United States
Department of
Agriculture

Forest Service

**Northeastern Forest
Experiment Station**

Research
Paper NE-493

1982

Height Prediction Equations for Even-Aged Upland Oak Stands

by Donald E. Hilt
and Martin E. Dale

The Authors

Donald E. Hilt received his bachelor's degree in forestry from Iowa State University in 1969, and a master of science degree in forestry from Oregon State University in 1975. He joined the Northeastern Forest Experiment Station in 1975 and worked as a statistician in the biometrics group at Upper Darby, Pennsylvania. In 1976 he transferred to Delaware, Ohio, where he is a research forester specializing in studies of growth and yield of managed upland oaks.

Martin E. Dale joined the Forest Service in 1957 and specialized in forest management and silviculture research at Berea, Kentucky. In 1973, he received a Ph. D. degree in forest biometry from Iowa State University, and since 1970 has been assigned to the Northeastern Forest Experiment Station's research unit on timber measurement and management planning at Delaware, Ohio. Currently he is specializing in studies of growth and yield of managed upland oaks.

Manuscript received for publication
15 June 1981

Abstract

Forest growth models that use predicted tree diameters or diameter distributions require a reliable height-prediction model to obtain volume estimates because future height-diameter relationships will not necessarily be the same as the present height-diameter relationship. A total tree height prediction equation for even-aged upland oak stands is presented. Predicted tree heights follow a biologically consistent progression with increasing dbh, age, and site index. The consistent progression of heights does not allow erratic or illogical volume increments. The proposed equation satisfactorily predicted the heights of 4,619 oak trees measured in six states.

Introduction

Diameter-growth models for individual trees, diameter distribution models, and stand table projection methods are all examples of forest growth modeling techniques that use predicted tree diameters or diameter distributions to estimate future stand characteristics. These types of forest growth models are receiving increased attention because they provide detailed information on the structure, and in some instances the species composition, of the future stand. Since the volume of the future stand is also of interest, heights must be assigned to predicted tree diameters or to the diameter classes of a predicted diameter distribution to obtain volume.

Methods for estimating future heights should be thoroughly investigated because future height-diameter relationships will not necessarily be the same as the present height-diameter relationship (Chapman and Meyer 1949). The preferred method of assigning future heights is to model *height growth* as a function of variables such as dbh, age, and site index. However, data necessary for the construction of a height-growth model are not available at the present time for the upland oak timber type. Reliable height growth data for the deliquescent-branching upland oaks can only be obtained through detailed stem analyses of felled trees. An alternate method for estimating future heights is to construct a general model that expresses *total tree height* as a function of variables such as dbh, age, and site index.

This paper presents a method for predicting total tree heights that follow a biologically consistent progression with increasing dbh, age, and site index. The consistent progression of heights does not allow erratic or illogical volume increments. The resulting height prediction equations are applicable to even-aged upland oak stands.

Data

Data for developing and testing the height equations were taken over a range of age and site conditions in unmanaged even-aged upland oak stands in Ohio, Kentucky, Missouri, Iowa, Illinois, and Indiana. A total of 2,306 felled-tree heights on 150 plots (stands) from an oak decay study were used in the analysis. Plot size was 0.08 ha (1/5-acre). These data were augmented with 2,313 standing-tree heights measured with a Spiegel-relaskop[1] on 158 0.2- and 0.4-ha (1/2- and 1-acre) growth and yield plots. Dbh and total height were measured on trees from all crown classes in both studies. Total age was determined by counting annual rings for felled trees, and increment borings for standing trees. Site index, the height attained by the average dominant and codominant oak at total age 50, was determined for each plot from Schnur's (1937) site index curves for upland oaks.

