 FEET	100 FEET	110 FEET	120 FEET
4	0.1	0.2	0.2	0.3	0.3	0.4	0.5	0.5	0.6	0.7
5	0.8	1.1	1.5	1.8	2.1	2.4	2.8	3.1	3.4	3.7
6	1.6	2.2	2.8	3.4	4.0	4.6	5.2	5.9	6.5	7.1
7	2.5	3.4	4.3	5.2	6.2	7.1	8.0	8.9	9.9	10.8
8	3.1	4.1	5.1	6.1	7.2	8.2	9.2	10.2	11.3	12.3
9	2.7	3.6	4.5	5.5	6.4	7.3	8.2	9.1	10.0	10.9
10	2.5	3.3	4.1	4.9	5.7	6.5	7.4	8.2	9.0	9.8
11	2.2	3.0	3.7	4.4	5.2	5.9	6.7	7.4	8.2	8.9
12	2.0	2.7	3.4	4.1	4.7	5.4	6.1	6.8	7.5	8.1
13	1.9	2.5	3.1	3.7	4.4	5.0	5.6	6.2	6.9	7.5
14	1.7	2.3	2.9	3.5	4.0	4.6	5.2	5.8	6.4	6.9
15	1.6	2.2	2.7	3.2	3.8	4.3	4.8	5.4	5.9	6.5
16	1.5	2.0	2.5	3.0	3.5	4.0	4.5	5.0	5.5	6.0
17	1.4	1.9	2.4	2.8	3.3	3.8	4.3	4.7	5.2	5.7
18	1.3	1.8	2.2	2.7	3.1	3.6	4.0	4.4	4.9	5.3
19	1.3	1.7	2.1	2.5	2.9	3.4	3.8	4.2	4.6	5.0
20	1.2	1.6	2.0	2.4	2.8	3.2	3.6	4.0	4.4	4.8
21	1.1	1.5	1.9	2.3	2.6	3.0	3.4	3.8	4.2	4.5
22	1.1	1.4	1.8	2.2	2.5	2.9	3.2	3.6	4.0	4.3
23	1.0	1.4	1.7	2.1	2.4	2.7	3.1	3.4	3.8	4.1
24	1.0	1.3	1.6	2.0	2.3	2.6	2.9	3.3	3.6	3.9
25	0.9	1.2	1.6	1.9	2.2	2.5	2.8	3.1	3.4	3.7
26	0.9	1.2	1.5	1.8	2.1	2.4	2.7	3.0	3.3	3.6
27	0.9	1.1	1.4	1.7	2.0	2.3	2.6	2.9	3.2	3.4
28	0.8	1.1	1.4	1.7	1.9	2.2	2.5	2.8	3.0	3.3
29	0.8	1.1	1.3	1.6	1.9	2.1	2.4	2.6	2.9	3.2
30	0.8	1.0	1.3	1.5	1.8	2.0	2.3	2.5	2.8	3.1

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Coefficients for five taper/volume models are developed for 18 Appalachian hardwood species. Each model can be used to estimate diameter at any point on the bole, height to any preselected diameter, and cubic foot volume between any two points on the bole. The resulting equations were tested on six sets of independent data and an evaluation of these tests is included. A wide variety of volume tables can be constructed with the models; some examples are given.

ODC: 524.1

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