External Validation of a Forest Inventory and Analysis Volume Equation and Comparisons With Estimates From Multiple Stem-Profile Models

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Abstract.—Sound estimation procedures are desideratum for generating credible population estimates to evaluate the status and trends in resource conditions. As such, volume estimation is an integral component of the U.S. Department of Agriculture, Forest Service, Forest Inventory and Analysis (FIA) program's reporting. In effect, reliable volume estimation procedures are fundamental to FIA's mission. Currently, FIA in the Southern Research Station (SRS) uses linear regression models of the form $V = \alpha + \beta(dbh^2 Ht) + \varepsilon$ to derive volume estimates. Although copious data have been used to estimate region- and species-specific parameters, little attention has been paid to external validation of the developed models. We investigated the validity of the SRS FIA volume equation (SRSvol) for cherrybark oak *(Quercus pagoda)* through external validation procedures using a stem analysis data set of 49 separate stems. The aggregate volume of measured Smalian bolts for each stem were compared with volume estimates derived from SRSvol for gross cubic-foot volume (VOLCFGRS in FIA Database 2.0). In addition, estimates from four stem-profile models (MAXvol, CLRKvol, FARvol, and WALTvol) were compared to those of SRSvol. The SRSvol model provided the most accurate and precise estimates of cherrybark oak volume. In addition, the stem-profile model FARvol proved to be the superior stem-profile model. Although the performance of the SRSvol model can be viewed as positive and supportive of the current SRS FIA volume estimation procedures, this study reflects results from one species. As such, additional external validation of the SRS FIA volume equations across multiple species is warranted. In addition, the results of this study suggest that as FIA as a national program proceeds towards nationally consistent estimation procedures, stem-profile models such as the FARvol model form may provide reasonable estimates with the additional benefits of allowing for regional and species-specific flexibility and for the incorporation of changing utilization specifications.

Introduction

The Forest Inventory and Analysis (FIA) program of the Forest Service Southern Research Station (SRS) is currently charged with producing State-level estimates of numerous forest resource values, levels, and trends for 13 southern States. Sound estimation procedures are desideratum for generating credible population estimates to evaluate the status and trends in resource conditions. As such, volume estimation is an integral component of FIA reporting. In effect, reliable volume estimation procedures are fundamental to FIA's mission. Currently, the SRS FIA uses volume estimators of the conventional linear regression model form $V = \alpha + \beta(dbh^2 Ht) + \varepsilon$ (Spurr 1952) to derive volume estimates (Zarnoch *et al.* 2003). Although copious data have been used to estimate region and species-specific parameters, little attention has been paid to publishing external validation of the developed models. As a result, published volume estimates must be taken at face value by external and internal FIA users who lack the appropriate information necessary for assessing confidence in the estimates.

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Numerous model forms and methods are used to estimate standing tree volume. For example, the former North Central Research Station FIA program uses nonlinear models developed by Hahn and Hansen (1991) and linear regression models developed by Hahn (1984). Furthermore, the former Northeastern Research Station FIA program uses nonlinear methods (Scott 1981) to derive cubic-foot volume estimates of standing trees. The Rocky Mountain Research Station FIA program uses multiple published (Edminster *et al.* 1980, 1982; Hann and Bare 1978; Myers 1964; Myers and Edminster 1972) and unpublished equations, including both linear and nonlinear model forms.

Advances in analytical techniques have led to additional, more complex methods for estimating volume in standing trees. Stem-profile (or stem-taper) models have been of particular interest to many researchers. For example, Clark *et al.* (1991) developed stem-profile equations for 58 southern tree species, for possible use by the Forest Service. Although the authors appeared to be pleased with the accuracy of the results, no comparison with the linear regression equations used by the SRS FIA was included. Zarnoch *et al.* (2003) compared the SRS individual tree linear regression methodology to taper models similar to taper models produced by Clark *et al.* (1991), and found no appreciable difference between the two methods on both an individual tree and statewide scale. No external validation was included, however. Clark *et al.* (1991) did include a validation of the developed models through cross-validation; however, true external validation with an independent data set is always optimal (Neter *et al.* 1996).