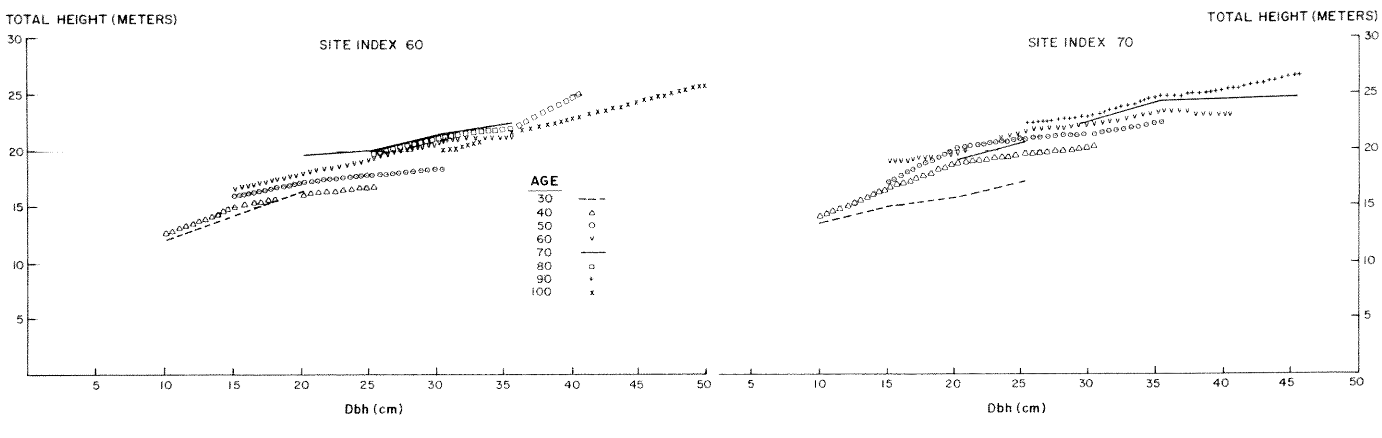
The data were divided for analysis according to species with similar growth patterns. The black oak group of 2,116 heights consisted of black oak (*Quercus velutina* Lam.), scarlet oak (*Q. coccinea* Muenchh.), and northern red oak (*Q. rubra* L.). The white oak group of 2,503 heights consisted of white oak (*Q. alba* L.) and chestnut oak (*Q. prinus* L.). Site index for both species groups ranged from approximately 50 to 80, age from 25 to 125 years, and dbh from 6.6 to 66 cm (2.6 to 26 inches).

Analysis

Mean black oak heights of various dbh x age x site index categories plotted in Figure 1 show that dbh, age, and site index are important factors related to tree height. Similar relationships were observed on other sites for black oak, and also for the white oak data. Our goal was to build the observed relation-

[1] The use of trade, firm, or corporation names in this publication is for the information and convenience of the reader. Such use does not constitute an official endorsement or approval by the U.S. Department of Agriculture or the Forest Service of any product or service to the exclusion of others that may be suitable.

Figure 1. Average heights for black oak by dbh, age, and site index.



ships into the height prediction model. The following conditions for a logical and consistent height prediction equation were proposed:

1. Height equals breast height (1.37 m) when dbh equals zero.
2. Height increases at a decreasing rate as dbh increases.
3. Height increases at a decreasing rate for a given dbh as age increases.
4. Height increases for a given dbh and age as site index increases.

Linear regression models for height-diameter-age equations were explored by Curtis (1967). Many of these proposed models, including site index, were fitted to the oak data. While many of the fitted equations produced R^2 values near 0.80, none fulfilled all of the above conditions. Some equations "peaked" within the data, predicting shorter heights for larger diameters. Other equations failed to produce reasonable maximum heights for a given age and site combination.

Meyer (1940) investigated a modified exponential height-diameter model of the form

$H = 1.37 + \alpha\,[1 - \exp(-\beta D)]$, where H = total height in meters, D = dbh in centimeters, and exp is the base of the natural logarithms. The intercept of this model is 1.37 m (breast height). Height increases at a decreasing rate as dbh increases, but can not exceed the asymptotic height, $\alpha + 1.37$ meters. The parameter β determines the rate at which the curve approaches the asymptotic height. Meyer found that this model provides an excellent fit to observed data for trees larger than about 5 inches dbh and older than 10 to 20 years. For practical purposes then, this curve is satisfactory for height-diameter equations for a given age and site index. It fulfills conditions 1 and 2 stated previously.

Meyer's equation works well for a specific stand at a given age and site index because α and β are usually estimated with a sample of dbh, and total height data. To obtain a general model that is suitable when sample heights are not available, we needed a method based on stand age and site index for estimating the parameters α and β.

The proposed height prediction model has the form

$$H_{ij} = 1.37 + \alpha_i\,[1 - \exp(-\beta_i D_{ij})], \qquad (1)$$

where H_{ij} = height of the jth tree in the i^{th} stand,
D_{ij} = dbh of the jth tree in the i^{th} stand,
α_i = asymptotic height parameter for the i^{th} stand,
and β_i = slope parameter for the i^{th} stand.

In this form the model is not useful as a height prediction model because it is stand-specific. One way to build a general height prediction equation would be to model the parameters α_i and β_i as functions of stand age and site index. The parameters of such a stochastic coefficients model could be estimated by the two-stage procedure proposed by Ferguson and Leech (1978). However, the two-stage least squares computer program for estimating the coefficients is not readily available. The approach we use in modeling the parameters is based on existing site index curves and stand tables for normal stands.

We first assume that the asymptotic height parameter, α_i, for any stand is related to the maximum tree height, and the maximum tree height is related to average height of the dominant and codominant trees in a stand (height of the site trees). Based on experience and investigation of the data, the maximum height is assumed to be a constant percentage taller than the mean height of the site trees. Since the asymptotic height for a given stand is $\alpha_i + 1.37$ meters, we can express α_i as a function of the mean height of the site trees for the stand, $\overline{H}_{si}$:

$$\alpha_i + 1.37 = k\,\overline{H}_{si},$$

or

$$\alpha_i = k\overline{H}_{si} - 1.37, \qquad (2)$$

where k is greater than one.

