A Multivariate Model and Statistical Method for Validating Tree Grade Lumber Yield Equations

by Donald W. Seegrist

USDA FOREST SERVICE RESEARCH PAPER NE-320
1975

NORTHEASTERN FOREST EXPERIMENT STATION
FOREST SERVICE, U.S. DEPARTMENT OF AGRICULTURE
6816 MARKET STREET, UPPER DARBY, PA. 19082
F. BRYAN CLARK, STATION DIRECTOR

THE AUTHOR

DONALD W. SEEGRIST is a biological statistician at the Northeastern Forest Experiment Station, Upper Darby, Pa. A graduate of George Washington University, Washington, D. C., in 1953 with a bachelor of science degree in zoology. He received his master's degree in 1959 and Ph.D. degree in zoology in 1965 at University of California, Berkeley. He majored in vertebrate zoology and ecology with primary emphasis on fisheries science. His minor was in applied statistics. In 1963, he joined the U. S. Forest Service as a biological statistician at the Pacific Southwest Forest and Range Experiment Station, Berkeley, Calif. In 1968, he joined the biometrics staff at the Northeastern Forest Experiment Station.

MANUSCRIPT RECEIVED FOR PUBLICATION 21 JANUARY 1975

A Multivariate Model and Statistical Method for Validating Tree Grade Lumber Yield Equations

ABSTRACT

Lumber yields within lumber grades can be described by a multivariate linear model. A method for validating lumber yield prediction equations when there are several tree grades is presented. The method is based on multivariate simultaneous test procedures.

Tree grading systems are designed to classify trees by value. Hardwood sawtimber grading systems have rules for determining tree grade and a set of equations or tables for predicting lumber yield by lumber grades within each tree grade.

When a grading system is developed, a sample of trees, the "index sample", is selected and a number of external characteristics are measured on each tree. Then the key variables —those that separate the trees into value classes—are determined. The grading rules are based on these key variables. In addition, a set of regression equations for predicting lumber yields by lumber grade is developed. The yields are usually functions of tree diameter and length.

The next step is to validate the regression equations. This is usually done by comparing the lumber yields predicted from the regression equations with the actual yields obtained from the second, or "validation" sample of trees. The validation should be done by comparing the lumber yield equations in the validation sample and the index sample.

The main purpose of this paper is to present this statistical method of validating the regression equations. A second purpose is to familiarize foresters with multivariate general linear models and multivariate analysis of variance (MANOVA). As an example I have used white oak graded with the Interim Hardwood Tree Grades for factory lumber.

White oak has eight lumber grades and three tree grades. Therefore there are 24 regression equations for predicting lumber volume. The independent variables are

$x_1 = 1$

$x_2 =$ square of the tree diameter (d^2)

$x_3 =$ tree height (h)

and $x_4 = d^2h$.

The variable $x_1 = 1$ allows for intercepts in the regression model.

The dependent variables are

$y_1 =$ volume of FAS lumber

$y_2 =$ volume of FASF lumber

$y_3 =$ volume of Select lumber

$y_4 =$ volume of 1C lumber

$y_5 =$ volume of 2C lumber

$y_6 =$ volume of SW lumber

$y_7 =$ volume of 3A lumber

and $y_8 =$ volume of 3B lumber.

The measurements on the k^{th} sample tree in the j^{th} tree grade in the i^{th} sample can be written in vector notation as

$$[\mathbf{x}_{ijk}, \mathbf{y}_{ijk}] = [1\ x_{2ijk}\ x_{3ijk}\ x_{4ijk}\ y_{1ijk} \ldots y_{8jjk}].$$

THE MULTIVARIATE REGRESSION MODEL

The usual procedure for developing the lumber volume prediction equations is to develop, one at a time, univariate multiple regression equations for lumber grade in each tree grade. However, the lumber volumes are known to be highly correlated and the correlation among lumber yields should be part of the statistical model. A multivariate multiple regression model can describe the vectors of lumber volumes. Morrison (*1967*). has given an excellent discussion of the multivariate general linear model and multivariate analysis of variance (MANOVA).

The multivariate regression model for the vector of lumber yields for k^{th} sample tree in the j^{th} tree grade in the i^{th} sample is

$$\mathbf{y}_{ijk} = \mathbf{x}_{ijk}\, \beta_{ij} + \varepsilon_{ijk}.$$

The model describing the set of measurements on the n_{ij} trees in the j^{th} tree grade of the i^{th} sample can be succinctly written in matrix notation as

$$\mathbf{Y}_{ij} = \mathbf{X}_{ij}\, \beta_{ij} + \varepsilon_{ij}$$

where $\mathbf{Y}_{ij}$ is a $n_{ij} \times 8$ matrix of dependent variables, $\mathbf{X}_{ij}$ is a $n_{ij} \times 4$ "design" matrix. β_{ij} is a 4×8 matrix of regression coefficients, and ϵ_{ij} is a $n_{ij} \times 8$ matrix of errors.

A model that combines the observations for the three tree grades and two samples is

$$\begin{bmatrix} \mathbf{Y}_{11} \\ \mathbf{Y}_{12} \\ \mathbf{Y}_{13} \\ \mathbf{Y}_{21} \\ \mathbf{Y}_{22} \\ \mathbf{Y}_{23} \end{bmatrix} = \begin{bmatrix} \mathbf{X}_{11} & & & & & \\ & \mathbf{X}_{12} & & & \mathbf{0} & \\ & & \mathbf{X}_{13} & & & \\ & & & \mathbf{X}_{21} & & \\ & \mathbf{0} & & & \mathbf{X}_{22} & \\ & & & & & \mathbf{X}_{23} \end{bmatrix} \begin{bmatrix} \beta_{11} \\ \beta_{12} \\ \beta_{13} \\ \beta_{21} \\ \beta_{22} \\ \beta_{23} \end{bmatrix} + \begin{bmatrix} \epsilon_{11} \\ \epsilon_{12} \\ \epsilon_{13} \\ \epsilon_{21} \\ \epsilon_{22} \\ \epsilon_{23} \end{bmatrix}$$

An example of the data needed for the model is given in figure 1. Symbolically the combined model can be written as

$$\begin{bmatrix} \mathbf{Y}_1 \\ \mathbf{Y}_2 \end{bmatrix} = \begin{bmatrix} \mathbf{X}_1 & 0 \\ 0 & \mathbf{X}_2 \end{bmatrix} \begin{bmatrix} \beta_1 \\ \beta_2 \end{bmatrix} + \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \end{bmatrix}.$$

It is assumed that

$$\text{and } E\,[\varepsilon_{ijk}] = 0 \text{ for all ijk}$$
$$E\,[\varepsilon'_{ijk}\, \varepsilon_{ijk}] = \Sigma \text{ for } ijk = ijk',$$
$$= 0 \text{ otherwise.}$$

Further, it is assumed the vectors of errors have a multivariate normal distribution.

