Allometric Equations for Predicting Puerto Rican Dry Forest Biomass and Volume

Thomas Brandeis[1], Matthew Delaney[2], Larry Royer[3], and Bernard Parresol[4]

Abstract.—We used forest inventory data, intensive tree measurement, destructive sampling in the field, and subsequent laboratory analyses to develop regression equations that estimate tree biomass, merchantable volume, and total volume for upland forests in Puerto Rican subtropical dry forest. Most parsimonious and additive biomass equations for mixed, dry forest species were fitted to data from 30 destructively sampled trees, and most parsimonious volume equations were fitted for mixed species, *Bucida buceras*, and *Bursera simaruba*. Before undertaking this study, we were unable to confidently estimate biomass and volume for these forests despite having current, detailed forest inventory data.

Introduction

The subtropical dry forest life zone on the Caribbean islands that constitute the Commonwealth of Puerto Rico is found in areas with rainfall between 600 to 1,100 mm per year at elevations less than 300 m (Ewel and Whitmore 1973). This forested life zone covers 15 percent of Puerto Rico, mainly along the south coast and over most of the outlying islands of Culebra and Vieques (Ewel and Whitmore 1973). Native tree species typically found in the upland deciduous dry forest in Puerto Rico and much of the Caribbean include *Bursera simaruba* (L.) Sarg., *Bucida buceras* L., *Gymnanthes lucida* Sw., *Exostema caribaeum* (Jacq.) J.A. Schultes, *Guaiacum officinale* L., *Guaiacum sanctum* L., *Pisonia albida* (Heimerl) Britt. ex Standl., *Pictetia aculeata* (Vahl) Urban, *Acacia macracantha* Humb. and Bonpl., *Capparis spp.,* and *Coccoloba spp.*, among many others (Ewel and Whitmore 1973, Little and Wadsworth 1989, Murphy *et al.* 1995).

Besides natural variability, disturbance has dominated subtropical dry forest development since European colonization of the islands; on mainland Puerto Rico, only 4 percent of the original dry forest remains (Murphy *et al.* 1995). Human uses of dry forest areas in Puerto Rico have included sugar cane production, livestock grazing, irrigated agricultural crops and fruit trees, urbanization, industrialization, and live-fire military exercises in parts of Vieques and Culebra. Naturalized species often dominate highly disturbed Caribbean subtropical dry forests and include *Prosopis juliflora* (Sw.) DC., *Parkinsonia aculeata* L., *Tamarindus indica* L., *Acacia farnesiana* (L.) Willd., *Melicoccus bijugatus* Jacq., and *Leucaena leucocephala* (Lam.) DeWit. (depending on authority, this *L. leucocephala* is considered either native or naturalized) (Ewel and Whitmore 1973, Little and Wadsworth 1989).

Carbon accounting, forest health monitoring, and sustainable management of these forests require an accurate assessment of the tree biomass and wood volume. Early inventories of Puerto Rico excluded dry forests even though they make up a substantial portion of the island's forests because they were not considered to have the productive capacity to support commercial wood production (Birdsey and Weaver 1982, Franco *et al.* 1997). Current inventories include all forest types regardless of their productive capacities, so resource reports will include estimates of live tree aboveground biomass (AGB) and wood volume and for forest types where these resources were not previously considered.

[1] Research Forester, U.S. Department of Agriculture (USDA) Forest Service, Southern Research Station, Forest Inventory and Analysis Unit, 4700 Old Kingston Pike, Knoxville, TN 37919. E-mail: tjbrandeis@fs.fed.us.

[2] Delaney Forestry Services, LLC, 1216 Calapooia Street, Albany, OR 97321. E-mail: matt@delaneyforestry.com.

[3] Forester, USDA Forest Service, Southern Research Station, Asheville, NC 28802. E-mail: lroyer@fs.fed.us.

[4] Mathematical Statistician, USDA Forest Service, Southern Research Station, Asheville, NC 28802. E-mail: bparresol@fs.fed.us.

No locally developed allometric equations for estimating AGB in Puerto Rico's dry forests have been available, so estimates were made using equations developed from international data sets, principally Brown (1997), whose equation uses diameter at breast height (d.b.h.) to estimate AGB, and Martínez-Yrízar *et al.* (1992), which uses basal area as the explanatory variable. The lack of predictive equations hinders accurately estimating subtropical dry forest wood volume as well. In the Puerto Rican forest inventories conducted in 1980 and 1990 (Birdsey and Weaver 1982, Franco *et al.* 1997), field crews took multiple diameter and height measurements along the bole of each tree so that merchantable stem volume inside bark could be calculated by applying a geometric formula to different bole sections.

