Defining Stem Profile Model for Wood Valuation of Red Pine in Ontario and Michigan With Consideration of Stand Density Influence on Tree Taper

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Abstract.—As part of the Canada-United States Great Lakes Stem Profile Modelling Project, established to support the local timber production process and to enable cross-border comparisons of timber volumes, here we present results of fitting Zakrzewski's (1999) stem profile model for red pine (*Pinus resinosa* Ait.) growing in Michigan, United States, and Ontario, Canada. The model was fitted as a system of simultaneous equations using a three-stage least squares regression method. Influence of stand density on red pine tree taper was explored using data from a spacing trial in Ontario.

Introduction

Twenty years ago, Reed and Green (1984: 977) wrote: "In the past, when merchantability standards were relatively stable, separate individual tree volume equations were developed for each set of merchantability limits. With rapidly changing standards, this approach becomes infeasible." This conclusion is still valid. A properly developed timber product estimation system should allow for the development of compatible and mathematically tractable models that are fitted with sound statistical procedures and are responsive to ever-changing utilization standards. The latter need is more and more important in an increasingly global timber trade network. Questions about the comparability of lumber prices and timber harvest structures (chiefly proportions of saw logs and pulp logs in harvested wood) between U.S. and Canadian timber markets highlight the rising importance of developing valid cross-border comparisons.

Stem profile models can provide a robust and systematic way for linking the raw commodity (wood) to wood products and thus should be useful for understanding differences in wood pricing systems and assessing potential growing timber stock value differences among markets. For example, the commercial software BUCK (http://www.forestyield.com) allows for consistent calculation of various product mixes (e.g., dimensional lumber and pulp wood volume) from raw wood volume, using an array of data inputs transformed via stem profile modelling. Consistent and tractable volume estimation can be accomplished in two steps: (1) develop stem profile models that can define the cross-sectional area and volume of tree stems or logs and (2) use analytic geometry and user-specific input variables describing wood product dimensions and sawing variables, such as saw blade width, to develop precise and consistent estimates of sawn volume (fig. 1) (for example, see information on the system Optitek [Forintek Canada Corp. 1994]). Step 2 can be accomplished with little error once Step 1 is accomplished. If board foot/cubic meter conversion factors were based on a common taper model, for example, conversions within a region of interest would be consistent. Further, once a common stem profile model is accepted to model raw volume, changes in utilization standards can be rapidly accommodated, allowing for computation of precise board (or other wood product) units, unlike typical board foot rules (Freese 1973) that confound assumptions regarding stem taper with peculiarities in sawing technology and assumptions about the size of trees that will be merchandized. Hence, a critical step in developing compatible estimates within a region of interest is developing a regionally valid stem profile model on which scaling conversions can be based. Losses in recoverable timber products caused by presence of cull or stem deformities could be accounted for during forest inventory procedures.

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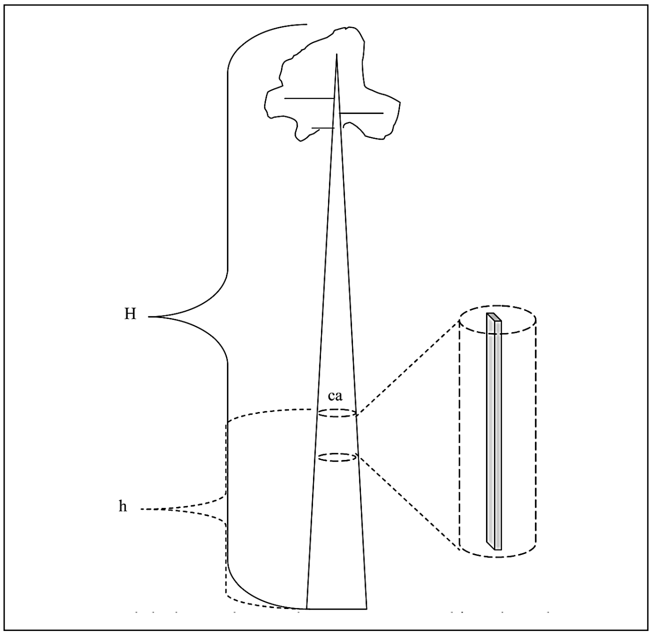
Figure 1.—*Stem profile models allow for the cross-sectional area (*ca*) of a tree stem or log to be assessed at any points along the stem length (h or H the total height of the tree), allowing precise estimates of stem volume. Once the cubic foot volume of a section of tree is defined by a stem profile model (dashed cylinder), the quantity of wood products of any dimension (shaded board) can be derived using analytical geometry and input variables describing the technique for wood processing (e.g., saw blade kerf).*