MULTIVARIATE LEAST SQUARES ESTIMATORS AND TESTS

Using multivariate least squares procedures, the multivariate normal equations are

$$\begin{bmatrix} \mathbf{X}_1'\mathbf{X}_1 & 0 \\ 0 & \mathbf{X}_2'\mathbf{X}_2 \end{bmatrix} \begin{bmatrix} \mathbf{b}_1 \\ \mathbf{b}_2 \end{bmatrix} = \begin{bmatrix} \mathbf{X}_1'\mathbf{Y}_1 \\ \mathbf{X}_2'\mathbf{Y}_2 \end{bmatrix}.$$

The solution of the normal equation is

$$\begin{bmatrix} \mathbf{b}_1 \\ \mathbf{b}_2 \end{bmatrix} = \begin{bmatrix} [\mathbf{X}_1'\mathbf{X}_1]^{-1}\ \mathbf{X}_1'\mathbf{Y}_1 \\ [\mathbf{X}_2'\mathbf{X}_2]^{-1}\ \mathbf{X}_2'\mathbf{Y}_2 \end{bmatrix}$$

The matrix of residuals is

$$\mathbf{E} = [\mathbf{Y}_1'\mathbf{Y}_1 - \mathbf{Y}_1'\mathbf{X}_1\mathbf{b}_1] + [\mathbf{Y}_2\mathbf{Y}_2 - \mathbf{Y}_2'\mathbf{X}_2\mathbf{b}_2].$$

With multivariate analysis of variance (MANOVA) one can test hypotheses of the form

$$\mathbf{A}\beta\mathbf{C}' = \mathbf{D}.$$

The hypothesis sums of products matrix is

$$\mathbf{H} = [\mathbf{AbC}' - \mathbf{D}]'\, [\mathbf{A}[\mathbf{X}'\mathbf{X}]^{-1}\, \mathbf{A}']^{-1}\, [\mathbf{AbC}' - \mathbf{D}].$$

The error sums of products matrix is

$$\mathbf{S} = \mathbf{CEC}'.$$

Several test statistics are available for testing multivariate hypotheses.

HYPOTHESIS TESTING OF THE TREE GRADE LUMBER VOLUME REGRESSION EQUATIONS

There are many relevant hypotheses about the tree-grade regression equations for the index and validation samples. These hypotheses can all be expressed as linear combinations of the parameters of an overall hypothesis. The overall hypothesis is that the sets of regression equations for the two samples are the same. If the overall hypothesis is accepted, differences between the two sets of regression equations are attributed to chance and the equations are valid.

If the overall hypothesis is rejected, further questions arise. Are the two sets of regression equations different for all tree grades or only for some tree grades? What tree-grade regressions are different for which lumber grades? Also, one might test whether the predicted lumber volumes are the same for an average size tree in the two samples. Are the predicted volumes of average size trees the same for all lumber grades or just some lumber grades? Hypotheses concerning the total lumber volume and total value are also relevant. If some of these hypotheses (for example, the equality of lumber yield predictions for average size trees) are accepted, the equations might be considered valid.

The overall and the relevant hypotheses for white oak are presented below as an example.

The Overall Hypothesis

Formally, the overall hypothesis that the regression equations for the two samples are the same for the three tree grades and eight lumber grades is

$$\begin{matrix} \beta_{11} = \beta_{21} & & \beta_{11} - \beta_{21} = \mathbf{0} \\ \beta_{12} = \beta_{22} & \text{or} & \beta_{12} - \beta_{22} = \mathbf{0} \\ \beta_{13} = \beta_{23} & & \beta_{13} - \beta_{23} = \mathbf{0}. \end{matrix}$$

In MANOVA notation, the overall hypothesis is

$$\Psi_o = [\mathbf{I}, \mathbf{-I}]\, \beta\, \mathbf{I} = \mathbf{0}$$

$$\text{where } \mathbf{I} = \begin{bmatrix} 1 & 0 & & 0 \\ 0 & 1 & & 0 \\ & & \cdots & \\ 0 & 0 & & 1 \end{bmatrix}, \beta = \begin{bmatrix} \beta_1 \\ \beta_2 \end{bmatrix},$$

$$\text{and } \mathbf{0} = \begin{bmatrix} 0 & 0 & & 0 \\ 0 & 0 & & 0 \\ & & \cdots & \\ 0 & 0 & & 0 \end{bmatrix}.$$

The Matrix **I** is called an identity matrix and **0** is called a null matrix. Note that $3 \times 8 = 24$ regressions for each sample are being compared. However, the overall hypothesis has 96 parameters since each regression equation has four coefficients.

Component Hypotheses

The overall hypothesis can be resolved into component hypotheses. A component hypothesis is the equality of the two sample regressions for one tree grade and one lumber grade. The component hypotheses depend on certain rows of the prematrix **[I, -I]** and one of the columns of the postmatrix **I**. For example, the hypothesis that the regression equations for tree grade 2 and select lumber are the same for the two samples is

$$\Psi_1 = [\mathbf{0\ I\ 0\ 0\ {-I}\ 0}]\, \beta \quad [0\,0\,1\,0\,0\,0\,0\,0]' = \mathbf{0}$$

where **0** and **I** are 4×4 matrixes. Altogether there are 24 possible component hypotheses for the white oak grading system. The dimensions of each component hypothesis are 4×1.

Predicted Lumber Volumes Conditional on Average Size Trees

The overall hypothesis implies all possible linear combinations of the overall hypothesis. One relevant linear combination is the predicted lumber volume for the average size tree. The predictions are made from the population mean vector. For example, the population mean vector for the j[th] tree grade is

$$\mu_j = [1, \mu_{2j}, \mu_{3j}, \mu_{4j}]$$

where μ_{2j} = population mean diameter squared,

μ_{3j} = population mean height,

and μ_{4j} = population mean diameter squared $\times$ height.

One hypothesis of interest is the equality of predicted lumber volume for the two samples conditional on the mean vector. The hypothesis is

$$\begin{aligned} \mu_1\, \beta_{11} &= \mu_1\, \beta_{21} \\ \mu_2\, \beta_{12} &= \mu_2\, \beta_{22} \\ \mu_3\, \beta_{13} &= \mu_3\, \beta_{23}\,. \end{aligned}$$

In MANOVA notation, the hypothesis is

$$\Psi_{21} = \begin{bmatrix} \mu_1 & \mathbf{0} & \mathbf{0} & -\mu_1 & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mu_2 & \mathbf{0} & \mathbf{0} & -\mu_2 & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mu_3 & \mathbf{0} & \mathbf{0} & -\mu_3 \end{bmatrix} \beta\, \mathbf{I} = \mathbf{0}.$$

Denoting the prefactor matrix in Ψ_{21} as $[\mu,-\mu]$, the hypothesis can be written as $[\mu,-\mu]\ \beta\ \mathbf{I}=\mathbf{0}$. Factoring μ out of the prefactor matrix gives $\mu\ [\mathbf{I},-\mathbf{I}]\ \beta\ \mathbf{I}=\mathbf{0}$, which shows that Ψ_{21} is a linear combination of the overall hypothesis. The dimensions of Ψ_{21} are 3×8 compared to 12×8 for Ψ_o.

One of the peculiarities of sawing logs is that the sawyer has considerable leeway in the way a log is cut. Therefore the proportions of lumber grades may vary. Consequently the lumber volume equations and predictions may differ from one sample to another because there is a difference in the way the logs were sawn. Although the lumber volume predictions may differ, certain combinations of lumber volume predictions should be the same.

White oak lumber grades can be divided into three groups, with the grades in each group having more or less the same value per board foot. The groups are (1) select lumber and better, (2) number one common (1C), and (3) 2C and poorer. Suppose one wants to test whether the predicted volumes of the most valuable (select and better), intermediate (1C), and least valuable (2C and poorer) lumber grades are the same. In MANOVA notation, the hypothesis is

$$\Psi_{22}=[\mu,-\mu]\ \beta\begin{bmatrix}1&1&1&0&0&0&0&0\\0&0&0&1&0&0&0&0\\0&0&0&0&1&1&1&1\end{bmatrix}'=\mathbf{0}.$$

One is also interested in hypotheses involving the total lumber volume. For example, the hypothesis that the predicted total lumber volumes for the two samples are the same is

$$\Psi_{23}=[\mu,-\mu]\ \beta\,\mathbf{1}'=\mathbf{0}$$

where $\mathbf{1}'=[1\ 1\ \ldots\ 1]'$. The vector $\mathbf{1}'$ is called a summation vector. The dimensions of Ψ_{21} are 3×1.

