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Maximum size–density relationships for mixed softwoods in the northeastern USA

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Abstract

The maximum size–density relationships or self-thinning lines were developed for three mixed-softwood climax forest habitats (hemlock-red spruce, spruce-fir, and cedar-black spruce) in the northeastern USA. The plot data were collected from an extensive data base used in growth studies from 1950 to 1970, and represented a wide range of species compositions, sites, management options, and densities. Plots having late successional species compositions with relative density index higher than 0.7 were selected as the most fully-stocked plots for model development. Reduced major axis regression was used to fit the self-thinning lines to the data. The differences between the three forest habitats were tested using the 95% confidence intervals of the coefficients of the self-thinning lines. The results indicated that there were no differences between the hemlock-red spruce and spruce-fir habitats for both relationships of the mean tree volume versus number of trees, and the quadratic mean tree diameter versus number of trees. Thus, these two habitats were combined to develop a single self-thinning line. However, the cedar-black spruce forest habitat required a separate self-thinning line. These maximum size–density relationships can be readily used to construct stand density management diagrams. © 2002 Elsevier Science B.V. All rights reserved.

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1. Introduction

Stocking charts and stand density management diagrams (DMDs) are useful tools for practicing foresters. They provide valuable information and guidelines on silvicultural treatments (e.g. thinning) and harvesting practices. They can visually display the consequences of stand stocking or density manipulations, and represent the relationships between stand density measures (e.g. number of trees or total basal area) and average tree sizes (e.g. mean tree volume or quadratic mean diameter) over time. Essentially, these tools are simple stand average models that graphically characterize growth, density and mortality at various stages of stand development. Forest managers can use them to assess stand conditions based on field surveys, and to rapidly design and evaluate alternative density management practices (Ernst and Knapp, 1985).

Traditionally, stocking charts have been primary thinning guides widely used by forest managers in the northeastern USA (e.g. Gingrich, 1967; Frank and Bjorkbom, 1973; Philbrook et al., 1973; Lancaster and Leak, 1978; Lancaster, 1985; Solomon and Leak, 1986). In recent years, DMDs have been constructed for pure or mixed softwood stands in this region (e.g., Newton and Weetman, 1993, 1994; Newton, 1997; Smith and Woods, 1997; Sturtevant et al., 1998; Wilson

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et al., 1999). Resource managers have found that DMDs not only allow point estimation of stand stocking but also serve as a stand monitoring system in which stand development and treatments can be traced through the stand management history (Kershaw and Fischer, 1991). DMDs can help forest managers to maximize wood fiber yield (Kershaw and Fischer, 1991; Dean and Jokela, 1992), and to assess favorable ecological conditions for vegetation (Barbour et al., 1997) and wildlife (Lilieholm et al., 1993; Hayes et al., 1997).

Before developing a DMD for a given forest habitat, it is necessary and crucial to establish the maximum size–density relationship which is ruled by the "−3/2 power law of self-thinning" (Jack and Long, 1996). Primarily, the −3/2 power law applies to two instances: (1) for a single species population of successive development stages, its growth trajectory over time will follow the self-thinning line. The plotted line describes the mean tree size and density relationship of the single stand at different points in time; and (2) for all populations of the same species composition, regardless of stand ages and site qualities, there is a maximum possible combination between the mean tree size and the number of trees per unit area defined by the −3/2 power law. This is known as the upper boundary self-thinning line (Jack and Long, 1996). It is a static relationship among stands observed at single points of time, and describes the upper limiting boundary of all possible combinations of mean tree size and density. Regardless of whether it should be touted as an ecological "law," the concept of self-thinning has been proven valuable in comprehending the dynamics and development of forest stands and their projection through time.

The self-thinning line has been commonly expressed by the following mathematical equations:

$$\ln(\mathrm{MV}) = a - 1.5\ln(N) \qquad (1)$$

(Drew and Flewelling, 1977)

$$\ln(W) = b - 0.5\ln(N) \qquad (2)$$

(Zeide, 1987)

$$\ln(\mathrm{QMD}) = c - 0.625\ln(N) \qquad (3)$$

(Jack and Long, 1996)

$$\ln(N) = d - 1.605\ln(\mathrm{QMD}) \qquad (4)$$

(Reineke, 1933)

where MV is mean tree volume, W total biomass or total volume, QMD the quadratic mean tree diameter, N the number of trees per unit area, ln the natural logarithm, the slope coefficients of the equations are theoretical values according to ecological rationales, and a, b, c and d are intercept constants. These four equations are mathematically compatible (Zeide, 1987). The intercepts (a, b, c and d) of the equations vary with species, but only within narrow logarithmic limits. The slope parameters are apparently consistent regardless of species, age, and site quality (Jack and Long, 1996). However, instead of being a constant, the slope coefficient of the self-thinning line has been found variable for different species (Zeide, 1987; Jack and Long, 1996; Bi and Turvey, 1997). While some researchers would use the theoretical or constant slope for the self-thinning line (e.g. Drew and Flewelling, 1977, 1979), others prefer to use observed data collected from fully-stocked forest stands to estimate both slope and intercept coefficients (e.g. Newton and Weetman, 1993; Wilson et al., 1999).

