/NT

An Individual-Tree Growth and Yield Prediction System for Even-Aged Natural Shortleaf Pine Forests

Thomas B. Lynch, *Department of Forestry, Oklahoma State University, Agricultural Hall Room 008C, Stillwater, OK 74078;* **Kenneth L. Hitch** *and* **Michael M. Huehschmann,** *Department of Forestry, Oklahoma State University, Agricultural Hall Room 008C, Stillwater, OK 74078; and* **Paul A. Murphy** *(deceased), Southern Research Station, University of Arkansas at Monticello, P.O. Box 3.516, UAM Monticello, AR 716.56-3516.*

ABSTRACT: *The development of a system of equations that model the growth and development of even-aged natural shortleaf* (Pinus echinata *Mill.) pine forests is described. The growth prediction system is a distance-independent individual-tree simulator containing equations that predict basal-area growth, survival, total and merchantable heights, and total and merchantable volumes for shortleaf pine trees. These equations were combined into a computer simulation program that predicts future states of shortleaf pine stands from initial stand descriptions. Comparisons of observed and predicted ending stand conditions in shortleaf pine research plots indicate the simulator makes acceptable forecasts of final stand attributes. South. J. Appl. For. 23(4):203–211.*

Predicting future stand conditions is an important element of forest management. It is perhaps surprising that, despite its economic importance and wide distribution, relatively little work has been done on growth and yield of shortleaf pine (*Pinus echinata* Mill.) compared to the other major southern pines. The Ozark and Ouachita National Forests contain extensive stands of natural shortleaf pine. Natural shortleaf pine stands are common on private nonindustrial lands, and to a lesser extent on industrial forestlands, of western Arkansas and eastern Oklahoma. The significance of the shortleaf pine resource in this area merits the development of growth models for shortleaf pine natural stands based on remeasured plots in managed stands representing a variety of ages, site qualities, and stand densities. The purpose of this paper is to report the development of a growth and yield simulator for shortleaf pine in even-aged natural stands.

Prior to 1985, most shortleaf pine growth studies were based on either temporary plots with a full-stocking assumption or inventory plots not intended to be the basis for forest growth models. The earliest growth and yield information for shortleaf pine stands is contained in USDA Miscellaneous Publication 50 (USDA Forest Service 1929), which provides site index curves and normal yield tables for fully stocked shortleaf pine natural stands. Later, Schumacher and Coile (1960) published growth and yield information based on 74 "well-stocked" temporary plots in the Piedmont of North Carolina. Since both the Miscellaneous 50 and the Schumacher and Coile yield tables were based on a normal stocking concept, adjustments are required for application to stands that are not at "normal" stocking levels.

Brinkman (1967) developed stand volume equations for naturally occurring shortleaf pine stands based on 57 stands in which periodic measurements were made at a 15 yr interval. He suggested procedures for predicting short-term growth based on basal area and height growth assumptions.

Murphy and Beltz (1981) and Murphy (1982) provided growth and yield models for natural shortleaf pine stands that can be applied to a range of stand densities. Their models use equations that predict yield per acre at the stand level and were based on Forest Inventory and Analysis data-covering a variety of ages and site indices-from the Southern Research Station. The Central States version of the TWIGS (Miner et al. 1989) individual-tree growth and yield simulation program also provides shortleaf pine growth and yield information, based on inventory data from Indiana, Illinois, and Missouri. Bolton and Meldahl (1990) used inventory data to develop an individual-tree simulation system for southern species, including shortleaf pine. The disadvantages of using inventory data for growth and yield modeling

NOTE: T.B. Lynch is the correspondence author, and he can be reached at (405) 744-5447; Fax: (405) 744-9693; E-mail: tlynch@okstate.edu. Approved for publication by the Director, Oklahoma Agricultural Experiment Station. Manuscript received April 27, 1998, accepted March 9, 1999.

include inadequate sampling of plots representing infrequent conditions, lack of plot isolation boundaries, limited knowledge of plot histories, and the fact that growth on these plots may not be typical of growth for managed stands (Murphy 1986).

Lynch et al. (1991) developed stand volume equations for natural shortleaf pine based on initial plot measurements from a shortleaf pine growth study. These can be combined with the basal area projection equations of Murphy and Beltz (1981) and Murphy (1982) to predict future yields.

