Maximum Size-Density Relationships for Mixed-Hardwood Forest Stands in New England

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Abstract.—Maximum size-density relationships were investigated for two mixed-hardwood ecological types (sugar maple-ash and beech-red maple) in New England. Plots meeting type criteria and undergoing self-thinning were selected for each habitat. Using reduced major axis regression, no differences were found between the two ecological types. Pure species plots (the species basal area greater than 70 percent of the total plot basal area) were identified for six major hardwood species, and corresponding size-density relationships were developed. Results showed that shade tolerant and intermediate species had similar intercept and slope coefficients for the maximum size-density line. No differences were found between each species and habitat types; however, shade intolerant species had smaller intercept and slope coefficients than others. The two ecological types were combined, and one maximum size-density relationship was constructed for mixed-hardwood stands.

Self-thinning is an important and fundamental process during stand development. As the individual trees in a stand grow in size, trees begin to compete for resources such as water, light, and mineral nutrients. Crowding or competition occurs when the density of a forest stand exceeds the level at which each tree can obtain its maximum growing space. Natural mortality results when the minimum growing space requirements of all trees exceed the resources of the site. The dynamic equilibrium between tree growth and death at crowding density is governed by the so-called -3/2 power law of self-thinning (Drew and Flewelling 1977). The law states that in logarithm scale the relationship between average tree size and stand density is a straight line (called the self-thinning line) for a fully stocked stand undergoing density-related mortality. The line has been commonly expressed by the following mathematical equations:

$$\ln(MV) = a - 1.5 * \ln(TPA) \quad [1]$$
(Drew and Flewelling 1977)

$$\ln(QMD) = b - 0.625 * \ln(TPA) \quad [2]$$
(Jack and Long 1996)

where MV is mean tree volume, QMD is quadratic mean tree diameter, TPA is number of trees per unit area, and a and b are constants. These equations are mathematically compatible (Zeide 1987). The intercepts (a and b) of the equations vary with species, but only within narrow logarithmic limits. The slope parameters are apparently consistent regardless of species, age, and site quality.

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Self-thinning has been observed in many types of plant forms (e.g., mosses, grasses, crops, shrubs, and trees), and in many types of plant populations (monocultures, mixtures, even-aged and uneven-aged stands). Primarily, the –3/2 power law applies to two instances: (1) for a single-species population of successive development stages, its growth trajectory over time will follow the -3/2 self-thinning line, which describes the mean tree size and density relationship of the single stand at different times; and (2) for all populations of the same species composition, regardless of stand ages and site qualities, there is a maximum possible combination between the mean tree size and the number of trees per unit area defined by the -3/2 power law. This is known as the upper boundary or maximum size-density relationship (Jack and Long 1996). It is a static relationship among stands observed at single points of time and describes the upper limiting boundary of all possible combinations of mean tree size and density.

One application of the -3/2 power law of self-thinning in forest management practices has been to derive relative stand density. It is usually expressed as a percent of absolute stand density to the reference level based on average maximum competition (Ernst and Knapp 1985). Although there are different expressions of relative stand density measures, such as Drew and Flewelling's (1979) relative density (RD) index and Reineke's (1933) stand density index (SDI), they are highly correlated since they all are a function of tree size and density (Jack and Long 1996). The maximum size-density relationship and relative stand density measures have been used to develop stand management diagrams (e.g., Dean and Jokela 1992, Drew and Flewelling 1979, Reineke 1933) and stocking charts (e.g., Gingrich 1967; Solomon and Leak 1969, 1986). Essentially, these tools are simple average stand models that graphically characterize growth, density, and mortality at various stages of stand development. Forest managers can use the stand density management diagrams to locate the results of different surveys of the stand and to rapidly design and evaluate alternative density management regimes. This practice will help them to maximize wood fiber yield (Dean and Jokela 1992, Kershaw and Fischer 1991) and to create favorable ecological conditions for vegetation (Barbour *et al.* 1997) and wildlife (Hayes *et al.* 1997, Lilicholm *et al.* 1993).

To date, most stand density management diagrams have been developed for western and southern coniferous species plantations or even-aged stands in North America (Newton 1997). Few studies have been conducted for deciduous or mixed forests. Kershaw and Fischer (1991) developed a stand density diagram for sawtimber-sized mixed upland hardwoods in the central United States. They found that the stand density management diagram is a useful tool that not only allows point estimation of stand stocking but also serves as a stand monitoring system in which stand development and treatments can be traced through the stand management history.

However, no maximum size-density relationships and stand density management diagrams have been published for mixed-hardwood forest stands in New England. The objective of this research project was to establish the maximum size-density relationships for the two hardwood ecological habitat types in this region: sugar maple-ash (SM-ASH) and beech-red maple (BE-RM) (Solomon *et al.* 1995). These relationships can be immediately used to establish the maximum limits of the stand density management diagrams.