The Great Lakes Stem Profile Modelling Project (GLSPMP) was established by the Ontario Ministry of Natural Resources, Ontario Forest Research Institute, in partnership with Michigan, Minnesota, and Wisconsin Departments of Natural Resources, the U.S. Department of Agriculture (USDA) Forest Service, and the support of Michigan State University, to improve local timber product estimates and to enable cross-border comparisons of productivity potential of forest sites within the region of interest. The objective of the project is to create a regional Stem Analysis Database Management System and to provide valid stem profile models for major commercial tree species for the Great Lakes Region. During the current softwood lumber dispute, Zakrzewski's (1999) taper model was verified by the U.S. Department of Commerce and extensively used to process timber measurement data from Ontario in support of cross-border comparisons of harvest structures. Here, we present results of fitting that stem profile model for red pine (*Pinus resinosa* Ait.) from Ontario and Michigan, using data and models developed as part of the GLSPMP. Volume tables most commonly used for red pine are those by Fowler (1997) in Michigan and Honer *et al.* (1983) in Ontario; however, the models used to produce the tables do not describe tree stem taper. We wanted to demonstrate the feasibility of developing statistically and legally defensible estimates of timber product volume across regional political borders. Our specific objective was to suggest one model for the both regions and examine the conditions under which one model is applicable.

The second objective of the presented study is to explore influence of a stand density on red pine taper using data from a spacing trial in Ontario referred to here as the Stiell trial. This influence has already been suggested in the literature, and accounted for either explicitly through including stand density measures into taper model (e.g., Sharma and Zhang 2004), or implicitly by accepting density-related tree slimness measures into the model (e.g., Zakrzewski 1999). Red pine was used to examine spacing effects on taper due to the availability of data and because red pine is genetically very uniform (Mosseler *et al.* 1992). Sharma and Zhang (2004) examined black spruce, jack pine, and balsam fir. Only black spruce required a density term.

Methods and Data

Stem Profile Models Specification

In the presented study Zakrzewski's (1999) stem profile model was defined. In taper-density relation study, Kozak's (2004) model was also used to examine consistency of the results. Zakrzewski's (1999) stem profile model is based on geometric foundations and describes either outside bark or inside bark cross-sectional areas (ca_z) along relative stem height locations z:

$$ca_z = K\,\frac{z^2 + \beta z^3 + \gamma z^4}{z - s} \qquad (1)$$

where z is defined by height location h of a cross-sectional area relative to total tree height H ($z = 1 - h / H$) and K is a tree-specific constant value calculated as:

$$K = \frac{C\,(z_0 - s)}{z_0^{\,2} + \beta z_0^{\,3} + \gamma z_0^{\,4}}$$

where C is a cross-sectional area, either outside or inside bark, respectively. For describing outside bark or inside bark cross-sectional areas along tree stem, determined by input diameter d_0 of height location h_0 (thus, $C = \pi\, d_0^2/40{,}000$), commonly d.b.h.; i.e., outside bark diameter D at breast height (BH). Therefore, $z_0 = 1 - h_0 / H$; β, γ and s are model constants. The equation may feature one, two, or no points of inflection, analytically calculable, without segmentation. In equation (1), $s = 2$ can be used as a predefined constant (Zakrzewski and MacFarlane 2006). A simple transformation of equation (1) provides an equation that describes diameters d_z along the tree stem:

$$d_z = (40{,}000\ \mathrm{ca}_z / \pi)^{0.5}.$$

The presented model is mathematically tractable, meaning that after integrating ca_z (eq. 1) over h, the exact volume of a total stem *(VOL)*, or any section of a stem, can be obtained. Total stem volume is calculated as:

$$VOL = KH[\frac{1}{2} + \frac{1}{4}\gamma + s^2 \ln(s - 1) + s^3\gamma + s^3\beta \ln(s - 1) + s^2\beta$$
$$+ s^4\gamma \ln(s - 1) + \frac{1}{2} s^2\gamma + s + \frac{1}{3} s\gamma \qquad (4)$$
$$+ \frac{1}{3}\beta + \frac{1}{2} s\beta - s^4\gamma \ln(s) - s^2 \ln(s) - s^3\beta \ln(s)]$$

The model (eq. 1) can be analytically solved to locate any reasonable merchantability (diameter) limit on a particular stem (Zakrzewski 1999).