The objective of this study was to develop the maximum size–density relationships for three mixed-softwood late successional species forest habitats in the northeastern USA. These relationships can be readily used to establish the upper boundary self-thinning line for DMDs. The forest habitats hemlock-red spruce (HE-RS), spruce-fir (SP-FIR), and cedar-black spruce (CE-BS) (Solomon et al., 1995) are forest land units based on ecological criteria defined by soil materials (or glacial process), land form, and successional climax vegetation sequence (Leak, 1982). These habitats are named according to the climax forest species and are equivalent to landtype phases in the National Hierarchical Framework of Ecological Units (McNab and Avers, 1994).

2. Data and methods

2.1. Data sources

Data were collected from a 6500 growth plot data base used in developing FIBER 3.0 (Solomon et al., 1995) for the three mixed-softwood forest habitats. The study area covers northern Maine, New Hampshire, Vermont, New Brunswick, and Nova Scotia.

Major softwood species include balsam fir (*Abies balsamea* [L.] Mill.), red spruce (*Picea rubens* Sarg.), black spruce (*Picea mariana* [Mill.] B.S.P.), white spruce (*Picea glauca* [Moench] Voss), eastern hemlock (*Tsuga canadensis* [L.] Carr.), northern white-cedar (*Thuja occidentalis* L.), and tamarack (*Larix laricina* [Du Roi] K. Koch). Associated hardwood species might include American beech (*Fagus grandifolia* Ehrh.), red maple (*Acer rubrum* L.), sugar maple (*Acer saccharum* Marsh.), and white ash (*Fraxinus americana* L.). Plots, 0.08 ha in size, were measured between 1959 and 1974 at 5-year intervals. The plot data used in this study were actually the combinations of different measurements over time of all available plots, and included wide ranges of the number of trees per unit area, total stand basal area, quadratic mean tree diameter, and mean tree volume. Summary statistics for these plots are shown in Table 1.

2.2. Procedure for selecting the most fully-stocked plots

Because there is no established statistical procedure for selecting appropriate plots to estimate the two coefficients of the maximum size–density relationships (Bi and Turvey, 1997), we chose the following approach to select the most fully-stocked plots with late successional species for each forest habitat: assuming the theoretical value of the slope for the ln(MV)–ln(*N*) relationship (Eq. (1)) is −1.5, the intercept coefficient was determined by $a = \ln(\text{MV}) + 1.5\ln(N)$ using the plot with the largest combination of ln(MV) and ln(*N*). The determined equation was then used to compute the maximum stand density ($N_{\max}$) for the MV of a given plot. The relative density index (RD) (Drew and Flewelling, 1979) was calculated by $N/N_{\max}$, where N is the current stand density (number of trees per unit area). The RD was computed for each plot, and we expected the plot with the largest combination of ln(MV) and ln(*N*) to have RD = 1.0. Next, all plots with RD > 0.7 were selected as the most fully-stocked plots for developing the maximum size–density relationship. Although this threshold (RD > 0.7) was arbitrary, we chose this for two reasons: (1) any stands with the relative density index higher than 0.7 should have been undergoing the self-thinning process and experiencing density-related mortality of late successional species, and (2) a sufficient number of plots would be obtained for fitting the maximum size–density relationship.

2.3. Fitting the self-thinning line to the data

The most fully-stocked plots of climax species selected in the above process were separately utilized to formulate the self-thinning line for each of the three mixed-softwood forest habitats. However, one particular problem when fitting the self-thinning relationship to data is that both dependent and independent

Table 1
Summary of stand variables for the three forest habitats of hemlock-red spruce (HE-RS), spruce-fir (SP-FIR), and cedar-black spruce (CE-BS)[a]