Useful growth and yield information can sometimes be obtained from thinning studies despite their often limited range of site qualities and ages. For example, several researchers (Brinkman et al. 1965, Sander and Rogers 1979, Rogers 1983, Rogers and Sander 1985) have reported results from a study on the Sinkin Experimental Forest in Missouri in which three replications of a control and four residual basal area levels were established in 30-yr-old even-aged natural shortleaf pine stands. Murphy et al. (1992) provided growth and yield information from a thinning study initiated by Frank Freese in the Ouachita National Forest. Wittwer et al. (1996) described the growth characteristics of thinned shortleaf pine natural stands on an industrial forest ownership in southeastern Oklahoma.

Smalley and Bailey (1974) developed models for shortleaf pine plantations which can be used to obtain yields by dbh classes. Murphy and Farrar (1985) used inventory data supplied by an industrial forestry organization to develop a growth and yield model for uneven-aged shortleaf pine stands. Additional information on shortleaf pine growth and yield is available from a summary by Murphy (1986).

Data

In 1985, the Forestry Department at Oklahoma State University (OSU), the USDA Forest Service Southern Research Station (USFS), and the Ouachita and Ozark National Forests undertook a cooperative study of growth in shortleaf pine natural stands. The original OSU-USFS study plan (Murphy 1988a) called for establishing plots in four categories each of age, basal area per acre, and site index (Table 1). A total of 192 plots were to be established, with three plots in each combination of the age, basal area, and site index categories. Of the 191 plots actually established, several were lost or made unusable for this analysis because of windstorms or failure to conduct thinning treatments. Thus, 183 plots remained for modeling after the second measurement. Plot locations on the Ozark and Ouachita National Forests ranged from areas north of Interstate Highway 40 near Russellville in western Arkansas to near Broken Bow in southeastern Oklahoma.

Table 1. Design-variable classes for natural, even-aged shortleaf pine growth study plots in western Arkansas and southeastern Oklahoma (after Murphy 1988a).

Design variable	Class midpoint	Class range
Basal area (ft^2/ac)	30	16–45
	60	46-75
	90	76–105
	120	106-135
Site index (ft at age 50 yr)	<56	<56
	60	56–65
	70	66-75
	>75	>75
Age (yr)	20	11–30
	40	31-50
	60	51–70
	80	71-90

Each plot consists of a 0.2 ac circular measurement plot, and a 33 ft wide circular buffer strip that received the same treatments for thinning and competing vegetation as the measurement plot. When established, each plot was thinned from below to a predetermined residual pine basal area, and chemical herbicide was applied to control competing vegetation. The tree number and breast height were marked, and dbh (in.) recorded for each residual pine in the measurement plot. Each measured tree was classified as dominant, codominant, intermediate, or suppressed. Total height (ft) and height to the base of the live crown (usually defined as the bottom live branch) were recorded for a representative sample of trees in each dbh class on the measurement plot. Ages of codominant and dominant pines on each plot were determined from increment cores. This information was used to estimate plot age and site index at a base age of 50 yr according to the Graney and Burkhart (1973) site index curves. Plots were established during the period from 1985–1988 and remeasured during the period 1990-1992. Measurement intervals were either 4 or 5 yr.

Data from a natural shortleaf pine thinning study initiated by Frank Freese in 1963-1964 were also used in this study. These plots also have a 0.2 ac measurement plot, and were originally thinned to 45, 65, 85, 105, or 125 ft^2/ac residual pine basal area. Of the 35 original plots, 25 still exist on the Ouachita National Forest. These remaining plots were thinned from below in 1988 to achieve a balanced design according to Freese's original basal area levels. Murphy (1988b) gives a more detailed description of the Freese study data. Before being thinned in 1988, stand basal area on several plots exceeded 125 ft^2/ac. In order to extend the range of stand density beyond the 120 ft^2/ac maximum of the OSU-USFS study described above, earlier measurement intervals from the Freese study plots were included when fitting basal-area-growth and mortality model parameters. Measurement intervals for these plots varied, but most were about 7 yr. By using the data from those earlier measurement periods, a total of 34 plots from the Freese study were available for analysis. Summary statistics for the combined 217 plots available for model construction are given in Table 2.