DATA AND METHODS

Plot data were collected from the database used in developing FIBER 3.0 (Solomon *et al.* 1995) for the two hardwood habitat types. The plots were actually the combinations of different measurements over time of all available plots and included wide ranges of the number of trees per acre, stand total volume, mean tree volume, stand total basal area, quadratic mean tree diameter, and other stand information (table 1).

Since there is no established statistical procedures for selecting appropriate plots to estimate the two coefficients of the maximum size-density relationship, we chose the following approach to select the most fully stocked plots: assuming the **slope** of the ln(MV)-ln(TPA) relationship is -1.5, the **intercept** was determined by $k = \ln(MV) + 1.5 * \ln(TPA)$ using the plot with the largest combination of ln(MV) and ln(TPA). The determined equation was then used to compute the maximum stand density (TPA_{max}) for the mean tree volume (MV) of a given plot. The relative density index (Drew and Flewelling 1979) was calculated by TPA/TPA_{max}, where TPA is the current stand density. The RD was computed for each plot, and the plots with $RD > 0.7$ were selected for developing the maximum size-density relationship. Although this threshold was arbitrary, we chose this for two reasons: (1) any stands with the relative density index of 0.7 or higher should have been undergoing the self-thinning process and experiencing density-related mortality, and (2) sufficient number of plots would be obtained for fitting the size-density relationships.

The selected plots were separately utilized to formulate the maximum size-density relationships for each of the two hardwood habitat types. The reduced major axis (RMA) regression (Leduc 1987) was used to estimate the two coefficients of the self-thinning line. Suppose a linear regression model $y = \alpha + \beta * x$, then the RMA slope coefficient is $\beta_{RMA} = \beta / r_{yx}$, where r_{yx} is the correlation coefficient between y and x. The standard error (SE) of

Table 1.—*Summary of stand variables for the two ecological habitat types of sugar maple-ash (SM-ASH) and beech-red maple (BE-RM)*

Habitat	No. plots	Variables	Mean	Std	Min.	Max.
SM-ASH	1,043	TPA	136	50	14	399
		BA	80.9	29.6	5.0	183
		QMD	10.6	1.7	6.6	16.0
		MV	16.7	6.1	5.7	40.1
BE-RM	728	TPA	152	75	7	580
		BA	68.7	33.1	1.4	172
		QMD	9.2	1.9	5.9	21.6
		MV	14.1	7.8	3.4	90.7

Note: Std - standard deviation, TPA - trees per acre, BA - stand total basal area (ft^2/acre), QMD - quadratic mean tree diameter (inch), and MV - mean tree volume (ft^3).

β_{RMA} is equal to the SE of β. The RMA intercept coefficient is $\alpha_{RMA} = \bar{y}\ \beta_{RMA} * \bar{x}$, and the SE of α_{RMA} is equal to the SE of α, where α and β are the ordinary least-squares estimates of the regression coefficients. The model development was conducted for both ln(MV)-ln(TPA) relationship (equation [1]) and ln(QMD)-ln(TPA) relationship (equation [2]). The 95 percent confidence interval (CI) was computed for each coefficient and used to compare the two hardwood habitat types.

Specific plots were identified for the major hardwood species: sugar maple (SM) (*Acer saccharum* Marsh.), American beech (BE) (*Fagus grandifolia* Ehrh.), red maple (RM) (*Acer rubrum* L.), yellow birch (YB) (*Betula alleghaniensis* Britton), paper birch (PB) (*Betula papyrifera* Marsh.), and aspen (AS) (*Populus tremuloides* Michx.). These pure species plots were defined as the basal area of the species greater than 70 percent of the total basal area of the plot. The proposed analysis procedure above was followed to develop the size-density relationship for each species, and 95 percent confidence interval (CI) of the coefficients was used to compare the differences between the species. Because only a small number of plots were available for the two shade intolerant species (paper birch and aspen), the plots of the two species were combined.

ANALYSIS RESULTS

Comparison of the Size-Density Relationships between the Two Hardwood Habitat Types

Table 2 indicated that the 95 percent CI of each coefficient for one habitat type included the corresponding coefficient for the other habitat type. Therefore, there is no difference between the two hardwood habitat types for both slope and intercept coefficients and for both ln(MV)-ln(TPA) and ln(QMD)-ln(TPA) relationships. Consequently, one maximum size-density relationship can be used for the two hardwood habitat types.

