Evaluating Ecoregion-Based Height-Diameter Relationships of Five Economically Important Appalachian Hardwood Species in West Virginia

John R. Brooks[1] **and Harry V. Wiant, Jr.**[2]

Abstract.—Five economically important Appalachian hardwood species were selected from five ecoregions in West Virginia. A nonlinear extra sum of squares procedure was employed to test whether the height-diameter relationships, based on measurements from the 2000 inventory from West Virginia, were significantly different at the ecoregion level. For all species examined, the null hypothesis was rejected indicating that at least one of the ecoregion specific parameters was not equal to zero. In addition, 56 percent of the paired ecoregion tests indicated significant height differences. Across all species and ecoregion combinations, average height error ranged from –3.6 to 7.6 ft for the statewide model.

Introduction

Height-diameter relationships are the driving force behind most tree volume, form, and weight relations. In a recent study by Jiang *et al.* (2004), yellow poplar (*Liriodendron tulipifera* L.) tree form and cubic foot volume were found to be statistically different between two major ecological regions in West Virginia. To further investigate whether the underlying height-diameter relationship also varied by ecoregion, the 2000 Forest Inventory and Analysis (FIA) data for West Virginia (Griffith and Widmann 2003) was used for evaluation. Five economically important Appalachian hardwood species were selected for study and included black cherry (*Prunus serotina* Ehrh.) (BC), red oak (*Quercus rubra* L.) (RO), red maple (*Acer rubrum* L.) (RM), sugar maple (*Acer saccharum* Marsh.) (SM), and yellow poplar (YP). Species specific measured total tree heights and diameters were used to fit the well-known Chapman-Richards growth model to determine whether the height-diameter relationship was statistically different by ecoregion. This technique has been employed for both jack pine (*Pinus banksiana* Lamb.) and black spruce (*Picea mariana* (Mill.) in Ontario (Peng *et al.* 2004, Zhang *et al.* 2002). Results from the jack pine study indicated that provincial models resulted in an average bias of 1 to 10 percent when applied to each ecoregion in Ontario (Zhang *et al.* 2002). The objectives of this study are to evaluate whether ecoregion-based diameter-height relations are statistically justified, to test for differences between ecoregions for the five hardwood species selected, and to evaluate the percent bias associated when a statewide model was compared to individual ecoregion models.

Methods

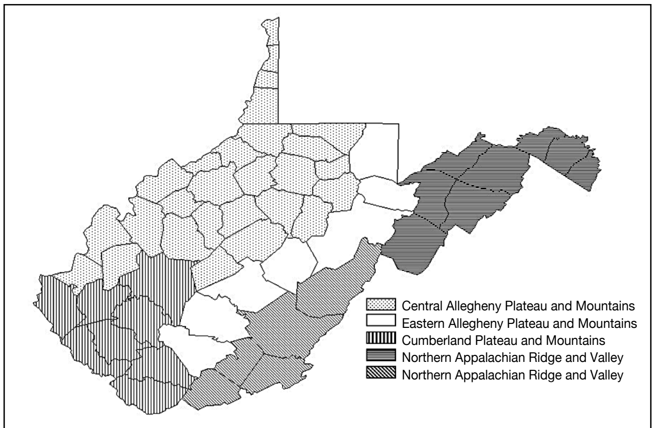
West Virginia was divided into five major ecoregions based on a combination of current subregions for Region III and IV (Bailey *et al.* 1994) and the land-based regions identified by the U.S. Soil Conservation Serivce (USDA Soil Conservation Service 1981).

This allocation was made on a county-by-county basis based on the county designation of the FIA plot location and the major subregion (by area) represented by each county. The following five major subregions employed are depicted in figure 1:

CALG: Central Allegheny Plateau.
CUPM: Cumberland Plateau and Mountains.
EALG: Eastern Allegheny Plateau and Mountains.

[1] Associate Professor Forest Biometrics, West Virginia University, 322 Percival Hall, Morgantown, WV 26506–6125. E-mail: jrbrooks@mail.wvu.edu.
[2] Joseph E. Ibberson Chair, Forest Resources Mgt., Penn State University, School of Forest Resources, 212 Ferguson Building, University Park, PA 16802. E-mail: hvw3@psu.edu.

Figure 1.—*Identification of the five ecological regions within West Virginia used to evaluate height-diameter relationships.*



NARV: Northern Appalachian Ridges and Valleys.
SARV: Southern Appalachian Ridges and Valleys.