Predicted Lumber Volume Conditional on Expected Tree Grade Proportions

Other relevant hypotheses involve the weighted average of the tree grades. The weights should be the true proportion of trees in each tree grade. One hypothesis is that there is no difference between the average lumber volume predictions for the two samples for each lumber grade. In MANOVA notation, the hypothesis is

$$\Psi_{31}=[\Pi_1\mu_1,\ \Pi_2\mu_2,\ \Pi_3\mu_3,-\Pi_1\mu_1,-\Pi_2\mu_2,-\Pi_3\mu_3]\beta\ \mathrm{I}=\mathbf{0}$$

where Π_1, Π_2, and Π_3 are the true proportion in tree grades 1, 2, and 3 respectively. Let the vector of proportions be $\Pi=[\Pi_1,\Pi_2,\Pi_3]$. The hypothesis Ψ_{31} can be written as $[\Pi\mu,-\Pi\mu]\ \beta\,\mathbf{I}=\mathbf{0}$.

Hypotheses Ψ_{31} tests the equality of the predicted lumber volumes for a stand of expected tree grade composition with average size white oaks. The hypothesis Ψ_{31} is also a linear combination of Ψ_o. The dimensions of Ψ_{31} are 1×8.

If we want to test the hypothesis that there is no difference in the total lumber yield, the postfactor is the summation matrix. The hypothesis is

$\Psi_{32}=[\Pi\mu,-\Pi\mu]\ \beta\ \mathbf{1}'=0$. The function Ψ_{32} is a scalar, that is, the dimensions of Ψ_{32} are 1×1. The hypothesis consists of one parametric function.

Predicted Lumber Values

Another set of relevant hypotheses involves the value of the lumber. Let $\mathbf{v}=[v_1\ v_2\ \ldots\ v_8]$ be the vector of average values for the eight white oak lumber grades. The hypothesis that the predicted total values of lumber for the two samples are the same for the three tree grades simultaneously is

$$\Psi_{41}=[\mu,-\mu]\ \beta\,\mathbf{v}'=\mathbf{0}.$$

This hypothesis tests the equality of the total predicted value for average size trees, for the three tree grades simultaneously.

The average size trees can also be weighted by their relative frequencies. The resulting hypothesis is a scalar hypothesis which tests the equality of the predicted total value of the lumber. The hypothesis is

$$\Psi_{42}=[\Pi\mu,-\Pi\mu]\ \beta\,\mathbf{v}'=0.$$

Weighting the tree grades by their relative frequencies gives more weight to the lower grades, which are more common, while weighting the lumber grades by their mean values gives more weight to the better tree grades, which are more valuable.

TESTING PROCEDURE

The overall hypothesis, that the 24 regression equations for the index sample are the same as those for the validation sample, is a matrix of 96 parameters. If there is a significant difference between the sets of equations, the overall hypothesis, as a whole, does not hold. Simultaneous test procedures (STP) will show which sets of parameters cause the hypothesis to be rejected (*Gabriel 1968*). A simultaneous test procedure rejects all hypotheses for which a particular statistic exceeds a single common critical value and accepts all others. A 5 percent level of an STP means that the probability of rejecting one or more true hypotheses out of all those tested is no more than 5 percent, whether the overall hypothesis or only some of its components are true. The significance level is said to be experimentwise rather than testwise.

Gabriel (*1956*) suggests several multivariate test statistics that can be used in a simultaneous test procedure.

$$\theta = C_1\,[S^{-1}\mathbf{H}]\ /\ [1 + C_1\,[S^{-1}\mathbf{H}]]$$
$$T^2 = n_e\ \mathrm{tr}\ [S^{-1}\mathbf{H}]$$
$$U = |S|\ /\ |S+\mathbf{H}|$$
and $V = \mathrm{tr}\ [[S+\mathbf{H}]^{-1}\,\mathbf{H}]$.

The notation $C_1[.]$ stands for the maximum characteristic root, tr [.] stands for the trace of a matrix, and $|.|$ stands for the determinant of a matrix.

For the white oak example, the maximum root statistic θ is used as an STP. The critical value, for a 5 percent level STP, is the upper 5 percent point of the null distribution of θ for the overall hypothesis. There are $p_o = 8$ lumber grades, $r_o = 3 \times 4 = 12$ parameters, and $n_e = 316$ degrees of freedom. The parameters of the null distribution of θ are $s = \min\ (p_o, r_o) = 8$, $m = 1/2\ (|r_o - p_o| - 1) = 3/2$, and $n = 1/2\ (n_e - p_o - 1) = 153.5$. The critical value of θ at the .05 level is 0.13329 (see *Pallai 1965*). The maximum root STP compares the calculated value of θ for any hypothesis, including the overall hypothesis, to the value 0.13329. Any hypothesis is rejected if the calculated θ exceeds 0.13329.

WHITE OAK TEST STATISTICS

Data on a sample of white oak trees were obtained from Leland Hanks of the Northeastern Forest Experiment Station. His findings will be reported elsewhere.

The number of sample trees in the two samples was

	Tree Grade			
Sample	*I*	*II*	*III*	*Total*
Index	108	122	75	305
Validation	8	13	14	35.

The validation sample is considered to be a random sample from the white oak population of interest. The sample mean vector and proportions are used to estimate the population mean vector and proportions. For example, the vector for the second sample for the j^{th} tree grade is

$$\bar{x}_{2j} = [1\ ,\ \overline{d^2}_{2j}\ ,\ \bar{h}_{2j}\ ,\ \overline{d^2h}_{2j}]$$

where $\overline{d^2}_{2j} = \Sigma_{k=1}^{n_{2j}}\ d_{2jk}{}^2/n_{2j}$

$$\bar{h}_{2j} = \Sigma_{k=1}^{n_{2j}}\ h_{2jk}\ /n_{2j}$$

and $\overline{d^2h}_{2j} = \Sigma_{k=1}^{n_{2j}}\ d^2h_{2jk}\ /n_{2j}$.

The mean vectors are

$$\mathbf{x}_{21} = [1.0 \quad 370.6 \quad 38.7 \quad 14435.8]$$
$$\mathbf{x}_{22} = [1.0 \quad 317.8 \quad 36.8 \quad 11821.2]$$
$$\mathbf{x}_{23} = [1.0 \quad 289.7 \quad 36.9 \quad 11215.8].$$

The proportion of logs in the j^{th} tree grade in the validation sample is $p_j = n_{2j}\ /\ (n_{21} + n_{22} + n_{23})$.

The proportions of trees in the validation sample are $p_1 = .2286$, $p_2 = .3714$, and $p_3 = .4000$. Multiplying the mean vectors by the proportions gives

$$p_1\,\mathbf{x}_{21} = [.2286 \quad 84.7 \quad 8.8 \quad 3298.6]$$
$$p_2\,\mathbf{x}_{22} = [.3714 \quad 118.0 \quad 13.7 \quad 4390.4]$$
$$p_3\,\mathbf{x}_{23} = [.4000 \quad 115.9 \quad 14.8 \quad 4486.3].$$

The average prices for the white oak lumber are

Grade	*$/fbm*
FAS	.24511
FASIF	.23439
SELECT	.22169
1C	.17004
2C	.09961
SW	.09808
3A	.08867
3B	.06094

The vector of average prices is

$$v = [0.24511 \ldots 0.06094].$$

To test the hypotheses involving the mean vector one can use $\bar{x}_{21}$, $\bar{x}_{22}$, and $\bar{x}_{23}$ in place of μ_1, μ_2, and μ_3. Likewise one can use p_1, p_2, and p_3 in place of Π_1, Π_2, and Π_3. The vector $\Pi\mu$ is replaced by $[p_1\bar{x}_{21}, p_2\bar{x}_{22}, p_3\bar{x}_{23}]$.

