The AFIS Tree Growth Model for Updating Annual Forest Inventories in Minnesota

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Abstract.—As the Forest Service moves towards annual inventories, states may use model predictions of growth to update unmeasured plots. A tree growth model (AFIS) based on the scaled Weibull function and using the average-adjusted model form is presented. Annual diameter growth for four species was modeled using undisturbed plots from Minnesota's Aspen-Birch and Northern Pine Inventory Units. On the validation database, the AFIS model showed increased precision over the recalibrated STEMS model by 5 percent for diameter predictions and 8 percent for stand basal area predictions. The average-adjusted model form has a number of desirable features.

The mission of the Forest Inventory and Analysis (FIA) program is to conduct periodic inventories of U.S. forestlands to furnish updated information about the resource. To provide more precise and timely forest inventory information than produced with the present FIA design, Congress has mandated that the USDA Forest Service move to annual inventories on a 5- to 7-year cycle. In the north central region, a part of the new inventory procedure is to replace the Stand and Tree Evaluation and Modeling System (STEMS) with a new model to better update unmeasured inventory plots.

Currently FIA inventories use variable-radius plots. Since this sampling design is not optimal for growth modeling, substantial bias and variability may be introduced by the sampling procedure. However, an improved model form may increase the accuracy and precision of growth estimates on inventory data. Further, it is important that the models be calibrated from the same data and sampling scheme that the model will be applied to. First, the STEMS tree growth model is recalibrated on FIA data to evaluate the effect of modeling the same model form on FIA data as opposed to non-inventory data (Lessard 2000). Second, I evaluate a new model form calibrated on the same FIA data used in the recalibration of STEMS. Third, these models are analyzed with respect to the uncertainty of their predictions (McRoberts *et al.* 2000).

In this paper I present a new individual tree growth model called AFIS (Annual Forest Inventory System). The objectives of the study are to (1) develop improved tree growth models on Minnesota's Aspen-Birch and Northern Pine Inventory Units, (2) calibrate the model with FIA inventory data using physiographic class as a measure of site productivity and the average-adjusted model form, and (3) compare the model predictions with the recalibrated model (called STEMS-FIA).

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BACKGROUND

Modeling Inventory Data

The North Central FIA unit uses a growth model-enhanced sampling design to inventory forests. To update previously measured plots, STEMS (Belcher *et al.* 1982), a distance-independent individual-tree growth model, was used to predict the growth of trees from the last inventory to the present. STEMS was calibrated from a variety of remeasured data sources—including research and industrial plots. Most were fixed-radius plots of 1/10 acre to 1/5 acre, representing small, fairly homogeneous areas. However, inventory plots often represent more diverse forest conditions than those used in the STEMS calibration. FIA survey plots were sampled using a variable-radius cluster sampling design covering a 1-acre sample area. Information from the cluster points is aggregated to represent the condition surrounding the center point. Complications arise when distinctly different forest conditions are represented by the aggregate stand condition. Variables derived from the tree lists, such as stand basal area, will misrepresent the local growing condition around each tree and hence have greater variability than those derived in the calibration of STEMS. However, since the new AFIS model is being developed from inventory data for an inventory application, a broader variable base is available than with the calibration of STEMS. The inclusion of these variables into the model may help account for more of the variability in growth estimates.

The traditional measure of site productivity used in growth models is site index. While measurement of site index is often imprecise and inaccurate (Bailey and Brooks 1994, McRoberts 1996, McRoberts *et al.* 1994), STEMS is sensitive to errors in site index (Gertner and

Dzialowy 1984). FIA inventories also record physiographic class (a measure of soil and water conditions that affect tree growth). Sites are classified into five classes from xeric to hydric, indicating their response to varying soil, climate, and topographic conditions. We have assumed that assignment to one of five classes is more precise than estimation of site index. This study will explore the substitution of physiographic class for site index as a measure of site productivity.

