

Predicting past and future diameter growth for trees in the northeastern United States

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Abstract: Tree diameter growth models are widely used in forestry applications, often to predict tree size at a future point in time. Also, there are instances where projections of past diameters are needed. A relative diameter growth model was developed to allow prediction of both future and past growth rates. Coefficients were estimated for 15 species groups that cover most tree species in the northeastern United States. Application of the model to independent data generally showed slight underprediction of growth, although the bias was negligible. Correlated observations were accounted for via a mixed-effects modeling approach, and an error function was specified to address heterogeneous variance. The models use a minimum amount of field-collected data, thus keeping data acquisition costs low and facilitating use in many forest growth applications.

Résumé : Les modèles de croissance en diamètre des arbres sont largement utilisés dans des applications reliées à la foresterie, souvent pour prédire la dimension des arbres à un moment donné dans le futur. Il y a aussi des situations où des projections du diamètre dans le passé sont nécessaires. Un modèle de croissance relative en diamètre a été développé afin de permettre la prédiction des taux de croissance tant futurs que passés. Les coefficients ont été estimés pour 15 groupes d'espèces qui incluent la plupart des espèces d'arbres dans le nord-est des États-Unis. L'application du modèle à des données indépendantes montre en général une légère sous-estimation de la croissance qui est par contre négligeable. Les observations corrélées ont été prises en compte en ayant recours à une approche de modélisation à effets mixtes et une fonction d'erreurs a été spécifiée pour tenir compte de l'hétérogénéité de la variance. Les modèles utilisent un minimum de données collectées sur le terrain, ce qui permet de garder le coût d'acquisition des données bas et facilite leur utilisation dans plusieurs applications de croissance de la forêt.

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Introduction

One of the most common and important measurements in forestry is tree diameter at breast height (DBH). Tree diameter is easy to measure and often is highly correlated with other tree attributes, such as crown characteristics and bole volume. The importance of tree diameter has led to numerous efforts to develop diameter (or basal area) growth models (Belcher et al. 1982; Amateis et al. 1989; Teck and Hilt 1991; Cao 2000; Schröder et al. 2002). A common modeling strategy is to formulate a potential or average growth component combined with a modifier function (Burkhart et al. 1987; Hilt and Teck 1988; Lessard et al. 2001). These modifier functions often are referred to as being distance dependent or distance independent, depending on whether the locations of competing trees are taken into account (Avery and Burkhart 2002, p. 371). Other researchers have modeled diameter growth directly with a single equation. These composite models estimate growth using tree- and stand-level variables as predictors (Wykoff 1990; Dolph 1992; Zhang et al. 2004).

Regardless of model form, it is common in diameter-growth research to standardize the time frame for the change in diameter. For instance, diameter growth over a measurement interval is divided by the interval length to obtain periodic annual diameter growth. This type of standardization approach assumes linear diameter growth over the period. The accuracy of this method depends on the measurement interval and the degree of nonlinearity in tree growth patterns. MacLean and Scott (1988) illustrate the unreliability of assuming either constant diameter growth or constant basal area growth. Other methods of interpolation over a measurement interval and their relation to growth modeling are described by McDill and Amateis (1993) and Cao et al. (2002). However, the complexity of these methods often limits their use for many practical forest growth applications. Martin and Ek (1984) avoided the interpolation issue by rewriting their model to estimate periodic growth. Periodic growth models are desirable because they allow direct prediction of growth over any reasonable time period without incorporating assumptions implicit to interpolation methods. In this paper, models are presented to estimate periodic diameter growth for tree species in the northeastern United States. These models allow for prediction of both future and past changes in tree diameter. Applications include growth projections and determination of tree sizes at earlier points in stand development. The ability to predict tree size at a previous point in time will facilitate proper assignment to classes of growth components (e.g., ingrowth — when a tree crosses a specified diameter threshold), a capability that has

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been lacking for accurate estimation of trends for northeastern forests.

Data

The data used in this study were collected by the northeastern unit of the USDA Forest Service's Forest Inventory and Analysis (NE-FIA) program. Growth was computed from a single interval for trees having DBH of 5.0 in. (12.7 cm) or larger on remeasured sample plots from Maine, Vermont, New Hampshire, Connecticut, Rhode Island, Massachusetts, New York, New Jersey, Delaware, Pennsylvania, Ohio, West Virginia, and Maryland. These data originated from subplot 1 of the current FIA plot design (Bechtold and Scott 2005), as most locations had transitioned to this design for the most recent measurement. Trees on subplot 1 are included in both previous and current plot designs. The plot radius is 24.0 ft (7.3 m). The plots had relatively uniform geographic spatial distribution, but sampling intensity varied by state, depending on whether the data were from the last periodic inventory or from the newer annual inventory in which full measurement cycles have not been completed for all states (McRoberts 2005). Plot-level data pertinent to this research included length of remeasurement period, latitude, longitude, and elevation. Longitude values are negative, as all locations are west of the prime meridian. Individual tree measurements vary by state and time of measurement, but included species, DBH, and compacted crown ratio (USDA Forest Service 2004).

These data were obtained from mapped plots where forested portions of plots were assigned to different conditions when there were changes in reserved status, forest type, stand size, owner group, regeneration status, or tree density (USDA Forest Service 2004). When these changes are observed, a boundary line is delineated to identify each unique condition, and this information can be used to "map" different conditions occurring over the plot area. A plot will have one or more conditions. Because of the potential for large differences among conditions, basal area per acre values were computed for each condition. Sometimes a condition occurred over a small portion of the plot, which could result in basal area per acre values that do not accurately reflect tree density for the condition.

Some species were combined into species groups to cover all forest tree species encountered and to maintain a sample size adequate to describe relationships between tree growth and predictor variables (Table 1). These groups predominantly were based on the aggregations used by Scott (1981), which were based on similarities in tree form. Table 2 shows a summary of the data used to fit the past diameter growth model. The data set used to fit the forward growth model was slightly smaller (about 10%), as some trees were dropped because of inability to obtain crown ratio values from the previous inventory. The attributes of these data are very similar to those of the data shown in Table 2, as these data comprise a large subset of those used in that summary. A validation data set was constructed by randomly withholding 20% of the observations.

Another independent data set was used to assess how model predictions would affect plot-level attributes of quadratic mean diameter and cubic-foot volume. These data rep-

resent the first cycle of remeasurement plots available in the NE-FIA region where data were collected under the FIA annual inventory system (McRoberts 2005). The attributes of these data are similar to those of the data used in model fitting (described above). The observations were collected between 1999 and 2003 from 2370 forested plots.