Model Development

A distance-independent individual-tree model was developed after remeasurements were completed in 1992. This type of model is based on individual-tree growth and allows for a great deal of flexibility in response to a variety of thinning scenarios. The basic components of the model system are individual-tree basal-area-growth and survival-probability equations, and compatible height-dbh and height-growth projection equations.

Table 2. Summary statistics for 217 natural, even-aged shortleaf pine growth and yield study plots in western Arkansas and southeastern Oklahoma.

Variable	Average	Stand. Dev.	Minimum	Maximum
Basal area (ft^2/ac)				
Initial	81.9	35.1	21.3	174.2
Mid-period	87.5	36.2	22.5	177.1
Final	93.0	37.9	14.9	180.0
Stand age (yr)				
Initial	55.2	18.7	18	93
Final	60.3	18.7	23	99
Site index (ft at age 50 yr)	62.2	10.7	38.9	87.1
Total volume (ft^3/ac)				
Initial	2,290	1,249	240	5,801
Final	2,736	1,359	657	6,342
Periodic mortality (trees/ac)	7.1	19.1	0	170

Individual-Tree Basal Area Growth

Hitch (1994) used preliminary remeasurement data (the Freese thinning study remeasurement was incomplete) from the OSU-USFS study described above to develop a basal area growth model for individual shortleaf pine trees. The final model presented below has a form very similar to Hitch's, but was derived from the complete dataset. Summary statistics for data used to fit parameters to an individual-tree basal area growth model are given in Table 3. The model is of the potential-modifier form in which a Chapman-Richards (Richards 1959) function, constrained by maximum tree size (Shifley and Brand 1984), represents potential tree growth. This potential is multiplied by a logistic modifier. The modifier function (Murphy and Shelton 1996) is constrained to take on values between 0 and 1 so that it reduces potential growth on the basis of variables representing stand and tree attributes. This general type of individual-tree basal area growth model is used in TWIGS (e.g., Miner et al. 1989) although TWIGS uses a different type of modifier. The following equation predicts basal area growth of individual shortleaf pine trees in even-aged natural stands:

$$G_i = \frac{b_1 B_i^{b_2} - \left(b_1 B_i / B_M^{1-b_2}\right)}{1 + \exp(b_3 + b_4 B_S + b_5 A + b_6 R_i + b_7 B_i)} \qquad (1)$$

where G_i is annual basal area growth (ft^2) of tree i; B_i is basal area (ft^2) of tree i; A is stand age; R_i is the ratio of quadratic mean stand diameter to the dbh of tree i; B_S is stand basal area (ft^2/ac); $B_M = 7.068384\ ft^2$, the maximum expected basal area for a shortleaf pine in managed stands; and $b_1, b_2, \ldots, b_7$ are coefficients to be estimated.

Nonlinear least squares techniques were used to estimate the coefficients (Table 4). Equation (1) had a mean square error (MSE) of $3.89 \cdot 10^{-5}$ and fit index of 0.62 (FI = 1 − SSE/SST, where SSE is the error sum of squares and SST is the total sum of squares). Table 5 contains the matrix of covariances and correlations among the coefficients.

Probability of Individual-Tree Survival

A method of modeling individual-tree survival is required to predict future stand conditions with a distance-independent individual-tree model. Hamilton (1974, 1976) described the use of a logistic model to predict the probability of individual-tree mortality. The OSU-USFS study adapted Hamilton's techniques to model the probability of individual shortleaf pine tree survival (relevant variables are summarized in Table 6).

Nonlinear regression techniques were used to fit parameters in the logistic model. The dependent variable was 1 for trees alive at both ends of the measurement interval, and 0 for trees that were alive at the first measurement but dead prior to the second. Iteratively reweighted least squares was used to achieve homogeneity of variance. The weight was the inverse of the variance $P^t(1 - P^t)$ where P is the annual probability of survival predicted by the logistic model and t is the number of years in the measurement period. An annual mortality prediction equation was desired; consequently, because the remeasurement intervals were not the same for all plots, the logistic model was raised to a power equal to the number of years in the measurement interval when fitting the parameters.