Despite having detailed and current forest inventory data, we have been unable to accurately estimate live tree AGB and merchantable stem volume for subtropical dry forest life zone in the forests of Puerto Rico. In this study, we outline the procedures taken to develop regression equations that use measurements of d.b.h. and H_T (total height) in the best parsimonious and additive models to estimate subtropical dry forest leaf, branch, bole, and total AGB. Volume equations were developed to estimate merchantable (stem volume to a 10-cm upper stem diameter) and total (stem volume in the entire stem) volume, inside bark, from measurements of d.b.h. and H_T.

Methods

Description of Study Area

We measured and destructively sampled trees in subtropical dry forest near the city of Ponce, PR (latitude 17°58'37.73" by longitude 66° 40'18.23") at a site about to be cleared for road construction. The site has an average rainfall of about 650 mm per year, according to the Southeast Regional Climate Center. A field crew visited the study area and installed a forest inventory plot nearby before destructive sampling. The study area was categorized as mature secondary forest and based on a single inventory plot installed at the site (see Brandeis [2003] for details on forest inventory sampling). A total of 7.1 m^2 per hectare of basal area on 2,749 stems per hectare of trees greater than or equal to 2.5 cm in d.b.h. was present. Tree species found on the inventory plot included *Bursera simaruba*, *Bucida buceras*, *Gymnanthes lucida*, *Bourreria succulenta* Jacq., *Krugiodendron ferreum* (Vahl) Urban, *Thouinia striata* Radlk, *Zanthoxylum caribaeum* Lam., *Reynosia uncinata* Urban, *Coccoloba microstachya* Willd., and *Thrinax morrisii* H. Wendl..

Sample Tree Selection, Harvesting, and Measurement

Preference was given to sampling trees that would be classified as growing stock according to the U.S. Department of Agriculture, Forest Service, Forest Inventory and Analysis guidelines. (Guidelines and measurement descriptions appear in the *FIA Field Data Collection Manual, Version 1.62, Supplement C for Puerto Rico and the Virgin Islands*, which can be downloaded at http://www.srs.fs.usda.gov/fia/manual/2.0_P2Manual.htm). To ensure sampling of healthy trees with more characteristic growth forms, we avoided trees with abnormal form, and trees classified as rough or rotten cull were sampled only when growing-stock trees were not present.

After each tree was felled, the field crew took detailed measurements following the methodology described in Cost (1978) used for developing volume equations in the southern United States. The measurements included diameter at the base or stump, d.b.h., diameters every 60 to 150 cm along the length of the stem depending on bole form, branch diameter and lengths, and total tree height. Bark thickness was sampled each time bole diameter was measured so that inside and outside bark merchantable volume could be estimated. The merchantable bole was defined as the main stem from a 30-cm-tall stump to a 10-cm upper stem diameter.

After the felled measurements were taken, the tree was separated into components and each component was weighed. The first component was the tree's crown, which consisted of leaves, small branches, and sometimes seeds and fruits. The second component was large branches (with a diameter greater than or equal to 2.5 cm). The third component was the main bole.

Subsamples were collected to determine the fresh-weight/dry-weight ratio for leaves and branches. The samples were then oven dried and a fresh-weight/dry-weight ratio was determined. For the large branches, small branches, and leaves, average fresh-weight/dry-weight ratios were applied to the total fresh-weight biomass of each of those components. Wood disc subsamples were cut from the main bole; one disc was collected from the top, one disc was collected from the middle, and a third was collected at the base of the bole. After all three discs were collected, detailed measurements of disc volume were taken, which consisted of two measurements of diameter and two measurements of thickness. After the measurements were taken, each of the discs was placed in a cloth bag and labeled. Oven-dry weight was determined by a commercial laboratory. To determine oven-dry weight of the bole, the density results from each of the three discs were applied to the fresh-weight values. After all of the oven-dry weights were determined, the total biomass of the tree was calculated by summing all components.

