Estimating the Quadratic Mean Diameter of Fine Woody Debris for Forest Type Groups of the United States

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Abstract.—The Forest Inventory and Analysis program of the Forest Service, U.S. Department of Agriculture conducts a national inventory of fine woody debris (FWD); however, the sampling protocols involve tallying only the number of FWD pieces by size class that intersect a sampling transect with no measure of actual size. The line intersect estimator used with those samples requires a measurement of the quadratic mean diameter (QMD). Published information regarding appropriate QMDs by FWD diameter class by species or forest types across the United States is lacking. Thus, the objective of this study is to employ a technique known as the graphical estimation (GE) method for estimating FWD QMDs for major forest types across the United States. Results indicate that the GE method, along with adaptations proposed in this study, allows for rapid estimation of FWD QMDs without additional fieldwork. The value (–1.81) of the scaling power function between the number of woody pieces by size class closely matches published scaling values (–2.00) between bole and fine branch diameters. Validation results indicate that estimates of QMD from the GE method differ substantially from published QMDs; however, published QMD results differ from one another to an even greater extent. Given the mixing of FWD from a diversity of tree and shrub species in the Nation's forests, this study concludes that a set of general QMDs for FWD across the United States should be used in the absence of empirically derived or power function-based QMD estimates.

A National Inventory of Fine Woody Debris

The Forest Service's Forest Inventory and Analysis (FIA) program conducts a national inventory of fine woody debris (FWD). FWD is defined by the FIA program as dead and downed woody debris in forests that is less than 3 in in diameter (USDA Forest Service 2006). Estimates of FWD volume and biomass based on the FIA inventory are highly desired by the fire and fuel and carbon science communities. FWD is a substantial component of fuel loadings and determines, to a large extent, fire behavior (Albini 1976, Burgan and Rothermel 1984, Deeming *et al.* 1977). FWD, as a component of the forest floor, is often reported as a forest carbon stock (IPCC *et al.* 1997, Smith *et al.* 2006).

FIA's sample protocol for FWD involves tallying the number of FWD pieces by size class that intersect a sampling plane. FIA protocols define three FWD size classes: small (0.01 to 0.24 in), medium (0.25 to 0.99 in), and large (1.00 to 2.99 in). The sample protocols do not require the measurement of the actual diameter of each FWD piece, only the number of pieces in each diameter class. In order to estimate FWD volume, the FWD quadratic mean diameter (QMD) for each size class is required. QMD is defined as the square root of the average squared diameter. Since the FWD volume estimator requires a QMD to be squared, any substantial errors in QMD approximation can lead to even larger errors in FWD volume estimates. Because the distribution of the number of FWD pieces as a function of FWD diameter is typically a negative exponential, using FWD size class midpoints in lieu of QMD may be inappropriate (solid arrows, fig. 1). Empirically derived FWD QMDs are available in the literature and offer a more defensible estimate of diameter. Unfortunately, published FWD QMDs are species specific and are only available for a very limited number of species and sites in the United States (e.g., see Nalder *et al.* 1997,

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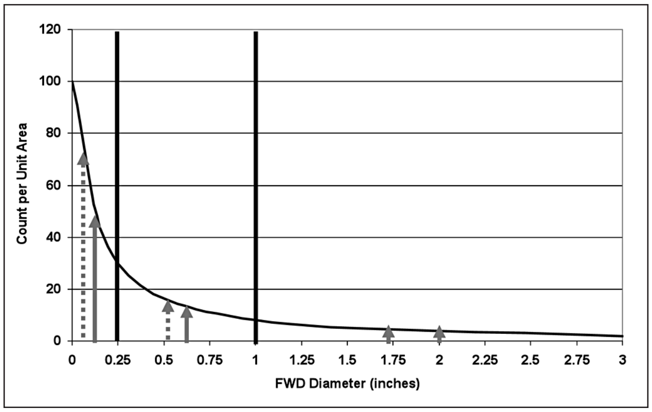
Figure 1.—*Hypothetical distribution of fine woody debris diameters.*



Note: Solid vertical lines define fine woody debris (FWD) size classes, solid arrows denote FWD diameter class midpoint, and dotted arrows denote midpoint of area under the FWD distribution curve.