Estimates of the regression coefficients and test statistics were computed with a modified version of the Multivariate General Linear Hypothesis Program (BMDX63) (*Dixon 1969*). A partial listing of the data and a printout of the computer output follow.

The computed value of the test statistic for the overall hypothesis is 0.10933, which is less than the critical value. Therefore, in this example, the overall hypothesis is accepted. The differences between the two sets of regression equations may be due to chance. Since the overall hypothesis is accepted, there is no need to test component hypotheses and other relevant linear combinations. The sets of regression equations based on the index sample are accepted.

LITERATURE CITED

Dixon, W. J.
1969. BMD BIOMEDICAL COMPUTER PROGRAMS, X-SERIES, SUPPLEMENT. Univ. Calif. Press, Berkeley and Los Angeles. 64-78.

Gabriel, K. R.
1968. SIMULTANEOUS TEST PROCEDURES IN MULTIVARIATE ANALYSIS OF VARIANCE. Biometrika 55: 489-504.

Morrison, D. F.
1967. MULTIVARIATE STATISTICAL METHODS. McGraw-Hill. New York. 338p.

Pallai. K. C. S.
1965. ON THE DISTRIBUTION OF THE LARGEST CHARACTERISTIC ROOT OF A MATRIX IN MULTIVARIATE ANALYSES. Biometrika 52:405-414.

APPENDIX

The first page presents a partial listing of the white oak data. The data is in the form for the multivariate regression model. Each line contains data from one tree. The first 12 columns contain values of the four independent variables for each of the three tree grades in the index sample. The next 12 columns contain the same data for the validation sample. The last eight columns are values of the lumber yields.

The second page shows the regression coefficients and the residual matrix. This is followed by the hypothesis matrices and test statistics for each of the hypotheses discussed in the paper.

PARTIAL LISTING OF WHITE OAK TEST DATA

1	324	45	14580	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	26	8	39	89	32	30	48
1	324	29	9396	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	16	18	0	45	16	0	19	59
1	289	45	13005	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	19	12	0	27	103	0	58	0
1	324	50	16200	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	16	8	0	73	72	0	87	17
1	361	37	13357	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	53	19	0	30	95	0	53	17
1	289	41	11849	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	23	59	0	19	55	18	36	31
1	289	43	12427	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	9	21	8	33	66	17	25	55
1	441	47	20727	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	43	10	0	85	70	9	91	69
1	289	41	11849	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	20	0	0	30	54	10	64	47
1	289	45	13005	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	9	0	30	50	0	97	69
0	0	0	0	1	169	29	4901	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	5	0	5	13	7	27	37
0	0	0	0	1	196	45	8820	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	11	38	0	24	58
0	0	0	0	1	196	45	8820	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	25	26	14	37	45
0	0	0	0	1	441	49	21609	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	22	0	4	87	229	29	70	55
0	0	0	0	1	400	56	22400	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	43	44	21	79	86	0	65	49
0	0	0	0	1	225	31	6975	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	6	5	20	47	0	27	27
0	0	0	0	1	361	41	14801	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	23	9	0	47	140	16	68	16
0	0	0	0	1	625	31	19375	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	10	34	119	168	0	45	27
0	0	0	0	1	196	41	8036	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	19	0	35	50	16	32	24
0	0	0	0	1	196	31	6076	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	30	8	0	38	37
0	0	0	0	0	0	0	0	1	225	31	6975	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	34	27	0	19	51
0	0	0	0	0	0	0	0	1	196	31	6076	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	18	41	0	56	8
0	0	0	0	0	0	0	0	1	169	30	5070	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	15	18	6	22	31
0	0	0	0	0	0	0	0	1	121	29	3509	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	13	8	6	39
0	0	0	0	0	0	0	0	1	144	33	4752	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	18	8	12	31
0	0	0	0	0	0	0	0	1	100	16	1600	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	5	24
0	0	0	0	0	0	0	0	1	289	47	13583	0	0	0	0	0	0	0	0	0	0	0	0	0	4	0	19	54	41	56	72
0	0	0	0	0	0	0	0	1	100	16	1600	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	5	0	0	5	24
0	0	0	0	0	0	0	0	1	324	49	15876	0	0	0	0	0	0	0	0	0	0	0	0	0	0	9	26	101	62	54	64
0	0	0	0	0	0	0	0	1	196	33	6468	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	12	17	12	5	62
0	0	0	0	0	0	0	0	0	0	0	0	1	484	47	22748	0	0	0	0	0	0	0	0	90	4	35	96	101	0	87	40
0	0	0	0	0	0	0	0	0	0	0	0	1	400	25	10000	0	0	0	0	0	0	0	0	22	17	0	81	14	10	21	36
0	0	0	0	0	0	0	0	0	0	0	0	1	400	39	15600	0	0	0	0	0	0	0	0	18	0	0	92	82	0	63	38
0	0	0	0	0	0	0	0	0	0	0	0	1	484	39	18876	0	0	0	0	0	0	0	0	71	0	38	83	104	11	77	25
0	0	0	0	0	0	0	0	0	0	0	0	1	256	45	11520	0	0	0	0	0	0	0	0	0	9	0	14	55	10	57	43
0	0	0	0	0	0	0	0	0	0	0	0	1	361	43	15523	0	0	0	0	0	0	0	0	8	8	0	80	77	0	76	45
0	0	0	0	0	0	0	0	0	0	0	0	1	324	41	13284	0	0	0	0	0	0	0	0	0	7	8	17	64	61	64	82
0	0	0	0	0	0	0	0	0	0	0	0	1	256	31	7936	0	0	0	0	0	0	0	0	4	0	0	31	27	27	29	48
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	441	45	19845	0	0	0	0	35	29	13	122	78	5	53	107
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	441	39	17199	0	0	0	0	37	0	4	79	77	31	67	81
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	441	27	11907	0	0	0	0	29	6	13	91	64	28	53	18
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	289	37	10693	0	0	0	0	17	5	0	14	64	0	41	27
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	361	43	15523	0	0	0	0	0	0	0	27	82	26	87	17
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	441	33	14553	0	0	0	0	64	0	0	108	65	11	53	33
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	289	59	17051	0	0	0	0	0	0	0	34	56	10	88	62
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	324	33	10692	0	0	0	0	22	0	23	36	111	12	48	15
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	289	31	8959	0	0	0	0	0	0	0	23	20	34	22	60
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	225	37	8325	0	0	0	0	0	33	7	4	40	0	50	36
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	256	31	7936	0	0	0	7	75	0	43	49
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	256	33	8448	0	0	0	0	50	0	72	52
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	144	31	4464	0	0	0	0	7	0	21	25
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	169	27	4563	0	0	0	0	15	6	25	33
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	196	37	7252	0	0	0	3	7	30	17	57
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	196	47	9212	0	0	0	0	13	0	24	94
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	225	33	7425	0	0	0	24	16	21	10	89
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	196	29	5684	0	0	0	25	18	16	23	45
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	324	33	10692	0	12	0	61	76	0	70	36
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	400	40	16000	0	0	0	27	108	24	87	59