Model Form

There are two conceptual approaches to model development (Wykoff 1990) based on how we view the world—either in terms of potentials and different degrees of realization of those potentials or as averages and adjustments to those averages. The first approach, the "potential-modifier," (Belcher *et al.* 1982, Hahn and Leary 1979, Holdaway 1984, Shifley 1987) was used in the STEMS model. Models based on this approach have a potential component and a modifier component that are modeled sequentially. First, the potential diameter growth of competition-free trees is modeled, and second, a competitive adjustment is applied to reduce the potential growth to that actually observed. Modeling growth of competition-free trees is not intuitively clear (nor is it an observable feature of the stand), and various researchers have chosen differing approaches. Problems with this two-stage model form include the difficulties in representing an unobservable tree characteristic; the lack of direct statistical methods to reliably estimate the precision of growth predictions; and the difficulty of comprehending the underlying model structure or incorporating additional variables into the model.

The second model form, the "average-adjusted" (or composite) approach, is presented in this paper. This approach attempts to include the most relevant factors influencing growth in an average expression and to adjust growth based on its deviation from the average condition (Lemmon and Schumacher 1962, West 1980, Wykoff 1990). Advantages of the average-adjusted model form include: it fits an observable quantity, it provides for statistical measures of precision and significance tests for variable entry, the features of the model are easier to understand, and it easily accommodates the inclusion of additional FIA variables.

DATA

The inventory data (using a 10-point cluster sample) came from Minnesota's Aspen-Birch and Northern Pine Inventory Units. At each cluster subpoint, trees ≥ 5.0 inches dbh are sampled with a 37.5 BAF prism. Smaller trees (1.0-5.0 inches dbh) are selected on 1/300-acre fixed-area plots on all 10 subpoints (before 1986, fixed-area plots at only the first three points were measured).

Four species were modeled—red pine (*Pinus resinosa* Ait.), balsam fir (*Abies balsamea* (L.) Mill.), quaking aspen (*Populus tremuloides* Michx.), and paper birch (*Betula papyrifera* Marsh.). Plot data initially recorded in the 1977 or 1990 inventories and remeasured in the first subsequent revisit were used. The database was restricted to undisturbed timberland plots with all trees having two diameter measurements. A systematic sample of every fourth plot was set aside for model validation, as described by Lessard (2000). Models were calibrated on the remainder of the plots.

METHODS

Model Fitting

The Weibull function was chosen because it is well suited to describing biological processes. It has been commonly used to represent the tree diameter distribution because of its flexibility (Bailey and Dell 1973). The model form simultaneously produces biologically reasonable and statistically accurate predictions. The usual expression of the three-parameter Weibull probability density function is:

$$f(X) = (C/B)\,((X-A)/B)^{C-1}\,e^{\left(-((X-A)/B)^{C}\right)} \quad [1]$$

If A=0, this reduces to a two-parameter Weibull function. When rescaled by a multiplier, it has the properties of a growth function and yields the modified three-parameter Weibull distribution function. In simplified expression, this equation forms the core diameter growth model used in this research:

$$E(GR) = b_1 DBH^{(b_3-1)}\,e^{\left(-b_2 DBH^{b_3}\right)} \quad [2]$$

Other researchers have proposed related functions: $E(GR) = b_1$ DBH exp ($-b_2$ DBH) (Leary 1970) and $E(GR) = b_1$ DBH^{b_3} exp ($-b_2$ DBH) (Zeide 1993).

A linear combination of stand and tree exploratory variables were entered in the exponent in [2] to further adjust growth, yielding:

$$E(GR) = b_1 DBH^{(b_3-1)}\,e^{\left(-b_2 DBH^{b_3} + ADJ\right)} \quad [3]$$

where *DBH* = average dbh over the interval, *GR* = average annual diameter growth, $ADJ = \sum b_i f_i(VAR_i)$, VAR_i = stand and tree variables, and the b_i's are regression parameters. The modified Weibull model, with additional terms in the exponent, is capable of incorporating the nonlinear effects of various predictor variables through transformations of the independent variables. Biologically constrained transformations were used to provide realistic growth effects. Various transformations were tested for each variable across species, and the one that tended to minimize sum of squares and provided logical biological behavior was selected. Transformations were standardized to yield a zero adjustment under "average" conditions. For ease, roughly average conditions across all species were used. The model has been geared to an inventory application—meaning that the maximum projection interval is assumed to be relatively short, and hence biological constraints on maximum stand basal area have not been modeled. This model form is general enough to be applied to all potential tree species and FIA predictor variables, thus forming the basis of a generalized tree-level modeling system.