Kozak' s model 02 (eq. [4] in Kozak 2004) model was used for comparison.

$$\hat{d}_i = a_0 D^{a_1} H^{a_2} X_i^{\,b_1 z_i^4 + b_2\left[1/e^{D/H}\right] + b_3 X_i^{0.1} + b_4\left[1/D\right] + b_5 H^{Q_i} + b_6 X_i} \qquad (5)$$

Where:

$\hat{\boldsymbol{d}}_{\boldsymbol{i}}$ = predicted inside bark diameter at height h_i from ground (cm)

$$\boldsymbol{X_i} = \left[1.0 - \left(\boldsymbol{h_i}/\boldsymbol{H}\right)^{1/3}\right] \Big/ \left[1.0 - \boldsymbol{p}^{1/3}\right]$$
$$\boldsymbol{Q_i} = \left[1.0 - \left(\boldsymbol{h_i}/\boldsymbol{H}\right)^{1/3}\right] \qquad (6)$$
$$\boldsymbol{p} = 1.3/\boldsymbol{H}$$

Kozak's model is one of the most flexible functions describing tree stem profiles (9 coefficients) and has low multicolinearity. The limitations of the above function come chiefly from a lack of full mathematical tractability of the model.

Data

Two data sets were used in the presented studies: one was used for defining a general stem profile model acceptable for Ontario and Michigan, another one was Ontario-specific and used for exploring influence of density on red pine's taper.

The data used for fitting general red pine stem profile models in this study consisted of measurements on 210 sample trees from Ontario and 128 sample trees from Michigan, for a total of 338 stems with 3,215 inside and outside bark diameters measured along the stems. The data were collected in Ontario by Honer (MacLeod 1978) and in Michigan by Van Dyck (2005). In both cases, d.b.h. was measured at 1.37 m. D.b.h.s in Ontario's data ranged from 6.5 to 65.5 cm, and in Michigan's data from 12.9 to 41.7 cm; heights ranged from 7.7 to 34.5 m, and from 8.5 to 26.1 m, respectively. The data were from red pine plantations, but information about planting densities was not available.

The Stiell trial data come from the Petawawa Research Forest from a spacing trial initiated by Will Stiell and described in Penner *et al.* (2001). Briefly, in 1953, a red pine spacing trial was established on abandoned fields near Chalk River, Ontario (46°00'N, 77°26'W) to examine the effects of initial planting density on the growth and yield of red pine. The experiment consists of two plantations (blocks 109 and 110) a few hundred metres apart. Block 109 is approximately 3.2 ha and block 110 is approximately 14.3 ha. The soil in one block is a deep fine to medium aeolian sand whereas the other block has medium water-laid sand with an aeolian sand cap. The soil moisture on both is similar and, as a result, both areas are considered to be in the same productivity class (Berry 1964). The study site was of high productivity for red pine with a site index of 24.4 m at 50 years (Stiell and Berry 1973).

Bareroot seedlings were machine planted in the spring of 1953 at square spacings of 1.2, 1.5, 1.8, 2.1, 2.4, 3.0, and 4.3 m. The average size of an area planted at a particular spacing was 1.6 ha.

In the fall of 2002, 50 growing seasons from establishment, 73 trees were selected from the buffers for destructive sampling, representing the range of d.b.h.s in the PSP (table 1) from the buffers. The diameter outside bark and bark thickness was measured at 0.3, 0.8, 1.0, 1.3, and every subsequent 2 m along the bole to the base of the live crown and then every 1 m in the live crown for an average of 16 measurements along the stem.