Without external validation and comparisons with externally validated alternative models, it is difficult to completely defend model or equation choice or maintain confidence that the optimal methodology has been chosen for FIA reporting. Currently, to the authors' knowledge, no published external validation, using an independent data set, of the SRS linear regression equations exists. In addition, a direct comparison with alternative models, using the same validation data set, such as taper equations developed by Clark *et al.* (1991), Farrar and Murphy (1987), Max and Burkhart (1976), and Walters and Hann (1986), may suggest the relative accuracy of the currently used methodology and provide insight into potentially needed change in estimation procedures.

Here, we investigate the strength of the SRS FIA volume equations currently used, specifically for predicting gross volume of cherrybark oak (*Quercus pagoda Raf.*), through external validation procedures using a stem analysis data set of 49 separate stems. In addition, alternate volume estimators are evaluated with the same data set. The three objectives of this article are to (1) evaluate current SRS FIA volume estimators with external validation procedures; (2) evaluate a series of alternate volume estimators, including recently developed stem-profile equations; and (3) compare the performance of the SRS FIA estimators with that of each of the alternative models.

Methods

Volume Estimators

Volume estimators used for comparative purposes included the SRS FIA linear regression model (SRSvol) currently used in data processing in the South, Max and Burkhart's (1976) segmented-profile polynomial model (MAXvol), the form-class segmented-profile model of Clark *et al.* (1991) (CLRKvol), Farrar and Murphy's (1987) taper function model (FARvol), and the taper equations of Walters and Hann (1986) that include live crown ratio as a predictor variable (WALTvol).

Data

Separate independent data sets were used to parameterize the models (parameter data set) for comparisons and evaluate predictions (evaluation data set). The parameter data set was used to parameterize each model form when published coefficients for cherrybark oak were not available (the SRS FIA linear regression coefficients, although not published, were made available by Larry Royer of the Forest Service SRS FIA program). When published coefficients were available for both pole and sawtimber, coefficients were applied accordingly. When coefficients were estimated with the parameter data set, however, one set of coefficients was developed due to limited data.

Data for the parameter data set represent a species-specific (cherrybark oak) subset of a larger data set of detailed felled tree data from across the South, collected by field crews from the old Southeastern Forest Experiment Station (Larry Royer and Tony Johnson provided the larger data set from which the parameter data set was taken). The data were obtained from a total of 81 cherrybark oak trees and collected sensu Cost (1978) modified for felled trees, for the development of volume equations for southern tree species. Tree height varied from 34 to 108 ft and diameter at breast height (d.b.h.) varied from 4 to 38 in.

Data for the evaluation data set were obtained from 49 felled cherrybark oak trees evenly distributed among four separate stands. Three stands were located on the Natchez Trace State Forest in western Tennessee (latitude 35.83333 by longitude -88.25833) and one stand was located on privately owned land in Dixon Springs, TN (latitude 36.35804 by longitude -86.050209). D.b.h. varied from 3 to 20 in and height varied from 19 to 99 ft. Selected trees were single stemmed, free of sweep or crook, and visibly undamaged. After felling, radial disks were removed from each tree at the stump, defined as 15 cm above ground level d.b.h. and every 1.22 m thereafter for the reminder of the stem. Diameter inside bark (d.i.b.), diameter outside bark (d.o.b.), and bark width were collected for each disk. Stem volume inside bark and stem volume outside bark were calculated for each tree using Smalian's formula. To achieve d.b.h. agreement between the parameter and evaluation data sets, all trees in the evaluation data set with a d.b.h. below 4 in (the minimum d.b.h. of the parameter data set) were removed. The resultant data set contained a total of 422 observations on 31 trees.

Validation Procedures

After satisfactory coefficients were either found in the literature or estimated using the parameter data set, gross cubic-foot volume was estimated for trees in the evaluation data set with each of the proposed models. The specifications for calculating gross cubic-foot volume followed definitions of gross volume outlined by the FIA Statistics Band (2006). Volume was calculated from a 1-ft stump to a minimum 4-in top d.o.b. Estimated volume was then compared to the aggregate Smalian-bolt volume for each tree under the same specifications. Mean prediction error ($\bar{e}$), standard deviation of the prediction, mean absolute difference (MAD), mean squared prediction error (MSPR), and 10-percent confidence limits were calculated for each model.