Model development

A wide range of attributes have been correlated with diameter growth rates. Often, stand- and tree-level measurements that are observed or computed are used as predictor variables in diameter growth models. Stand-level predictors often include age, site productivity, stand density, and stand size (Burkhart et al. 1987; Teck and Hilt 1991; Andreassen and Tomter 2003). Initial tree size, crown attributes, and social position descriptors often are used to tailor the prediction for a given tree (Cole and Lorimer 1994; King and Arner 1999; Zhang et al. 2004). In some instances, ancillary data (e.g., elevation) also have been useful (Wykoff 1990). In keeping with other modeling efforts aimed at mixed-species and largely uneven-aged stands, age and site index were not considered to be viable predictor variables (Wykoff 1990; Monserud and Sterba 1996; Schröder et al. 2002).

Several predictor variables were identified for possible inclusion in the model. NE-FIA collects forest inventory information at various levels of detail, which provides many potential contributors to diameter growth prediction. Because the model needed to be fit to a number of species groups, the modeling strategy was to find a parsimonious model form that adequately describes the observed variability in relative diameter growth, defined as

$$[1] \quad \Delta D_R = \frac{D_2 - D_1}{D_1}$$

where

ΔD_R is the relative periodic increment

D_1 is DBH at initial inventory (in.; 1 in. = 2.54 cm)

D_2 is DBH at subsequent inventory (in.)

Relative diameter growth was chosen as the dependent variable because it creates a frame of reference that improves model prediction accuracy when compared with untransformed diameter change (e.g., large trees tend to have relatively small values of relative change).

A number of potential predictor variables were evaluated. Those considered but not included in the final model were slope, aspect, temperature, precipitation, and various formulations of distance-independent competition indices. Variables retained in the final model are shown in the form of the composite model:

$$[2] \quad \Delta D_{R_{ijk}} = P_i^{\beta_0} \beta_1^{CR_{ijk}} \times \left(\beta_2 LAT_i + \beta_3 LNG_i + \frac{\beta_4}{1000} ELEV_i + \beta_5 M_{ijk} \right) \times \exp(\beta_6 BA_{ijk} + \beta_7 BAAC_{ij}) + \epsilon_{ijk}$$

where

i is the i th sample plot

j is the j th condition on i th sample plot

Table 1. Species groups and within-group species composition.

Group	Species	<i>n</i>	Percentage of group
1	Eastern white pine (<i>Pinus strobus</i> L.)	1686	90.89
	Red pine (<i>Pinus resinosa</i> Ait.)	169	9.11
2	Black spruce (<i>Picea mariana</i> (Mill.) BSP)	238	11.13
	Blue spruce (<i>Picea pungens</i> Engelm.)	2	0.09
	Red spruce (<i>Picea rubens</i> Sarg.)	1702	79.61
	White spruce (<i>Picea glauca</i> (Moench) Voss)	196	9.17
3	Balsam fir (<i>Abies balsamea</i> (L.) Mill.)	1615	100
4	Eastern hemlock (<i>Tsuga canadensis</i> (L.) Carrière)	1952	100
5	Douglas fir (<i>Pseudotsuga menziesii</i> (Mirb.) Franco)	1	0.1
	Norway spruce (<i>Picea abies</i> (L.) Karst.)	26	2.51
	Scots pine (<i>Pinus sylvestris</i> L.)	36	3.48
	Table Mountain pine (<i>Pinus pungens</i> Lamb.)	18	1.74
	Virginia pine (<i>Pinus virginiana</i> Mill.)	269	25.99
	Jack pine (<i>Pinus banksiana</i> Lamb.)	10	0.97
	Larch (introduced) (<i>Larix</i> sp. Mill.)	3	0.29
	Loblolly pine (<i>Pinus taeda</i> L.)	201	19.42
	Pitch pine (<i>Pinus rigida</i> Mill.)	346	33.43
	Shortleaf pine (<i>Pinus echinata</i> Mill.)	14	1.35
	Tamarack (native) (<i>Larix laricina</i> (Du Roi) K. Koch)	111	10.72
6	Atlantic white-cedar (<i>Chamaecyparis thyoides</i> (L.) BSP)	158	11.27
	Bald-cypress (<i>Taxodium distichum</i> (L.) L. Rich.)	1	0.07
	Eastern redcedar (<i>Juniperus virginiana</i> L.)	83	5.92
	Northern white-cedar (<i>Thuja occidentalis</i> L.)	1160	82.74
7	Sugar maple (<i>Acer saccharum</i> Marsh.)	2609	100
8	Balsam poplar (<i>Populus balsamifera</i> L.)	34	1.46
	Bigtooth aspen (<i>Populus grandidentata</i> Michx.)	249	10.73
	Black ash (<i>Fraxinus nigra</i> Marsh.)	92	3.96
	Eastern cottonwood (<i>Populus deltoides</i> Bartr. ex Marsh.)	11	0.47
	Green ash (<i>Fraxinus pennsylvanica</i> Marsh.)	75	3.23
	Quaking aspen (<i>Populus tremuloides</i> Michx.)	376	16.2
	Swamp cottonwood (<i>Populus heterophylla</i> L.)	5	0.22
	White ash (<i>Fraxinus americana</i> L.)	937	40.37
	Yellow-poplar (<i>Liriodendron tulipifera</i> L.)	542	23.35
	Black cherry (<i>Prunus serotina</i> Ehrh.)	739	100
10	Gray birch (<i>Betula populifolia</i> Marsh.)	78	2.55
	Paper birch (<i>Betula papyrifera</i> Marsh.)	1133	37.1
	River birch (<i>Betula nigra</i> L.)	7	0.23
	Sweet birch (<i>Betula lenta</i> L.)	692	22.66
	Yellow birch (<i>Betula alleghaniensis</i> Britt.)	1144	37.46
11	American beech (<i>Fagus grandifolia</i> Ehrh.)	1448	100
12	Bitternut hickory (<i>Carya cordiformis</i> (Wangenh.) K. Koch)	57	1.68
	Black oak (<i>Quercus velutina</i> Lam.)	510	15.04
	Black-gum (<i>Nyssa sylvatica</i> Marsh.)	245	7.23
	Cherrybark – swamp red oak (<i>Quercus falcata</i> var. <i>pagodaefolia</i> Ell.)	1	0.03
	Hickory (<i>Carya</i> sp. Nutt.)	37	1.09
	Mockernut hickory (<i>Carya tomentosa</i> (Poir.) Nutt.)	119	3.51
	Northern red oak (<i>Quercus rubra</i> L.)	1365	40.27
	Pignut hickory (<i>Carya glabra</i> (Mill.) Sweet)	224	6.61
	Pin oak (<i>Quercus palustris</i> Muenchh.)	13	0.38
	Scarlet oak (<i>Quercus coccinea</i> Muenchh.)	408	12.04
	Shagbark hickory (<i>Carya ovata</i> (Mill.) K. Koch)	163	4.81
	Shellbark hickory (<i>Carya laciniosa</i> (Michx. f.) Loud.)	1	0.03
	Shingle oak (<i>Quercus imbricaria</i> Michx.)	3	0.09
	Southern red oak (<i>Quercus falcata</i> var. <i>falcata</i> Michx.)	34	1
	Sweetgum (<i>Liquidambar styraciflua</i> L.)	185	5.46
	Willow oak (<i>Quercus phellos</i> L.)	25	0.74

Table 1 (concluded).