Forest habitat	No. of plots	Variables	Mean	S.D.	Minimum	Maximum
HE-RS	348	*N*	4949	9.0	2.2	46.8
		BA	22.9	9.0	2.2	46.8
		QMD	24.7	4.3	17.0	39.6
		MV	0.41	0.19	0.12	1.20
SP-FIR	4559	*N*	583	281	17	1766
		BA	21.0	9.7	0.3	51.3
		QMD	21.6	3.2	14.8	44.8
		MV	0.22	0.12	0.01	1.3
CE-BS	850	*N*	559	265	17	1522
		BA	24.4	13.3	0.3	64.4
		QMD	23.2	4.0	13.8	36.7
		MV	0.28	0.11	0.05	0.91

[a] S.D.: standard deviation, *N*: trees per ha, BA: total basal area (m^2/ha), QMD: quadratic mean tree diameter (cm), and MV: mean tree volume (m^3).

variables in the Eqs. (1)–(4) are subject to random errors (Zeide, 1987). A stochastic independent variable results in biased and inefficient estimates of ordinary least-squares (OLS) parameters (Kmenta, 1986). Reduced major axis (RMA) regression has been proven as a popular alternative technique for fitting bivariate relationships (Leduc, 1987). The model fitting procedure is as follows: suppose a linear regression model $y = \alpha + \beta x$, where α and β are the OLS estimates of the two coefficients. In the case of the Eq. (1) y is ln(MV) and x is ln(N), and in the case of the Eq. (3) y is ln(QMD) and x is ln(N). The RMA slope coefficient is $\beta_{RMA} = \beta/|r_{yx}|$, where r_{yx} is the Pearson correlation coefficient between y and x. The standard error (S.E.) of β_{RMA} is equal to the S.E. of β. The RMA intercept coefficient is $\alpha_{RMA} = \bar{y} - \beta_{RMA}\bar{x}$, and the S.E. of α_{RMA} is equal to the S.E. of α. The S.E.s are essential for constructing confidence intervals for the coefficients. This model fitting procedure was conducted for both the ln(MV)–ln(N) relationship and the ln(QMD)–ln(N) relationship. The 95% confidence interval was computed for each coefficient, and used to compare the equality between the three mixed-softwood forest habitats.

2.4. Obtaining the maximum or upper boundary self-thinning line

The size-density relationship developed by the RMA regression technique is considered as the "average" self-thinning line because it passes through all data points and allows for some plots to be above the line (Smith and Woods, 1997). To obtain the "maximum" or upper boundary self-thinning line, the "average" self-thinning line is moved parallel upward to the plot which has the highest relative density index (RD = 1.0). This procedure is holding the RMA slope coefficient and recalculating the intercept coefficients by using the ln(MV) or ln(QMD) and ln(N) of the plot with the highest RD. This "corrected" intercept coefficient is proven to be consistent (Schmidt, 1986).

3. Results

The estimated RMA slope and intercept coefficients were provided in Table 2 for both the ln(MV)–ln(N) relationships and the ln(QMD)–ln(N) relationships. The 95% confidence intervals of the coefficients were also shown in parentheses. If a coefficient of any forest habitat falls between the lower and upper limits of the corresponding confidence interval of another forest habitat, we would conclude that the difference between the two forest habitats is not statistically significant.

The analysis results indicated that the slope and intercept coefficients for the cedar-black spruce (CE-BS) forest habitat fell outside of corresponding confidence intervals for either the hemlock-red spruce (HE-RS) forest habitat or the spruce-fir (SP-FIR) forest habitat. This was true for both ln(MV)–ln(N) and the ln(QMD)–ln(N) relationships. Thus, we concluded that the difference between the CE-BS forest habitat and the other two habitats was statistically

Table 2
Comparison of the slope and intercept coefficients of the maximum size–density line for the ln(MV)–ln(N) and ln(QMD)–ln(N) relationships between the three forest habitats of hemlock-red spruce (HE-RS), spruce-fir (SP-FIR), and cedar-black spruce (CE-BS)

Habitat	No. of plots[a]	Slope (β_{RMA})	Intercept (α_{RMA})
ln(MV)–ln(N)			
HE-RS	34	−1.37 (−1.49, −1.25)[b]	8.16 (7.37, 8.95)
SP-FIR	78	−1.41 (−1.49, −1.33)	8.44 (7.86, 9.02)
CE-BS	47	−1.57 (−1.74, −1.40)	9.73 (8.55, 10.91)
ln(QMD)–ln(N)			
HE-RS	34	−0.57 (−0.62, −0.52)	7.01 (6.69, 7.33)
SP-FIR	78	−0.52 (−0.57, −0.48)	6.72 (6.45, 7.00)
CE-BS	47	−0.68 (−0.75, −0.61)	7.92 (7.41, 8.43)

[a] The number of plots selected as the most fully-stocked plots.

[b] Numbers in parentheses are the lower and upper limits of the 95% confidence interval.