Comparison of the Size-Density Relationships between the Pure Species Stands

Because the available number of plots for the pure species stands was limited, the 95 percent CI's for these plots were wider than those for the two habitat types (table 3). In general, there was no difference between the shade tolerant or intermediate species such as SM, BE, RM, and YB, or between each species and the two hardwood habitat types for both slope and intercept coefficients and for both ln(MV)-ln(TPA) and ln(QMD)-ln(TPA) relationships. However, the two shade intolerant species (PB and AS) had smaller slope and intercept coefficients than other species (but not statistically significant). This result agreed with other findings (Jack and Long 1996). However, this was not true for the ln(MV)-ln(TPA) relationship. We suspect that the difference might be due to the computation of tree volumes for the species and the small number of data plots.

In summary, one maximum size-density relationship can be established for the mixed-hardwood forest stands in New England. The selected plots from the two habitat types were combined to estimate the upper boundary line

Table 2.—*Comparison of the slope and intercept coefficients of the maximum size-density line between the two hardwood habitat types of sugar maple-ash (SM-ASH) and beech-red maple (BE-RM)*

Habitat	No. Plots	Slope β_{RMA}		Intercept α_{RMA}	
		ln(MV)-ln(TPA)			
SM-ASH	60	1.38	(1.27, 1.49)	10.24	(10.24, 10.80)
BE-RM	123	1.41	(1.33, 1.49)	10.34	(9.88, 10.80)
		ln(QMD)-ln(TPA)			
SM-ASH	60	0.62	(0.57, 0.67)	5.71	(5.45, 5.97)
BE-RM	123	0.60	(0.56, 0.64)	5.52	(5.30, 5.74)

Note: Numbers in the parentheses are the 95 percent confidence interval. MV - mean tree volume (ft^3), QMD - quadratic mean diameter (inch), and TPA - trees per acre.

Table 3.—*Comparison of the slope and intercept coefficients of the maximum size-density line between the six hardwood species of sugar maple (SM), beech (BE), red-maple (RM), yellow birch (YB), paper birch (PB), and aspen (AS)*

Species	No. plots	Slope β_{RMA}		Intercept α_{RMA}	
		ln(MV)-ln(TPA)			
SM	41	1.41	(1.24, 1.58)	10.24	(9.38, 11.08)
BE	24	1.31	(0.98, 1.64)	9.56	(7.85, 11.27)
RM	16	1.31	(0.95, 1.67)	9.78	(7.79, 11.77)
YB	18	1.29	(0.92, 1.66)	9.66	(7.75, 11.57)
PB+AS	14	1.38	(1.10, 1.66)	10.13	(8.52, 11.74)
		ln(QMD)-ln(TPA)			
SM	41	0.63	(0.56, 0.70)	5.71	(5.33, 6.09)
BE	24	0.63	(0.47, 0.79)	5.59	(4.79, 6.39)
RM	16	0.63	(0.45, 0.81)	5.70	(4.71, 6.69)
YB	18	0.56	(0.36, 0.76)	5.29	(4.27, 6.31)
PB+AS	14	0.46	(0.36, 0.56)	4.67	(4.10, 5.24)

Note: Numbers in the parentheses are the 95 percent confidence interval. MV - mean tree volume (ft^3), QMD - quadratic mean diameter (inch), and TPA - trees per acre.

using the RMA technique. The resulting models were:

$$\ln(MV) = 10.86 - 1.43 * \ln(TPA) \qquad [3]$$

$$\ln(QMD) = 6.08 - 0.66 * \ln(TPA) \qquad [4]$$

where the slope coefficients were the RMA coefficients, and the intercept coefficients were recalculated using the MV, QMD, and TPA of the plot with the largest relative density index. The observed data of the two habitat types and the pure species stands are plotted for both ln(MV)-ln(TPA) and ln(QMD)-ln(TPA) relationships (figs. 1 to 10). The maximum size-density relationships (equations [3] and [4]) are also shown as the upper boundary line in the graphics.