Group	Species	n	Percentage of group
13	Chestnut oak (<i>Quercus prinus</i> L.)	942	100
14	American basswood (<i>Tilia americana</i> L.)	245	9.01
	American elm (<i>Ulmus americana</i> L.)	109	4.03
	American holly (<i>Ilex opaca</i> Ait.)	33	1.22
	American hornbeam (<i>Carpinus caroliniana</i> Walt.)	19	0.7
	American mountain-ash (<i>Sorbus americana</i> Marsh.)	7	0.26
	Osage-orange (<i>Maclura pomifera</i> (Raf.) Schneid.)	3	0.11
	Paulownia (<i>Paulownia tomentosa</i> (Thunb.) Sieb. & Zucc. ex Steud.)	1	0.04
	Ailanthus (<i>Ailanthus altissima</i> (Mill.) Swingle)	4	0.15
	Apple (<i>Malus</i> sp. Mill.)	46	1.7
	Basswood (<i>Tilia</i> sp. L.)	3	0.11
	Black locust (<i>Robinia pseudoacacia</i> L.)	175	6.46
	Black maple (<i>Acer nigrum</i> Michx. f.)	1	0.04
	Black walnut (<i>Juglans nigra</i> L.)	69	2.55
	Black willow (<i>Salix nigra</i> Marsh.)	5	0.18
	Boxelder (<i>Acer negundo</i> L.)	38	1.4
	Buckeye (<i>Aesculus</i> sp. L.)	2	0.07
	Bur oak (<i>Quercus macrocarpa</i> Michx.)	2	0.07
	Butternut (<i>Juglans cinerea</i> L.)	9	0.33
	Catalpa (<i>Catalpa</i> sp. Scop.)	2	0.07
	Cherry, plum (<i>Prunus</i> sp. L.)	11	0.41
	Chinkapin oak (<i>Quercus muehlenbergii</i> Engelm.)	8	0.3
	Chokecherry (<i>Prunus virginiana</i> L.)	2	0.07
	Common persimmon (<i>Diospyros virginiana</i> L.)	9	0.33
	Cucumbertree (<i>Magnolia acuminata</i> L.)	53	1.96
	Eastern hophornbeam (<i>Ostrya virginiana</i> (Mill.) K. Koch)	208	7.64
	Eastern redbud (<i>Cercis canadensis</i> L.)	3	0.11
	Flowering dogwood (<i>Cornus florida</i> L.)	12	0.44
	Hackberry (<i>Celtis occidentalis</i> L.)	7	0.26
	Hawthorn (<i>Crataegus</i> sp. L.)	16	0.59
	Honeylocust (<i>Gleditsia triacanthos</i> L.)	1	0.04
	Magnolia (<i>Magnolia</i> sp. L.)	2	0.07
	Mountain magnolia (<i>Magnolia fraseri</i> Walt.)	13	0.48
	Mountain maple (<i>Acer spicatum</i> Lam.)	1	0.04
	Pin cherry (<i>Prunus pensylvanica</i> L. f.)	25	0.92
	Post oak (<i>Quercus stellata</i> Wangenh.)	12	0.44
	Sassafras (<i>Sassafras albidum</i> (Nutt.) Nees)	161	5.95
	Serviceberry (<i>Amelanchier</i> sp. Medic.)	38	1.4
	Slippery elm (<i>Ulmus rubra</i> Muhl.)	56	2.07
	Sourwood (<i>Oxydendrum arboreum</i> (L.) DC.)	68	2.51
	Striped maple (<i>Acer pensylvanicum</i> L.)	37	1.37
	Sugarberry (<i>Celtis laevigata</i> Willd.)	1	0.04
	Swamp chestnut oak (<i>Quercus michauxii</i> Nutt.)	13	0.48
	Swamp white oak (<i>Quercus bicolor</i> Willd.)	25	0.92
	Sweetbay (<i>Magnolia virginiana</i> L.)	4	0.15
	Sycamore (<i>Platanus occidentalis</i> L.)	41	1.51
	Water oak (<i>Quercus nigra</i> L.)	12	0.44
	Water tupelo (<i>Nyssa aquatica</i> L.)	1	0.04
	White oak (<i>Quercus alba</i> L.)	1067	39.29
	Willow (<i>Salix</i> sp. L.)	7	0.26
	Yellow buckeye (<i>Aesculus octandra</i> Marsh.)	26	0.96
15	Red maple (<i>Acer rubrum</i> L.)	5302	99.44
	Silver maple (<i>Acer saccharinum</i> L.)	30	0.56

Table 2. Summary of data by species group.

Group	n	Measurement interval (years)			DBH (in.)			Basal area (ft ² /acre)			Latitude (degrees)		Longitude (degrees)		Elevation (ft)	
		Min.	Mean	Max.	Min.	Mean	Max.	Min.	Mean	Max.	Min.	Max.	Min.	Max.	Min.	Max.
1	1855	3	9.1	17	5	11.4	40.7	3.7	163.3	445.8	37.37	46.75	-82.26	-67.31	13	3038
2	2138	3	7.1	17	5	8.6	25.2	3.6	119.7	359.3	38.19	47.35	-80.38	-67.04	0	4282
3	1615	3	6.8	17	5	7.7	18.7	3.8	114.2	359.3	39.04	47.31	-79.47	-67.04	0	4023
4	1952	3	10.1	17	5	10.0	33.8	5.2	151.7	436.3	37.38	46.53	-82.36	-67.46	59	3865
5	1035	3	11.6	15	5	9.6	23.4	3.6	105.3	359.3	37.32	47.21	-82.52	-67.46	0	2946
6	1402	3	7.4	15	5	8.7	30.1	2.4	151.9	359.3	37.56	47.35	-84.72	-67.10	0	2759
7	2609	3	10.9	17	5	10.2	41.7	4.0	115.2	349.4	37.29	47.35	-84.72	-67.40	33	4239
8	2321	3	10.2	17	5	10.7	30.3	3.7	119.3	381.2	37.32	47.22	-84.60	-67.17	7	4239
9	739	3	11.6	16	5.1	11.1	29.5	5.0	125.6	359.3	37.31	46.01	-84.04	-68.26	10	4239
10	3054	3	9.5	17	5	9.1	31.9	3.6	112.0	368.1	37.30	47.41	-82.14	-67.04	0	4285
11	1448	3	10.0	17	5	9.4	34.5	3.7	113.8	295.3	37.30	47.29	-82.29	-67.54	7	4232
12	3390	3	11.8	17	5	11.0	41.6	3.7	114.3	368.1	37.24	45.27	-84.76	-67.94	0	4239
13	942	8	12.3	15	5.2	11.0	35	11.5	112.6	335.5	37.32	43.61	-83.35	-71.72	69	3370
14	2715	3	12.1	17	5	9.9	43.4	4.3	103.9	368.1	37.29	47.16	-84.72	-67.15	0	4101
15	5332	3	10.3	17	5	9.6	44.9	3	116.3	352.6	37.31	47.24	-84.76	-67.09	0	4232