Model Fitting and Validation Methods

There are a few optional approaches to defining equation (1). One can use a traditional regression fitting technique, either nonlinear or linear fit to benefit from the intrinsic linearity of equation (1). Use of the regression procedures and their respective weaknesses (Gregoire *et al.* 2000) can be, however, avoided. If an individual tree total height, diameter at known tree height, and a tree stem volume or volume of a height location specific section of the stem are known, the coefficient β of equation (1) can be calculated analytically (Zakrzewski 2004) for that tree (in such case it is required that $s = 2$ or substituted with H, and coefficient γ set to 1). This approach was applied to "translate" Gevorkiantz and Olsen (1955) volume tables into stem profile models for Michigan State inventories. In fact, there is no need to transform the one-equation taper model fitted for one endogeneous variable (e.g., diameter) to predict response of a different variable (e.g., volume). A system of simultaneous equations may be fit, duly recognizing that many variables in the system may be interdependent, and thus, the classical least squares rules could be biased (Judge *et al.* 1988). We chose this method to define the general stem profile model for Ontario and Michigan, so equation (1) has been transformed to predict four individual tree measures:

(1) Outside bark cross-sectional areas along tree stem ca_{ob}.
(2) Cross-sectional areas of bark along tree stem ca_{bark}.
(3) Height location of randomly selected inside bark diameter h_{diam} (the equation defining this height is not shown here, but is presented in Zakrzewski [1999]).
(4) Inside bark volume of a stem section vol_{log} of variable length (0.45 to 31 m, 9 m average) including possibility for total tree volume.

Outside and inside bark cross-sectional areas were based on 5 to 11 sectional diameter measurements per stem; most often 8 to 10. Inside bark cross-sectional area ca_{ib} was not an endogeneous variable in the system; however, a bark cross-sectional area model (right-hand side of the equation) was formulated as the difference between outside bark cross-sectional area and a form of equation (1) describing inside bark cross-sectional areas. Accepting one set of coefficients of equation (1) to define bark thickness along a stem was a deliberate simplification.

From a range of relative height locations between 2 m and 80 percent of the total tree height, inside bark diameters were randomly selected. Height location of these diameters h_{diam} was the endogeneous variable in the system of equations.

To define inside bark volume of variable length logs (vol_{log}, endogeneous variable in the system), height location of the base of the log and the location of the top cross-sectional area of the log were selected randomly for each stem. For about 10 percent of tree stems, log volume was defined as total tree volume (with no stump). Volumes were calculated as a sum of sectional volumes estimated using Smalian's formula (Avery and Burkhart 1994). The ultimate objective of fitting the model was to ensure acceptable predictions of tree stem parameters of interest to users of the model; i.e., a combination of low bias and high precision of predictions. Three-stage least squares (3SLS) was applied to account for both a simultaneity bias and

Table 1.—*Stiell trial spacing data are summarized by initial spacing.*

Initial spacing (m)	Number of trees	Quadratic mean d.b.h. (cm)	Top height (m)	Volume (m^3)
1.5 x 1.5	8	18.6	20.9	0.31
1.8 x 1.8	17	19.2	21.1	0.32
2.1 x 2.1	18	22.5	24.5	0.48
2.4 x 2.4	15	25.6	24.5	0.62
3.0 x 3.0	15	29.0	24.9	0.78

d.b.h. = diameter at breast height.

contemporaneous correlation (LeMay 1990, Van Deusen 1988). SAS Institute's PROC MODEL was used to fit the system of equations with 3SLS algorithms.

In the presented study, autocorrelation of errors was not accounted for during the model fitting procedure.

Similar to the task of model fitting, the model performance test requires examining both the method and the data. The fitted equation (1); i.e., a general model for Ontario and Michigan, was applied to obtain prediction errors for the 338 trees described above (following Kozak and Kozak 2003). An equivalence test (Wellek 2003) was applied to examine optional regions of indifference for estimates of stem volumes. Traditionally, a null difference hypothesis would assume that the respective mean errors produced by model predictions are equal to 0, and the respective tests indicate whether sufficiently strong evidence exists to question this equality. The hypothesis we examined was different: namely, we insisted that the mean tree volume prediction error is not equal to 0. Such a hypothesis can be rejected if existing differences fall in the user-specified region of indifference. This subjectivity allows a practical evaluation of the modelling tool and helps decide if the expected errors are negligible for the model's user (Robinson and Froese 2004). The region of indifference was defined as ε, a measure relative to standard deviation of prediction errors. Using this measure, a noncentrality parameter of F-distribution was calculated as $\psi 2 = n\,\varepsilon^2$, and then a cut-off statistic $\acute{C}$ was generated for comparison with the t-value. The value of cut-off is the square root of the α-quantile of the noncentral F distribution with degrees of freedom ν_1 = 1 and $\nu_2 = n - 1$. The cut-off value $\acute{C}$ can be easily obtained using the SAS software (function FINV): $\acute{C}$ = sqrt(FINV(α, ν_1, ν_2, ψ^2)). If the t-value is smaller than the cut-off value, the hypothesis of dissimilarity can be rejected (we used $\alpha = 0.05$).