1999; Roussoupolos and Johnson 1973; Van Wagtendonk *et al.* 1996). Given the predominantly mixed species condition of forests across the Nation, along with hundreds of tree and shrub species, empirically deriving FWD QMDs for every FIA FWD transect may be unfeasible. Hence, a substantial knowledge gap exists regarding FWD QMDs in forests of the United States.

An Alternative: The Graphical Estimation Method

Van Wagner (1982) was one of the first to identify the limitation of empirically derived FWD QMDs. As an alternative to conducting fieldwork for every population of interest (unique forest type across the country), Van Wagner (1982) proposed the graphical estimation (GE) method for deriving QMDs that involve no additional fieldwork. Van Wagner (1982) assumed that the distribution of FWD diameters follows a power law:

$$y = ax^b \tag{1}$$

where y is the number of FWD pieces, x is the diameter, and a and b are constants. Then, the theoretical QMD for a diameter class can be calculated analytically as (equation 5 of Van Wagner 1982):

$$(QMD)^2 = \int_{x_1}^{x_2} ax^{b+2}dx \Big/ \int_{x_1}^{x_2} ax^b dx = \frac{(b+1)\left(x_2^{b+3} - x_1^{b+3}\right)}{(b+3)\left(x_2^{b+1} - x_1^{b+1}\right)} \tag{2}$$

where QMD is the quadratic mean diameter and x_1 and x_2 are the lower and upper diameter class limits, respectively.

To estimate the QMD for each diameter class, the coefficient b has to be estimated from the data. To this effect, Van Wagner (1982) log-transformed the simple power equation (equation 1). He estimated b as the slope of the regression of the log of Y, the number of intersections per unit sample line and diameter class, on the log of X, the midpoint of the diameter class. The diameter classes were normalized to a unit width by dividing the frequency by the width of the diameter class. Thus, the slope of the regression model is used to determine the midpoint of the area under the estimated FWD diameter distribution (dotted arrows, fig. 1). Van Wagner's original hypothesis (1982) was only demonstrated for a few selected species, assuming that users would only be able to manually fit the GE models.

The goal of this study was to explore the possibility of using Van Wagner's GE method to estimate FWD QMDs across the United States with specific objectives including: (1) estimating FWD QMDs by forest type in the United States using a nonlinear power function to estimate b (equation 1) in Van Wagner's GE method (1982), (2) validating FWD QMD estimates where published FWD QMD estimates are available, and (3) discussing future options for approximating FWD QMDs when estimating FWD volumes in a national inventory of FWD.

Data and Analysis

A total of 9,788 observations from single forest condition FIA plots across the United States were included in this study. Each observation consisted of FWD and coarse woody debris (CWD) tally counts along 10 ft of the 150-degree transect (for further sample protocol information, see USDA 2006, Woodall and Williams 2005). FWD with transect diameters less than 0.25 in and 0.25 in to 1.00 in (1 and 10 hr, respectively) were tallied separately on a 6-ft slope-length transect (14 ft to 20 ft on the 150-degree transect) and standardized to a 10-ft length by multiplying counts by 1.666. FWD with transect diameters of 1.00 to 2.99 in (100-hr) were tallied on a 10-ft slope-length transect (14 ft to 24 ft on the 150-degree transect) (for more information on fuel class definitions, see Deeming *et al.* 1977). CWD, along the 10-ft section of the 150-degree transect with transect diameters within the size classes of 3.0 to 8.9 in and 9.0 to 27.0 in, were included as additional woody debris size classes. Tally counts, standardized to a 10-ft sample transect by woody size class, were averaged by forest type group.