Note: 1 in. = 2.54 cm; 1 ft = 0.3048 m; 1 ft²/acre = 0.2296 m²/ha.

k is the k th tree on j th condition on i th sample plot

ΔD_{ijk} is the relative periodic increment

P_i is the length of measurement interval (years)

CR_{ijk} is the compacted crown ratio (%)

LAT_i is latitude (degrees)

LNG_i is longitude (degrees)

$ELEV_i$ is the elevation above sea level (ft; 1 ft = 0.3048 m)

M_{ijk} is a mortality indicator (0, live tree; 1, tree died between measurements)

BA_{ijk} is the tree basal area at initial inventory (ft²; 1 ft² = 0.0929 m²)

$BAAC_{ij}$ is the condition-level basal area per acre at initial inventory (ft²/acre; 1 ft²/acre = 0.2296 m²/ha)

ε_{ijk} is an error term

β_0 – β_7 are parameters to be estimated

When growth projection time interval is not standardized (e.g., annualized), a necessary model input is length of growth projection (P). Tree-level factors important to prediction of relative diameter change include crown ratio (CR) and basal area (BA). CR is an indicator of growth potential, and BA facilitates definition of relative growth. A tree mortality indicator (M) was used to account for potential slower growth rates for trees suffering mortality during the projection period. Stand-level predictor variables include condition-level basal area per acre (BAAC), latitude (LAT), longitude (LNG), and elevation (ELEV). BAAC was used to describe the level of competition for resources, and LAT, LNG, and ELEV were included to account for environmental influences. BAAC values from forested plots with no mapping information could also be used for prediction, with the possible loss of predictive accuracy when plot- and condition-level BAAC are dissimilar. Parameter estimation was accomplished using the SAS NLMIXED procedure (SAS Institute Inc. 2003), and significance of estimates was determined at the 0.05 level.

Initial regression analyses were performed using this model and plots of residuals (observed–predicted) were evaluated for model adequacy. Although there were no systematic trends noted, the plot of residuals versus predicted values indicated heterogeneity of variance. To account for this, the error distribution was described as

$$[3] \quad \varepsilon_{ijk} \sim \{0, \sigma^2 [\delta_1^{BA_{ijk}} \times \exp(\delta_2 P_i + \delta_3 BAAC_{ij})]\}$$

where

σ^2 is the model error variance

δ_1 – δ_3 are parameters to be estimated

others variables are as previously defined.

This methodology permits heterogeneous variance among individual trees, plot conditions, and projection lengths. A similar method of describing model variance was taken by Valentine and Gregoire (2001), who specified within- and among-tree variance for taper equations.

Correlations among growth rates for trees located within a plot condition are another issue. Correlated observations violate the least-squares regression assumption of independence. Ignoring this circumstance still provides unbiased estimates of model parameters; however, the estimate of model error is biased (Swindel 1968; Sullivan and Reynolds 1976). This is a cause for concern because there is a direct

effect on inferences for estimated model parameters. With biased error estimates, model coefficients that are not statistically significant may be included in the model. Even worse, relevant predictor variables may be dropped from the model because of a false indication of nonsignificance. Correlations among observations were accounted for in the model-fitting process by incorporating random-effects parameters into the model (Gregoire and Schabenberger 1996). This mixed-effects modeling methodology allows for indirect specification of the variance-covariance matrix used to estimate model parameters (Gregoire et al. 1995). The mixed-effects model is specified as

$$[4] \quad \Delta D_{R_{ijk}} = P_i^{\beta_0} (\beta_1 + \theta_{ijk})^{CR_{ijk}} \\ \times \left(\beta_2 \text{LAT}_i + \beta_3 \text{LNG}_i + \frac{\beta_4}{1000} \text{ELEV}_i + \beta_5 M_{ijk} \right) \\ \times \exp[\beta_6 \text{BA}_{ijk} + (\beta_7 + \theta_{2ijk}) \text{BAAC}_{ij}] + \varepsilon_{ijk}$$

where

θ_{hijk} 's are random-effects parameter

$\theta_h \sim N(0, \sigma_h^2)$, $h = 1, 2$

other variables are as previously defined.

This formulation allows coefficients for BAAC and CR to vary from tree to tree, essentially providing a model tailored to each tree in the fitting data. Thus, the accuracy of prediction for these trees is improved. However, in most practical applications for prediction the random-effects parameters are set to their expected value of zero. Thus, the predictive accuracy for new observations is roughly the same as would be obtained without addition of random-effects parameters. Methodologies have been proposed to predict random effects for new observations (Fang and Bailey 2001), but they require additional information and computation. The primary appeal for a mixed-effects model is to obtain improved estimates of model variance when observations are correlated.

The work described above is consistent with most diameter-growth research, where the primary application is to predict future growth based on current conditions. However, there are situations where it is desirable to determine past rates of growth that led to the current conditions. For instance, the estimation procedures outlined by Scott et al. (2005) assume that any reversion to forest land occurs at the midpoint of the plot measurement interval. To properly account for growth, tree diameters at the interval midpoint are needed and must be computed from current values. To facilitate such computations, model [4] also was fitted to the data where the most recent inventory data formed the basis for variables whose values change over time (i.e., CR, BA, BAAC). Under this data formulation, the definition of ΔD_R in eq. 1 is changed to reflect the change in basis:

$$[5] \quad \Delta D_R = \frac{D_2 - D_1}{D_2}$$

where all variables are as previously defined.

Although diameter growth is now being projected into the past, the measurement interval (P) remains a positive value for prediction purposes.

Results

Models for both future and past diameter growth were fitted for each species group listed in Table 2. As expected, not all parameter estimates were significant at the 95% confidence level. For species groups where this occurred, nonsignificant parameters were removed and the model was recalibrated using only significant predictors.