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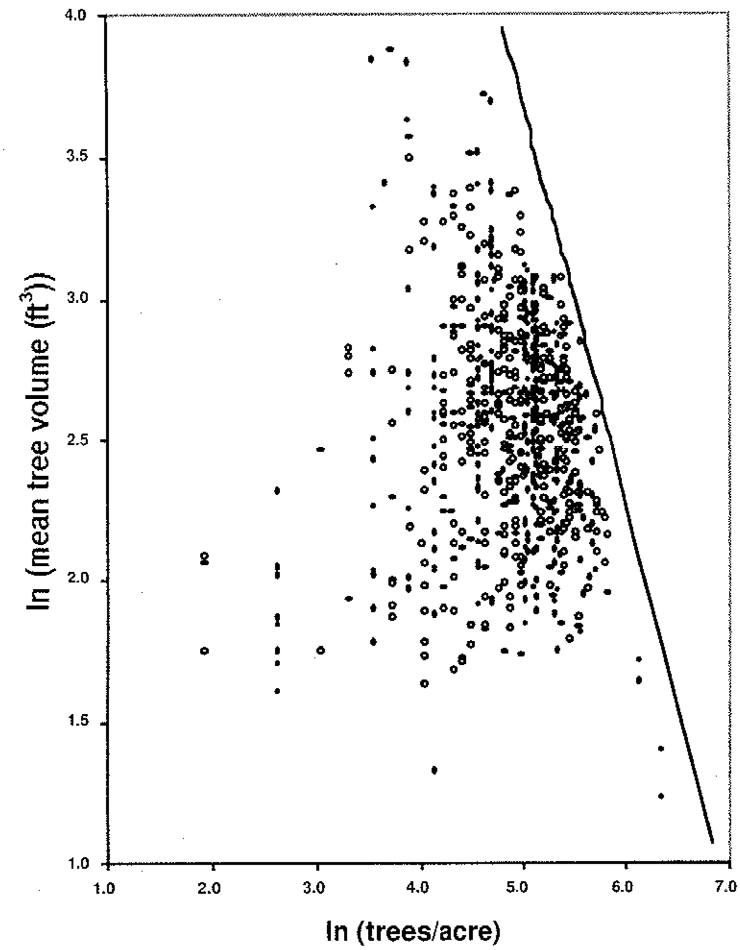

Figure 1.—*The maximum size-density relationship, ln (Mean Tree Volume (ft^3)) = 10.86 - 1.43 *ln (Trees Per Acre), for the sugar maple - ash ecological type.*

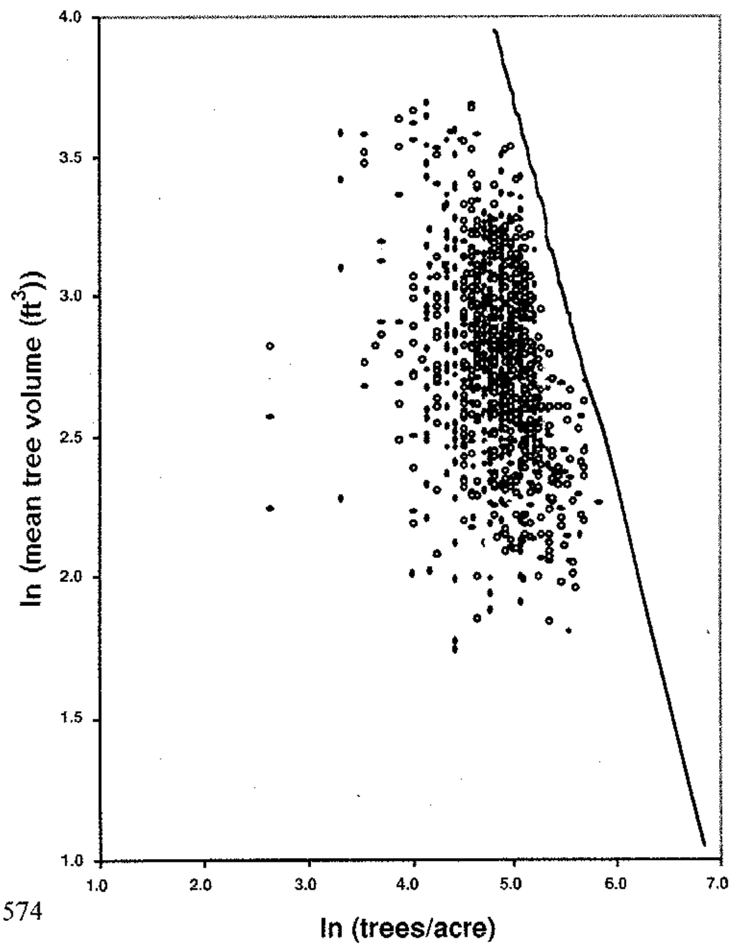

Figure 2.—*The maximum size-density relationship, ln (Mean Tree Volume (ft^3)) = 10.86 - 1.43 *ln (Trees Per Acre), for the beech — red maple ecological type.*