A simple power function formulated as a nonlinear model (equation 3) was fitted by forest type group in order to estimate QMD^2 by forest type group across the United States:

$$E(y) = ax^b \quad (3)$$

where E(.) is an expected value, y is the number of FWD pieces tallied in a particular size class, x is the diameter of the size class, and a and b are coefficients to be estimated. The b in equation 3 was used to solve for QMD in equation 2.

QMDs were validated by comparing selected forest type group QMD^2s to empirically derived QMD^2s (ponderosa pine, Douglas-fir, western larch; see table 1) and determining a mean absolute and relative difference. Because more than one published QMD^2 was available by species, a mean and coefficient of variation among published empirical estimates (Brown and Roussopoulos 1974, Roussopoulos and Johnson 1973, Ryan and Pickford 1978, Sackett 1980, Van Wagtendonk *et al.* 1996) were determined for each validation species.

Estimates of QMD^2 by Forest Type Group

The approximate R-square for the fitted nonlinear model exceeded 0.80 for all forest type groups. Estimates of b (equation 2) used to determine the midpoint of the area under the nonlinear curve (Van Wagner 1982) did not vary greatly by forest type group (table 2). Consequently, estimated QMD^2s did not vary greatly by forest type group (table 2). QMD^2s for the smallest FWD did not vary to any degree, while QMD^2s for the largest FWD varied from 2.7 to 3.3. The mean QMD^2s for all forest type groups in the United States was 0.018, 0.262, and 3.101 for increasing FWD size classes, respectively. If the square of the midpoint of the diameter class is used, the estimates of QMD^2 would have been 0.016, 0.384, and 3.98.

Validation of QMD^2s

The differences between the selected forest type groups' QMD^2 estimate and the empirically derived QMD^2s for selected species were substantial (table 3). The differences in estimates may be attributed to the possible inability of the GE method to adequately represent natural interspecific variation in FWD sizes. The predicted QMD^2s are based on an estimated mean. This conclusion, however, cannot be strongly supported by the validation results since it may be inappropriate to compare forest type group results from a wide geographic area to individual species from a particular site. Across the United States, the mixing of FWD from a diversity of tree and shrubs species obscures interspecific differences, thus a substantial difference in FWD QMDs between forest type groups should not be expected. Furthermore, the coefficients of variation between empirical studies themselves often exceeded differences between empirical studies and the GE method results (table 3). For example, although this study's QMD^2 estimates for small Douglas-fir FWD varied from the literature by 67 percent, the coefficient of variation among the published QMD^2s exceeded 115 percent.

Table 1.—*Means of published estimates of FWD QMD^2s used in validation procedures.*

Forest type	Small FWD		Medium FWD		Large FWD	
	Mean QMD^2	Citations	Mean QMD^2	Citations	Mean QMD^2	Citations
All types	0.045	All	0.451	All	3.976	All
Ponderosa	0.087	All	0.434	All	4.285	BR 1974, S 1980,V 1996
Douglas-fir	0.049	All	0.419	All	2.880	BR 1974, S 1980,V 1996
Western larch	0.078	BR 1974, RP 1978	0.335	BR 1974, RP 1978	1.472	BR 1974

FWD = fine woody debris. QMD = quadratic mean diameter.
Sources: Brown and Roussopoulos 1974 (BR 1974), Ryan and Pickford 1978 (RP 1978), Sackett 1980 (S 1980), Van Wagtendonk et al. 1996 (V 1996)

Table 2.—*Estimates of QMD² by forest type group across the United States based on the graphical estimation method.*