These models were used to predict relative diameter growth for the validation data, and the resulting residuals were analyzed to evaluate model performance.

The mean residuals from the validation data for both the future and past projection models are almost all positive (Table 3), indicating a net underprediction of growth. However, the magnitude of error compared with the mean relative rate of growth is small (e.g., 5%–10%), and generally, the magnitude of error is proportional to rate of growth. Conversely, the median residuals are generally negative in value, suggesting slight overprediction of growth when the influence of extreme values is removed. These residual patterns are due to a small number of trees that grew at accelerated rates in relation to tree- and stand-level conditions, which had the effect of creating a positive mean residual and a negative median residual. This is probably due to one or more factors that contribute to particularly favorable microsite conditions for a given tree or group of trees. This same phenomenon was noted by Lessard et al. (2001) and appears to be a factor in the results of Zhang et al. (2004).

To obtain optimal estimates of parameters, the models were recalibrated using all available data. The final estimates of model parameters and variance components are given in Tables 4 and 5. Model parameter estimates not statistically significant at the 95% confidence level are denoted with zeros. Because the dependent variable is unitless, the only modifications necessary for application to data measured in metric units are for coefficients of elevation (ELEV), individual-tree basal area (BA), and condition-level basal area per acre (BAAC). For ELEV measured in metres, the reported coefficients should be multiplied by 3.281, for BA in square metres the conversion would be a factor of 10.764, and for BAAC computed in square metres per hectare the coefficient translation is accomplished via multiplication by 4.359.

An examination of performance for the final models was accomplished using independent data from Maine. These measurements were not included in the data used to estimate the final model parameters. The future growth model was applied at the time of initial measurement, and results were compared with observed data from the most recent measurement. Similarly, the past growth model used information from the most recent measurement to project diameters for comparison with initial values. Application of the models to these data was performed by setting the random-effects parameters to their expected value of zero. Assessments were made for quadratic mean diameter (DBH_q) and cubic-foot volume per acre prediction. To compute individual tree volumes, a single-entry volume equation based on DBH² was fitted using the predicted volumes (Scott 1981) at the initial measurement. This was done because Scott's equations use a merchantable height estimate, which was not available in these growth projections. Only one tree appeared in group

Table 3. Residual analysis from validation data for future and past growth models by species group.

Group	Future growth model					Past growth model				
	<i>n</i>	Mean residual	SD	Median residual	Interquartile range	Mean residual	SD	Median residual	Interquartile range	Mean annual residual
										ΔD_R
1	340	0.009	0.116	0.000	0.117	0.001	0.155	0.005	0.076	0.001
2	429	0.006	0.060	-0.001	0.058	0.001	0.077	-0.001	0.049	0.001
3	298	0.010	0.087	-0.008	0.085	0.001	0.129	-0.006	0.062	0.001
4	330	0.002	0.092	0.003	0.106	0.000	0.148	0.001	0.062	0.000
5	211	0.018	0.139	-0.007	0.105	0.002	0.168	0.012	0.095	0.002
6	258	-0.001	0.056	-0.007	0.038	0.000	0.057	-0.002	0.043	0.000
7	416	0.010	0.107	-0.004	0.106	0.001	0.138	0.002	0.082	0.001
8	370	0.018	0.130	-0.006	0.093	0.002	0.160	-0.006	0.079	0.002
9	115	0.038	0.160	0.005	0.141	0.003	0.197	-0.007	0.132	0.003
10	569	0.007	0.083	-0.006	0.079	0.001	0.110	-0.005	0.063	0.001
11	242	0.005	0.090	-0.009	0.096	0.000	0.138	-0.007	0.062	0.000
12	567	0.023	0.142	-0.011	0.133	0.002	0.185	-0.008	0.088	0.002
13	163	-0.009	0.070	-0.014	0.096	-0.001	0.131	-0.007	0.070	-0.001
14	481	0.009	0.102	-0.004	0.103	0.001	0.135	-0.005	0.094	0.001
15	861	0.007	0.110	-0.011	0.103	0.000	0.138	-0.002	0.085	0.000

Note: SD, standard deviation; ΔD_R , relative periodic increment.

13 (chestnut oak), so this group was not included in the analysis.

Results for the future diameter growth model show underprediction of both DBH_q and volume per acre for all groups except group 3 (Table 6). Differences between predicted and observed DBH_q were mostly near 0.1 in. (0.25 cm), and deviations in volume were on the order of 15 ft³/acre (1.05 m³/ha) or less. Bias over the projection period for DBH_q are generally near 1%, while differences in volume were primarily less than 3%. Projection period percent bias was computed as (observed - predicted)/observed. Computation of average annual bias shows most groups under 0.3% for DBH_q and under 1.0% for volume predictions. The percent annual bias is equal to the projection period bias divided by projection length. These results compare favorably with levels of bias for other efforts aimed at prediction of future diameter or basal area growth (Andreassen and Tomter 2003 (and references therein); Zhang et al. 2004), where basal area growth prediction bias ranges from roughly 10% to 50%.

Predictions of past DBH_q and volume were consistent with the future projections in that, generally, the change over the projection period was underestimated and biases were of similar magnitude (Table 7). Also consistent with future projections was the behavior of individual species groups. Group 5 (pine) had notably poorer predictive accuracy than other groups for both future and past projections. The only instance of consistent overprediction of future and past growth occurred with group 3 (balsam fir).

Discussion

For the future projection models, all of the nonsignificant parameter estimates were associated with latitude, longitude, and elevation. Where this situation occurs, there is no influence from these predictors on tree growth rates or the data lack geographic extent to capture the effect. When significant, the effect of latitude was not consistent across all groups. Negative estimates were associated with groups 7 (sugar maple), 9 (black cherry), 13 (chestnut oak), 14 (miscellaneous hardwood), and 15 (red-silver maple), suggesting that growth rates decrease as location moves northward. Better growth in northern latitudes is provided for groups 1 (red-white pine), 11 (beech), and 12 (oak-hickory). For red-white pine and beech groups, this is reasonable, as the natural range of these species extends well beyond northeastern United States. The reason for this outcome in oak-hickory is not readily apparent. Significant parameter estimates for effect of longitude were all negative, indicating better growth is attained in western areas of the region. This outcome is likely due to a number of factors, including soil type, topography, and distribution of the species within the region. Elevation was a significant predictor for only 4 of 15 species groups. Group 6 (cedar) had a positive parameter estimate, as the species presence in mountainous areas of West Virginia is closer to the middle of the natural range of occurrence (Harlow et al. 1991). Increased elevation was detrimental to growth for groups 5 (pines), 8 (poplars), and 13 (chestnut oak), likely because of factors such as cooler temperatures and relatively dry, shallow soils. These species

Table 4. Estimated parameters and variance components for future diameter growth model [4] with variance specified in [3] by species group.