The predictive capability of the chosen model was evaluated with the calculated MSPR (Neter *et al.* 1996):

$$\text{MSPR} = \frac{\sum_{i=1}^{n}\left(Yi - \hat{Yi}\right)^2}{n} \tag{1}$$

where

Yi is the value of the response variable in the *i*th validation case,

$\hat{Yi}$ is the predicted value for the *i*th validation case based on the development data set,

and *n* is the number of cases in the evaluation data set.

In addition, the relative error in prediction (RE%) and the prediction coefficient of determination (R_p^2) were calculated for each model:

$$R_p^2 = 1 - \frac{\lfloor \sum (Yi - \hat{Yi})^2 \rfloor}{\sum (Yi - \bar{y})^2} \tag{2}$$

where

Yi is the value of the response variable in the *i*th validation case,

$\hat{Yi}$ is the predicted value for the *i*th validation case based on the development data set,

and $\bar{y}$ is the mean of *n* observations.

Finally, individual two-tailed t-tests were preformed to detect significant differences between the cumulative Smalian-bolt volume and each of the volume estimates.

Results and Discussion

Model Performance

Electing a "best" model purely from the interpretation of the calculated validation statistics and diagnostic tests appeared straightforward. Most of the validation statistics supported the SRSvol model as the better performing estimator. In addition, the plurality of the diagnostic tests supported the SRSvol model. The lowest MAD, mean absolute percent prediction error (MAPPE), MSPR, RE%, $\bar{e}$ and standard deviation, and standard error of the $\bar{e}$ resulted from the application of the SRSvol model (table 1). The SRSvol model also produced the largest prediction coefficient of determination, or R_p^2 of 0.98, followed by the CLRKvol, FARvol, WALTvol, and MAXvol models (0.96, 0.92, 0.87, and 0.65, respectively). A simple nonweighted ranking of the calculated validation statistics and diagnostic tests used in this study indicated a separation between the SRSvol and the FARvol models from the remaining three models. The procedure ranked the SRSvol model as the best model, followed by the FARvol, CLRKvol, MAXvol, and WALTvol models.

Overall, the SRSvol and FARvol models appeared to produce the most accurate estimates with $\bar{e}$ of -1.21 and -1.62 and MAD of 1.42 and 1.71, respectively. In addition, the MAPPE was smallest for the SRSvol and FARvol models (table 1). All of the models had a tendency to overpredict, with the exception of the MAXvol model, which tended toward the underprediciton of individual stem volume. When comparing the predicted and measured volumes of the collective data set, the MAXvol model underpredicted total volume and the SRSvol, FARvol, WALTvol and CLRKvol models overpredicted it (fig. 1). The SRSvol, FARvol, and CLRKvol models appeared much more accurate than the WALTvol and MAXvol models.

Comparable to the SRSvol model's accuracy, the SRSvol model appeared to promulgate the greatest precision. Each of the estimates of model precision, MSPR, RE%, and R_p^2 indicated the SRSvol model as having greatest precision of the five tested models, followed by the FARvol model and then the CLRKvol model (table 1).

Although the validation statistics and the nonweighted ranking of the models suggested that the SRSvol and FARvol models were superior, the results from both the t-tests and f-tests concluded that the predicted volume was statistically different from the measured volume (table 1) for all models. However,

Table 1.—*External validation statistics for five volume estimation procedures for estimating cubic-foot volume in cherrybark oak.*