The 2000 FIA data for West Virginia (Griffith and Widmann 2003) was used as the base data. A subset of the individual tree data was selected based on trees with measured diameters and total heights and the species identified previously. The ecoregion classification was added based on the county designation of the FIA plot location. The resultant dataset included 1,379 BC, 2,083 RO, 5,725 RM, 3,826 SM, and 3,714 YP.

The Chapman-Richards growth function was selected to model the nonlinear relationship between tree diameter and height due to its biologically interpretable coefficients (Pienaar and Turnbull 1973) and its documented flexibility for modeling height-diameter relationships in forest tree species (Fang and Bailey 1998; Huang *et al.* 1992; Pienaar and Shiver 1980, 1984). The Chapman-Richard function is a three parameter model of the form

$$H = 4.5 + \alpha\left(1 - Exp\{-\beta * D\}^{\gamma}\right) \tag{1}$$

where:
H = total tree height (ft).
D = diameter at breast height (d.b.h.) (in).
α, β, r = asymptote, scale and shape parameters.

To test for differences between the overall model (state) and each ecoregion, a nonlinear extra sum of squares procedure was employed (Neter *et al.* 1996). This procedure involves the use of dummy variables for the ecoregions in the full model form, while the reduced model form is represented by a three-parameter model representing the height-diameter relationship across all ecoregions (statewide). The full model form uses the following indicator variable (r_i) approach to represent the five ecoregions:

If ecoregion = EALG, r_1 = 1, all other r_i = 0.
If ecoregion = CUPM, r_2 = 1, all other r_i = 0.
If ecoregion = NARV, r_3 = 1, all other r_i = 0.
If ecoregion = SARV, r_4 = 1, all other r_i = 0.
If ecoregion = CALG, all r_i = 0.

The form of the full model for each species tested is

$$H = 4.5 + \left(\alpha + \sum_{i=1}^{4}\alpha_i r_i\right)\left[1 - Exp\left\{-\left(\beta + \sum_{i=1}^{4}\beta_i r_i\right)D\right\}\right]^{\left(\gamma + \sum_{i=1}^{4}\gamma_i r_i\right)} \tag{2}$$

where:
H = total height for a specific species (ft).
r_i = indicator variable for region r_i, i = 1.4.
D = tree d.b.h. for a specific species (in).
α, β, r = parameters to be estimated from the data.

The full model form has 15 parameters and an error sum of squares (SSE_F) with N-15 degrees of freedom (df_F), where N is the total number of trees for each species-specific test. The form of the reduced model is that of equation (1) and has three parameters and an error sum of squares (SSE_R) with N-3 degrees of freedom (df_R). The full model test has the following null and alternative hypotheses for each of the five species:

H_0: $\alpha_1 = \alpha_2 = \alpha_3 = \alpha_4 = \beta_1 = \beta_2 = \beta_3 = \beta_4 = \gamma_1 = \gamma_2 = \gamma_3 = \gamma_4 = 0$

and
H_a: at least one parameter is not equal to 0.

Rejecting the null hypothesis would indicate that the height-diameter relationship is not the same for all ecoregions. Failure to reject the null hypothesis would indicate that the reduced model form (equation [1]) could be applied to all ecoregions. These tests were conducted independently for each of the five species investigated.

In addition, similar test were conducted for each of the 10 pairwise ecoregion comparisons for each of the five hardwood species investigated. The same indicator variable approach was applied to the specific ecoregion test where the full model was of the form

$$H = 4.5 + \left(\alpha + \alpha_1 r_1\right)\left[1 - Exp\left\{-\left(\beta + \beta_1 r_1\right) D\right\}\right]^{\left(\gamma + \gamma_1 r_1\right)} \quad (3)$$

and the reduced model form is that of equation (1). The full model form has six parameters to be estimated and an error sum of squares (SSE_F) with N-6 degrees of freedom (df_F). The reduced model is the same as previously identified. For each species tested, the full model test has the following null and alternative hypotheses:

$H_0 : \alpha_1 = \beta_1 = \gamma_1 = 0$

and

H_a: at least one parameter is not equal to 0.

Rejecting the null hypothesis would indicate that the height-diameter relationship is not the same for both ecoregions tested. Failure to reject the null hypothesis would indicate that the reduced model form (equation [1]) could be applied to both ecoregions for that species. These tests were conducted independently for each of the five species investigated.