Group	β_0	β_1	β_2	β_3	β_4	β_5	β_6
1	0.878 909	1.011 245	0.000 526	0	0	-0.013 147	-0.229 599
2	0.712 278	1.012 648	0	-0.000 333	0	-0.017 561	-0.880 359
3	0.824 870	1.011 810	0	-0.000 402	0	-0.010 811	-1.111 420
4	0.814 258	1.007 323	0	-0.000 396	0	-0.014 732	-0.389 773
5	1.423 649	1.003 472	0	-0.000 105	-0.001 219	-0.006 135	-0.483 440
6	1.044 842	1.011 152	0	-0.000 090	0.001 225	-0.002 899	-0.765 228
7	0.739 105	1.006 620	-0.000 443	-0.000 720	0	-0.025 543	-0.455 677
8	0.974 299	1.002 874	0	-0.000 338	-0.001 841	-0.017 935	-0.162 309
9	1.092 324	1.007 052	-0.000 555	-0.000 489	0	-0.011 004	-0.323 671
10	0.854 890	1.007 102	0	-0.000 349	0	-0.016 414	-1.005 767
11	0.907 462	1.006 347	0.000 539	0	0	-0.016 505	-0.809 807
12	0.713 109	0.998 422	0.001 126	0	0	-0.026 907	-0.171 425
13	0.734 374	0.997 010	-0.001 366	-0.001 321	-0.004 721	-0.019 390	-0.284 927
14	0.720 911	1.001 557	-0.000 720	-0.000 786	0	-0.020 346	-0.329 420
15	0.773 920	1.004 973	-0.000 557	-0.000 747	0	-0.022 861	-0.520 040

Note: $\Delta D_{ijk} = P_i^{\beta_0} (\beta_1 + \theta_{1ijk})^{CR_{ijk}} \times \left(\beta_2 LAT_i + \beta_3 LNG_i + \frac{\beta_4}{1000} ELEV_i + \beta_5 M_{ijk} \right) \times \exp[\beta_6 BA_{ijk} + (\beta_7 + \theta_{2ijk}) BAAC_{ij}] + \varepsilon_{ijk}$
 $\varepsilon_{ijk} \sim N\{0, \sigma^2[\delta_1^{BA_{ijk}} \times \exp(\delta_2 P_i + \delta_3 BAAC_{ij})]\}$

Table 5. Estimated parameters and variance components for past diameter growth model [4] with variance specified in [3] by species group.

Group	β_0	β_1	β_2	β_3	β_4	β_5	β_6
1	0.933 035	1.021 482	0.000 257	0	0	-0.005 399	-0.099 140
2	0.804 798	1.018 835	0	-0.000 160	0	-0.007 635	-0.437 110
3	0.923 259	1.017 689	-0.000 268	-0.000 355	-0.001 094	-0.002 620	-0.315 822
4	0.883 965	1.011 413	0.000 329	0	0	-0.007 047	-0.200 491
5	1.229 973	1.011 141	0	-0.000 082	0	-0.004 929	0
6	1.186 289	1.016 165	0	-0.000 054	0	-0.001 354	-0.538 209
7	0.881 443	1.013 129	-0.000 211	-0.000 331	-0.000 885 5	-0.008 924	-0.227 245
8	0.885 465	1.015 237	0	-0.000 211	-0.000 856 9	-0.007 203	0
9	1.084 245	1.010 913	0	-0.000 121	0	-0.006 800	0
10	1.021 052	1.014 734	0	-0.000 140	-0.000 746	-0.004 597	-0.431 619
11	0.922 177	1.011 145	0.000 311	0	0	-0.007 960	-0.288 444
12	0.707 828	1.005 732	0.000 674	0	0	-0.011 176	-0.038 387
13	0.756 939	1.009 653	-0.000 439	-0.000 498	-0.001 843 7	0	-0.162 899
14	0.639 391	1.007 645	-0.000 517	-0.000 582	0	-0.012 649	0
15	0.799 012	1.015 157	-0.000 199	-0.000 310	0	-0.009 966	-0.103 281

Note: $\Delta D_{ijk} = P_i^{\beta_0} (\beta_1 + \theta_{1ijk})^{CR_{ijk}} \times \left(\beta_2 LAT_i + \beta_3 LNG_i + \frac{\beta_4}{1000} ELEV_i + \beta_5 M_{ijk} \right) \times \exp[\beta_6 BA_{ijk} + (\beta_7 + \theta_{2ijk}) BAAC_{ij}] + \varepsilon_{ijk}$
 $\varepsilon_{ijk} \sim N\{0, \sigma^2[\delta_1^{BA_{ijk}} \times \exp(\delta_2 P_i + \delta_3 BAAC_{ij})]\}$

can occupy these poorer sites, but often exhibit slow growth rates.

The remaining model parameters were significant with consistent signs across all groups. As expected, parameter estimates associated with projection length (P) and crown ratio (CR) were positive, indicating that growth rates increase as values of these predictors increase. Longer crowns usually produce more leaf area, which increases photosynthetic capability, which, in turn, often accelerates growth rates. Similarly, parameter estimates for tree basal area (BA) and condition basal area per acre ($BAAC$) were negative, resulting in less relative diameter growth as these values be-

come larger. Because of the formulation of ΔD_R and tree growth patterns with age, larger trees (BA) tend to have less growth relative to their size than smaller trees. Increasing competition from neighboring trees, as indicated by larger values of $BAAC$, also produces slower growth rates. Negative estimates for the mortality indicator (M) are associated with reduced growth rates for trees suffering mortality during the projection period. Most of these trees are suppressed and are dying because of their inability to compete for resources.