Biomass Modeling

We took two approaches to modeling these data. In the first, we sought the best parsimonious models for the leaf, woody, and total tree biomass. In the second, we invoked the principle of additivity, meaning the predicted leaf and woody biomass sum to give exactly the total biomass predicted from the total biomass equation. This process is accomplished by imposing across equation constraints and fitting the resultant system of equations with nonlinear, seemingly unrelated regressions (NSUR).

Only two independent variables were available for modeling: diameter *(D)* and height *(H)*. This situation leads to expressions having the functional form Y = f (D, H). We selected the best biomass component equations based on scatterplots of the data and running all possible regressions on combinations of the variables.

For the mixed, dry forest species data ($N = 26$), we settled on the following equations:

$$\begin{aligned} \hat{w}_{\text{leaf}} &= b_1 D^{b_2} \\ \hat{w}_{\text{woody}} &= b_1 (D^2 H)^{b_2} \\ \hat{w}_{\text{total}} &= b_1 (D^2 H)^{b_2} \end{aligned} \qquad (1)$$

where
$\hat{w}$ = biomass in oven-dry kilograms.

Scatterplots of the residuals revealed significant heteroscedasticity (as expected). One may want to assume equation errors that are multiplicative to derive log-linear models. The logarithmic transformation tends to stabilize heteroscedastic variance (if σ_ε is proportional to E[y]; Neter *et al.* [1985]) and is an alternative to deriving weights for each equation. Thus, for the dry forest data, we have the following biomass equations:

$$\begin{aligned} \ln \hat{w}_{\text{leaf}} &= b_1' + b_2 \ln D \\ \ln \hat{w}_{\text{woody}} &= b_1' + b_2 \ln\left(D^2 H\right) \\ \ln \hat{w}_{\text{total}} &= b_1' + b_2 \ln\left(D^2 H\right) \end{aligned} \qquad (2)$$

where
ln = the natural logarithm,
$\hat{w}$ = biomass in oven-dry kilograms,

and $b_1' = \ln b_1$.

Parresol (1999, 2001) has discussed the problem of forcing additivity on a set of tree biomass functions. Because of the additivity restriction, the inherent model for $\hat{w}_{\text{total}}$ cannot be linearized. Thus, NSUR must be used as opposed to linear, seemingly unrelated regressions. The resulting system of additive equations for the mixed, dry forest species data includes following:

$$\begin{aligned} \ln \hat{w}_{\text{leaf}} &= \ln b_{11} + b_{12} \ln D \\ \ln \hat{w}_{\text{woody}} &= \ln b_{21} + b_{22} \ln\left(D^2 H\right) \\ \ln \hat{w}_{\text{total}} &= \ln\left[b_{11} D^{b_{12}} + b_{21} (D^2 H)^{b_{22}}\right] \end{aligned} \qquad (3)$$

This system was fitted with PROC MODEL in SAS software (SAS Institute 1993).

Volume Modeling

Using the detailed measurements taken on each tree, volume estimates (in cubic meters) were directly calculated by applying the formula for the volume of a conic frustum to bole sections and summing these section volumes for total and merchantable volumes. Total volume is defined here as the inside bark portion of the tree's stem between a 30-cm-tall stump and the tip of the tree's stem without a minimum upper diameter merchantability limit. Merchantable volume is defined here as the inside bark portion of the tree's stem between a 30-cm-tall stump and a 10-cm upper stem diameter (outside bark). Both total and merchantable volumes exclude wood volume in branches and only refer to main bole volume.

To estimate inside bark merchantable stem volume, the diameters for all sections were converted from outside bark to inside bark. Diameters inside bark at stump, breast, saw-log top, and pole-top heights were then calculated with the following formula for hardwoods:

$$BR = \frac{D_{BH} - D_{BT}}{D_{BH}} \tag{4}$$

where
BR = bark ratio,
D_{BH} = d.b.h. outside bark,
and D_{BT} = double bark thickness.

Section heights and inside and outside bark diameters were used in the formula for a conic frustum to calculate the wood volume of individual sections:

$$V_{SEC} = \{H_{SEC} \cdot [D_{IB1}^{\,2} + (D_{IB1} \cdot D_{IB2}) + D_{IB2}^{\,2}] \cdot 0.00007854\}/3 \tag{5}$$

where
V_{SEC} = section volume in m^3,
H_{SEC} = section height in meters,
D_{IB1} = diameter in cm inside bark at one end of section,
D_{IB2} = diameter in cm inside bark at other end of section,
and 0.00007854 is a constant derived from the expression

$$D_i^{\,2} \cdot [\pi/(4 \cdot 10{,}000)] \tag{6}$$

where
D_i = section diameter in centimeters (Husch *et al.* 1993).