Explanatory variables obtained from the FIA measured variables are diameter (DBH), crown ratio code (CR), crown class (CC), site index (SI), and physiographic class (PHY). Coding and definition of these variables is given in Hansen *et al.* (1992). Calculated or derived variables are plot basal area per acre (BA), number of trees per acre (NT), average stand diameter (AD), relative tree diameter (REL=DBH/AD), and basal area in trees larger than the subject tree (BAL). CC, NT, BAL, and PHY are variables not used in the original STEMS model.

Competition among trees within a stand affects individual tree growth. In distance-independent models, measures of overall crowding within a stand are represented by BA and NT, and measures of relative competitiveness of an individual tree to the neighboring trees are expressed by REL and BAL. CC also provides knowledge of competitive status.

Transformations of the predictor variables, together with interactions, were used to explore relationships between the independent variable set and diameter growth. Many interactions were tested and they predicted reasonable growth behavior. But while increasing the complexity of the model, they did not practically improve it. The Weibull model [3] was fit using nonlinear least squares; variables entered the equations only if significant at p=0.05. The final model selected is:

$$E(GR) = b_1 DBH^{(b_3 - 1)} e^{\left(-b_2 DBH^{b_3} + ADJ\right)} \qquad [4]$$

where

$$\begin{aligned} ADJ = {} & b_4 * (CR - 4) + \exp(b_5 * (BAL - 50)) - 1 \\ & + b_6 * \log((DBH / AD) / 1.5) \\ & + \exp(b_7 * (NT - 900.)) - 1 \\ & + b_8 * (BA - 100) + b_9 * (AD - 5) \\ & + b_{10} * (\exp(CC / 3) - 2.718) \\ & + b_{11} * (PHY - 5) + b_{12} * (PHY - 5)^2. \end{aligned}$$

Note that DBH also enters the equation in the ADJ term through relative diameter, expressed as DBH/AD.

We can demonstrate graphically how the AFIS model in [4] behaves. When REL is expressed as DBH/AD, it produces a set of average diameter curves of diameter growth by diameter. Diameter increment increases to a maximum and then slowly decreases, approaching zero as the tree matures. Figure 1a shows how the AFIS model would grow red pine trees of a given DBH found growing in a young stand of AD = 4 inch for CR values of 2 (low), 5 (medium), and 8 (high). To account for the remaining variables, CC, BA, and NT in the model have been set to values concomitant with a 4-inch AD stand, and BAL has been expressed as a function of DBH. Figure 2a shows the same graph for a mature red pine stand of AD = 12 inch. The plots portray the range of DBH values typically found on a 4-inch and 12-inch AD stand, respectively.

Figures 1b and 2b show how the same conditions are modeled by STEMS-FIA. There are some obvious differences between the models used in AFIS and STEMS-FIA. The growth assigned a tree due to its relative position in the stand is handled differently by the two models. The STEMS-FIA model applies an ascending relative diameter influence with increasing DBH regardless of stand maturity. The AFIS model allows for the understory trees on a mature stand to demonstrate faster growth than the larger overstory trees, which are diminishing in growth. Scatterplots of the data indicate that this is reasonable growth behavior. CR is entered additively in STEMS and multiplicatively in AFIS. The differences in CR effect are more important on the edges of the CR and DBH range. For the largest trees (unaffected by competition), the models vary greatly in how they apply growth, due both to the CR effect and the underlying shape of the functions. The predicted diameter growth curve with the STEMS-FIA model is concave downward, whereas the AFIS model has an inflection point that provides for a slower approach to zero for the largest trees. Since there are very few trees in the inventory database above 25 inches, it is impossible to determine from the data which model is better.