BMDX63 - MULTIVARIATE GENERAL LINEAR HYPOTHESIS - REVISED MAY 10, 1968
HEALTH SCIENCES COMPUTING FACILITY, UCLA

OAK LEE HANKS DATA

NO. OF INDEPENDENT VARIABLES	24
NO. OF DEPENDENT VARIABLES	8
NO. OF OBSERVATIONS	340
NO. OF HYPOTHESES	9

REGRESSION COEFFICIENTS $B=(X'X)^{-1}\ X'Y$

1	-49.74578	38.61154	-22.20332	121.45298	-55.98549	12.07271	-45.71794	12.14341
2	0.21179	-0.00547	0.04969	-0.21281	0.09960	-0.08540	0.11320	-0.00190
3	0.38402	-1.31154	0.45801	-4.99858	1.29745	-0.37565	1.83333	0.26926
4	-0.00124	0.00257	-0.00048	0.01455	0.00283	0.00347	-0.00126	0.00073
5	-3.97260	-27.02114	5.66302	-108.02191	-18.51471	-24.13804	16.63381	30.66055
6	0.04218	0.12346	-0.00714	0.38703	0.04071	0.01612	-0.04201	-0.06572
7	-0.06012	0.56331	-0.10537	1.25788	-0.04723	0.86514	0.04420	0.07194
8	0.00010	-0.00177	0.00035	-0.00081	0.00600	0.00105	0.00328	0.00185
9	1.76068	-4.38396	-2.56438	81.81661	7.69042	-22.39903	-20.19143	-17.86638
10	0.00727	0.04861	0.00444	-0.10565	0.01809	0.04556	0.08689	0.03783
11	-0.18492	-0.02890	0.13489	-3.97969	-0.85289	0.57048	0.89217	1.71251
12	0.00054	-0.00047	-0.00018	0.01117	0.00736	0.00134	0.00042	-0.00055
13	124.60510	-72.15117	37.22445	20.35130	-79.85826	102.18312	-63.53607	103.85335
14	-0.33902	0.23511	-0.13842	0.18170	0.06830	-0.21169	0.05584	-0.20968
15	-5.48285	1.99242	-1.90425	-2.13923	1.89379	-1.29430	2.17140	-0.56725
16	0.01663	-0.00600	0.00679	0.00395	0.00323	0.00286	0.00124	0.00280
17	-3.36754	32.11777	-3.60426	-8.82129	10.08515	-42.13702	-107.03810	205.72110
18	0.12444	-0.10843	0.04530	0.16771	0.09155	0.19896	0.29871	-0.69673
19	-1.02305	-0.80666	0.02598	-2.04251	-0.07262	0.94057	3.67817	-5.74273
20	0.00146	0.00323	-0.00058	0.00629	0.00186	-0.00365	-0.00622	0.02278
21	-9.73343	23.14028	-9.73343	68.76658	-61.25674	51.28313	-59.09157	9.73165
22	0.05620	-0.06781	0.05620	-0.00915	0.37403	-0.14646	0.40238	-0.31801
23	0.16855	-0.80949	0.16855	-3.00000	0.75424	-1.58855	1.48929	1.49619
24	-0.00108	0.00259	-0.00108	0.00590	-0.00194	0.00580	-0.00614	0.00849

RESIDUAL SUM OF PRODUCT MATRIX E=Y'Y-Y'XB

1	1649.08744	269.59309	62.73361	-383.67557	-626.05759	-1082.12031	-65.02085	-324.88452
2	269.59309	1190.56573	-29.86247	180.38170	-377.88049	-449.30010	158.50438	-193.41953
3	62.73361	-29.86247	238.80577	-13.42369	34.24962	3.27135	-145.39851	-116.03618
4	-383.67557	180.38170	-13.42369	6162.23977	842.40438	-398.33432	-504.28024	-1613.92685
5	-626.05759	-377.88049	34.24962	842.40438	5171.43888	-195.91976	-40.43333	-1471.34936
6	-1082.12031	-449.30010	3.27135	-398.33432	-195.91976	5167.14675	-575.64761	550.25962
7	-65.02085	158.50438	-145.39851	-504.28024	-40.43333	-575.64761	2243.37720	43.67915
8	-324.88452	-193.41953	-116.03618	-1613.92685	-1471.34936	550.25962	43.67915	3005.00838

OUTPUT FOR HYPOTHESIS NUMBER 1 (I,-I)BI OVERALL

A MATRIX

1	1.0000	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
	-1.0000	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
2	0.0	1.0000	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
	0.0	-1.0000	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
3	0.0	0.0	1.0000	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
	0.0	0.0	-1.0000	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
4	0.0	0.0	0.0	1.0000	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
	0.0	0.0	0.0	-1.0000	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
5	0.0	0.0	0.0	0.0	1.0000	0.0	0.0	0.0	0.0	0.0	0.0	0.0
	0.0	0.0	0.0	0.0	-1.0000	0.0	0.0	0.0	0.0	0.0	0.0	0.0
6	0.0	0.0	0.0	0.0	0.0	1.0000	0.0	0.0	0.0	0.0	0.0	0.0
	0.0	0.0	0.0	0.0	0.0	-1.0000	0.0	0.0	0.0	0.0	0.0	0.0
7	0.0	0.0	0.0	0.0	0.0	0.0	1.0000	0.0	0.0	0.0	0.0	0.0
	0.0	0.0	0.0	0.0	0.0	0.0	-1.0000	0.0	0.0	0.0	0.0	0.0
8	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.0000	0.0	0.0	0.0	0.0
	0.0	0.0	0.0	0.0	0.0	0.0	0.0	-1.0000	0.0	0.0	0.0	0.0
9	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.0000	0.0	0.0	0.0
	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	-1.0000	0.0	0.0	0.0
10	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.0000	0.0	0.0
	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	-1.0000	0.0	0.0
11	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.0000	0.0
	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	-1.0000	0.0
12	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.0000
	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	-1.0000

C MATRIX

1	1.0000	0.0	0.0	0.0	0.0	0.0	0.0	0.0
2	0.0	1.0000	0.0	0.0	0.0	0.0	0.0	0.0
3	0.0	0.0	1.0000	0.0	0.0	0.0	0.0	0.0
4	0.0	0.0	0.0	1.0000	0.0	0.0	0.0	0.0
5	0.0	0.0	0.0	0.0	1.0000	0.0	0.0	0.0
6	0.0	0.0	0.0	0.0	0.0	1.0000	0.0	0.0
7	0.0	0.0	0.0	0.0	0.0	0.0	1.0000	0.0
8	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.0000

D MATRIX

1	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
2	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
3	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
4	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
5	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
6	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
7	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
8	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
9	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
10	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
11	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
12	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

CONTRAST ABC'-D

1	-174.3509	110.7627	-59.4278	101.1017	23.8728	-90.1104	17.8181	-91.7099
2	0.5508	-0.2406	0.1881	-0.3945	0.0313	0.1263	0.0574	0.2078
3	5.8669	-3.3040	2.3623	-2.8594	-0.5963	0.9186	-0.3381	0.8365
4	-0.0179	0.0086	-0.0073	0.0106	-0.0004	0.0006	-0.0025	-0.0021
5	-0.6051	-59.1389	9.2673	-99.2006	-28.5999	17.9990	123.6719	-175.0605
6	-0.0823	0.2319	-0.0524	0.2193	-0.0508	-0.1828	-0.3407	0.6310
7	0.9629	1.3700	-0.1313	3.3004	0.0254	-0.0754	-3.6340	5.8147
8	-0.0014	-0.0050	0.0009	-0.0071	0.0041	0.0047	0.0095	-0.0209
9	11.4941	-27.5242	7.1691	13.0500	68.9472	-73.6822	38.9001	-27.5980
10	-0.0489	0.1164	-0.0518	-0.0965	-0.3559	0.1920	-0.3155	0.3558
11	-0.3535	0.7806	-0.0337	-0.9797	-1.6071	2.1590	-0.5971	0.2163
12	0.0016	-0.0031	0.0009	0.0053	0.0093	-0.0045	0.0066	-0.0090

CEC'

