Compatible Taper Algorithms for California Hardwoods

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Abstract.—For 13 species of California hardwoods, cubic volume equations to three merchantability standards had been developed earlier. The equations predict cubic volume from the primary bole, forks, and branches, but do not differentiate between the sources of the wood. The Forest Inventory and Analysis (FIA) program needed taper equations that are compatible with the volume equations. An algorithm (Flewelling 2004) predicts various numbers of solid wood pieces representing the primary bole and branches, each piece having a predicted inside bark and outside bark profile. No actual data on branch size distribution was used.

Introduction

The Forest Inventory and Analysis (FIA) program and other sections of the U.S. Department of Agriculture (USDA) Forest Service have been using a wide variety of volume equations and taper systems, often developed for different standards. It would be beneficial to standardize taper methodologies because they are generally independent of utilization conventions. An unpublished report on available West Coast volume and taper equations (Flewelling *et al*. 2003) does not cite any profile equations for these hardwood species; several options for conditioning taper equations to match specified volume equations are cited. The special difficulties with merchantable wood in multiple stems or branches, however, are not commonly a consideration in compatibility between taper and volume equations. Furthermore, the National Volume Equation Library (NVEL), a standardized software package maintained by the USDA Forest Service Forest Management Service Center, does not include protocols for handling prediction equations that specifically deal with multiple stems. Hence, no road map dictates exactly how to proceed.

The NVEL framework generally requires that a taper equation be integratable to volume, and should be able to give diameter inside bark at any height. When volume is from multiple stems or branches, a taper function can not be used for both of those purposes. A true solution for a multistemmed tree would require that each pathway from the ground to a merchantable diameter limit be estimated in terms of height, total length, diameter, and perhaps straightness. A model of that complexity would be extraordinarily difficult to construct under the best of circumstances. As there are no real data available, the realism of any attempt to describe piece-size distribution is necessarily limited.

Pillsbury and Kirkley (1984) sampled 13 species of hardwoods throughout their native range in California, all of which had a diameter at breast height (d.b.h.) of 5 inches or greater. The species, along with the FIA species codes, are listed in table 1. Decadent trees and trees with major defects were avoided. Exacting measurements were made with the Spiegel Relaskop, with an objective of accurately determining shape and volume for all trees, even the most deliquescent. In addition, bark thicknesses were sampled at various heights. Three types of cubic volumes were computed for each sample tree. Regression equations were developed to predict these volumes as functions of d.b.h., total height, and species. The following three volumes are all in cubic feet:

TVOL Total aboveground volume of wood and bark, including the stump, but excluding foliage.

CV4 Volume of wood from a 1-foot stump to a 4-inch small-end outside bark diameter. Excludes the bark and foliage. This volume is referred to as WVOL in Pillsbury and Kirkley (1984).

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CV9 Volume of wood from a 1-foot stump to a 9-inch small-end outside bark diameter. Excludes the bark and foliage. Every segment included must be from straight merchantable sections at least 8-feet long. This sawlog volume is referred to as SVOL in Pillsbury and Kirkley (1984).

Table 1.—*Species list.*

FIA code	Common name	Scientific name
312	Bigleaf maple	*Acer macrophyllum*
361	Pacific madrone	*Arbutus menziesii*
431	Giant chinkapin	*Castanopsis chrysophylla*
631	Tanoak	*Lithocarpus densiflorus*
801	Coast live oak	*Quercus agrifolia*
805	Canyon live oak	*Quercus chrysolepis*
807	Blue oak	*Quercus douglasii*
811	Engelmann oak	*Quercus engelmannii*
815	Oregon white oak	*Quercus garryana*
818	California black oak	*Quercus kelloggii*
821	California white oak	*Quercus lobata*
839	Interior live oak	*Quercus wislizeni*
981	California laurel	*Umbellularia californica*

FIA = Forest Inventory and Analysis.

The differences in volume between the primary bole and the entire tree including forks and branches can be substantial, yet most taper-based approaches to volume modeling would ignore the contributions of the forks and branches. An indication of the magnitude of the differences in approaches can be obtained by comparing Pillsbury and Kirkley's (1984) predictions with predictions derived from primary-bole data. For an oak in the deep South with a d.b.h. of 24 inches and height of 90 feet, Clark and Souter (1996) estimate primary-bole CV4 and CV9 as 97.9 and 94.9 cubic feet, respectively. Greater CV4 volumes are predicted by the Pillsbury and Kirkley equations for all the species they considered, with a median prediction of 130 cubic feet. Another difference is that the CV9 to CV4 ratio is almost 1 in the foregoing primary-bole example, and is much smaller for the Pillsbury and Kirkley equations. For example, in Oregon white oak, the predicted CV4 and CV9 are 159 and 86 cubic feet, respectively, implying some 73 cubic feet of volume between 4 inches and 9 inches in diameter. That difference corresponds to about 24 pieces of wood that taper from 9 inches to 4 inches in outside bark diameter; the computation here assumes that each such piece is 3 cubic feet, which is the difference between CV4 and CV9 in the example from Clark and Souter. Hence, Oregon white oak seems to have significant amounts of volume apart from the principal bole, and that volume can be distributed among many different braches.