To look at spacing effects, equation (1) was fit using ordinary least squares nonlinear regression (PROC NLIN), predicting inside bark cross-sectional from d.b.h. (outside bark) and total height. A combined model was fit to trees from all spacings and then separate models were fit by spacing. Kozak's model (Eq. [5]) was modified to predict cross-sectional area and then fit to the same data for comparisons with equation (1).

Results

Results of Fitting the General Model

The 3SLS coefficients for the proposed regional stem profile model and associated statistics are presented in table 2. The constant $s = 2$ was used in the model, thus only two regression coefficients were estimated. Coefficient γ for red pine was significantly different from unity; however, setting this coefficient to 1 would be conceivable without affecting the predictive power of the model. Points of inflection for red pine were determined to occur on the average at 29 and 70 percent of the trees' relative length. The system weighted statistics for R^2 were 96 percent for outside bark cross-sectional areas (root mean squared error [RMSE] = 0.0109 m^2), 93 percent for height location of given diameter (RMSE = 1.28 m), 98 percent for log volume (RMSE = 7.2 percent), and 25 percent for bark cross-sectional area (RMSE = 0.00006 m^2). Reported goodness of fit measures were calculated using McElroy's (1977) multi-equation analog of Buse's (1973) result (Judge *et al.* 1985).

Using the coefficients of the model, inside bark diameter prediction errors were calculated using the same data set used for the model fitting (table 3). Lack of fit statistics indicated a negative bias (mean error of prediction in table 3) in diameter predictions in the bottom 30 percent of relative height ranging from -0.6 to -0.1 cm, and in the top 30 percent, ranging from -0.7 to -0.3 cm. In the middle of the stems, bias was positive, ranging from 0.05 to 0.6 cm. Standard deviation of errors ranged from 0.8 to 2.9 cm in the top of stems. When the data were split into Michigan and Ontario subsets, the average standard deviation of diameter prediction errors for Ontario (2 cm) was twice that for Michigan (1 cm). Residuals of the

Table 2.—*Stem profile model 3SLS coefficients.*

Coefficient	Coefficient estimate	Coefficient standard error
β	– 1.8412	0.010
γ	0.9516	0.009

3SLS = three-stage least squares.

Table 3.—*Model performance statistics (absolute and relative (%) errors) for the proposed regional red pine stem profile model.*

Region	Variable	N	Mean error	Standard error of mean	Standard deviation of error	MIN error	MAX error
Combined	dob_z [cm]	3,215	– 0.16	0.03	2.09	– 11.34	9.61
	dib_z [cm]		– 0.13	0.03	1.79	– 11.05	8.42
	h_{diam} [m]	338	0.17	0.02	1.28	– 3.55	5.16
	Vol_{log} [%]		– 0.69	0.72	13.28	– 71.25	34.57
	Vol_{tot} [%]		0.14	0.51	9.43	– 61.04	21.63
Ontario	dob_z [cm]	2,218	– 0.27	0.05	2.34	– 11.34	9.66
	dib_z [cm]		– 0.28	0.04	2.02	– 11.05	8.42
	h_{diam} [m]	210	0.13	0.03	1.41	– 3.55	5.16
	Vol_{log} [%]		– 1.48	1.08	15.71	– 71.25	34.57
	Vol_{tot} [%]		– 0.95	0.74	10.81	– 61.04	21.63
Michigan	dob_z [sq. m]	997	0.08	0.04	1.36	– 7.09	6.73
	dib_z [cm]		0.18	0.03	1.06	– 6.19	5.42
	h_{diam} [m]	128	– 0.26	0.03	0.93	– 2.69	4.01
	Vol_{log} [%]		0.61	0.68	7.71	– 33.69	26.43
	Vol_{tot} [%]		1.94	0.54	6.22	– 21.81	12.72

equations were strongly correlated in case of cross-sectional area outside bark and cross-sectional area of bark (93 percent). Due to the model formulation, predicted diameter outside bark (d_{ob}) is always larger than diameter inside bark. In terms of bark thickness, the standard deviation of prediction errors was, on average, about 0.35 cm. Not unexpectedly, bark thickness was significantly underestimated at the bottom part of stems.