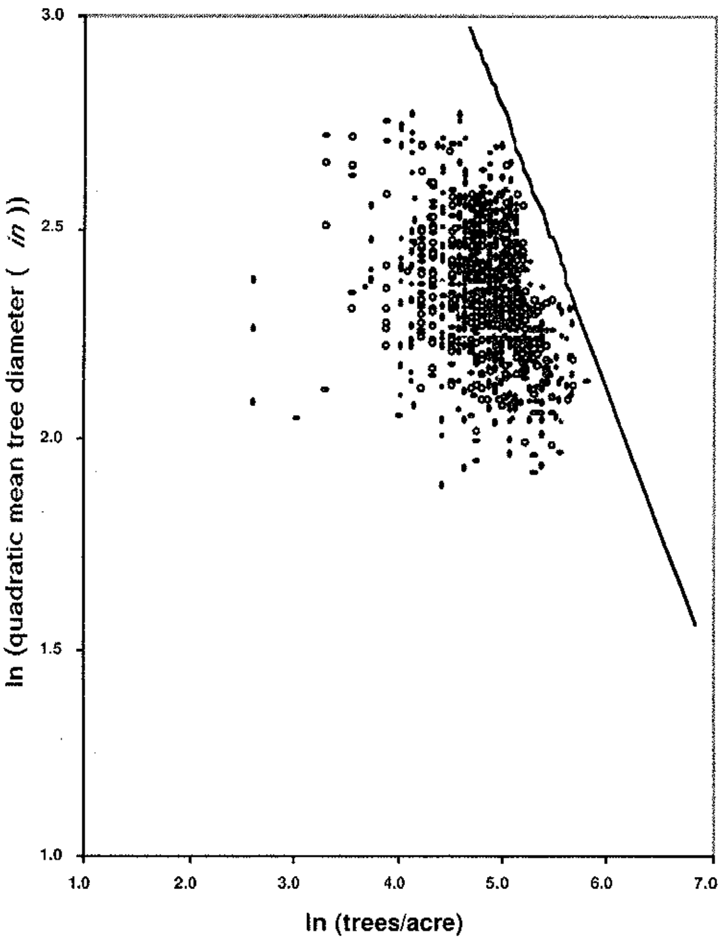

Figure 3.—*The maximum size-density relationship, ln (Quadratic Mean Tree Diameter (in)) = 6.08 - 0.66 * ln (Trees Per Acre), for the sugar maple – ash ecological type.*

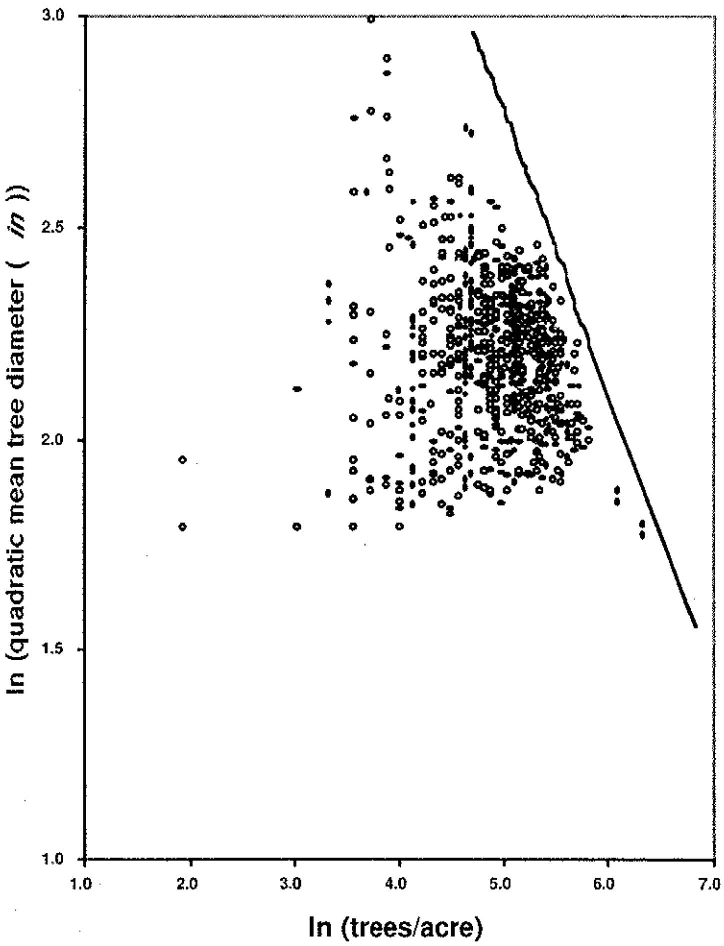

Figure 4.—*The maximum size-density relationship, ln (Quadratic Mean Tree Diameter (in)) = 6.08 - 0.66 * ln (Trees Per Acre), for the beech – red maple ecological type.*