	Models				
	SRSvol	**MAXvol**	**CLRKvol**	**WALTvol**	**FARvol**
$\bar{e}$	– 1.21	2.26	– 1.76	– 3.49	– 1.62
Std Dev	1.34	1.61	2.38	2.98	1.53
Std Err	0.25	0.3	0.44	0.55	0.28
MSE	1.77	1.42	2.66	2.44	1.95
SSE	47.88	38.45	71.88	65.92	52.71
MPPE	– 13.19	32.42	– 3.66	– 19.62	– 13.54
MAPPE	15.09	25.92	17.99	23.25	15.89
MAD	1.42	2.29	2.07	3.59	1.71
MSPR	3.19	7.65	8.59	20.77	4.88
RE%	15.11	27.94	24.79	37.78	20.59
Rp^2	0.98	0.65	0.96	0.87	0.92
t-test	4.88	7.57	– 3.98	– 6.32	– 5.71
(P-value)	*< 0.0001*	*< 0.0001*	*0.0004*	*< 0.0001*	*< 0.0001*
f-test	1052.35	904.41	980.24	1250.84	1051.91
(P-value)	*< 0.0001*	*< 0.0001*	*< 0.0001*	*< 0.0001*	*< 0.0001*
X^2	4.18	5.4	4.89	4.07	4.16
(P-value)	*0.12*	*0.07*	*0.09*	*0.13*	*0.12*
Pearson	0.987	0.985	0.987	0.989	0.987
(P-value)	*< 0.0001*	*< 0.0001*	*< 0.0001*	*< 0.0001*	*< 0.0001*
AIC	18.54	12.18	30.32	27.81	21.33

AIC = Akaike's Information Criterion. $\bar{e}$ = prediction error. MAD = mean absolute difference. MAPPE = mean absolute percent prediction error. MPPE = mean percent prediction error. MSE = mean square error. MSPR = mean square error of prediction. Pearson = Pearson correlation coefficient. (P-value) = associated P-value of the previous test statistic. RE% = relative error in prediction. Rp^2 = prediction coefficient of determination. SSE = error sum of squares. Std Dev = standard deviation of the prediction error. Std Err = standard error of the prediction error. t-test = paired t-test. X^2 = chi-square test of independence.

the X^2 test of independence was not significant for any of the five models. Furthermore, multiple thresholds for acceptable growth and yield model performance have been published. Freese (1960) suggested a plus or minus 7.16-percent error and Pilsbury *et al.* (1995) estimated that a plus or minus 12-percent error was an appropriate line on which to base model choice. Moreover, Huang *et al.* (2003) posited that a plus or minus 10- to 20-percent error implied a “reasonable” model. No model in this study preformed adequately, according to the standard set by Freese (1960) or Pilsbury *et al.* (1995), and only the SRSvol, FARvol, and CLRKvol models preformed reasonably, according to the Huang (2003) standard.

Composite validation statistics, such as previously discussed, do not always provide all necessary information with respect to model performance across the population of interest. For example, many of the composite statistics will not adequately convey how a volume model specifically operates as stem diameter changes. Graphical evaluation of model predictions provides a very effective method to gain this form of insight (Huang *et al.* 2003).

Fluctuations in prediction error for each of the five models varied as d.b.h. increased (fig. 2). The MAXvol model consistently underpredicted and the SRSvol and FARvol models appeared to consistently overpredict. The remaining models underpredicted for small diameter stems and overpredicted for large diameter stems (WALTvol and CLRKvol). The impact of changing diameter on prediction error, as a percentage of total stem volume, also varied drastically among the five tested models (fig. 3). The estimates produced by the CLRKvol, MAXvol, and WALTvol models appeared more sensitive to changes in diameter with errors ranging from approximately -10 to 60 percent, -10 to -35 percent, and -40 to 25 percent of the measured volume for the MAXvol, WALTvol, and CLRKvol models, respectively. The SRSvol and FARvol models appeared much less sensitive to stem diameter changes (fig. 3). The use of models that minimize the variability in prediction error among diameter classes will decrease the overall error in the

Figure 1.—*Total predicted volume (in cubic feet) for all cases by five volume models and associated difference between predicted and measured volume for cherrybark oak (*Quercus pagoda Raf.*).*