While tree sizes along tree stems are a major focus in stem profile modelling, predictions of wood volumes are ultimately of most interest to forest practitioners. Using the obtained coefficients, total tree (without stump) volume (Vol_{tot}, table 3) predictions were examined (input variables were the same as those used for model fitting). For the whole data set, relative error was, on average, 0.14 percent, with standard deviation of 9.43 percent relating to individual tree predictions. For Michigan, respective values were 1.94 percent and 6.22 percent, and for Ontario 0.95 percent and 10.81 percent. Average relative error in log volume (Vol_{log}, table 3) was 0.69 percent, with standard deviation 13.28 percent. For Michigan, respective values were 0.61 percent and 7.71 percent, and for Ontario 1.48 percent and 15.71 percent. Bias was negligible.

Results of Fitting the Stand-Density-Specific Model

Both models fit the data well (table 4, fig. 2) with the Kozak model having lower bias. The combined model (all spacings in a common model) was compared to separate models (fig. 2b) by spacing by comparing the error sums of squares for the combined model to the pooled error sums of squares from the separate models.

$$F = \frac{\left(SSE_{combined} - \sum_{spacing} SSE_{spacing}\right) \Big/ \left(dfe_{combined} - \sum_{spacing} dfe_{spacing}\right)}{SSE_{combined} \Big/ dfe_{combined}}$$

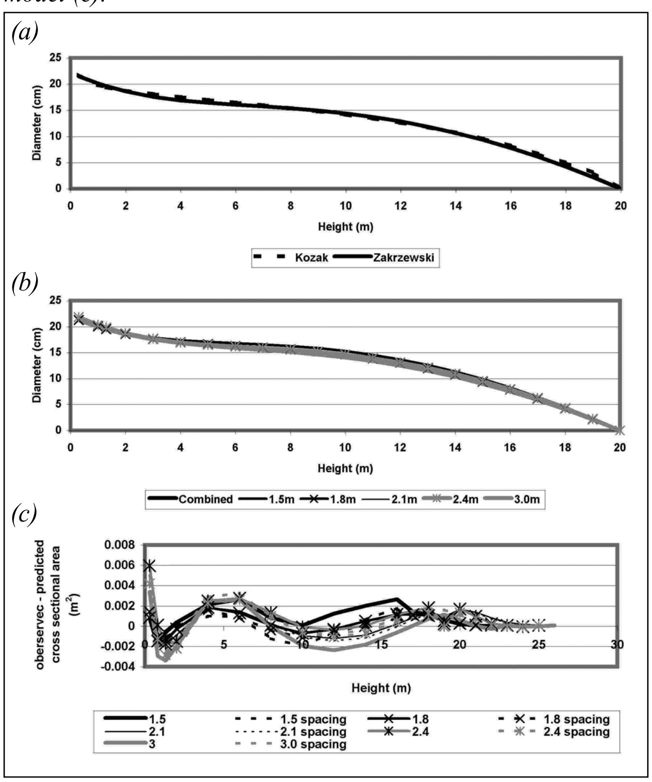
For both the Zakrzewski and the Kozak model, the hypothesis of a common taper model across all spacings was rejected (p < 0.0001 for both models). When the narrowest and widest spacings were removed, a common model fit the data as well as separate spacing models indicating a single model is adequate for spacings of 1.8 to 2.4 m. Spacings of 1.5 and 3.0 m are better predicted using separate models. As seen in figure 2b, the narrower spacings have larger cross-sectional areas above breast height for the same d.b.h. and total tree height, leading to higher volume. This is similar to the results of Sharma and Zhang (2004) for black spruce.