Forest type group	b	QMD²		
		Small	Medium	Large
White/red/jack pine	– 1.9106	0.017	0.254	3.043
Spruce/fir	– 1.8861	0.018	0.256	3.058
Longleaf/slash pine	– 1.2514	0.020	0.310	3.457
Loblolly/shortleaf pine	– 1.5277	0.019	0.286	3.279
Pinyon/juniper	– 1.8785	0.018	0.257	3.062
Douglas-fir	– 2.1937	0.016	0.233	2.879
Ponderosa pine	– 1.4478	0.019	0.293	3.330
Fir/spruce/mountain hemlock	– 2.0054	0.017	0.247	2.987
Lodgepole pine	– 1.9952	0.017	0.248	2.993
Hemlock/Sitka spruce	– 1.8624	0.018	0.258	3.072
Western larch	– 1.6936	0.018	0.272	3.175
Redwood	– 1.7481	0.018	0.267	3.141
Other western softwoods	– 1.6366	0.019	0.277	3.210
California mixed conifer	– 1.6981	0.018	0.271	3.172
Oak/pine	– 1.5927	0.019	0.280	3.238
Oak/hickory	– 1.6870	0.018	0.272	3.179
Oak/gum/cypress	– 1.5227	0.019	0.286	3.282
Elm/ash/cottonwood	– 1.5926	0.019	0.280	3.238
Maple/beech/birch	– 1.7763	0.018	0.265	3.124
Aspen/birch	– 1.6121	0.019	0.279	3.226
Alder/maple	– 1.6665	0.018	0.274	3.192
Western oak	– 1.9738	0.017	0.249	3.005
Tanoak/laurel	– 2.0071	0.017	0.247	2.986
Other western hardwoods	– 2.5979	0.015	0.206	2.661
All types	– 1.8143	0.018	0.262	3.101

QMD = quadratic mean diameter.

Table 3.—*Mean absolute (Abs) and relative difference (Rel. diff.) between estimates of FWD QMD²s derived from this study's GE method and mean of published QMD²s (n = number of empirically estimated QMD²s) along with the coefficient of variation between the published QMD²s (CV).*

Forest type group	Small FWD				Medium FWD				Large FWD			
	Abs. (inches)	Rel. diff. (percent)	n	CV	Abs. (inches)	Rel. diff. (percent)	n	CV	Abs. (inches)	Rel. diff. (percent)	n	CV
All groups	0.028	48.6	5	NA	0.190	39.4	5	NA	1.293	33.1	3	NA
Ponderosa	0.069	78.2	4	83.1	0.141	32.5	4	34.89	0.955	22.3	3	70.26
Douglas-fir	0.033	67.3	4	115.6	0.186	44.4	4	44.4	0.001	0.0	3	56.2
Western larch	0.060	76.9	2	98.8	0.063	18.7	2	64.9	1.703	115.7	1	NA

FWD = fine woody debris. GE = graphical estimation. QMD = quadratic mean diameter.

Future Options

Budgetary constraints most likely limit the ability of large-scale forest inventories to actually measure the line intersect diameter of FWD pieces. Therefore, tallying the number of FWD pieces by size class has become the widely accepted method for inventorying FWD and estimating volume. Van Wagner's (1982) GE method provides a theoretical approach for estimating FWD QMDs by defined populations (e.g., forest types). Our study demonstrated application of Van Wagner's approach for a national inventory of FWD. Substantial disparity exists between the estimated QMDs and the empirically derived QMDs, however. Therefore, three options for the future include (1) empirically measuring FWD QMDs by major forest type group across the United States, (2) using the GE method to theoretically determine FWD QMDs across the United States, and (3) exploring the possibility of using stem or branch scaling factors to determine FWD QMDs. The value (–1.81) of the scaling power function between the number of woody pieces by size class across the United States closely matches published scaling values (–2.00 in Enquist 2002) between bole and fine branch diameters. Therefore, refining understanding of the scaling function between tree boles, limbs, and fine twigs may afford a new technique for estimating FWD QMDs.

Overall, although empirically derived QMDs may be superior in all cases, their determination is currently prohibitive for the entire United States because of budgetary limitations. Given the mixing of FWD from trees and shrubs in all the various forest types across the United States, it is suggested that generic FWD QMD defaults, such as those presented in this study, may be used in estimation procedures until either empirical or scaling information supplants these defaults.

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