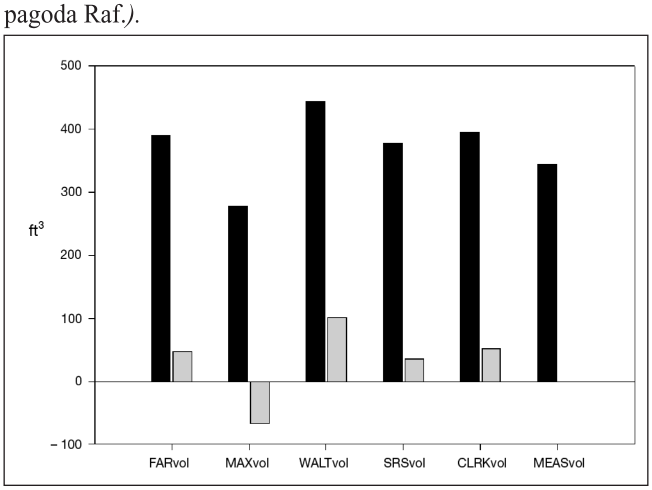


Figure 2.—*Regression of prediction error by diameter class for five volume estimation methods.*

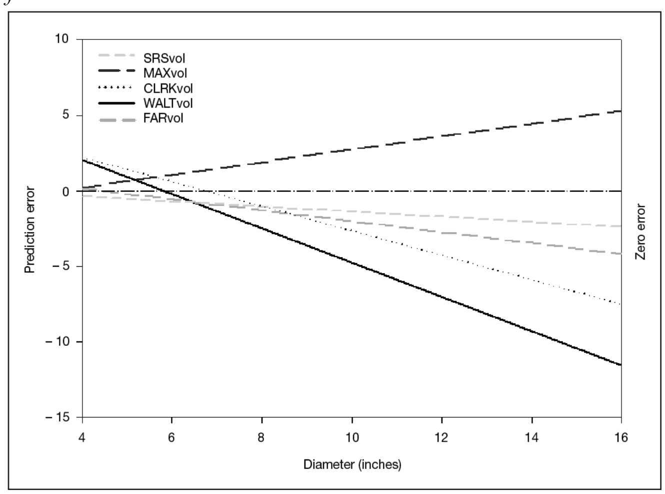
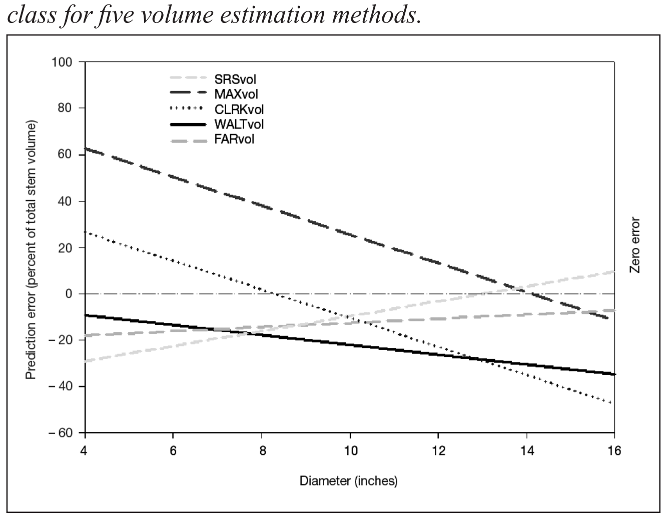


Figure 3.—*Regression of percent prediction error by diameter class for five volume estimation methods.*

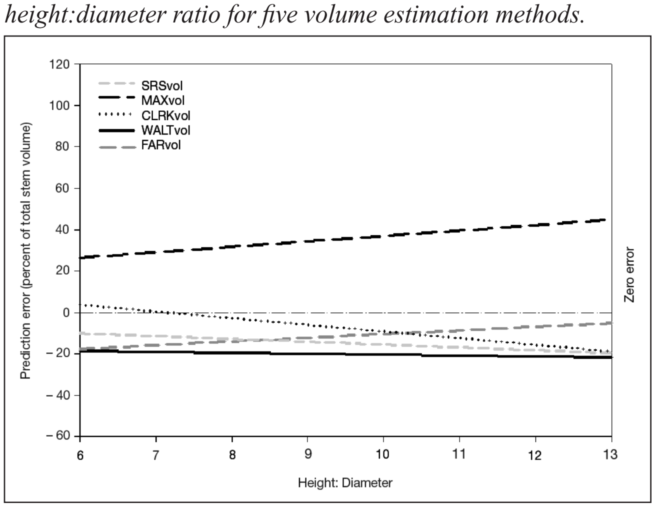


cumulative volume estimates. Even though both the SRSvol and FARvol models appeared to be less sensitive to changes in diameter, the FARvol model was superior to the SRSvol model in this respect and appeared to demonstrate increasing accuracy within the larger diameter classes of the evaluation data set (fig. 3).