Verification and Validation of Models

The AFIS model was tested by inserting model equations and parameters into a validation version of the STEMS tree growth projection system. The newly developed model was first verified against the calibration data, and the data were analyzed for trends in diameter residuals against predictor variables. If no significant trends were found, validation of the model was performed. Statistics evaluated included annualized mean errors, root mean square error (RMSE), model efficiency (as measured by relative improvement in RMSE) and r-square values. Model efficiency is defined as RMSE (STEMS-FIA) /

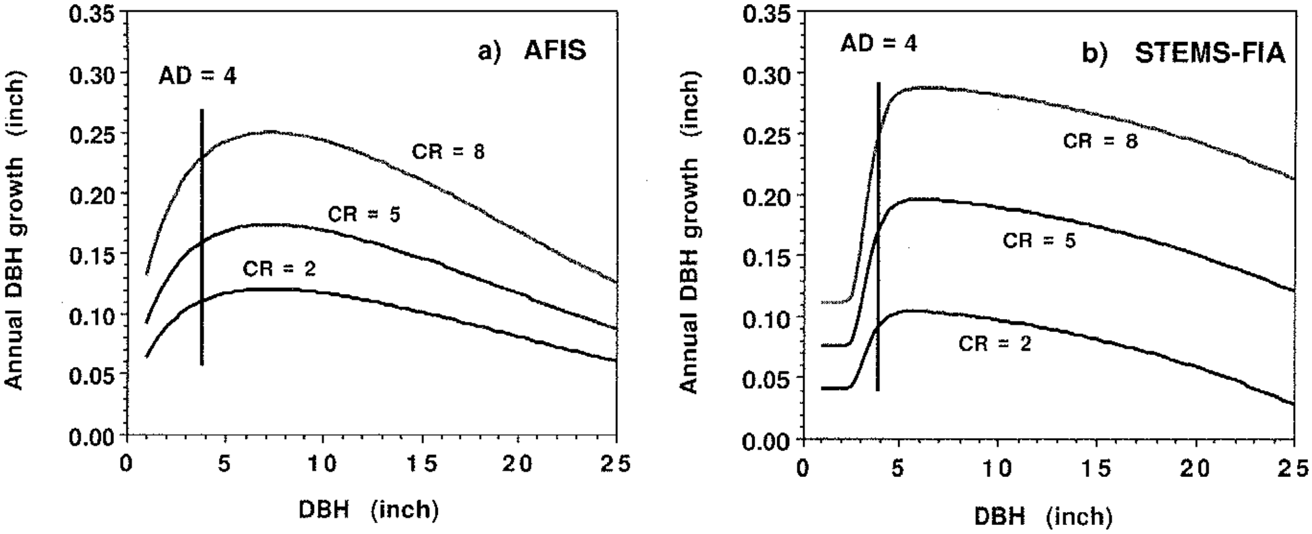

Figure 1.—*Diameter growth curves for red pine trees growing on a young stand (4-inch average diameter) as modeled by a) AFIS and b) STEMS-FIA. Low, moderate, and high crown ratio (CR) values are given.*

RMSE (AFIS). This provides a measure of the increase in precision (or decrease in residual variability) of the model over the STEMS-FIA model. Note that RMSE is a statistical measure that incorporates both bias and variability.

Accuracy and precision of the predictions were evaluated for tree diameter and stand basal area. The tree-level analysis evaluates the growth component of the model, whereas the stand-level analysis evaluates the combination of the growth and mortality components. Tree-level statistics were provided on all trees observed and predicted alive over the growth interval for the target species. Plot-level statistics were derived for plots of the corresponding cover types. The model verification and validation include a comparison of the AFIS model with the STEMS-FIA model (Lessard 2000). Both models used the STEMS mortality equations with localized mortality adjustments based on their modeled growth (Smith 1983). Since both AFIS and STEMS-FIA were calibrated on local data, neither used diameter adjustments to localize the model.