We next condition the height equation to pass through the point $(\overline{D}_{si}, \overline{H}_{si})$ for a given stand, where $\overline{D}_{si}$ is the mean dbh of the site trees in the i^{th} stand. The condition that has to be met is

$$\overline{H}_{si} = 1.37 + \alpha_i\,[1 - \exp(-\beta_i \overline{D}_{si})].$$

Solving for β_i gives

$$\beta_i = -\ln\,[(\alpha_i + 1.37 - \overline{H}_{si})/\alpha_i]\,\overline{D}_{si}^{-1}. \qquad (3)$$

Substituting the value of α_i from (2) into (3),

$$\beta_i = -\ln\,[(k\,\overline{H}_{si} - \overline{H}_{si})/(k\,\overline{H}_{si} - 1.37)]\,\overline{D}_{si}^{-1}. \qquad (4)$$

Substituting the right hand sides of equations (2) and (4) into equation (1) gives a conditioned height model which is written as

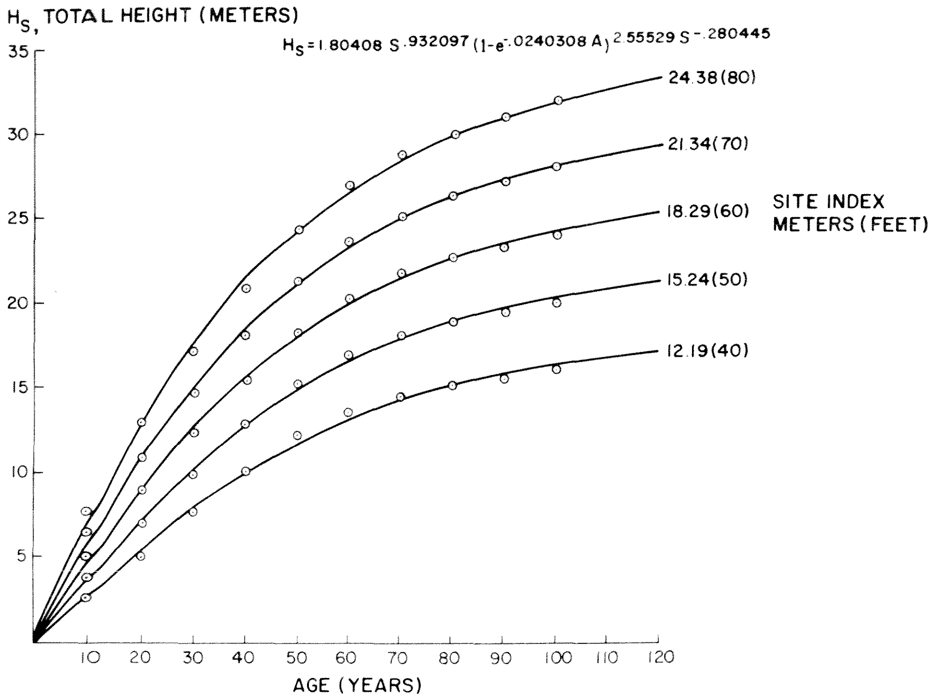
$$H_{ij} = 1.37 + (k\,\overline{H}_{si} - 1.37) \times \{1 - \exp\,[\ln[(k\,\overline{H}_{si} - \overline{H}_{si})/(k\,\overline{H}_{si} - 1.37)]\,\overline{D}_{si}^{-1} D_{ij}]\}, \qquad (5)$$

The conditioned height model is nonlinear with one unknown parameter, k. The model is still stand-specific and not useful as a height prediction equation unless the mean height and dbh of the site trees can be modeled as functions of stand age and site index.

We used values read from Schnur's site index curves to model the relationship between $\overline{H}_s$, stand age and site index. The mean height of the site trees, regardless of oak species, can be read directly from Schnur's curves for any age and site index. We fitted Schnur's curves to a modified Richards' growth function (Ek 1971 and Payandeh 1974). The mean height of the site trees can be estimated as a function of stand age (A) and site index (S) with the following equation:

$$\overline{H}_s = 1.80408\,S^{.932097}\,[1 - \exp(-.0240308\,A)]^{2.55529\,S^{-.280445}} \qquad (6)$$

Figure 2. Nonlinear equations for Schnur's site index curves. Circles represent points read from Schnur's curves.



The equation fitted the site curve data very well (Fig. 2). The R^2 was greater than 0.99. Since $\overline{H}_s$ is a function of stand age, equation (6) can be used to project $\overline{H}_s$ to some future age.