The coefficients for past projection models are similar to those for future projections in that the preponderance of

β_7	δ_1	δ_2	δ_3	σ^2	σ_1^2	σ_2^2
-0.003 358	0.437 276	-0.003 641	0.216 911	0.001 752	0.000 078	0.000 004
-0.004 636	0.560 406	-0.003 342	0.186 338	0.000 631	0.000 118	0.000 014
-0.003 533	0.305 666	-0.004 235	0.190 900	0.001 269	0.000 073	0.000 014
-0.003 814	0.299 223	-0.005 792	0.176 670	0.001 648	0.000 057	0.000 012
-0.006 412	0.240 925	-0.002 014	0.285 277	0.000 424	0.000 064	0.000 021
-0.001 250	0.359 015	-0.002 023	0.272 353	0.000 214	0.000 061	0.000 002
-0.005 929	0.311 670	-0.004 657	0.157 149	0.001 618	0.000 167	0.000 023
-0.004 388	0.245 028	-0.000 981	0.188 544	0.001 525	0.000 261	0.000 015
-0.004 314	0.261 881	-0.011 374	0.212 424	0.003 866	0.000 169	0.000 024
-0.004 985	0.886 407	-0.003 704	0.134 560	0.001 164	0.000 117	0.000 024
-0.003 457	0.686 395	-0.006 304	0.149 882	0.001 798	0.000 033	0.000 006
-0.003 849	0.269 801	-0.002 587	0.216 853	0.001 112	0.000 134	0.000 011
-0.004 311	0.428 776	-0.002 555	0.118 745	0.001 377	0.000 087	0.000 018
-0.003 987	0.807 569	-0.005 286	0.129 932	0.002 105	0.000 253	0.000 038
-0.004 567	0.777 774	-0.003 562	0.177 475	0.001 209	0.000 195	0.000 020

β_7	δ_1	δ_2	δ_3	σ^2	σ_1^2	σ_2^2
-0.002 818	0.798 170	-0.003 536	0.200 433	0.000 685	0.000 055	0.000 006
-0.003 562	0.922 368	-0.004 846	0.160 785	0.000 519	0.000 074	0.000 006
-0.001 998	0.662 840	-0.003 349	0.204 761	0.000 569	0.000 016	0.000 002
-0.002 577	0.512 382	-0.003 212	0.156 498	0.000 635	0.000 040	0.000 003
-0.005 162	1.067 624	-0.003 839	0.194 636	0.000 370	0.000 037	0.000 016
-0.001 343	0.646 021	-0.001 957	0.255 918	0.000 158	0.000 038	0.000 008
-0.004 038	0.646 382	-0.005 984	0.164 153	0.000 880	0.000 069	0.000 006
-0.003 211	0.611 102	0	0.151 537	0.000 723	0.000 203	0.000 005
-0.003 193	0.339 046	-0.007 426	0.195 041	0.001 459	0.000 149	0.000 006
-0.003 384	0.683 437	-0.005 255	0.174 214	0.000 620	0.000 102	0.000 007
-0.002 768	0.627 137	-0.004 348	0.175 276	0.000 781	0.000 062	0.000 007
-0.002 925	0.769 942	-0.002 703	0.171 682	0.000 613	0.000 078	0.000 004
-0.002 594	0.759 340	0	0.155 762	0.000 329	0.000 094	0.000 008
-0.003 337	1.019 248	-0.005 011	0.104 539	0.001 329	0.000 162	0.000 016
-0.002 904	0.855 430	-0.003 194	0.136 978	0.000 922	0.000 148	0.000 013

nonsignificant parameter estimates is associated with the environmental predictors latitude, longitude, and elevation. Again, latitude had both positive and negative effects on growth rates. Magnitude and direction of influence were similar to those reported for future projections, except for groups 3 (balsam fir), 4 (eastern hemlock), and 9 (black cherry). For groups 3 and 4, latitude had a significant effect: negative in the case of group 3 and positive for group 4. These effects are consistent with the growth patterns of these species (Harlow et al. 1991). Latitudinal influence on growth for group 9 was nonsignificant when fitting the past projection model. Again longitudinal effects consistently fa-

vored western areas of the region. Results by species group were similar to those of the future projection model, except that group 4 (eastern hemlock) had a nonsignificant parameter estimate. Significant effects of elevation were consistently negative, with groups 3 (balsam fir), 7 (sugar maple), and 10 (black cherry) showing a significant response and groups 5 (pines) and 6 (cedar) having no response to changes in elevation.

A perplexing outcome was the nonsignificance of the estimated β_6 parameter (BA) for groups 5 (pines), 8 (poplar), 9 (black cherry), and 14 (miscellaneous hardwood), although this parameter was almost significant for groups 5 and 8

Table 6. Comparison between predicted and observed quadratic mean diameter and volume per acre for future diameter projection by species group.

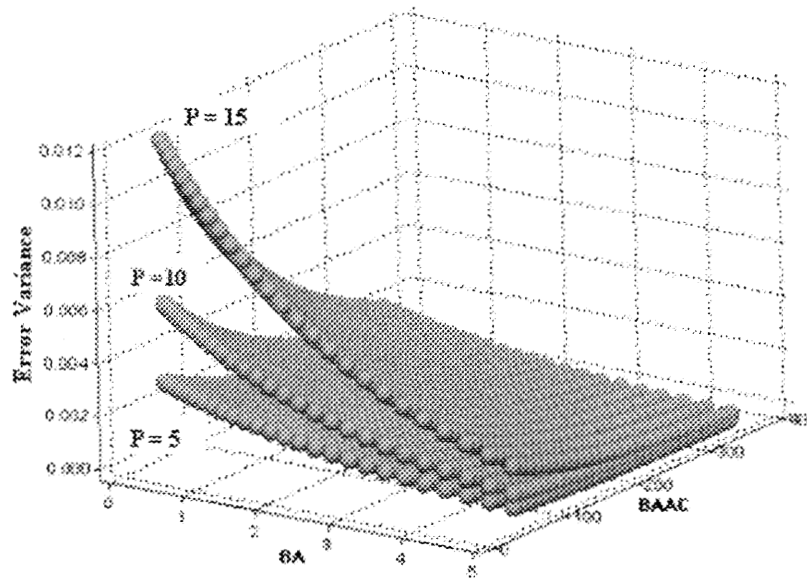
Group	n	Mean projection length (years)	Quadratic mean diameter (in.)				Volume (ft ³ /acre)							
			Predicted mean	Predicted SD	Observed mean	Observed SD	Periodic bias (%)	Mean annual bias (%)	Predicted mean	Predicted SD	Observed mean	Observed SD	Periodic bias (%)	Mean annual bias (%)
1	159	4.4	12.31	5.02	12.44	4.99	1.00	0.23	1003.8	1320.8	1019.7	1351.6	1.56	0.36
2	398	4.5	8.41	2.00	8.54	2.02	1.49	0.33	497.2	904.7	512.7	940.1	3.02	0.66
3	374	4.5	7.44	1.46	7.33	1.33	-1.51	-0.34	257.8	320.0	247.9	298.1	-4.01	-0.90
4	152	4.3	10.13	3.14	10.23	3.10	1.00	0.23	561.1	871.2	573.9	912.7	2.23	0.52
5	32	4.4	10.13	3.74	10.42	3.78	2.72	0.61	255.6	244.6	272.7	256.4	6.27	1.42
6	203	4.6	9.94	2.94	10.02	2.98	0.84	0.18	575.5	689.0	589.4	708.9	2.37	0.51
7	163	4.4	10.27	3.57	10.39	3.54	1.13	0.26	509.7	606.2	524.9	624.4	2.89	0.66
8	209	4.5	9.24	3.03	9.36	2.92	1.26	0.28	385.9	515.6	400.4	546.6	3.62	0.81
9	28	4.3	7.39	1.66	7.46	1.57	0.96	0.22	112.1	206.8	114.2	204.1	1.87	0.44
10	425	4.4	8.97	3.00	9.00	3.05	0.38	0.09	291.8	368.7	295.5	380.6	1.25	0.28
11	148	4.3	8.28	1.89	8.33	1.83	0.62	0.14	286.2	354.9	286.6	357.4	0.14	0.03
12	73	4.3	10.33	3.60	10.46	3.47	1.23	0.29	472.0	599.3	480.4	619.3	1.75	0.41
14	108	4.3	6.94	2.18	6.97	2.11	0.39	0.09	61.2	69.7	61.7	69.3	0.80	0.18
15	390	4.4	8.86	2.42	8.96	2.41	1.06	0.24	382.5	438.5	393.0	455.4	2.68	0.61