1	1649.0874	269.5931	62.7336	-383.6756	-626.0576	-1082.1203	-65.0209	-324.8845
2	269.5931	1190.5657	-29.8625	180.3817	-377.8805	-449.3001	158.5044	-193.4195
3	62.7336	-29.8625	238.8058	-13.4237	34.2496	3.2713	-145.3985	-116.0362
4	-383.6756	180.3817	-13.4237	6162.2398	842.4044	-398.3343	-504.2802	-1613.9269
5	-626.0576	-377.8805	34.2496	842.4044	5171.4389	-195.9198	-40.4333	-1471.3494
6	-1082.1203	-449.3001	3.2713	-398.3343	-195.9198	5167.1467	-575.6476	550.2596
7	-65.0209	158.5044	-145.3985	-504.2802	-40.4333	-575.6476	2243.3772	43.6792
8	-324.8845	-193.4195	-116.0362	-1613.9269	-1471.3494	550.2596	43.6792	3005.0084

HYPOTHESIS SUM OF PRODUCT MATRIX, $H=(ABC'-D)'(A(X'X)^{-1}\ A')^{-1}\ (ABC'-D)$

1	54.8037	-30.0177	19.6861	-2.2543	-3.7798	-19.0558	19.3578	-6.2211
2	-30.0177	40.9538	-14.8201	-0.3642	9.5295	16.2217	-28.5347	-4.9734
3	19.6861	-14.8201	10.3884	-3.1874	2.2620	-9.6905	12.4153	-10.0533
4	-2.2543	-0.3642	-3.1874	73.8244	48.4253	51.0869	-5.7491	-62.2763
5	-3.7798	9.5295	2.2620	48.4253	82.5368	40.8542	10.5372	-106.0128
6	-19.0558	16.2217	-9.6905	51.0869	40.8542	76.8403	-22.7490	-51.2851
7	19.3578	-28.5347	12.4153	-5.7491	10.5372	-22.7490	51.2437	-29.3219
8	-6.2211	-4.9734	-10.0533	-62.2763	-106.0128	-51.2851	-29.3219	178.5239