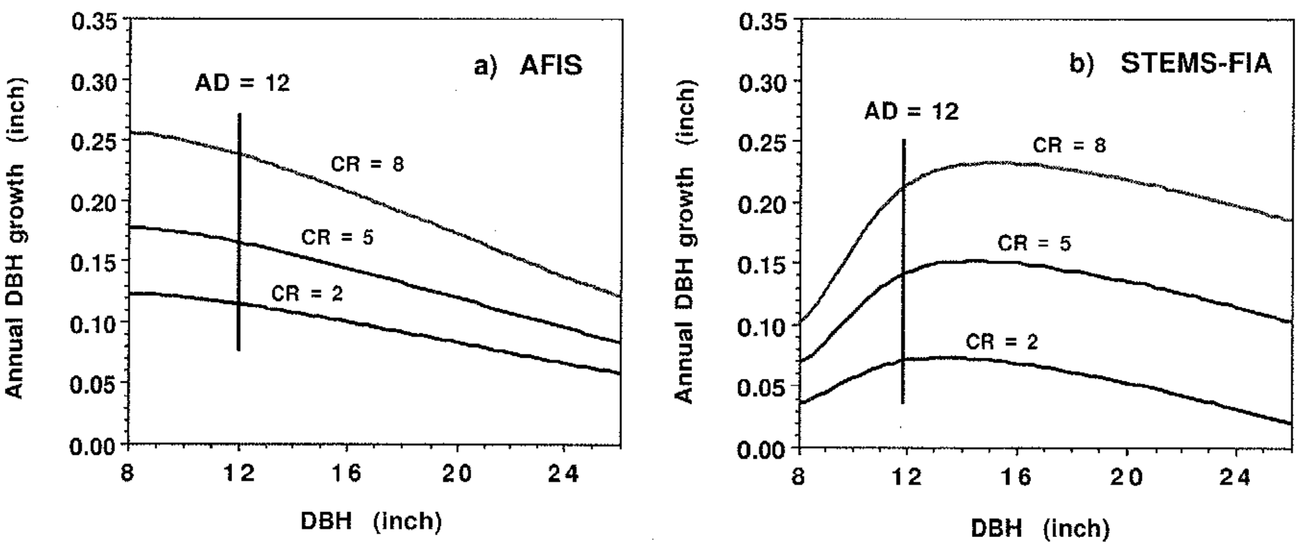

Figure 2.—*Diameter growth curves for red pine trees growing on a mature stand (12-inch average diameter) as modeled by a) AFIS and b) STEMS-FIA. Low, moderate, and high crown ratio (CR) values are given.*

Tests of Model Improvements

Tests were also made of the suitability of using (1) PHY in place of SI as a measure of site productivity and (2) the average-adjusted versus potential-modifier model form. First, site productivity measures were tested by replacing PHY with SI in the AFIS model form and re-estimating the parameters. SI was entered using the linear transformation: SI-60. When the appropriate SI was not recorded for the tree species, SI was set to 60, thus effectively dropping its influence from the equation. This model is called AFIS-SI. Second, the model form was tested by comparing the AFIS-SI model with STEMS-FIA (both use SI, but have a different model form).

RESULTS

Parameter values and transformations of the predictor variables for the AFIS model are given in table 1. Results of the model comparisons on the calibration and validation databases are given for the tree-level analysis (table 2) and the plot-level analysis (table 3). No obvious trends in residuals plotted against predictor variables for the AFIS model were found, except for some mild curvilinear trends and fluctuations in the largest trees (representing only a fraction of the trees).

Accuracy of the models, pertaining to both the calibration/validation databases, showed minimal differences between the STEMS-FIA and AFIS models for DBH predictions (tables 2, 4) and stand BA predictions (tables 3, 5). Improvement in precision of the models as measured by the model efficiency statistic (EFF) show that DBH predictions for the AFIS model were improved on all species on the calibration database and on three of four species on the validation database. At the stand level, BA predictions were improved on three of four species on both databases. For the AFIS model on the validation database, r^2 values ranged from .084 to .562 for annualized DBH growth predictions and .350 to .817 for annualized BA growth predictions. R^2 values for plot-level BA growth showed stronger correlations than for tree-level DBH growth. Since we are more concerned with the higher level attributes of the model, this is encouraging.