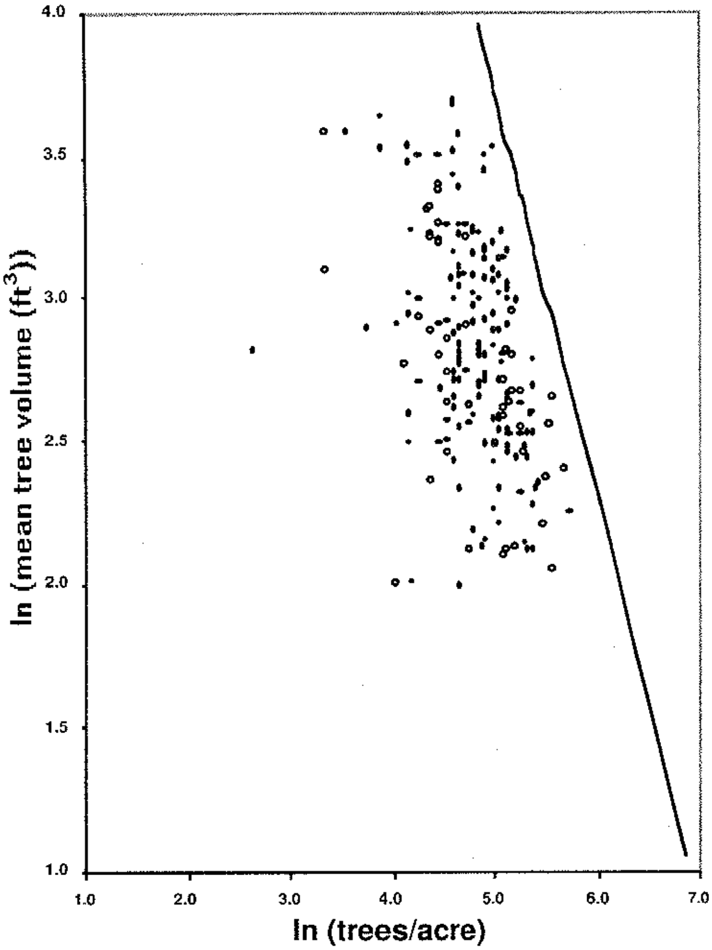

Figure 5.—*The maximum size-density relationship, ln (Mean Tree Volume (ft³)) - ln (Trees Per Acre), for stands having > 70 percent square feet of basal area in sugar maple.*

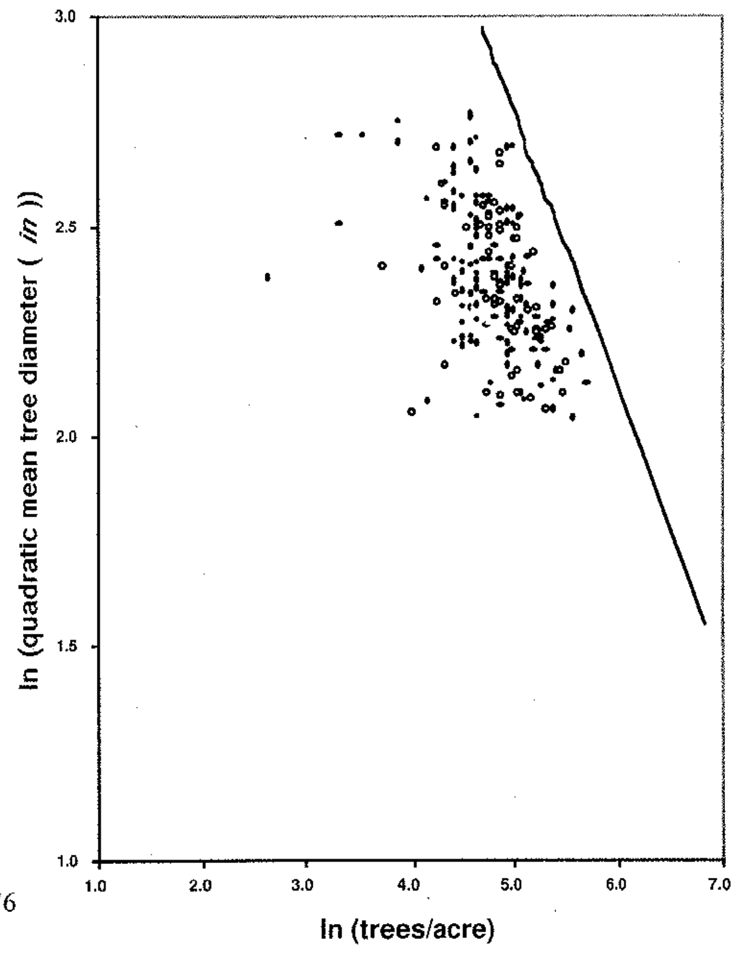

Figure 6.—*The maximum size-density relationship, ln (Quadratic Mean Tree Diameter (in)) – ln (Trees Per Acre), for stands having > 70 percent square feet of basal area in sugar maple.*