Using height-diameter ratio as an analogue for stem form, the MAXvol and CLRKvol models were more influenced by changes in stem form than the WALTvol, SRSvol, and FARvol models (fig. 4). Differences in stem form are a result of stem developmental history and can vary widely in a large sample population, such as that collected by FIA. As such, additional error may propagate through volume estimates through the impact of variable stand histories. This error can be removed through the development of models that lessen, if not remove, such influence. The WALTvol model, although a consistently poor predictor, appeared resistant to the influence of changing stem form (figure 4), indicating the possibility of using stem-profile models to reduce volume estimate error.

Overall, although the FARvol model represented the "best" performing stem-profile model, the cherrybark oak volume equation currently used by the SRS FIA (SRSvol) consistently outperformed the other four models. SRSvol was the only model in which the 95-percent prediction interval completely contained the 1-to-1 line for the range of volumes represented by the evaluation data set (fig. 5). The performance of the SRS-vol model in this external validation study should give users of

Figure 4.—*Regression of percent prediction error by height:diameter ratio for five volume estimation methods.*



FIA data confidence in the volume estimates generated by the SRS FIA; however, the analysis in this study is restricted by the limited independent data available and that only one species is represented.

Evaluation of Four Tested Stem-Profile Models

1. ***WALTvol***

Description of Equation. This is a variation on the Max and Burkhart (1976) three-part polynomial equation that uses a percentage of crown base as the join point for the upper stem.

Parameterization. This equation is very long and complicated. Mistakes are easily made when entering the equation due to its complexity. The alpha parameter needs to be hand estimated before regression estimation and needs to be iterated and reregressed until hand-estimated and regression-estimated alpha agree. This process can be very time consuming or can appear to be impossible in some cases.

Pros. The upper join parameter (alpha) is flexible and incorporates crown ratio into the equation in a logical way. The equation produces very nice taper plots.

Cons. This is a very long and complicated model. Alpha is needed to set indicator variables before regression and needed within the model. Iterative steps need to be made until the hand-estimated and regression-estimated alpha values are equal.