If data are collected from a large number of stands, observed values of $\overline{D}_{si}$ could be used to model the relationship between $\overline{D}_s$, stand age, and site index. Although we had a large amount of data, the trees used to determine the site index of a plot could not be identified. To circumvent this problem, we used the mean diameter of the trees in the upper 20 percent of the diameter distribution from Schnur's stand tables as our estimate of $\overline{D}_s$ for a given stand. Our experience in upland oaks has shown that the site index trees are generally in the upper 20 percent of the diameter distribution for the trees in a stand. A field test of this assumption on 32 permanent growth and yield plots revealed that on the average 75 percent of the site trees selected on a given plot were in the upper 20 percent of the diameter distribution for that plot. Values of $\overline{D}_s$ calculated from Schnur's stand tables increase with increasing stand age and site index (Figs. 3a-3b). We used a modified Richards' function to model the relationship between the mean diameter of the site trees, stand age (A), and site index (S). The resulting equation for black oaks,

$$\overline{D}_s = 5.49927\,S^{.744034}\,[1 - \exp(-.0192593\,A)]^{1.25342}, \quad (7)$$

had an R^2 greater than 0.99.

White oaks are generally smaller in diameter than black oaks for a given age and site index. Therefore, we fitted separate curves for white oaks based on Schnur's stand tables. The resulting equation for white oak,

$$\overline{D}_s = 6.40146\,S^{.631893}\,[1 - \exp(-.0227614\,A)]^{1.21892}, \quad (8)$$

also had an R^2 greater than 0.99.

We calculated values of $\overline{H}_{si}$ and $\overline{D}_{si}$ for each stand with equations (6), (7), and (8). These values, along with the observed values of H_{ij} and D_{ij} were used to estimate the value of the parameter k in the conditioned height model, equation (5). Our initial estimate of the asymptotic height ($k\,\overline{H}_s$) for trees at a given stand age and site index, based on experience and investigation of the data, was 10 percent greater than the average height of the site trees. That is, K = 1.1. Our final estimate of k was determined with an iterative fitting routine that minimized the percentage differences between estimated and actual heights (estimated height less actual height × 100/ actual height).

Results

Black oaks. A k value of 1.07 resulted in a model that fitted the black oak data well. The average of the percentage differences for the 2116 black oak trees was −0.15 percent with a standard deviation of 9.05 percent, and the calculated R^2 was 0.79. The fitted curves are shown in Figures 4a-4c.

The average percentage differences were then tabulated by age and site index categories to check for bias in certain parts of the curve

Figure 3a. Average diameters of the largest 20 percent of the trees for black oak, by age and site index. Circles represent points calculated from Schnur's stand tables.

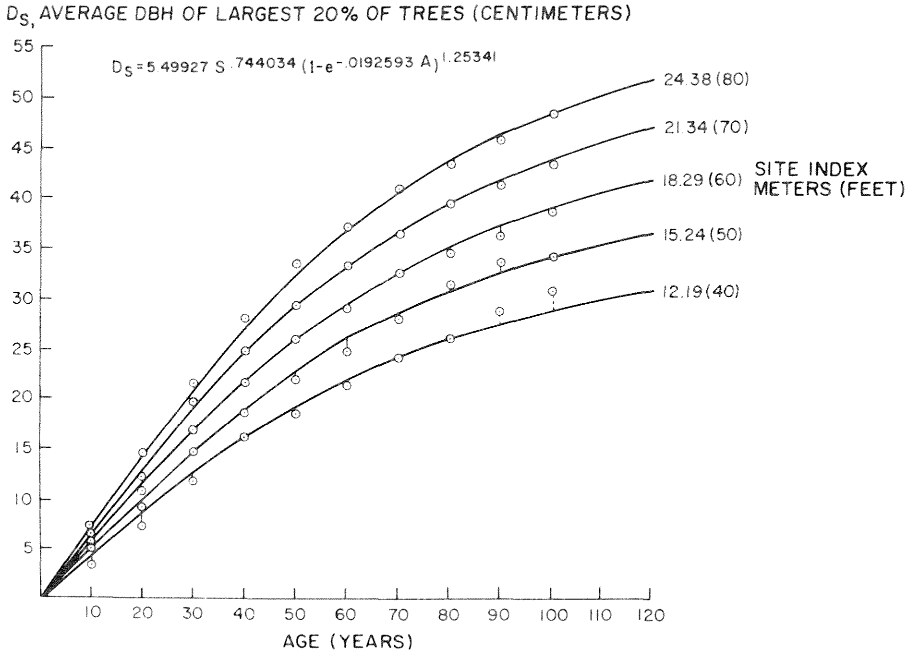


Figure 3b. Average diameters of the largest 20 percent of the trees for white oak, by age and site index.