Results

Figure 1 shows total AGB (in oven-dry kilograms) by stem d.b.h. for each species sampled. The coefficients and fit statistics for the most parsimonious, natural logarithm-transformed models for mixed, dry forest species are given in table 1. The coefficients and fit statistics for additive models fitted to the mixed, dry forest species are presented in tables 2 and 3. The number of trees sampled, equation coefficients, mean square error (MSE) and r-squared statistic for estimating total and merchantable volume in cubic meters for mixed, dry forest species, *Bucida buceras*, and *Bursera simaruba* for models using tree d.b.h. and total tree height are given in table 4.

Figure 1.—*Total aboveground biomass (in oven-dry kilograms) by stem diameter at breast height (d.b.h.) for each species sampled.*

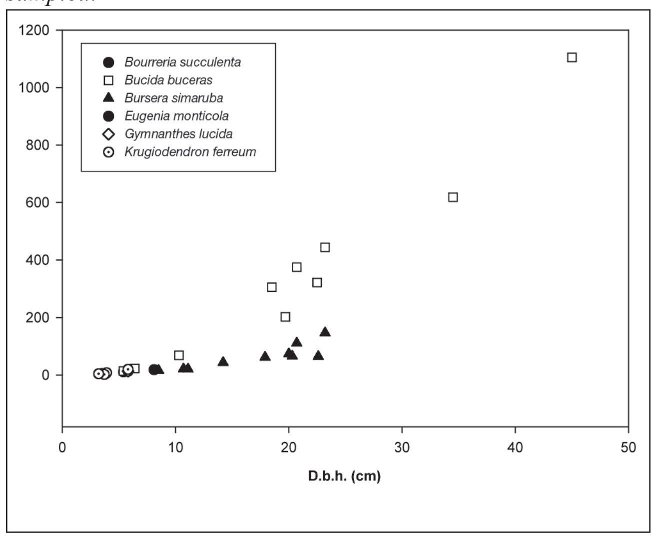

Table 1.—*Number of trees sampled, equation coefficients, mean square error (MSE), and r-squared statistic for the most parsimonious leaf, woody, and total aboveground biomass models for mixed, dry forest species.*

Component	Trees Number	a	b	MSE	r^2
Leaf	26	– 1.75242	1.71833	0.57199	0.75980
Woody	26	– 2.87503	0.92900	0.27738	0.92500
Total	26	– 1.94371	0.84134	0.25252	0.91750

Table 2.—*Additive equation for aboveground biomass models for mixed, dry forest species.*

b_{11}	b_{12}	b_{21}	b_{22}
0.307631	1.540044	0.072847	0.899279

Table 3.—*Number of trees sampled, mean square error (MSE), and r-squared statistic for additive leaf, woody, and total aboveground biomass models for mixed, dry forest species.*

Component	Trees Number	MSE	r^2
Leaf	26	0.5938	0.7403
Woody	26	0.2729	0.9232
Total	26	0.2488	0.9153

Table 4.—*Number of trees sampled, equation coefficients, mean square error (MSE), and r-squared statistic for estimating total and merchantable volume in cubic meters for mixed, dry forest species,* Bucida buceras*, and* Bursera simaruba *for models using tree diameter at breast height (in centimeters) and tree height (in meters).*

Species	Trees	Inside bark				Outside bark			
		a	b	MSE	r^2	a	b	MSE	r^2
	Number								
Total volume[a]									
Mixed species	17	– 0.0173	0.000047	0.0011	0.9906	– 0.0114	0.000046	0.0010	0.9916
Bucida buceras	9	0.0001	0.000047	0.0010	0.9952	0.0002	0.000046	0.0010	0.9950
Bursera simaruba	8	0.0370	0.000028	0.0003	0.8536	0.0354	0.000030	0.0004	0.8312
Merchantable volume[b]									
Mixed species	17	– 0.0593	0.000046	0.0020	0.9826	– 0.0576	0.000045	0.0019	0.9828
Bucida buceras	9	– 0.0697	0.000046	0.0024	0.9888	– 0.0688	0.000046	0.0023	0.9890
Bursera simaruba	8	0.2890	0.000025	0.0009	0.5553	0.0285	0.000023	0.0009	0.5620