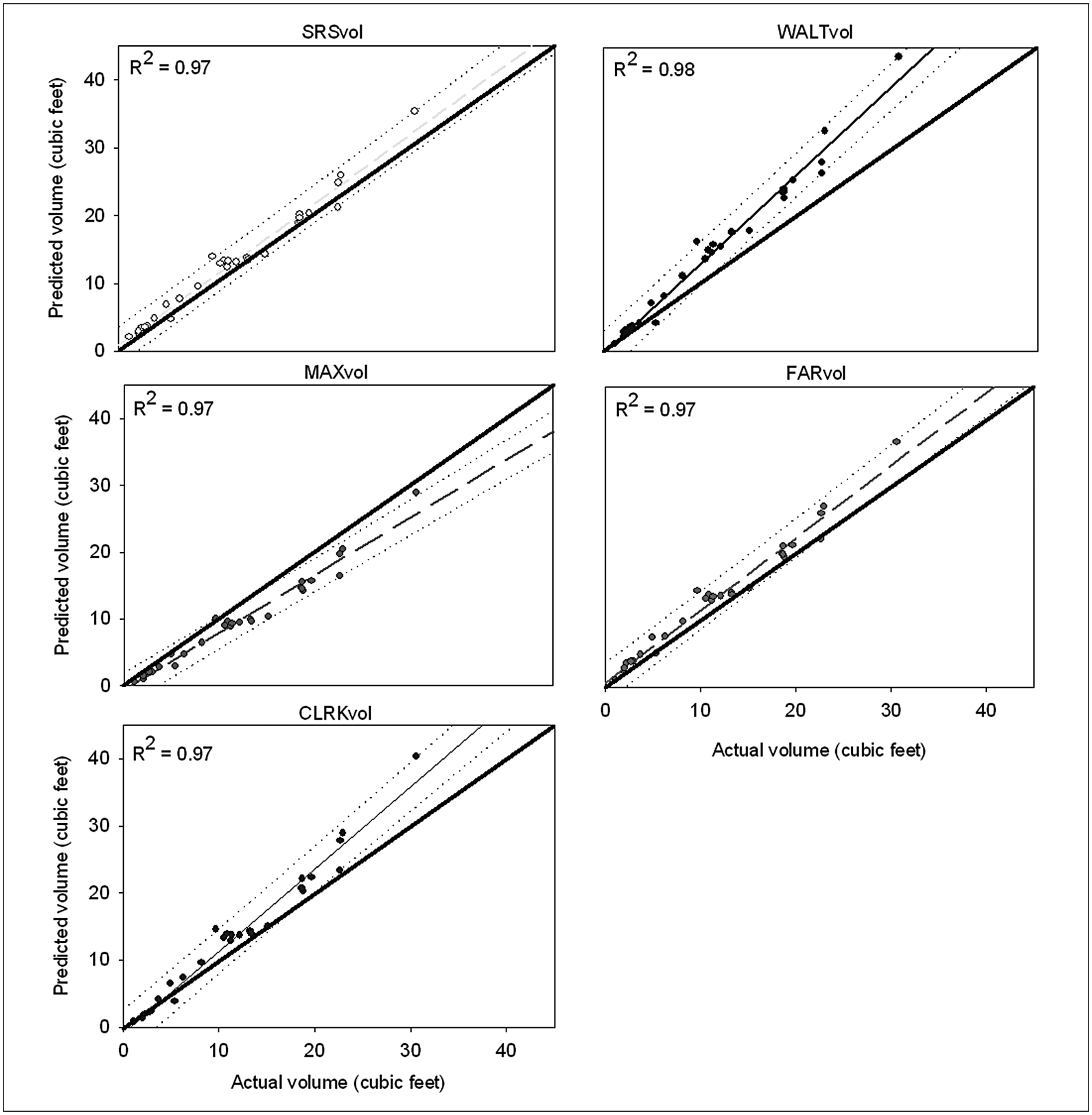
Results. Model performance was poorest of the five that we examined. The model produced the largest standard deviation (2.98) and the largest mean difference (-3.49). This model tends to overpredict volume more than any of the five tested models.

2. ***MAXvol***

Description of Equation. This is a three-part equation using a quadratic-quadratic-quadratic form.

Parameterization. The equation is fairly simple and easy to code; as a result, mistakes are less likely. Like the WALTvol equation, an alpha parameter is present that is hand estimated to set an indicator variable before regression and is estimated in the regression. This hand-estimated alpha needs to be iterated until it agrees with the regression-estimated

Figure 5.—*Actual compared with predicted volume for five volume estimation methods with 95-percent prediction intervals and associated coefficient of determination for cherrybark oak (*Quercus pagoda Raf.*).*



value. Finding matching $alpha_{xx}$ can be time consuming or impossible.

Pros. The equation creates very nice realistic taper profile curves.

Cons. The indicator variable requires iteration, which is time consuming and complicated.

Results. This model predicted fourth best of the five examined models. It had the third smallest standard deviation (1.61) but the second largest mean difference (2.26). This model tends to predict volume too conservatively and produced the only overall conservative prediction.

3. ***CLRKvol***

Description of Equation. The CLRKvol model uses form-class measurement at 17.3 ft to aid in fitting the segmented-profile equation.

Parameterization. Iteration from preregression parameter estimations and outputted regression estimates can be difficult to stabilize.

Pros. Incorporating form class into the model helps to produce nice fits. Model flexibility allows for many forms of the model to be used in analysis.

Cons. The paper repeats parameter names in multiple parts of the paper, which creates some confusion. One must use existing parameters to estimate a form class if one has not been measured.

Results. The model predicted the third best of the five examined models. It had the fourth smallest standard deviation (2.38) and the third smallest mean difference (-1.76). This model tends to overpredict volume but predicts well.