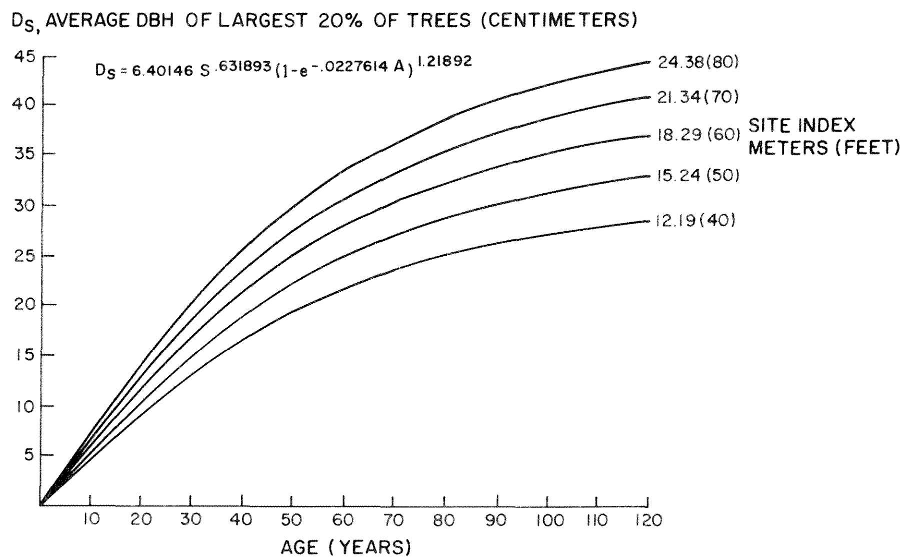


Figure 4a. Height-diameter-age curves for black oak, site index 15.24 m (50 feet). Circles represent the point ($\bar{D}_s$, $\bar{H}_s$) that curve is forced through.

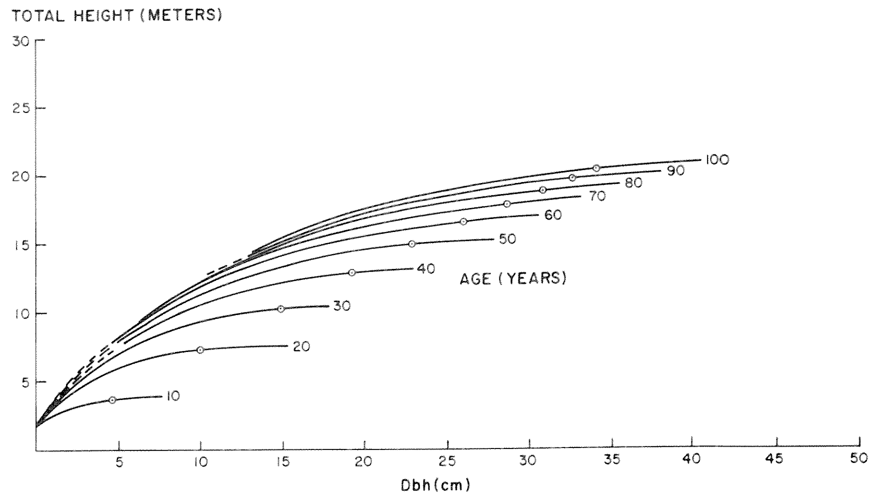


Figure 4b. Black oak site index 18.29 m (60 feet).

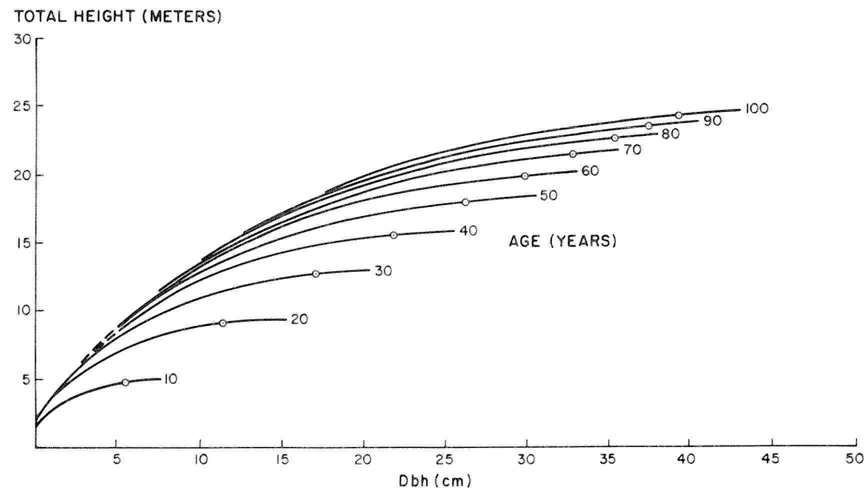
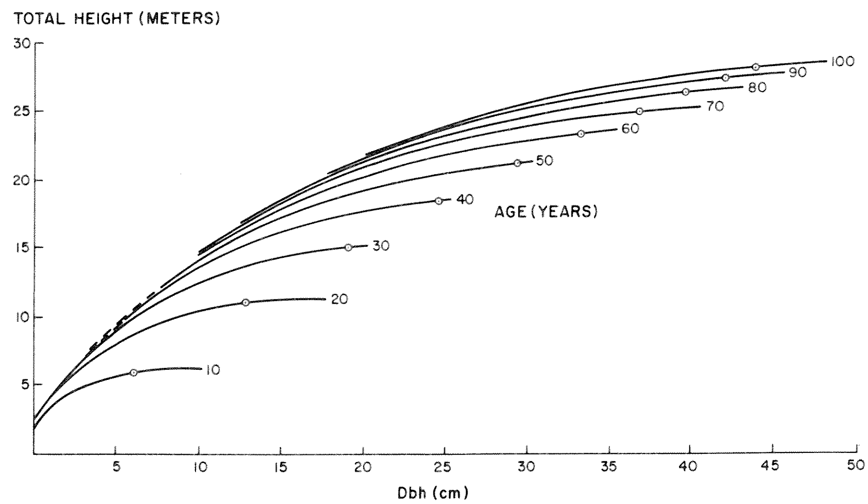