The significance of the full and reduced model comparisons are based on an F-test of the form

$$F = \frac{\dfrac{SSE_R - SSE_F}{df_R - df_F}}{\dfrac{SSE_F}{df_F}}$$

It is possible that significant differences can be identified but these differences may not be of practical significance. To evaluate the magnitude of the differences between using the ecoregion specific and statewide models, the mean height prediction error ($\bar{\varepsilon}$), standard deviation of the prediction error (S_e), and the prediction bias as a percent of mean actual height (Bias %) were calculated and defined as follows:

$$\bar{\varepsilon} = \frac{\sum_{i=1}^{m}\left(\hat{H}_i - H_i\right)}{m} \quad (4)$$

$$S_e = \sqrt{\frac{\sum_{i=1}^{m}\left(\varepsilon_i - \bar{\varepsilon}\right)^2}{m-1}} \quad (5)$$

$$\text{Bias (\%)} = \frac{\bar{\varepsilon}}{\bar{H}} * 100 \quad (6)$$

where:

m = number of trees for each species.
H_i = measured height of tree i.
$\hat{H}_i$ = predicted height of tree i.
$\bar{H}$ = mean of observed tree heights.

Two comparisons were made. One compared the ecoregion specific prediction equation and actual total height and one compared the statewide model and the actual height measurement.

Results

An initial test was conducted to determine whether the height-diameter relationship for each of five economically important Appalachian hardwood species could be modeled with a single three-parameter Chapman-Richards growth model. For all species tested, the null hypothesis was rejected, indicating that at least one of the ecoregion parameters was not equal to zero. P-values for this test ranged from < 0.0001 (SM, RM, and RO) to 0.0054 (BC) (table 1). Results of the initial statewide test led to further comparisons of individual ecoregion models. The same full and reduced model approach was employed to test differences between the 10 combinations of the five ecoregions identified (fig. 1). Of the 10 comparisons, the null hypothesis was rejected in 3 comparisons for BC and 7 comparisons for RO and RM when using a single comparison alpha value of 0.05 (table 2). Rejection of the null hypothesis indicates that the height-diameter relationship between the two ecoregions tested are not the same.

Existence of statistically significant differences between the ecoregions tested does not dictate that these differences are of practical significance. For each of the five species examined, the average error ($\bar{\varepsilon}$), standard deviation of the prediction error (S_e), and percent bias $\left(Bias\%\right)$ between the actual total height and predicted total height based on the statewide based height model was examined (table 3). Across all species and ecoregion combinations, average height error ranged from –3.6 to 7.6 feet

Table 1.—*Results of the full and reduced model comparisons indicating whether a single height-diameter model could be used across all five ecoregions examined.*

Species test	Full model		Reduced model		N	F-value	P-value
	df_F	SSE_F	df_R	SSE_R			
BC	1,364	227,591	1,376	232,308	1,379	2.3558	0.0054
RO	2,068	305,301	2,080	352,005	2,083	26.3630	< 0.0001
RM	5,710	705,396	5,722	714,415	5,725	6.0839	< 0.0001
SM	3,811	500,292	3,823	512,491	3,826	7.7439	< 0.0001
YP	3,699	609,529	3,711	615,036	3,714	2.7850	0.0009

BC = black cherry; RM = red maple; RO = red oak; SM = sugar maple; YP = yellow poplar.

Table 2.—*P-values associated with the full and reduced model tests for height-diameter relations between all combinations for the five ecoregions in West Virginia.*

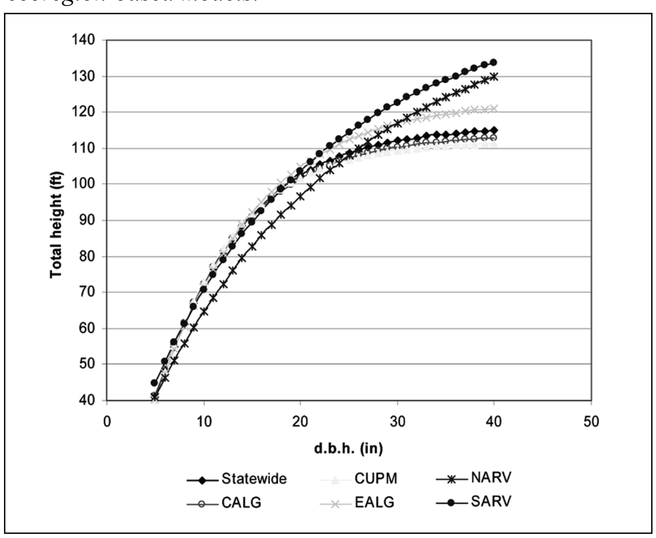
Ecoregion test	Species				
	BC	RO	RM	SM	YP
CUPM vs CALG	0.2158	0.0816	0.0832	0.6316	0.6467
EALG vs CALG	0.0562	0.0991	0.4161	**0.0013**	**0.0455**
NARV vs CALG	0.1460	**< 0.0001**	**< 0.0001**	**< 0.0001**	**0.0025**
SARV vs CALG	0.1031	**< 0.0001**	**< 0.0001**	0.6064	0.2071
EALG vs CUPM	0.2745	0.6139	**0.0093**	0.0668	0.0718
NARV vs CUPM	**0.0136**	**< 0.0001**	0.0553	**< 0.0001**	**0.0007**
SARV vs CUPM	0.1992	**0.0109**	**0.0231**	0.3407	**0.0360**
NARV vs EALG	**0.0129**	**< 0.0001**	**< 0.0001**	**< 0.0001**	**0.0018**
SARV vs EALG	0.1477	**0.0003**	**< 0.0001**	0.1108	0.1050
NARV vs SARV	**0.0071**	**< 0.0001**	**0.0002**	**< 0.0001**	**0.0038**