Across both data sets, the species most consistently improved in precision (for both DBH and BA predictions) was balsam fir. Quaking aspen was improved on DBH predictions, but not necessarily on BA predictions. Paper birch predictions were better for BA than DBH. Some of these differences may be due to problems in the mortality model included in the stand-level analysis.

Results of the tests of model improvements (a substitute measure of site productivity and the average-adjusted model from) are given in tables 4 (tree-level) and 5 (plot-level). Calibration and validation results have been

Table 1.—*Model parameter estimates for the AFIS model equation [4] for four species in the Aspen-Birch and Northern Pine Units of northern Minnesota*

	Estimates			
Parameters	Red pine	Balsam fir	Quaking aspen	Paper birch
b1	1.00000*	1.00000*	1.00000*	1.00000*
b2	0.34122	0.54916	0.38574	0.73029
b3	0.70717	0.60225	0.58417	0.35746
b4	0.11923	0.15509	0.11609	0.14361
b5	-0.00585	-0.00094	-0.00532	-0.00432
b6	1.08781	1.26480	1.11420	0.97978
b7	-0.00026	-0.00030	-0.00027	-0.00026
b8	0.00000*	0.00000*	0.00385	0.00252
b9	0.13040	0.21274	0.16785	0.15905
b10	-0.05979	-0.03575	-0.09480	-0.08142
b11	-0.04390	-0.09880	0.00000	0.00000
b12	0.00000	0.00000	-0.05926	-0.05805

* Parameter not significant at p = .05 and set to 1.0 or 0.0 as appropriate.

Table 2.—*The annualized diameter prediction errors for the STEMS-FIA and AFIS models on the calibration and validation databases*

Species	Model	Number of trees	Mean residual[1]	RMSE	EFF[2]	r-sq[3]	Change in r-sq
Calibration			*Inches*	*Inches*			
Red pine	STEMS-FIA	1,360	.000	.0689		.437	
Red pine	AFIS	1,360	-.001	.0673	1.02	.454	.017
Balsam fir	STEMS-FIA	5,909	.000	.0573		.304	
Balsam fir	AFIS	5,909	.001	.0546	1.05	.366	.062
Quaking aspen	STEMS-FIA	15,383	.002	.0717		.172	
Quaking aspen	AFIS	15,383	-.000	.0670	1.07	.217	.045
Paper birch	STEMS-FIA	5,461	-.001	.0486		.117	
Paper birch	AFIS	5,461	.001	.0477	1.02	.130	.013
Validation							
Red pine	STEMS-FIA	501	.013	.0731		.560	
Red pine	AFIS	501	.011	.0734	1.00	.562	.002
Balsam fir	STEMS-FIA	1,924	.001	.0599		.327	
Balsam fir	AFIS	1,924	.001	.0576	1.04	.383	.056
Quaking aspen	STEMS-FIA	5,348	.004	.0725		.157	
Quaking aspen	AFIS	5,348	-.001	.0680	1.07	.173	.016
Paper birch	STEMS-FIA	1,708	.003	.0666		.075	
Paper birch	AFIS	1,708	.004	.0657	1.01	.084	.009

[1] Positive residuals indicate an underprediction.
[2] Efficiency of the new model is calculated by EFF = RMSE (STEMS-FIA) / RMSE (AFIS).
[3] The r^2 (r-sq) is calculated as the square of the coefficient of correlation between predicted and observed annualized growth.