DETERMINANT(CEC') = .21634385255963D+27 DETERMINANT(CEC'+H) = .27610118023100D+27
U-STATISTIC = .783567250 DEGREES OF FREEDOM 8 12 316
F-STATISTIC = .803372383 DEGREES OF FREEDOM 96 2092.
MAX CHARACTERISTIC ROOT OF H*(E INVERSE) .12274766
HECK'S THETA WITH PARAMETERS S,M,N 8 1.5 153.5 .10932791

```
OUTPUT FOR HYPOTHESIS NUMBER   2  (0,I,0,0,-I,0)B(00100000)'  COMPONENT 2,3

A MATRIX
  1    0.0     0.0     0.0     0.0     1.0000   0.0      0.0      0.0      0.0   0.0   0.0   0.0
       0.0     0.0     0.0     0.0    -1.0000   0.0      0.0      0.0      0.0   0.0   0.0   0.0
  2    0.0     0.0     0.0     0.0     0.0      1.0000   0.0      0.0      0.0   0.0   0.0   0.0
       0.0     0.0     0.0     0.0     0.0     -1.0000   0.0      0.0      0.0   0.0   0.0   0.0
  3    0.0     0.0     0.0     0.0     0.0      0.0      1.0000   0.0      0.0   0.0   0.0   0.0
       0.0     0.0     0.0     0.0     0.0      0.0     -1.0000   0.0      0.0   0.0   0.0   0.0
  4    0.0     0.0     0.0     0.0     0.0      0.0      0.0      1.0000   0.0   0.0   0.0   0.0
       0.0     0.0     0.0     0.0     0.0      0.0      0.0     -1.0000   0.0   0.0   0.0   0.0

C MATRIX
  1    0.0     1.0000  0.0     0.0     0.0      0.0      0.0      0.0

D MATRIX
  1    0.0
  2    0.0
  3    0.0
  4    0.0

CONTRAST  ABC'-D
  1  -59.1389
  2    0.2319
  3    1.3700
  4   -0.0050

CEC'
  1 1190.5657

                                                          -1    -1
HYPOTHESIS SUM OF PRODUCT MATRIX, H=(ABC'-D)'(A(X'X)   A')   (ABC'-D)
  1    9.6245

DETERMINANT(CEC')    =  1190.5657320470       DETERMINANT(CEC'+H) =  1200.1902413705
U-STATISTIC          =  .991980791           DEGREES OF FREEDOM   1   4 316
F-STATISTIC          =  .638638735           DEGREES OF FREEDOM   4   316.
MAX CHARACTERISTIC ROOT OF H*(E INVERSE)   .80839798D-02
HECK'S THETA WITH PARAMETERS S,M,N     1    1.0  157.0  .80191531D-02
```

```
OUTPUT FOR HYPOTHESIS NUMBER   3  (X,-X)BI     PREDICTED VOLUMES

A MATRIX
  1     1.0000  370.6000   38.7000   14435.8000   0.0        0.0        0.0          0.0        0.0       0.0       0.0          0.0
       -1.0000 -370.6000  -38.7000  -14435.8000   0.0        0.0        0.0          0.0        0.0       0.0       0.0          0.0
  2     0.0       0.0       0.0          0.0      1.0000   317.8000    36.8000   11821.2000     0.0       0.0       0.0          0.0
        0.0       0.0       0.0          0.0     -1.0000  -317.8000   -36.8000  -11821.2000     0.0       0.0       0.0          0.0
  3     0.0       0.0       0.0          0.0      0.0        0.0        0.0          0.0        1.0000  286.7000   36.9000   11215.8000
        0.0       0.0       0.0          0.0      0.0        0.0        0.0          0.0       -1.0000 -286.7000  -36.9000  -11215.8000

C MATRIX
  1     1.0000    0.0       0.0          0.0      0.0        0.0        0.0          0.0
  2     0.0       1.0000    0.0          0.0      0.0        0.0        0.0          0.0
  3     0.0       0.0       1.0000       0.0      0.0        0.0        0.0          0.0
  4     0.0       0.0       0.0          1.0000   0.0        0.0        0.0          0.0
  5     0.0       0.0       0.0          0.0      1.0000     0.0        0.0          0.0
  6     0.0       0.0       0.0          0.0      0.0        1.0000     0.0          0.0
  7     0.0       0.0       0.0          0.0      0.0        0.0        1.0000       0.0
  8     0.0       0.0       0.0          0.0      0.0        0.0        0.0          1.0000

D MATRIX
  1     0.0       0.0       0.0          0.0      0.0        0.0        0.0          0.0
  2     0.0       0.0       0.0          0.0      0.0        0.0        0.0          0.0
  3     0.0       0.0       0.0          0.0      0.0        0.0        0.0          0.0

CONTRAST  ABC'-D
  1    -1.2561   17.4285   -3.2203      -2.6593    6.5268     1.0317   -10.1939     -12.2693
  2    -7.4015    5.9361   -1.3219       8.0379    5.1968    12.6175    -5.9701      -7.8766
  3     2.6024    0.2888    1.2420       8.3686   11.9328    11.0054     0.0041     -18.9316

CEC'
  1 1649.0874  269.5931   62.7336    -383.6756 -626.0576-1082.1203   -65.0209    -324.8845
  2  269.5931 1190.5657  -29.8625     180.3817 -377.8805 -449.3001   158.5044    -193.4195
  3   62.7336  -29.8625  238.8058     -13.4237   34.2496    3.2713  -145.3985    -116.0362
  4 -383.6756  180.3817  -13.4237    6162.2398  842.4044 -398.3343  -504.2802   -1613.9269
  5 -626.0576 -377.8805   34.2496     842.4044 5171.4389 -195.9198   -40.4333   -1471.3494
  6-1082.1203 -449.3001    3.2713    -398.3343 -195.9198 5167.1467  -575.6476     550.2596
  7  -65.0209  158.5044 -145.3985    -504.2802  -40.4333 -575.6476  2243.3772      43.6792
  8 -324.8845 -193.4195 -116.0362  -1613.9269-1471.3494  550.2596    43.6792    3005.0084

                                                        -1   -1
HYPOTHESIS SUM OF PRODUCT MATRIX, H=(ABC'-D)'(A(X'X)  A')  (ABC'-D)
  1     7.1630   -6.5336    1.7771      -4.1432   -1.5171   -7.5971     5.9946       2.2670
  2    -6.5336   25.8966   -4.8964       2.4161   12.1080   10.2481   -16.8435     -21.3580
  3     1.7771   -4.8964    1.1196       0.5635   -0.6354   -0.6184     3.2671       1.3955
  4    -4.1432    2.4161    0.5635      15.7916   14.7638   21.7901    -3.5556     -22.7171
  5    -1.5171   12.1080   -0.6354      14.7638   22.1574   22.7624    -8.3403     -35.8318
  6    -7.5971   10.2481   -0.6184      21.7901   22.7624   31.9569    -9.3986     -35.7215
  7     5.9946  -16.8435    3.2671      -3.5556   -8.3403   -9.3986    11.5638      14.3831
  8     2.2670  -21.3580    1.3955     -22.7171  -35.8318  -35.7215    14.3831      58.2039

DETERMINANT(CEC')     =  .21634385255963D+27     DETERMINANT(CEC'+H) =  .23261580764499D+27
U-STATISTIC           =  .930047929             DEGREES OF FREEDOM    8    3 316
F-STATISTIC           =  .946088135             DEGREES OF FREEDOM   24   897.
MAX CHARACTERISTIC ROOT OF H*(E INVERSE)    .51512456D-01
HECK'S THETA WITH PARAMETERS S,M,N      3    2.0  153.5  .48988917D-01
```

```
OUTPUT FOR HYPOTHESIS NUMBER   4  (X,-X)B(I 0) PREDICTED VOLUMES SUBTOTALS

A MATRIX
  1    1.0000  370.6000   38.7000   14435.8000   0.0       0.0        0.0         0.0        0.0      0.0        0.0        0.0
      -1.0000 -370.6000  -38.7000  -14435.8000   0.0       0.0        0.0         0.0        0.0      0.0        0.0        0.0
  2    0.0       0.0       0.0          0.0      1.0000   317.8000    36.8000   11821.2000   0.0      0.0        0.0        0.0
       0.0       0.0       0.0          0.0     -1.0000  -317.8000   -36.8000  -11821.2000   0.0      0.0        0.0        0.0
  3    0.0       0.0       0.0          0.0      0.0       0.0        0.0         0.0        1.0000   286.7000   36.9000   11215.8000
       0.0       0.0       0.0          0.0      0.0       0.0        0.0         0.0       -1.0000  -286.7000  -36.9000  -11215.8000

C MATRIX
  1    1.0000    1.0000    1.0000       1.0000   0.0       0.0        0.0         0.0
  2    0.0       0.0       0.0          0.0      1.0000    0.0        0.0         0.0
  3    0.0       0.0       0.0          0.0      0.0       1.0000     1.0000      1.0000

D MATRIX
  1    0.0       0.0       0.0
  2    0.0       0.0       0.0
  3    0.0       0.0       0.0

CONTRAST  ABC'-D
  1  10.2928     6.5268   -21.4315
  2   5.2505     5.1968    -1.2291
  3  12.5017    11.9328    -7.9221

CEC'
  1 9412.1921 -127.2841-4730.9457
  2 -127.2841 5171.4389-1707.7025
  3-4730.9457-1707.702510452.1146

                                                          -1    -1
HYPOTHESIS SUM OF PRODUCT MATRIX, H=(ABC'-D)'(A(X'X)   A')   (ABC'-D)
  1  28.3378    24.7193   -27.7273
  2  24.7193    22.1574   -21.4098
  3 -27.7273   -21.4098    40.2507

DETERMINANT(CEC')    =  363331605918.09         DETERMINANT(CEC'+H) =  366811190320.20
U-STATISTIC          =  .990513921            DEGREES OF FREEDOM    3    3 316
F-STATISTIC          =  .333256364            DEGREES OF FREEDOM    9   764.
MAX CHARACTERISTIC ROOT OF H*(E INVERSE)   .74028003D-02
HECK'S THETA WITH PARAMETERS S,M,N     3   -0.5  156.0  .73484016D-02
```

```
OUTPUT FOR HYPOTHESIS NUMBER   5  (X,-X)B1'     PREDICTED VOLUMES TOTAL

A MATRIX
  1    1.0000   370.6000   38.7000   14435.8000    0.0       0.0       0.0         0.0       0.0       0.0       0.0        0.0
      -1.0000  -370.6000  -38.7000  -14435.8000    0.0       0.0       0.0         0.0       0.0       0.0       0.0        0.0