Figure 4c. Black oak site index 21.34 m (70 feet).



(Table 1). Most of the percentage differences, especially for those age × site index categories with a large number of trees in them, were within ± 4 percent. There was a slight trend from negative to positive bias with increasing age and site index.

The percentage differences were also tabulated by age, site index, and dbh as illustrated in Table 2 for site index 70. Except for the slight trend with age, no significant bias was detectable. Differences for other site index categories were similar.

White oak. The value of k for white oak was 1.12. The resulting average percentage difference for the 2503 white oak trees was 0.42 percent with a standard deviation of 10.0 percent, and the calculated R^2 was 0.81. The magnitude and trends of the percentage differences were similar to those for black oak. The fitted curves are shown in Figures 4d–4f.

We feel that the general height model does a good job of predicting individual tree heights. Conditioning the height model to pass through the point ($\overline{D}_s$, $\overline{H}_s$) guarantees logical height predictions for a given stand. All of the conditions stated previously for a logical and consistent height prediction equation were met.

Table 1.—Average percentage differences in height by age × site index categories for black oak.

Age	Site index class: 15.24 m (50 feet)		18.29 m (60 feet)		21.34 m (70 feet)		24.38 m (80 feet)	
	No. of trees	Average % deviation	No. of trees	Average % deviation	No. of trees	Average % deviation	No. of trees	Average % deviation
30	—	—	39	– 10.3	169	3.5	116	8.8
40	—	—	282	– 4.7	284	– 3.2	73	– 1.7
50	6	– 7.3	91	– 1.7	272	– 1.6	79	3.9
60	23	– 4.0	76	– 2.0	121	0.7	72	– .8
70	21	– 3.2	59	– .9	55	5.0	33	3.6
80	13	– 13.9	29	3.0	15	8.6	5	2.4
90	16	– 4.6	10	8.6	30	4.4	—	—
100	25	– 1.4	18	9.2	11	9.9	20	14.0

[1] Percentage difference = [(estimated height-actual height)/actual height] × 100. Only those age × site categories with five or more trees are shown.

Table 2.—Average percentage differences of age × dbh categories for black oak, site index 21.34 m (70 feet).[1]

Age	Dbh Class: 10.16 cm (4 inches)	15.24 cm (6 inches)	20.32 cm (8 inches)	25.40 cm (10 inches)	30.48 cm (12 inches)	35.56 cm (14 inches)	40.64 cm (16 inches)	45.72 cm (18 inches)
30	1.9	3.4	4.7					
40	0.0	– 2.8	– 4.4	– 3.2	– 1.7			
50	10.4	2.5	– 5.2	– 3.2	1.1	– 2.3		
60		– 2.9	1.3	– .9	2.3	– .6		
70			11.3	6.5	4.1	1.3	3.1	5.8
80					9.0	6.4		
90					7.8	6.8	3.6	– .3
100						5.6	11.9	

[1] Percentage difference = [(estimated height-actual height)/actual height)] × 100. Only those age × dbh categories with five or more trees are shown.

Figure 4d. Height-diameter-age curves for white oak, site index 15.24 m (50 feet). Circles represent the point ($\bar{D}_s$, $\bar{H}_s$) that curve is forced through.