Several models were compared based on goodness of fit criteria, including the chi--square statistic. The following model was selected to predict probability of survival in natural shortleaf pine stands:

$$P_i = \frac{1}{1 + \exp\left[-\left(b_0 + \frac{b_1}{R_i} + b_2 B_S + b_3 H_D\right)\right]} \qquad (2)$$

Table 3. Summary statistics for 8,928 shortleaf pine trees from natural, even-aged stands in western Arkansas and southeastern Oklahoma used to develop an individual-tree basal area growth model.

Variable	Mean	Stand. Dev.	Minimum	Maximum
Tree basal area (ft^2)				
Initial	0.39	0.36	0.01	3.25
Mid-period	0.42	0.38	0.01	3.38
Final	0.45	0.40	0.01	3.52
Mid-period ratio of quadratic mean diameter (in.) to individual-tree dbh (in.)	1.09	0.32	0.44	4.24

Table 4. Parameter estimates and standard errors for a distance-independent individual-tree basal area growth model for even-aged natural shortleaf pine fitted to data from western Arkansas and southeastern Oklahoma.

Coefficient	Estimate	Stand. Err.
b_1	0.0714278055	0.00090
b_2	0.4803818382	0.00779
b_3	-3.236284358	0.07528
b_4	0.015766456	0.00032
b_5	0.027880079	0.00087
b_6	1.294524159	0.06085
b_7	-1.212688088	0.04147

where P_i is the probability of annual survival for tree i; H_D is the average height (ft) of dominants and codominants; b_0, b_1, . . . , b_3 are coefficients to be estimated; and other variables are as defined in Equation (1) above.

Parameter estimates for Equation (2), with corresponding standard errors are given in Table 7. Table 8 contains the matrix of correlations and covariances among the coefficients. Mortality rates observed in the OSU-USFS study were low, probably because the thinning from below at the time of plot installation removed the trees most likely to die. Several of the Freese study plots had not been thinned since installation in the 1960s and had achieved higher basal area levels than observed on the OSU-USFS study. Nevertheless, overall mortality rates observed in the combined data set were low. Of 9,238 trees present at the first measurement, 3 10 died leaving 8,928 live trees remaining at the second measurement. Use of compound interest formulas indicate that this would correspond to an annual mortality rate of between 0.7 and 0.8% for a measurement period of 4 to 5 yr, although the calculation is not exact since the measurement interval was not the same on all plots. Additional information regarding the mortality function above is available in Lynch et al. (in press).

Site Index

Site index and average height of dominants and codominants were estimated from the polymorphic site index curves developed by Graney and Burkhart (1973) for natural shortleaf pine in the Ouachita Mountains where most of the OSU-USFS study data were obtained. Graney and Burkhart's equation is:

$$H_D = [20.975 + 1.2113S] \left[1 - \exp\{-(0.012362 + 0.00013639S)A\}\right]^{1.0018} \quad (3)$$

where S is height (ft) at age 50 (site index base age 50 yr); A is age in years; and the other variables are defined above.

Height Prediction and Projection

Individual-tree heights are needed in order to obtain total and merchantable tree volumes for a complete distance-independent individual-tree model. Equations relating dbh to height and age or time in stand development have been developed by Curtis (1967) and Lenhart and Clutter (197 1) among many others. A more complete discussion of previous work is given by Lynch and Murphy (1995).

Table 5. Matrix of correlations and covariances for the shortleaf pine basal-area growth modifier fitted to data from natural, even-aged stands in western Arkansas and southeastern Oklahoma. Values above the diagonal are correlations; those below are covariances.

Coefficient	b_3	b_4	b_5	b_6	b_7
b_3		$-6.373 \cdot 10^{-1}$	$4.957 \cdot 10^{-2}$	$-8.136 \cdot 10^{-1}$	$-3.111 \cdot 10^{-1}$
b_4	$-1.550 \cdot 10^{-5}$		$-8.725 \cdot 10^{-2}$	$2.656 \cdot 10^{-1}$	$1.464 \cdot 10^{-1}$
b_5	$3.239 \cdot 10^{-6}$	$-2.447 \cdot 10^{-8}$		$-4.190 \cdot 10^{-1}$	$-8.190 \cdot 10^{-1}$
b_6	$-3.727 \cdot 10^{-3}$	$5.222 \cdot 10^{-6}$	$-2.213 \cdot 10^{-5}$		$5.013 \cdot 10^{-1}$
b_7	$-9.711 \cdot 10^{-4}$	$1.962 \cdot 10^{-6}$	$-1.508 \cdot 10^{-5}$	$1.265 \cdot 10^{-3}$	

a The correlation and covariance among potential-growth parameters b_1 and b_2 of Equation (1) are, respectively, $9.327 \cdot 10^{-1}$ and $6.521 \cdot 10^{-6}$. Correlations and covariances among modifier and potential parameters are not presented because they were estimated separately.