BC = black cherry; CALG = Central Allegheny Plateau; CUPM = Cumberland Plateau and Mountains; EALG = Eastern Allegheny Plateau and Mountains; NARV = Northern Appalachian Ridges and Valleys; RM = red maple; RO = red oak; SARV = Southern Appalachian Ridges and Valleys; SM = sugar maple; YP = yellow poplar.

for the statewide model. The average percent bias ranged from –5.3 to 12.1 percent. Average percent bias for the ecoregion-based models ranged from –0.06 to 0.04 percent.

To visually examine the differences in height curves by ecological region, height curves based on each ecoregion for YP as well as the statewide model are displayed in figure 2. For YP, very little difference can be ascertained from trees less than 20-inches d.b.h. For other species (SM, RO and RM), visual separation can be discerned by the 15-inch class. For BC, this separation occurs by the 10-inch class.

Figure 2.—*YP total height prediction based on statewide and ecoregion-based models.*



CALG = Central Allegheny Plateau; CUPM = Cumberland Plateau and Mountains; EALG = Eastern Allegheny Plateau and Mountains; NARV = Northern Appalachian Ridges and Valleys; SARV = Southern Appalachian Ridges and Valleys; YP = yellow poplar.

Table 3.—*Prediction error and model fit statistics for five species in each of the five ecological regions within West Virginia.*

				Ecoregion specific model			Statewide model			
Ecoregion	Species	N	$\overline{H}$	$\hat{H}$	S_e	Bias%	$\hat{H}$	Mean error	S_e	Bias%
CALG	BC	471	57.7558	57.7566	12.58	0.00	58.5705	0.8147	12.62	1.39
CUPM	BC	48	62.1875	62.1860	11.20	0.00	59.7648	– 2.4227	11.87	– 4.05
EALG	BC	450	64.3356	64.2974	13.50	– 0.06	63.2847	– 1.0509	13.58	– 1.66
NARV	BC	138	54.2174	54.2069	12.17	– 0.02	57.4757	3.2583	12.62	5.67
SARV	BC	272	60.7353	60.7195	12.91	– 0.03	59.8830	– 0.8523	13.07	– 1.42
CALG	RM	1,718	57.4499	57.4373	10.97	– 0.02	57.1181	– 0.3319	10.98	– 0.58
CUPM	RM	743	54.5303	54.5242	11.05	– 0.01	55.1978	0.6675	11.09	1.21
EALG	RM	1,853	59.6487	59.6419	11.48	– 0.01	59.2049	– 0.4438	11.53	– 0.75
NARV	RM	525	53.2762	53.2913	10.65	0.03	55.3875	2.1113	10.94	3.81
SARV	RM	886	55.9029	55.9107	10.86	0.01	55.5765	– 0.3264	11.01	– 0.59
CALG	RO	476	71.6555	71.6588	12.12	0.00	68.0394	– 3.6160	12.78	– 5.31
CUPM	RO	280	69.2571	69.2694	12.02	0.02	67.4228	– 1.8343	12.16	– 2.72
EALG	RO	436	73.0803	73.0723	13.12	– 0.01	70.8743	– 2.2060	13.38	– 3.11
NARV	RO	398	55.3719	55.3753	12.46	0.01	62.9613	7.5895	15.53	12.05
SARV	RO	493	64.3753	64.3729	10.93	0.00	64.7406	0.3654	11.00	0.56
CALG	SM	1,498	56.1195	56.1106	11.11	– 0.02	55.9268	– 0.1927	11.12	– 0.34
CUPM	SM	410	58.2659	58.2399	12.22	– 0.04	57.4155	– 0.8504	12.26	– 1.48
EALG	SM	992	59.2218	59.2340	11.67	0.02	58.2318	– 0.9900	11.79	– 1.70
NARV	SM	417	53.7770	53.7723	11.97	– 0.01	57.7511	3.9741	12.80	6.88
SARV	SM	509	57.7819	57.8012	10.85	0.03	57.7023	– 0.0796	10.86	– 0.14
CALG	YP	1,316	74.5266	74.4997	13.10	– 0.04	74.5036	– 0.0230	13.11	– 0.03
CUPM	YP	1,030	71.3990	71.3782	12.41	– 0.03	71.1402	– 0.2588	12.43	– 0.36
EALG	YP	1,027	76.0448	76.0245	13.15	– 0.03	75.8475	– 0.1973	13.20	– 0.26
NARV	YP	93	60.9462	60.9714	11.12	0.04	65.3363	4.3900	12.29	6.72
SARV	YP	248	71.5282	71.5141	12.14	– 0.02	71.7536	0.2254	12.31	0.31