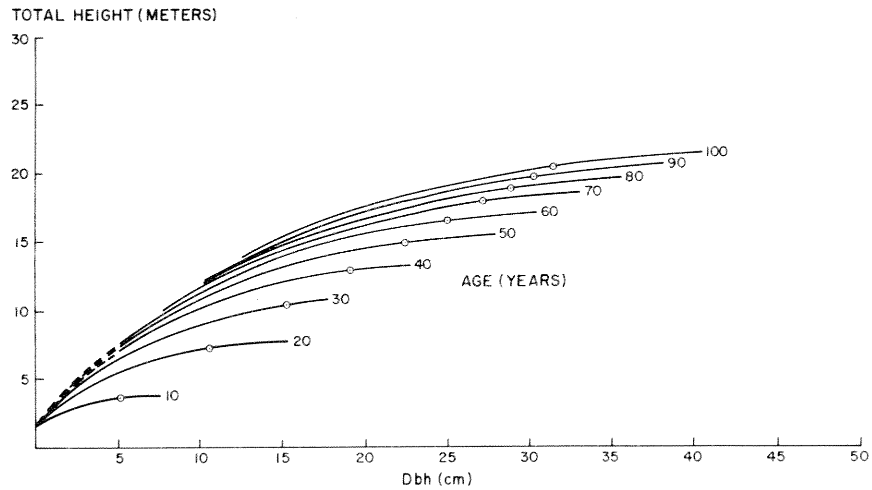


Figure 4e. White oak site index 18.29 m (60 feet).

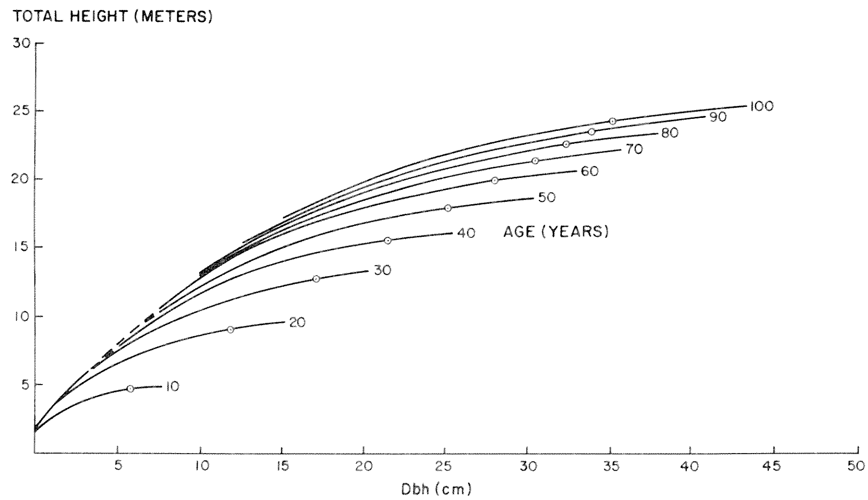
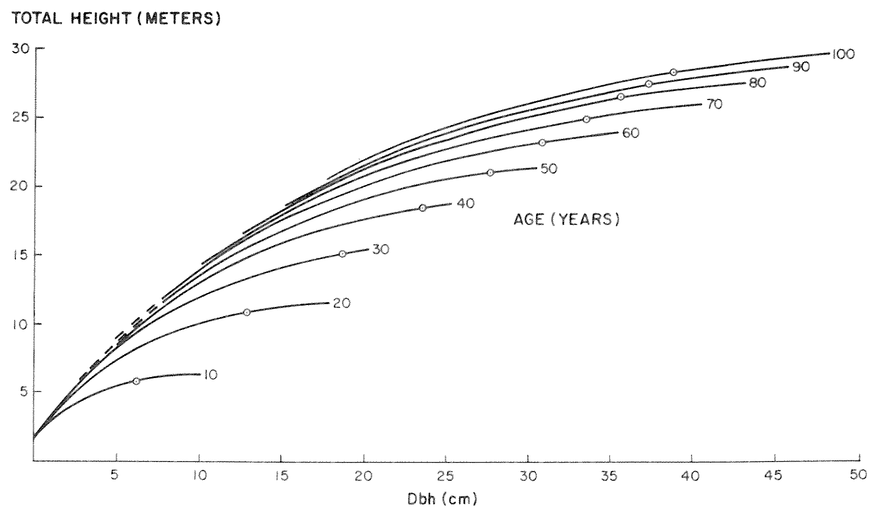


Figure 4f. White oak site index 21.34 m (70 feet).



Thinned Stands

The proposed height equation predicts heights of trees from unmanaged stands very well. However, many growth simulation models are developed from managed stands. Increased diameter growth of residual trees after thinning would necessarily alter the height-diameter relationship at future ages if height growth is not affected. However, the effect of thinning on height growth of residual trees may neutralize any change caused by increased diameter growth, or it may even augment the change. Changes in the height-diameter relationship due to thinning are beyond the scope of this paper. Our intention here is to determine whether the height equations developed in this paper for unmanaged stands perform satisfactorily for thinned stands as well.