Table 4.—*The error sums of squares and degrees of freedom are given by spacing for the Zakrzewski model and the Kozak model.*

Model	Spacing	N	d.b.h. (cm)	Total height (m)	Diameter inside bark (d.i.b.) (cm)	Combined		Separate	
						Predicted d.i.b. (cm)	Mean d.i.b. error (cm)	Predicted d.i.b. (cm)	Mean d.i.b. error (cm)
Zakrzewski	1.5	108	18.74	21.15	12.43	12.38	0.52	12.77	0.13
	1.8	236	19.39	21.10	12.88	12.62	0.26	12.68	0.20
	2.1	296	22.60	24.50	14.83	14.57	0.26	14.65	0.19
	2.4	214	25.61	24.62	16.87	16.49	0.41	16.69	0.21
	3	202	28.95	24.79	19.41	19.23	– 0.02	18.94	0.28
Kozak	1.5	108	18.74	21.15	12.43	12.74	0.17	12.92	– 0.01
	1.8	236	19.39	21.10	12.88	12.95	– 0.07	12.93	– 0.05
	2.1	296	22.60	24.50	14.83	15.13	– 0.30	14.91	– 0.08
	2.4	214	25.61	24.62	16.87	16.88	0.02	16.98	– 0.08
	3	202	28.95	24.79	19.41	19.38	– 0.16	19.33	– 0.11

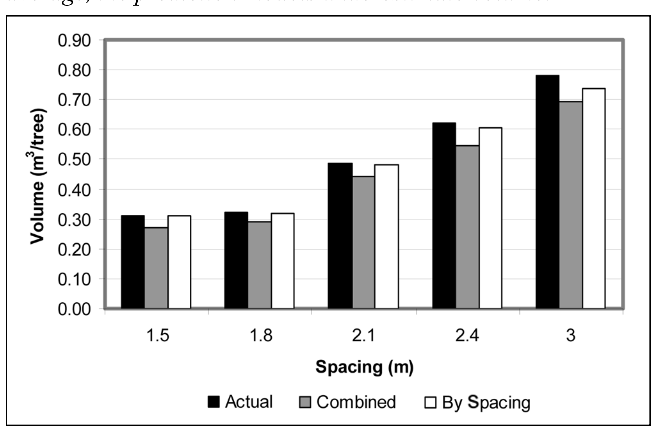
d.b.h. = diameter at breast height; d.i.b. = diameter inside bark.

Figure 2.—*The Kozak and Zakrzewski models for all spacings combined are compared for a tree with a total height of 20 m and a d.b.h. of 20 cm (a). The predictions are very close with the Kozak model having increased flexibility due to the inclusion of additional parameters. The combined model is compared to separate models by spacings for the Zakrzewski model (b). The errors along the stem are given by spacing for the Zakrzewski model (c).*



Although the taper model was fit to cross-sectional area, stem volume is key variable of interest. The Zakrzewki taper model was used to predict total stem volume and compared to the actual volume (fig. 3). The actual volume was calculated using Smalian's formula using the taper data. The taper model consistently underestimated total volume regardless of whether separate models were calibrated by spacing or whether the combined model was used. The taper model errors vary along the stem (fig. 2c) but the influence of cross-sectional area errors on volume are much greater near the base of the stem. This illustrates one of the benefits of a tractable model. Volume errors can be minimized directly by fitting the volume form of the taper model or volume and cross-sectional area errors can be minimized simultaneously.

Figure 3.—*The volume predictions are given by spacing for the combined model and separate models by spacing. Note, on average, the prediction models underestimate volume.*



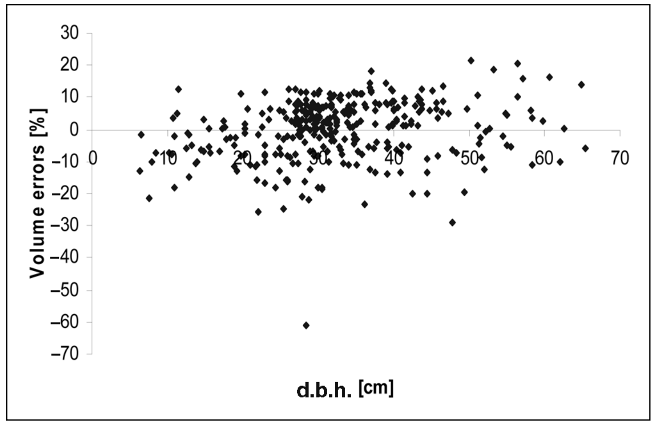
Validation of the Performance of the Models

From the results presented in table 3, it is evident that the stem profile model yields acceptably low errors in log and tree volume prediction. For the purpose of cross-border comparisons, however, we must ask if we can use the same set of coefficients (the same stem profile model) to estimate wood volume in Michigan and Ontario. The answer is conditional based on user-specified criteria of equivalence.