BC = black cherry; CALG = Central Allegheny Plateau; CUPM = Cumberland Plateau and Mountains; EALG = Eastern Allegheny Plateau and Mountains; NARV = Northern Appalachian Ridges and Valleys; RM = red maple; RO = red oak; SARV = Southern Appalachian Ridges and Valleys; SM = sugar maple; YP = yellow poplar.

Discussion

The analysis indicates that statistical differences in the height-diameter relationship exist between the ecoregions identified in the current study. These differences, however, are only in the magnitude of 4 to 8 feet. Whether these differences are of practical significance depends on the practitioner's willingness to accept this magnitude of error. Use of ecoregion-based height models reduces the height prediction error and the data suggest that this difference is significant in at least half of the ecoregion-based tests for the hardwood species investigated. The effect of these height-diameter differences on individual tree volume differences is currently unknown.

Literature Cited

Bailey, R.G.; Avers, P.E.; King, T.; McNab, W.H., eds. 1994. Ecoregions and subregions of the United States [1:7,500,000]. Washington, DC: U.S. Department of Agriculture, Forest Service.

Fang, Z.; Bailey, R.L. 1998. Height-diameter models for tropical forests on Hainan Island in southern China. Forest Ecology and Management. 110: 315-327.

Griffith, D.M.; Widmann, R.H. 2003. Forest statistics for West Virginia: 1989 and 2000. Res. Bull. NE-157. Washington, DC: U.S. Department of Agriculture, Forest Service. 119 p.

Huang, S.; Titus, S.J.; Wiens, D.P. 1992. Comparison of nonlinear height-diameter functions for major Alberta tree species. Canadian Journal of Forest Research. 22: 1297-1304.

Jiang, L.; Brooks, J.R.; Wang, J. 2004. Compatible taper and volume equations for yellow-poplar in West Virginia. Forest Ecology and Management. 213: 399-409.

Neter, J.; Kutner, M.H.; Nachtsheim, C.J.; Wasserman, W. 1996. Applied linear statistical models. New York: McGraw-Hill. 1048 p.

Peng, C.; Zhang, L.; Zhou, X.; Dang, Q.; Huang, S. 2004. Developing and evaluating tree height-diameter models at three geographic scales for black spruce in Ontario. Northern Journal of Applied Forestry. 21(2): 83-92.

Pienaar, L.V.; Shiver, B.D. 1980. Dominant height growth and site index curves for loblolly pine plantations in the Carolina flatwoods. Southern Journal of Applied Forestry. 4(1): 54-59.

Pienaar, L.V.; Shiver, B.D. 1984. The effect of planting density on dominant height in unthinned slash pine plantations. Forest Science. 30(4): 1059-1066.

Pienaar, L.V.; Turnbull, K.J. 1973. The Chapman-Richards generalization of von Bertalanffy's growth model for basal area growth and yield in even-aged stands. Forest Science. 19: 2-22.

U.S. Department of Agriculture (USDA) Soil Conservation Service. 1981. Land resource regions and major land resource areas of the United States. U.S. Department of Agriculture Handbook 296. Washington, DC: U.S. Department of Agriculture, Soil Conservation Service. 156 p.

Zhang, L.; Peng, C.; Huang, S.; Zhou, X. 2002. Evaluation of ecoregion-based tree height-diameter models for jack pine in Ontario. The Forestry Chronicle. 78: 530-538.