Table 3.—*The annualized stand basal area prediction errors are the STEMS-FIA and AFIS models on the calibration and validation databases (see footnotes of table 2)*

Cover type	Model	Number of plots	Mean residual	RMSE	EFF	r-sq	Change in r-sq
Calibration			*Sq ft/ac*	*Sq ft/ac*			
Red pine	STEMS-FIA	62	.219	2.055		.621	
Red pine	AFIS	62	-.320	1.992	1.03	.645	.024
Balsam fir	STEMS-FIA	228	.604	2.008		.370	
Balsam fir	AFIS	228	-.430	1.879	1.07	.437	.067
Quaking aspen	STEMS-FIA	1,296	.315	2.053		.471	
Quaking aspen	AFIS	1,296	-.352	2.135	.96	.466	-.005
Paper birch	STEMS-FIA	247	.467	1.820		.177	
Paper birch	AFIS	247	-.328	1.724	1.06	.187	.010
Validation							
Red pine	STEMS-FIA	25	-.515	1.694		.887	
Red pine	AFIS	25	.506	1.696	1.00	.817	-.070
Balsam fir	STEMS-FIA	62	.246	2.270		.288	
Balsam fir	AFIS	62	-.210	2.050	1.11	.421	.133
Quaking aspen	STEMS-FIA	443	.241	1.931		.424	
Quaking aspen	AFIS	443	-.265	1.815	1.06	.533	.109
Paper birch	STEMS-FIA	71	.534	1.949		.234	
Paper birch	AFIS	71	-.517	1.753	1.11	.350	.116

Table 4.—*The annualized diameter prediction errors for STEMS-FIA, AFIS-SI, and AFIS (with physiographic class in place of SI) models on the calibration and validation databases (see footnotes of table 2)*

Model	Number of trees	Mean residual	RMSE	EFF	r-sq	Change in r-sq
		Inches	*Inches*			
Calibration						
STEMS-FIA	28,113	.0011	648		.292	
AFIS-SI	28,113	-.0006	0615	1.05	.347	.048
AFIS	28,113	.0001	0612	1.06	.347	.055
Validation						
STEMS-FIA	9,481	.0034	0691	.266		
AFIS-SI	9,481	-.0020	.0659	1.05	.308	.042
AFIS	9,481	.0013	.0659	1.05	.308	.042

Table 5.—*The annualized stand basal area prediction errors for STEMS-FIA, AFIS-SI, and AFIS (with physiographic class in place of SI) models on the calibration and validation databases (see footnotes of table 2)*

Model	Number of plots	Mean residual	RMSE	EFF	r-sq	Change in r-sq
Calibration		*Sq ft/ac*	*Sq ft/ac*			
STEMS-FIA	1,833	-.331	1.976		.449	
AFIS-SI	1,833	-.320	2.039	.97	.458	.009
AFIS	1,833	-.358	2.049	.96	.456	.007
Validation						
STEMS-FIA	601	-.225	1.974		.437	
AFIS-SI	601	-.210	1.810	1.09	.543	.106
AFIS	601	-.257	1.829	1.08	.536	.099

averaged across all trees and hence are weighted in favor of quaking aspen (with 15,383 trees). PHY (the physiographic class), used as an indicator of site productivity, performed similarly to SI. The AFIS-SI model showed moderate improvement over the potential-modifier model (used in STEMS-FIA).

In summation, for all four species on the validation database, precision of DBH predictions improved by 5 percent and stand BA predictions by 8 percent for the AFIS model over the STEMS-FIA model.

CONCLUSIONS

The development and evaluation of a new model form, applied to four species on the Aspen-Birch and Northern Pine Inventory Units of Minnesota, was undertaken. The model, AFIS, was based on the modified three-parameter Weibull function. The model was fit in one step as a linear combination of transformed variables in the exponent reflecting departures from the average condition. The results indicate that the AFIS model performs moderately better than the STEMS-FIA model. The AFIS model design also has many desirable features: it uses a simple, flexible model form; allows for the complex relationships between variables to be expressed by simple biologically reasonable transformations; provides for inclusion of additional inventory variables (including climatic variables), significance testing of variable entry and precision estimates; and is easy to calibrate. However, for those who believe strongly in the biological concept of "potential" growth in plant dynamics and are less interested in precision and climatic questions, the potential-modified approach may be appropriate.

In the test of using physiographic class in place of site index, no difference was found. However, when the precision of site productivity measures is considered, physiographic class may well out-perform site index.

The AFIS model will be retained for further study in both the upcoming incorporation of a climatic component into the growth model and the precision analysis (McRoberts *et al.* 2000).

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