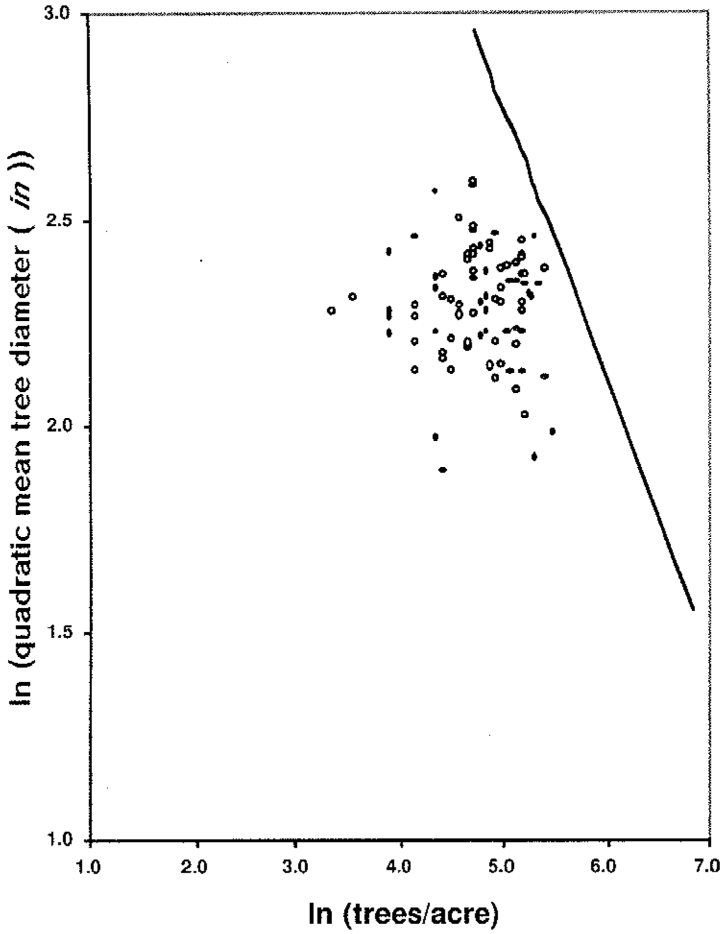

Figure 7.—*The maximum size-density relationship, ln (Quadratic Mean Tree Diameter (in)) – ln (Trees Per Acre), for stands having > 70 percent square feet of basal area in beech.*

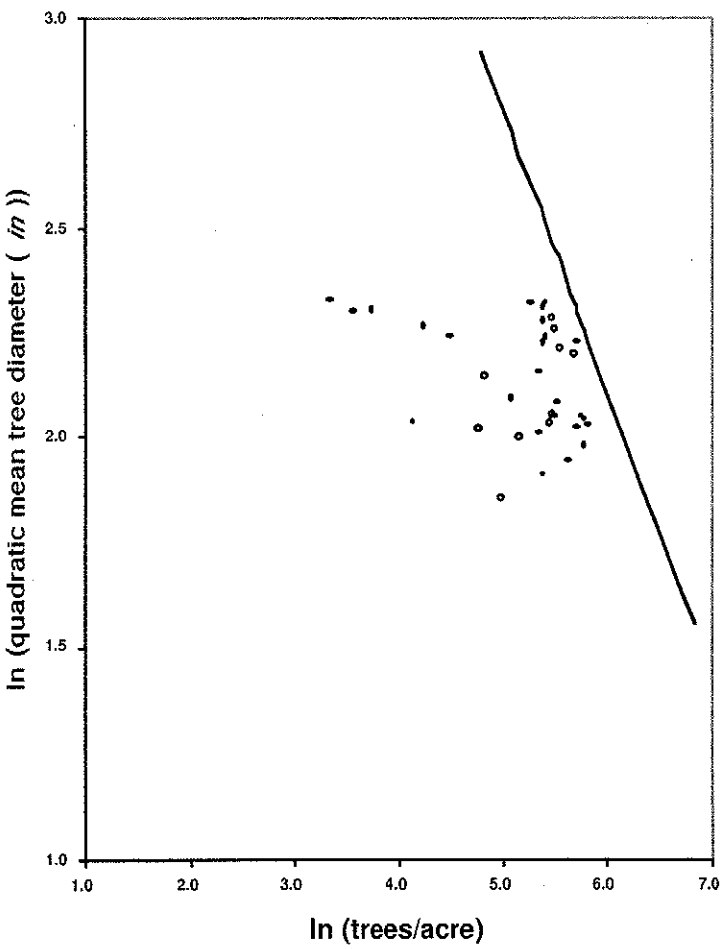

Figure 8.—*The maximum size-density relationship, ln (Quadratic Mean Tree Diameter (in)) – ln (Trees Per Acre), for stands having > 70 percent square feet of basal area in red maple.*