Table 6. Summary of data for 9,238 individual shortleaf pine trees from natural, even-aged stands in western Arkansas and southeastern Oklahoma used to fit parameters to a logistic survival model.

Variable	Average	Stand. Dev.	Minimum	Maximum
Quadratic mean diameter (in.)	7.10	3.31	3.10	17.90
Dbh (in.)	7.51	3.73	1.1	24.4
Ratio of quadratic mean diameter to dbh	1.10	0.33	0.44	4.42

Table 7. Parameter estimates for a logistic model of survival probability for individual shortleaf pine trees fitted to data from natural, even-aged stands in western Arkansas and southeastern Oklahoma.

Coefficient	Estimate	Stand. Err.
b_0	2.912370652	0.44483
b_1	4.789284600	0.39415
b_2	-0.015129972	0.00310
b_3	-0.006680302	0.00465

Table 8. Matrix of correlations and covariances for the shortleaf pine survival model fitted to data from natural, even-aged stands in western Arkansas and southeastern Oklahoma. Values above the diagonal are correlations; those below are covariances.

Coefficient	b_0	b_1	b_2	b_3
b_0		$-5.995 \cdot 10^{-1}$	$-6.770 \cdot 10^{-1}$	$-5.042 \cdot 10^{-2}$
b_1	$-1.051 \cdot 10^{-1}$		$1.319 \cdot 10^{-1}$	$-2.997 \cdot 10^{-1}$
b_2	$-9.336 \cdot 10^{-4}$	$1.612 \cdot 10^{-4}$		$-3.846 \cdot 10^{-1}$
b_3	$-1.043 \cdot 10^{-4}$	$-5.494 \cdot 10^{-4}$	$-5.545 \cdot 10^{-6}$	

Lynch and Murphy (1995) developed a compatible height prediction and projection system for individual shortleaf pine trees based on the OSU-USFS plots and last two measurements of Freese study plots discussed above. Their equation for predicting either current or future individual shortleaf pine tree heights is:

$$(H_i - 4.5) = 3.072887(H_D - 4.5)^{0.790356} \exp(-2.491153 D_i^{-0.940809}) \quad (4)$$

where H_i is total height (ft) of tree i; D_i is dbh (in.) of tree i; and H_D is as defined above.

Prediction of future tree heights may be more accurate if information from previous tree heights can be used as a predictive variable. Since the shortleaf pine data set contained two height measurements, it was possible to develop the following projection equation that can be used to predict future heights based on previously measured heights:

$$(H_{2i} - 4.5) = (H_{1i} - 4.5)\left(\frac{H_{D2} - 4.5}{H_{D1} - 4.5}\right)^{0.790356} \exp[-2.491153(D_{2i}^{-0.940809} \quad D_{1i}^{-0.940809})] \quad (5)$$

where H_{1i} and H_{2i} are time 1 and 2 total heights (ft) of tree i; H_{D1} and H_{D2} are time 1 and 2 average total heights (ft) of dominants and codominants; and D_{1i} and D_{2i} are time 1 and 2 dbh's (in.) of tree i.

Equation (5) uses measured height at time 1, if available, to predict future heights at time 2, leading to better predictions than could be obtained with Equation (4). Equations (4) and (5) are compatible in the sense that when Equation (4) is used to generate a height at time 1, the predicted height at time 2 given by Equation (5) is the same as would be predicted by Equation (4).

Equation (4) had a fit index of 0.95 and MSE of 20.02 ft^2, while Equation (5) had a fit index of 0.98 and MSE of 8.53 ft^2. Lynch and Murphy (1995) present additional details concerning the model construction and parameter estimation of Equations (4) and (5).