Sample tree heights were measured in 1977 on four different thinning studies, 16 years after thinning. The thinning method used is best described as "free"—the marker was free to remove trees from all crown classes. The degree of thinning was controlled by reducing the basal area or the stocking level to the desired percentage. Stocking percent (Gingrich 1967) less than 50 represents a heavily thinned stand, 50–75 percent a medium thinning, and 75+ percent a light thinning. Sample tree heights were predicted by using the height models developed in this paper, and the average percentage differences were tabulated by study and stocking level (Table 3).

The height equations predicted the sample tree heights satisfactorily. Even though some of the study x stocking level categories had a limited number of trees, none of the average percentage differences exceeded 10 percent. Although not conclusive, these results indicate that thinning did not dramatically alter the height-diameter relationship present in unthinned stands. The effects of thinning on the height-diameter relationship may become apparent when thinned stands are observed after a period longer than 16 years.

Discussion

Tree heights estimated by the method presented in this paper follow a logical and consistent progression with increasing tree dbh, age, and site index. The conditions imposed on the height equations prevent erratic and illogical tree-height predictions that sometimes occur with traditional regression techniques. The equations can be inserted with only a few programming statements into many forest-growth computer routines.

The height equations are intended for use with growth models for upland oaks that involve the prediction of tree diameters or diameter distributions to estimate future stand characteristics. We do not advocate using these height equations for existing stands of known age and site index. Height-diameter equations should be constructed for such stands from sample tree heights and diameters.

While we have demonstrated that the height equations perform satisfactorily for thinned stands, changes in the height-diameter relationship due to thinning need to be investigated more thoroughly in future studies.

Table 3. Average percentage differences for stands 16 years after thinning.[1]

Thinning study	Predominant species	Initial avg. age (years)	Avg. site index	Stocking level (percent)					
				<50		50–75		75+	
				No. of trees	Average % deviation	No. of trees	Average % deviation	No. of trees	Average % deviation
1	White oak	34	70	56	−3.4	38	−9.5	38	−3.3
2	Black, scarlet oak	34	73	104	6.9	112	3.0	39	6.9
3	Black, scarlet oak	62	64	64	0.9	37	6.4	17	0.8
4	White oak	80	64	172	−1.8	119	−6.7	—	—

[1] Percentage difference = [(estimated height-actual height)/actual height)] × 100. Only those age × stocking categories with five or more trees are shown.

Literature Cited

Chapman, H. H., and W. H. Meyer
1949. Forest mensuration. 1st ed. McGraw-Hill, New York, 522p, illus.

Curtis, R. O.
1967. Height-diameter and height-diameter-age equations for second growth Douglas-fir. For. Sci. 13:365–375.

Ek, A. R.
1971. A formula for white spruce site index curves. Univ. Wis. For. Res. Note 161. 2p.

Ferguson, I. S. and J. W. Leech.
1978. Generalized least squares estimation of yield functions. For. Sci. 24:27–42.

Gingrich, S. F.
1967. Measuring and evaluating stocking and stand density in upland hardwood forests in the Central States. For. Sci. 13:38–52.

Meyer, A. H.
1940. A mathematical expression for height curves. J. For. 38:415–420.

Payandeh, B.
1974. Formulated site index curves for major timber species in Ontario. For. Sci. 20:143–144.

Schnur, L. G.
1937. Yield, stand, and volume tables for even-aged upland oak forests. U.S. Dep. Agric. Tech. Bull. 560. 87p.

Hilt, Donald E. and Martin E. Dale. Height prediction equations for even-aged upland oak stands. Broomall, Pa.: Northeast. For. Exp. Stn.; 1982; USDA For. Serv. Res. Pap. NE-493, 9p.

A total tree height prediction equation for even-aged upland oak stands is presented. Predicted tree heights follow a logical and consistent progression with increasing dbh, age, and site index.

ODC 522.2, 522.31, 561.1

Keywords: Height prediction; upland oaks

☆U.S. GOVERNMENT PRINTING OFFICE: 1982-505-012:3

Headquarters of the Northeastern Forest Experiment Station are in Broomall, Pa. Field laboratories are maintained at:

- Amherst, Massachusetts, in cooperation with the University of Massachusetts.
- Berea, Kentucky, in cooperation with Berea College.
- Burlington, Vermont, in cooperation with the University of Vermont.
- Delaware, Ohio.
- Durham, New Hampshire, in cooperation with the University of New Hampshire.
- Hamden, Connecticut, in cooperation with Yale University.
- Morgantown, West Virginia, in cooperation with West Virginia University, Morgantown.
- Orono, Maine, in cooperation with the University of Maine, Orono.
- Parsons, West Virginia.
- Princeton, West Virginia.
- Syracuse, New York, in cooperation with the State University of New York College of Environmental Sciences and Forestry at Syracuse University, Syracuse.
- University Park, Pennsylvania, in cooperation with the Pennsylvania State University.
- Warren, Pennsylvania.