Equivalence tests, introduced to forestry by Robinson and Froese (2004), provide foresters with a sound and practical foundation to address the issue of model suitability for a desired purpose. Hypothesized dissimilarity between the observations and the model predictions is rejected if the existing differences fall in the region of indifference specified by model users from Michigan and Ontario. If the hypothesis is rejected, further efforts to improve the precision of the model's predictive power could be considered as a purely academic exercise, or worth pursuing only for tasks demanding higher accuracy and precision.

To test the scenario of inside bark total tree stem volume prediction (Vol_{tot}) being more practical for model application, an estimate of inside bark diameter at the *BH* level was used as the model's input. To reduce d.b.h. (outside bark diameter) to the inside bark diameter at the *BH* level, the double bark thickness was predicted from the sample tree based regression model. The resulting volume errors were similar to those obtained using variable location input diameters. Tree volume errors were plotted against d.b.h. values and did not show any consistent pattern (fig. 4).

Figure 4.—*Inside-bark tree volume prediction errors estimated for red pine data from Michigan and Ontario using proposed regional stem profile model.*



d.b.h. = diameter at breast height.

To evaluate the presented results, a paired *t*-test of equivalence was performed. Normal probability plots indicated normality of distribution of errors for the joint data sets and for the individual regions. The criteria (suggested acceptable errors of 0.03 or 0.06 m^3) were used to test the base for the rejection of dissimilarity hypothesis. Those were translated to different values of ε depending on the estimate of standard deviation of the volume prediction errors (ε values are obtained by dividing a criterion value by the respective standard deviation). Results indicate that the fitted model is acceptable if the model users in Michigan and Ontario can tolerate expected errors of total stem volume estimates not greater than around 50 percent of expected standard deviation of errors, i.e., 0.06 m^3 (table 5). In practical terms, this can be expressed as around 4.7 percent volume error for a larger number of stems (a half of relative standard deviation reported in table 3 for the combined regions).

Table 5.—*Statistical summary of equivalence test (at $\alpha = 0.05$) for red pine total tree volume prediction errors.*

Region	Criterion (m^3)	N	Mean error	Standard deviation	ε	*t*-value	Cut-off value $\bar{C}$	Hypothesis of dissimilarity
Combined	0.06	338	0.027	0.120	0.495	4.10	7.41	Rejected
	0.03				0.247		2.90	Not rejected
Ontario	0.06	210	0.033	0.150	0.399	3.28	4.12	Rejected
	0.03				0.199		1.25	Not rejected
Michigan	0.06	128	0.015	0.039	1.501	4.40	14.76	Rejected
	0.03				0.750		6.72	Rejected

Conclusions

As part of the Great Lakes Stem Profile Modelling Project, the general stem profile model developed by Zakrzewski (1999) was fitted for red pine in Michigan, United States, and Ontario, Canada, and its predictive performance examined. Results indicate much greater variability in red pine stem profiles in Ontario than in Michigan, which relates to differences in the data sets (perhaps stand density) defined by characteristic differences in stem form among species, across ecosystems, and under different growing conditions. While local variation in tree stem form can be accounted for by fitting (localizing) models (e.g., MacFarlane [2004] used ecological classification systems to define ecologically referenced height-diameter models), suitable regional models may be adequate for defining equivalent comparisons across larger regions (e.g., species neutral, composite volume equations developed by Gevorkiantz and Olsen [1955]).

Our results suggest that cross-border comparisons of timber inventories can be directly addressed using regional stem profile models. The fact that in this study stem profile model error was generally insensitive to the use of variable location input diameters demonstrates how the taper model can accommodate measurements taken from any reasonable portion of a stem or log, an important consideration in a model developed to combine data from different forest.

The Zakrzewski model and Kozak's taper model led to similar conclusions about the effect of tree density (stems/ha) on taper. Based on the data used here, for moderate initial spacings (1.8 to 2.4 m), a single taper model is adequate. For narrower (1.5 m) and wider (3.0 m) spacings, separate taper models should be calibrated or the taper model modified to explicitly incorporate the density effect. Fitting only the cross-sectional area formulation of the taper model led to underprediction of volume, on average. A mathematically tractable taper model allows fitting of volume directly either on its own or simultaneously with cross-sectional area.

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