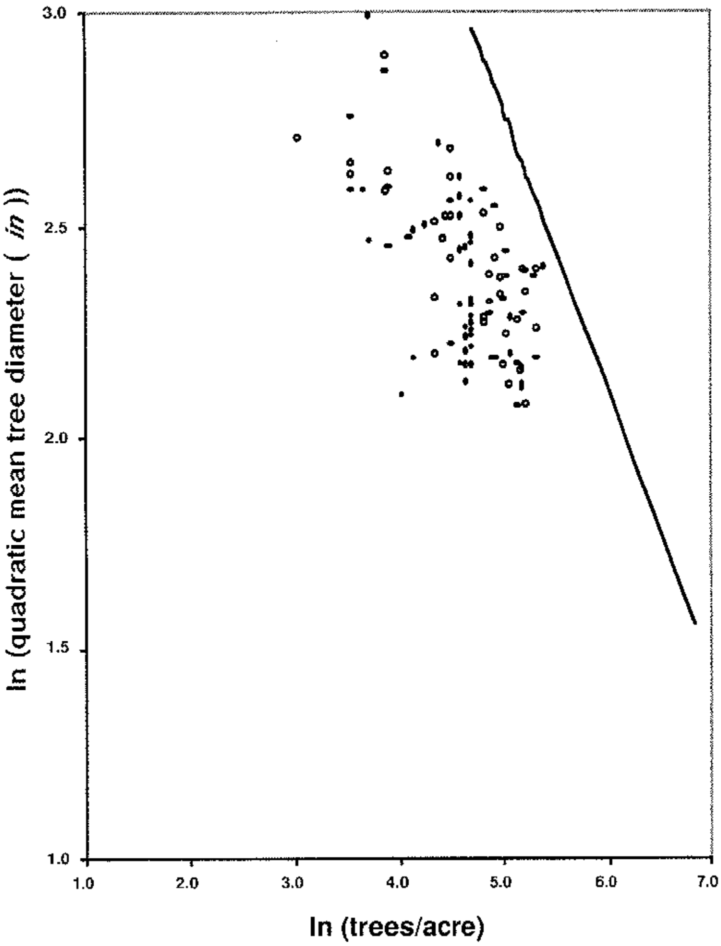

Figure 9.—*The maximum size-density relationship, ln (Quadratic Mean Tree Diameter (in)) – ln (Trees Per Acre), for stands having > 70 percent square feet of basal area in yellow birch.*

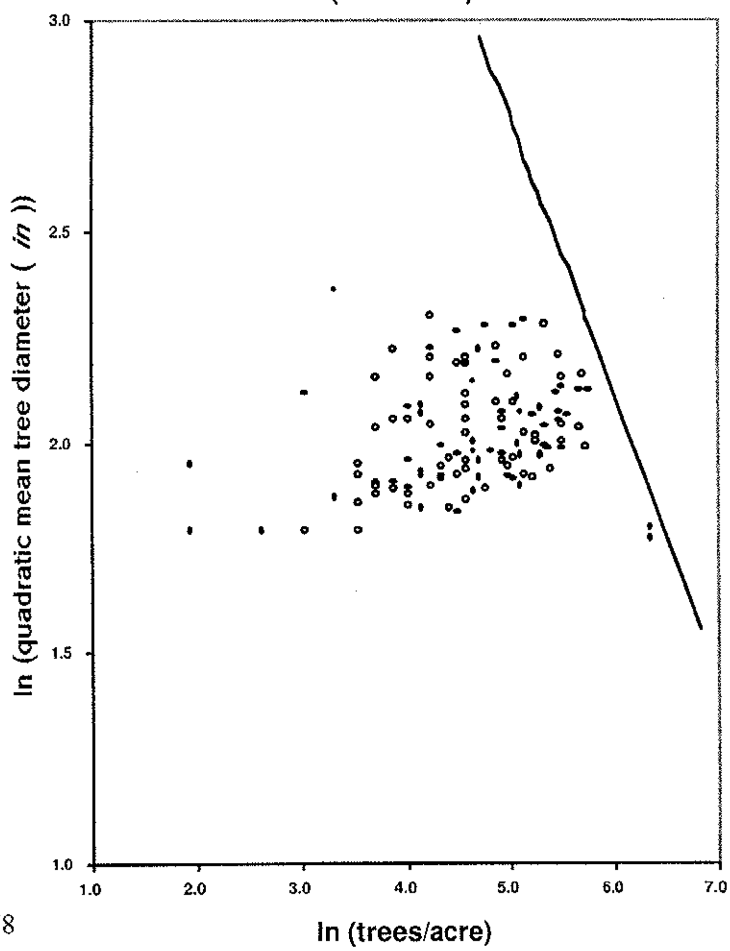

Figure 10.—*The maximum size-density relationship, ln (Quadratic Mean Tree Diameter (in)) – ln (Trees Per Acre), for stands having > 70 percent square feet of basal area in paper birch and aspen plots combined.*

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