[a] *Inside bark portion of the tree's stem between a 30-cm-tall stump and the tip of the tree's stem without a minimum upper diameter merchantability limit.*

[b] *Inside bark portion of the tree's stem between a 30-cm-tall stump and a 10-cm upper stem diameter.*

Conclusions

Although our ability to accurately estimate merchantable stem volume and live tree AGB for upland forests in the subtropical dry forest life zone in Puerto Rico and the U.S. Virgin Islands has been improved, much work remains to be done. This sample is small and needs to be expanded to include more species and more of the larger (d.b.h. greater than 30 cm) trees. This work should be seen as part of an ongoing process, and the equations presented here will be refined as additional sampling adds to the data available.

Acknowledgments

We thank James Bentley, Vincent Few, Tony Johnson, Humfredo Marcano, and Luis Ortíz of the Forest Service, Southern Research Station, Forest Inventory and Analysis program; Ross Hammons of the Tombigbee National Forest; Eileen Helmer and Ivan Vicéns of the Forest Service International Institute of Tropical Forestry; and Esther Rojas of the Puerto Rican Conservation Foundation.

Literature Cited

Birdsey, R.A.; Weaver, P.L. 1982. The forest resources of Puerto Rico. Res. Bull. SO-85. New Orleans, LA: U.S Department of Agriculture, Forest Service, Southern Forest Experiment Station. 59 p.

Brandeis, T.J. 2003. Puerto Rico's forest inventory: adapting the Forest Inventory and Analysis program to a Caribbean island. Journal of Forestry. 101: 8-13.

Brown, S. 1997. Estimating biomass and biomass change of tropical forests: a primer. FAO For. Pap. 134. Rome, Italy: Food and Agriculture Organization of the United Nations. 55 p.

Cost, N.D. 1978. Multiresource inventories—a technique for measuring volumes in standing trees. Res. Pap. SE-196. Asheville, NC: U.S. Department of Agriculture, Forest Service, Southeastern Forest Experiment Station. 18 p.

Ewel, J.J.; Whitmore, J.L. 1973. The ecological life zones of Puerto Rico and the U.S, Virgin Islands. Res. Pap. ITF-18. Rio Piedras, PR: U.S. Department of Agriculture, Forest Service, Institute of Tropical Forestry. 72 p.

Franco, P.A.; Weaver, P.L.; Eggen-McIntosh, S. 1997. Forest resources of Puerto Rico, 1990. Res. Bull. SRS-22. Asheville, NC: U.S. Department of Agriculture, Forest Service, Southern Research Station. 45 p.

Husch, B.; Miller, C.I.; Beers, T.W. 1993. Forest Mensuration. 3rd ed. Malabar, FL: Krieger Publishing. 402 p.

Little, E.L.; Wadsworth, F.H. 1989. Common trees of Puerto Rico and the Virgin Islands. 2nd ed. Agricultural Handbook No. 249. Washington, DC: U.S. Department of Agriculture, Forest Service. 556 p.

Martínez-Yrízar, A.; Sarukhán, J.; Pérez-Jiménez, A.; Rincón, E.; Maass, M.; Solís-Magallanes, A.; Cervantes, L. 1992. Above-ground phytomass of a tropical deciduous forest on the coast of Jalisco, México. Journal of Tropical Ecology. 8: 87-96.

Murphy, P.G.; Lugo, A.E.; Murphy, A.J.; Nepstad, D.C. 1995. The dry forests of Puerto Rico’s south coast. In: Lugo, A.E.; Lowe, C., eds. Tropical forests: management and ecology. New York: Springer-Verlag: 178-209.

Neter, J.; Wasserman, W.; Kutner, M.H. 1985. Applied linear statistical models. 2nd ed. Homewood, IL: Irwin Professional Publishing: 137-138.

Parresol, B.R. 1999. Assessing tree and stand biomass: a review with examples and critical comparisons. Forest Science. 45: 573-593.

Parresol, B.R. 2001. Additivity of nonlinear biomass equations. Canadian Journal of Forest Research. 31: 865-878.

SAS Institute. 1993. SAS/ETS user’s guide. Version 6. 2nd ed. Cary, NC: SAS Institute.