Crown Ratio Estimation

A representative subsample of trees from each OSU-USFS study plot was selected for developing an individual-tree crown ratio prediction model. The data set consisted of 3,132 shortleaf pine trees on which total height and height to live crown (generally defined as the first live branch) were measured. The crown ratios in the data set ranged from 0.03 to 0.80, with a mean of 0.36 and standard deviation of 0.10. The crown ratio equation had the form:

$$CR_i = 1 - \exp\left[-\left(b_0 + \frac{b_1}{A}\right)\left(\frac{D_i}{H_i}\right)^{b_2}\right] \quad (6)$$

where CR_i is crown ratio of tree i; b_0, b_1, and b_2 are parameters to be estimated; and other variables are as defined above.

Table 9. Parameter estimates for a shortleaf pine crown ratio model, based on 3,131 individual shortleaf pine trees from natural, even-aged stands in western Arkansas and southeastern Oklahoma.

Coefficient	Estimate	Stand. Err.
b_0	2.03470146	0.08399
b_1	25.27922541	1.80157
b_2	0.95968104	0.02219

Equation (6) had a fit index of 0.42 and MSE of $5.62 \cdot 10^{-3}$. Parameter estimates and their corresponding standard errors are given in Table 9. Table 10 contains the matrix of correlations and covariances among the coefficients. The form of Equation (6) was previously used by Dyer and Burkhart (1987), and is constrained to give estimates between zero and one.

Volume and Weight Estimation

Total volumes and merchantable volumes to any desired upper-stem diameter limit can be estimated by integrating the taper functions in Exhibit A (on the next page) that Farrar and Murphy (1987) developed for natural shortleaf pine in Arkansas and Louisiana.

Both cubic and bd ft volumes can be computed. The system uses three sets of coefficients: one set if $CR < 0.36$, a second if $0.36 \leq CR < 0.50$, and a third if $CR \geq 0.50$. The crown ratio prediction Equation (6) is used to determine which set of coefficients is appropriate for a given tree.

No weight factors are available for natural shortleaf pine in western Arkansas and eastern Oklahoma. However, approximate green densities are obtained from forming the ratio of the shortleaf pine weight and volume equations of Saucier et al. (1981) for a tree of a given dbh and height. This density is then multiplied by the appropriate cubic foot volume supplied by the taper curves of Farrar and Murphy (1987).

A Distance-Independent Individual-Tree Simulator

Huebschmann et al. (1998) incorporated Equations (1) through (7) to create a Shortleaf Pine Stand Simulator (SLPSS), written in Microsoft QuickBasic, that simulates growth and yield in even-aged natural shortleaf pine stands. The basic input to the simulator consists of current stand conditions in the form of either a stand table (number of trees by dbh classes) or inventory data from field plots. If stand table data are input on a per-acre basis, the simulator multiplies the number of trees in a dbh class by 10 (essentially simulating 10 ac) and uniformly distributes the trees in tenth-in. increments within that dbh class.

Table 10. Matrix of correlations and covariances for the shortleaf pine crown ratio model fitted to data from natural, even-aged stands in western Arkansas and southeastern Oklahoma. Values above the diagonal are correlations; those below are covariances.

Coefficient	b_0	b_1	b_2
b_0		$2.749 \cdot 10^{-1}$	$9.293 \cdot 10^{-1}$
b_1	$4.160 \cdot 10^{-2}$		$5.918 \cdot 10^{-1}$
b_2	$1.732 \cdot 10^{-3}$	$2.366 \cdot 10^{-2}$	

Exhibit A

$$d_i = \begin{cases} D_i\left(h_i/4.5\right)^n & \text{if } h_s \le h_i \le 4.5 \quad \text{(7a)} \\ \text{or} & \\ D_i X/\left(H_i - 4.5\right) + b_1 XZ/H_i^2 + b_2 D_i XZ/H_i^2 & \\ + b_3 D_i^2 XZ/H_i^2 + b_4 XZ\left(2H_i - h_i - 4.5\right)/H_i^3 & \text{if } 4.5 \le h_i \le H_i \quad \text{(7b)} \end{cases}$$

where d_i is predicted stem diameter (in.), either ob or ib, at height h_i of tree i; n and $b_1, b_2, \ldots, b_4$ are parameters given in Table 11; h_i is height (ft) above ground line; D_i is diameter (in.) at breast height: D_{ob} if d_i is diameter ob, or D_{ib} if d_i is diameter ib ($D_{ib} = g_1 + g_2 D_{ob}$ where g_1 and g_2 are parameters given in Table 11); h_s is stump height (ft) above ground line; H_i is total tree height (ft), ground line to tip of bud; X is ($H_i - h_s$); and Z is ($h_i - 4.5$).