  2    0.0        0.0       0.0          0.0        1.0000   317.8000   36.8000   11821.2000  0.0       0.0       0.0        0.0
       0.0        0.0       0.0          0.0       -1.0000  -317.8000  -36.8000  -11821.2000  0.0       0.0       0.0        0.0
  3    0.0        0.0       0.0          0.0        0.0       0.0       0.0         0.0      1.0000   286.7000   36.9000   11215.8000
       0.0        0.0       0.0          0.0        0.0       0.0       0.0         0.0     -1.0000  -286.7000  -36.9000  -11215.8000

C MATRIX
  1    1.0000     1.0000    1.0000       1.0000     1.0000    1.0000    1.0000      1.0000

D MATRIX
  1    0.0
  2    0.0
  3    0.0

CONTRAST  ABC'-D
  1   -4.6120
  2    9.2182
  3   16.5124

CEC'
  111903.8812

                                                            -1   -1
HYPOTHESIS SUM OF PRODUCT MATRIX, H=(ABC'-D)'(A(X'X)  A')  (ABC'-D)
  1   41.9102

DETERMINANT(CEC')    =  11903.881158397        DETERMINANT(CEC'+H) =  11945.791400978
U-STATISTIC          =  .996491611            DEGREES OF FREEDOM    1    3 316
F-STATISTIC          =  .370851159            DEGREES OF FREEDOM   3    316.
MAX CHARACTERISTIC ROOT OF H*(E INVERSE)    .35207208D-02
HECK'S THETA WITH PARAMETERS S,M,N     1     0.5  157.0  .35083689D-02
```

```
OUTPUT FOR HYPOTHESIS NUMBER   6  (PX,-PX)BI   WEIGHTED AVERAGE PREDICTED VOLUMES

A MATRIX
  1     0.2286    84.7000     8.8000     3298.6000     0.3714  118.0000   13.7000    4390.4000     0.4000  115.9000    14.8000    4486.3000
       -0.2286   -84.7000    -8.8000    -3298.6000    -0.3714 -118.0000  -13.7000   -4390.4000    -0.4000 -115.9000   -14.8000   -4486.3000

C MATRIX
  1    1.0000     0.0        0.0           0.0        0.0        0.0        0.0           0.0
  2    0.0        1.0000     0.0           0.0        0.0        0.0        0.0           0.0
  3    0.0        0.0        1.0000        0.0        0.0        0.0        0.0           0.0
  4    0.0        0.0        0.0           1.0000     0.0        0.0        0.0           0.0
  5    0.0        0.0        0.0           0.0        1.0000     0.0        0.0           0.0
  6    0.0        0.0        0.0           0.0        0.0        1.0000     0.0           0.0
  7    0.0        0.0        0.0           0.0        0.0        0.0        1.0000        0.0
  8    0.0        0.0        0.0           0.0        0.0        0.0        0.0           1.0000

D MATRIX
  1    0.0        0.0        0.0           0.0        0.0        0.0        0.0           0.0

CCNTRAST  ABC'-D
  1   -2.2950     6.6620    -0.9013        5.7945     7.7268     9.6018    -5.0440      -12.7308

CEC'
  1 1649.0874   269.5931    62.7336     -383.6756 -626.0576-1082.1203   -65.0209     -324.8845
  2  269.5931  1190.5657   -29.8625      180.3817 -377.8805 -449.3001   158.5044     -193.4195
  3   52.7336   -29.8625   238.8058      -13.4237   34.2496    3.2713  -145.3985     -116.0362
  4 -383.6756   180.3817   -13.4237     6162.2398  842.4044 -398.3343 -504.2802    -1613.9269
  5 -626.0576  -377.8805    34.2496      842.4044 5171.4389 -195.9198  -40.4333    -1471.3494
  6-1082.1203  -449.3001     3.2713     -398.3343 -195.9198 5167.1467 -575.6476      550.2596
  7  -65.0209   158.5044  -145.3985     -504.2802  -40.4333 -575.6476 2243.3772       43.6792
  8 -324.8845  -193.4195  -116.0362    -1613.9269-1471.3494  550.2596   43.6792     3005.0084
                                                          -1      -1
HYPOTHESIS SUM OF PRODUCT MATRIX, H=(ABC'-D)'(A(X'X)  A')  (ABC'-D)
  1    1.5669    -4.5486     0.6154       -3.9563    -5.2756    -6.5558     3.4439        8.6922
  2   -4.5486    13.2042    -1.7865       11.4846    15.3146    19.0308    -9.9973      -25.2325
  3    0.6154    -1.7865     0.2417       -1.5538    -2.0720    -2.5748     1.3526        3.4138
  4   -3.9563    11.4846    -1.5538        9.9890    13.3202    16.5525    -8.6953      -21.9465
  5   -5.2756    15.3146    -2.0720       13.3202    17.7624    22.0725   -11.5951      -29.2654
  6   -6.5558    19.0308    -2.5748       16.5525    22.0725    27.4285   -14.4087      -36.3668
  7    3.4439    -9.9973     1.3526       -8.6953   -11.5951   -14.4087     7.5692       19.1042
  8    8.6922   -25.2325     3.4138      -21.9465   -29.2654   -36.3668    19.1042       48.2179

DETERMINANT(CEC')     =  .21634385255963D+27     DETERMINANT(CEC'+H) =  .22596525890724D+27
U-STATISTIC           =  .957420826              DEGREES OF FREEDOM    8    1 316
F-STATISTIC           =  1.71776104              DEGREES OF FREEDOM   8    309.
MAX CHARACTERISTIC ROOT OF H*(E INVERSE)    .44472751D-01
HECK'S THETA WITH PARAMETERS S,M,N     1     3.0  153.5  .42579140D-01
```

```
OUTPUT FOR HYPOTHESIS NUMBER   7  (PX,-PX)B1'  WEIGHTED AVERAGE PREDICTED VOLUMES TOTAL

A MATRIX
  1    0.2286    84.7000     8.8000     3298.6000     0.3714  118.0000    13.7000     4390.4000     0.4000  115.9000    14.8000     4486.3000
      -0.2286   -84.7000    -8.8000    -3298.6000    -0.3714 -118.0000   -13.7000    -4390.4000    -0.4000 -115.9000   -14.8000    -4486.3000

C MATRIX
  1    1.0000     1.0000     1.0000        1.0000     1.0000    1.0000     1.0000        1.0000

D MATRIX
  1    0.0

CONTRAST  ABC'-D
  1    8.8140

CEC'
  111903.8812
                                                           -1    -1
HYPOTHESIS SUM OF PRODUCT MATRIX, H=(ABC'-D)'(A(X'X)  A')  (ABC'-D)
  1   23.1125

DETERMINANT(CEC')    =  11903.881158397        DETERMINANT(CEC'+H) =  11926.993629831
U-STATISTIC          =  .998062134            DEGREES OF FREEDOM    1   1 316
F-STATISTIC          =  .613554657            DEGREES OF FREEDOM   1   316.
MAX CHARACTERISTIC ROOT OF H*(E INVERSE)   .19415912D-02
HECK'S THETA WITH PARAMETERS S,M,N     1   -0.5  157.0  .19378288D-02
```

```
OUTPUT FOR HYPOTHESIS NUMBER    8   (X,-X)BV      PREDICTED VALUE TOTAL

A MATRIX
  1     1.0000  370.6000    38.7000   14435.8000     0.0          0.0         0.0          0.0          0.0         0.0         0.0          0.0
       -1.0000 -370.6000   -38.7000  -14435.8000     0.0          0.0         0.0          0.0          0.0         0.0         0.0          0.0
  2     0.0       0.0        0.0          0.0        1.0000     317.8000     36.8000   11821.2000     0.0         0.0         0.0          0.0
        0.0       0.0        0.0          0.0       -1.0000    -317.8000    -36.8000  -11821.2000     0.0         0.0         0.0          0.0
  3     0.0       0.0        0.0          0.0        0.0          0.0         0.0          0.0          1.0000    286.7000     36.9000   11215.8000
        0.0       0.0        0.0          0.0        0.0          0.0         0.0          0.0         -1.0000   -286.7000    -36.9000  -11215.8000

C MATRIX
  1     0.2451    0.2344     0.2217       0.1700     0.0996       0.0981      0.0887       0.0609

D MATRIX
  1     0.0
  2     0.0
  3     0.0

CONTRAST  ABC'-D
  1     1.7108
  2     1.3967
  3     3.5186

CEC'
  1  301.9434

                                                               -1     -1
HYPOTHESIS SUM OF PRODUCT MATRIX, H=(ABC'-D)'(A(X'X)  A')  (ABC'-D)
  1     1.8250

DETERMINANT(CEC')     =  301.94344595354         DETERMINANT(CEC'+H) =  303.76845448555
U-STATISTIC           =  .993992090             DEGREES OF FREEDOM    1    3 316
F-STATISTIC           =  .636657715             DEGREES OF FREEDOM    3   316.
MAX CHARACTERISTIC ROOT OF H*(E INVERSE)    .60442065D-02
HECK'S THETA WITH PARAMETERS S,M,N      1     0.5  157.0  .60078935D-02
```

```
OUTPUT FOR HYPOTHESIS NUMBER   9  (PX,-PX)BV   WEIGHTED AVERAGE PREDICTED VALUE TOTAL

A MATRIX
  1    0.2286    84.7000     8.8000     3298.6000     0.3714  118.0000    13.7000     4390.4000     0.4000  115.9000    14.8000     4486.3000
      -0.2286   -84.7000    -8.8000    -3298.6000    -0.3714 -118.0000   -13.7000    -4390.4000    -0.4000 -115.9000   -14.8000    -4486.3000

C MATRIX
  1    0.2451     0.2344     0.2217        0.1700     0.0996    0.0981     0.0887        0.0609

D MATRIX
  1    0.0

CONTRAST  ABC'-D
  1    2.2728

CEC'
  1  301.9434
                                                                   -1    -1
HYPOTHESIS SUM OF PRODUCT MATRIX, H=(ABC'-D)'(A(X'X)  A')  (ABC'-D)
  1    1.5368

DETERMINANT(CEC')    =  301.94344595354        DETERMINANT(CEC'+H) =  303.48027623389
U-STATISTIC          =  .994935930            DEGREES OF FREEDOM    1   1 316
F-STATISTIC          =  1.60839081            DEGREES OF FREEDOM   1   316.
MAX CHARACTERISTIC ROOT OF H*(E INVERSE)   .50897951D-02
HECK'S THETA WITH PARAMETERS S,M,N     1   -0.5  157.0  .50640203D-02
```