Algorithm

An algorithm has been developed that predicts multiple wood pieces for a tree of a given species, d.b.h., and total height. Each predicted piece has a predicted length and predicted inside bark taper function. Outside bark diameters are obtained by using the inside to outside bark diameter relationships in Pillsbury and Kirkley (1984). The merchandizing of the individual pieces can be performed to any standards after the piece sizes and their profiles have been predicted.

The method makes explicit use of the TVOL and CV4 equations from Pillsbury and Kirkley (1984) in addition to their bark thickness relationships. It also uses several newly developed empirical equations, and makes use of profile equations developed for the primary boles of white oak in the South (Clark *et al*. 1991), and of bigleaf maple in British Columbia (Kozak 1994). The full algorithm and coefficients are presented by Flewelling (2004). The coefficients were set for each species by an optimization procedure with an objective of a close agreement between the three predictions from the volume equations and the corresponding volumes inferred from the taper equations for the inferred wood segments.

A single example is given to illustrate the algorithm. Consider a Pacific madrone with a d.b.h. of 17 inches and a total height of 50 feet. The algorithm predicts the taper curves for the segments listed in table 2. The summed TVOL and CV4 in the table are in exact agreement with predictions from the Pillsbury and Kirkley (1984) equations. The summed CV9, 22.66 cubic feet, falls short of the 27.70 cubic feet predicted by the volume equation. The profile for each segment is a scaled copy of a portion of the predicted bole profile for a white oak with the same d.b.h. and total height. The lower bole segment has a diameter-scale factor set such that the scaled inside bark diameter at breast height, plus the double bark thickness from the Pillsbury and Kirkley equation, equals the desired d.b.h.

Table 2.—*Counts and predicted volumes of wood segments predicted for Pacific madrone with a d.b.h. of 17 inches and a total height of 50 feet. Volumes of the segments are per piece.*

Segment	Count	Length (ft)	TVOL (ft^3)	CV4 (ft^3)	CV9 (ft^3)
Lower main	1.00	20.5	26.10	22.66	22.66
Upper main	1.00	29.5	6.41	5.86	0.00
Large branches	2.48	20.3	4.41	4.03	0.00
Small branches	13.25	7.3	0.21	0.00	0.00
Total	17.73		46.29	38.52	22.66

d.b.h. =diameter at breast height.

The upper bole segment has a diameter-scale factor set at .8254, and an implying discontinuity in the profile at 20.5 feet. The large branch segments use the same diameter scale factor as for the upper bole, and a height-scale factor of .6878. The small branch segments have large-end outside bark diameters of 4 inches; their profiles are the same as for the small-diameter end portions of the large branch segments. The scale factors and the length of the lower bole segment are from species-dependent equations that depend on d.b.h., total height, and species.

Results

The algorithm has been tested for the full range of d.b.h.s and heights in the Pillsbury and Kirkley (1984) data. TVOL is always recovered exactly. CV4 is usually recovered exactly. The predicted trends of CV9 versus d.b.h. and total height are similar to those from the volume equations, with discrepancies that are usually not more than 10 percent for trees with d.b.h.s more than 15 inches. For lesser d.b.h.s, the percentage discrepancies can be larger. The results are plausible for d.b.h.s as low as 4.5 inches, and for heights as low as 20 feet.

Discussion

Sawlog volume equations and taper equations make predictions that are essentially different and sometimes incompatible at the level of an individual tree. For the smaller tree sizes, the volume equation prediction of CV9 is often for a value that is smaller than the lowest possible volume of an 8-foot sawlog. Arguably the volume equation may be ill conditioned. On the other hand, for d.b.h.s of about 10 inches the volume equation may be predicting a CV9 value that is a valid expectation, even though that may be less than the minimum volume of a single 8-foot sawlog. The taper equations can predict zero volume, or an amount of volume that corresponds to a sawlog of 8 feet or more, but they can not predict intermediate CV9 volumes. This is one of several reasons for differences in CV9 predictions between the volume equations and the algorithm.

The three volume equations do fall into the correct rank order within the range of the data. They are not, however, always in reasonable accord. In the more extreme extrapolations, TVOL may be predicted to be less than CV4. Hence, any taper system that is forced to exactly replicate the equation-predicted volumes would sometimes produce absurd results or totally fail.

One weakness of this project is that none of the original data were available. If the three volume statistics for each tree had been available, the optimization procedure could have minimized the errors between volumes inferred from the taper equations and actual volumes, which would have been a more satisfactory approach as it would bypass any lack of fit in the Pillsbury and Kirkley (1984) equations. The lack of original data also precluded any check on the reasonableness of the predicted numbers and sizes of branches.

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