4. ***FARvol***

Description of Equation. This is a two-part model, below and above breast height. Farrar and Murphy (1987) recommend separating the data into three crown ratio classes and fitting unique parameters for each class.

Parameterization. This is a simple equation that is easy to code and fit. The model predicts real diameters from real heights (not in relative space).

Pros. The model produces good estimates and is very easy to apply. In addition, it produces estimates in real space. No parameter is present that needs to be hand estimated before regression, iteration, and refit.

Cons. The equation originally applied to shortleaf pine. The current study applies the equation to hardwoods.

Results. The model predicted the second best of all the five. The standard deviation is the second smallest (1.53) and the mean difference was also the second smallest (-1.62). The small standard deviation reduces the t-test value, making this model the second closest to 0. This model tends to overpredict volume but predicted very well in the current study.

Conclusions

The five models examined in this study vary greatly in user friendliness, accuracy, and method. Each model had its strong and weak points; however, the SRSvol model and FARvol model clearly outperformed the others with the superior model being SRSvol. Although the simple form of the SRSvol model is clearly the easiest to apply, the FARvol model is uncomplicated relative to the other stem-profile models in this study and could be easily applied in large-scale estimation efforts such as that of FIA. As a result, the FARvol model form may be appropriate for further testing as interest increases and efforts proceed in developing nationally consistent FIA estimation procedures.

The FIA volume equation is the easiest to use because a taper profile equation step is not involved. This model uses simple linear regression to directly predict volume from d.b.h. and height. The four other models predict a taper profile equation as an intermediate step before volume is estimated. Volume is derived from the taper profile by inserting heights at every 10th ft into the equation and outputting diameter estimates accordingly. The 10th-ft diameter estimations are converted into “discs” and summed to yield total volume for each tree. The intermediate taper profile step is difficult to apply at first but goes very quickly after the appropriate algorithm and code have been written. The largest problem with three of the taper models examined in this study is an iterative process required to fit the model. These models require the user to estimate a parameter related to a join point or an indicator variable that is also a parameter fit in the model. If the preregression parameter does not match the regression estimated parameter, then the preregression parameter must be changed slightly and the regression rerun. This procedure must be repeated until the preregression parameter is equal to the regression found parameter. In some cases, this procedure can take tremendous time or is impossible. This process was not discussed in any of the model citations and no advice was given on how to find fits efficiently. The FARvol equation does not have any iterative parameters, which makes it the easiest taper equation to use.

The five studied models each have unique assumptions and model forms. Indepth discussion of the model forms is beyond the scope of the article but would make for a very interesting comparison. The largest difference between the taper and FIA volume procedures is the taper profile intermediate steps. Taper profiles provide very good flexibility to make changes to volume prediction criteria, such as small-end diameter limits. Taper profiles have great potential to provide information on wood quality, wood value, and growth modeling. A need exists to develop models that provide the flexibility associated with taper profile equations with a stronger emphasis on usability and simplicity.

Although the evaluation data set used in this study is not truly representative of the population of cherrybark oaks sampled across the 13 southern States, the results do suggest that the currently used SRSvol model form is appropriate. Certain stem-profile models, such as the FARvol model, may be potentially applicable if the form of a stem-profile model is desired at the national level. As such, it appears that FIA may benefit from pursuing, through multispecies testing, alternative estimation procedures that have reliable performance across regions. Concomitantly, it may prove advantageous to the national FIA program to pursue nationally consistent volume models. As seen here, and in other studies cited in this work, stem-profile models could be a workable alternative that has the potential to improve FIA volume estimates while providing for region-specific and species-specific flexibility and temporally and geographically shifting utilization specifications. In addition, the performance of the SRSvol lends a certain degree of credibility to the SRS FIA estimation procedures.

Acknowledgments

The authors gratefully acknowledge Larry Royer and Tony Johnson for providing data, Wayne Clatterbuck and Dave Larsen for their reviews and numerous discussions that greatly improved this article, and Sonja Oswalt and other FIA personnel for additional reviews. We also thank the Forest Service Southern Research Station Center for Bottomland Hardwoods Research for funding this research and the Tennessee Division of Forestry and Mr. Thomas Beasley for granting access to their property and allowing destructive sampling of trees.

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