Note: SD, standard deviation; 1 in. = 2.54 cm; 1 ft³/ac = 0.0700 m³/ha.**Table 7.** Comparison between predicted and observed quadratic mean diameter and volume per acre for past diameter projection by species group.

Group	n	Mean projection length (years)	Quadratic mean diameter (in.)				Volume (ft ³ /acre)							
			Predicted mean	Predicted SD	Observed mean	Observed SD	Periodic bias (%)	Mean annual bias (%)	Predicted mean	Predicted SD	Observed mean	Observed SD	Periodic bias (%)	Mean annual bias (%)
1	159	4.4	11.66	4.85	11.61	4.95	-0.44	-0.10	922.6	1285.8	909.8	1253.3	-1.41	-0.32
2	398	4.5	8.15	2.02	8.04	2.03	-1.38	-0.30	469.3	919.4	456.7	885.7	-2.76	-0.61
3	374	4.5	6.76	1.26	6.79	1.29	0.47	0.10	201.4	260.0	204.8	262.4	1.65	0.37
4	152	4.3	9.65	3.05	9.57	3.13	-0.92	-0.21	518.0	894.6	507.7	855.0	-2.03	-0.47
5	32	4.4	10.10	3.69	9.91	3.73	-1.85	-0.42	255.8	246.0	244.0	237.3	-4.85	-1.10
6	203	4.6	9.75	2.95	9.67	2.93	-0.92	-0.20	553.7	675.6	539.4	655.2	-2.66	-0.58
7	163	4.4	9.97	3.48	9.83	3.56	-1.35	-0.31	482.4	588.4	466.6	572.5	-3.39	-0.77
8	209	4.5	8.86	2.84	8.78	2.99	-0.86	-0.19	356.1	510.7	345.8	482.1	-2.97	-0.67
9	28	4.3	7.17	1.53	7.17	1.64	0.06	0.01	102.3	187.5	103.4	194.8	1.08	0.25
10	425	4.4	8.68	3.04	8.66	3.04	-0.26	-0.06	274.5	364.7	272.2	354.4	-0.86	-0.19
11	148	4.3	7.90	1.76	7.85	1.87	-0.54	-0.12	252.6	329.2	252.7	326.9	0.05	0.01
12	73	4.3	9.83	3.36	9.69	3.52	-1.36	-0.32	421.8	586.0	414.1	570.8	-1.85	-0.43
14	108	4.3	6.73	2.02	6.72	2.11	-0.24	-0.06	55.4	63.8	55.2	64.9	-0.30	-0.07
15	390	4.4	8.56	2.33	8.49	2.40	-0.83	-0.19	355.3	430.5	348.4	419.3	-1.97	-0.45

Note: SD, standard deviation; 1 in. = 2.54 cm; 1 ft³/ac = 0.0700 m³/ha.

Fig. 1. Estimated error variance from [3] for 5, 10, and 15 year future projection lengths (P) for species group 12. BA, basal area; BAAC, condition-level basal area per acre.



($0.05 < p < 0.10$). Given that the dependent variable was relative diameter growth (ΔD_R), it was expected that a tree size predictor variable (BA) would always be important. One possible cause of nonsignificance of the β_6 estimate is that values of ΔD_R for past growth are smaller and have less dispersion than those for future growth. Thus, ΔD_R is more constant, and the importance of BA in defining relative growth rates is diminished.

Shifts in significance of estimated parameters that occur when changing from future to past projections are puzzling, as one would expect that factors influencing growth would affect both past and future rates. An underlying cause may be change in the basis of relativity between the future and past model-fitting data. For a given rate of change, the relative change is larger for forward projections. This creates more variability in the dependent variable, but more importantly, creates differences in variability between the fitting data sets. Depending on how these differences arise, the ability of predictors to explain the variation may increase, stay about the same, or decrease. Hence, the significant predictors for future projections may not match with those for past projections. There could be a number of alternative explanations as well. Further study is needed to fully understand the phenomenon.

The estimated parameters for the variance model can be used to examine trends in variance related to projection length (P), basal area per acre (BAAC), and tree basal area (BA). Figure 1 depicts the trend surface of species group 12 for projection lengths of 5, 10, and 15 years. The variability in ΔD_R is fairly flat for the 5-year projection with a trend toward increasing variability as projection length increases. Additionally, conditions where trees are relatively small and stand density is low create higher variability. These circumstances likely occur in immature stands where trees are subjected to varying levels of competition. Growth rates become quite variable as some trees assume dominance and others

become suppressed. In contrast, there is low variability for large trees and stands having high basal area per acre values. In these mature stands, growth rates are more consistent, as the stand is not in a rapid development phase.

Conclusion

Models to estimate both future and past relative diameter growth were developed and fitted to data from 15 species groups occurring in the northeastern United States. Correlations among growth rates for trees on the same plot condition were accounted for via a mixed-effects modeling approach. Additionally, a function was specified to account for heterogeneous variance among individual trees, plot conditions, and growth projection lengths. The models described can be used with a minimal amount of data. The only field-collected parameters needed are DBH, crown ratio, and for past projections, an indicator of mortality. Latitude, longitude, and elevation can be obtained in the field via GPS or acquired from one of many other sources (maps, GIS, etc.) at a later time. For future projections, mortality assumptions are needed and may be obtained via a mortality model or other methods.

With the exception of group 3 (balsam fir), application to independent data showed that growth rates were slightly underpredicted for both future and past projections. However, the magnitude of bias was minimal for most species groups. A wide range of flexibility is afforded across species, forest conditions, and projection lengths, which should make the models suitable for many applications. With this in mind, users are cautioned to avoid extrapolation, primarily for tree sizes, projection lengths, and geographic areas not represented in the model fitting data.

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