Each tree (or group of trees in a dbh-class increment) is grown on a year-by-year basis. Equation (1) estimates each tree's yearly basal area increment. Equation (2) determines each tree's probability of survival. A tree survives the year if its probability of survival exceeds the value of a uniformly distributed random number (restricted to the interval 0 to 1) generated for that tree.

Equation (4) or (5) estimates each tree's total height, and Equation (6) calculates its crown ratio. The height and crown ratio estimates determine which of Farrar and Murphy's (1987) taper functions [Equation (7) in Exhibit A above] is used to compute the tree's volume.

The simulator is capable of conducting low or free thinning to specified levels of residual stand basal area. Other types of thinning can be accomplished by specifying a desired residual stand table.

The simulator was evaluated by using the OSU-USFS and Freese thinning study plots. Stand conditions at the initial measurement for each plot were supplied to the simulator which predicted conditions at the second measurement (usually 4 to 5 yr subsequent to the initial measurement). Plot simulations were repeated 10 times. Observed conditions at the second measurement were compared to average predicted conditions. Residual (observed minus predicted) values of stand basal area (ft^2/ac) are plotted against the basic study design variables of stand basal area, age, and site index in Figure 1. Residual values of total volume inside bark (ft^3/ac) are similarly plotted in Figure 2. The basic results for basal area and volume are similar. In general, no obvious trends are evident in the basal area residuals. This tends to confirm acceptable performance on the data used to develop the stand simulator. Figure 2 seems to indicate that the simulator may tend to underestimate volume in stands having 120 ft^2/ac or more of basal area. The charts of basal area and volume residuals against initial stand basal area show positive residuals for stands having initial basal area greater than 150 ft^2/ac. This might show some tendency to underpredict basal area and volume for heavily stocked stands. However, only four plots in the data set had initial basal area in excess of 150 ft^2/ac, all in mature stands from the Freese thinning study.

A summary of residual statistics including plot averages, standard deviations of these differences, and the minimum and maximum differences, is given in Table 12 for a variety of stand attributes. Negative differences for quadratic mean diameter and basal area per acre indicate some tendency to overpredict those attributes. All other differences-including trees per acre, and cubic and board-foot volumes per acre-showed positive average differences, indicating a tendency to underpredict those attributes.

The standard deviations in Table 12 can be used to obtain standard errors based on the number of plots. The ratio of average difference to standard error is 1.72 for number of trees per acre, -1.57 for quadratic mean diameter, and -0.27 for stand basal area. The hypothesis of a zero mean difference cannot be rejected when comparing these values to an approximate t-value of 2. The ratio of average difference to standard error is greater than 2 (usually 3 to 4) for all the volume differences. However, the average differences for the volume variables are a modest proportion of typical stand volume. The standard deviations indicate that the simulator is reasonably precise for 4 to 5 yr projections. These comparisons should be appraised in light of the fact that the data set

Table 11. Parameter estimates for shot-deaf pine lower and upper stem taper functions by outside bark, inside bark, and three crown ratio (CR) classes (from Farrar and Murphy 1987).

	Outside bark			Inside bark		
Coefficient	$CR < 0.36$	$0.36 \le CR < 0.50$	$CR \ge 0.50$	$CR < 0.36$	$0.36 \le CR < 0.50$	$CR \ge 0.50$
n	-0.13253541	-0.11988464	-0.11799179	-0.12195134	-0.10905991	-0.11164159
g_1				-0.372176	-0.406013	-0.534799
g_2				0.936758	0.930204	0.935277
b_1	25.385423	19.513315	4.995668	19.473495	13.933809	-1.878825
b_2	2.279039	1.772916	2.091531	2.066904	1.593209	1.954822
b_3	-0.044477	-0.026344	-0.027642	-0.038933	-0.020028	-0.023757
b_4	-23.637118	-18.120387	-10.484750	-17.738120	-12.575889	-3.780569

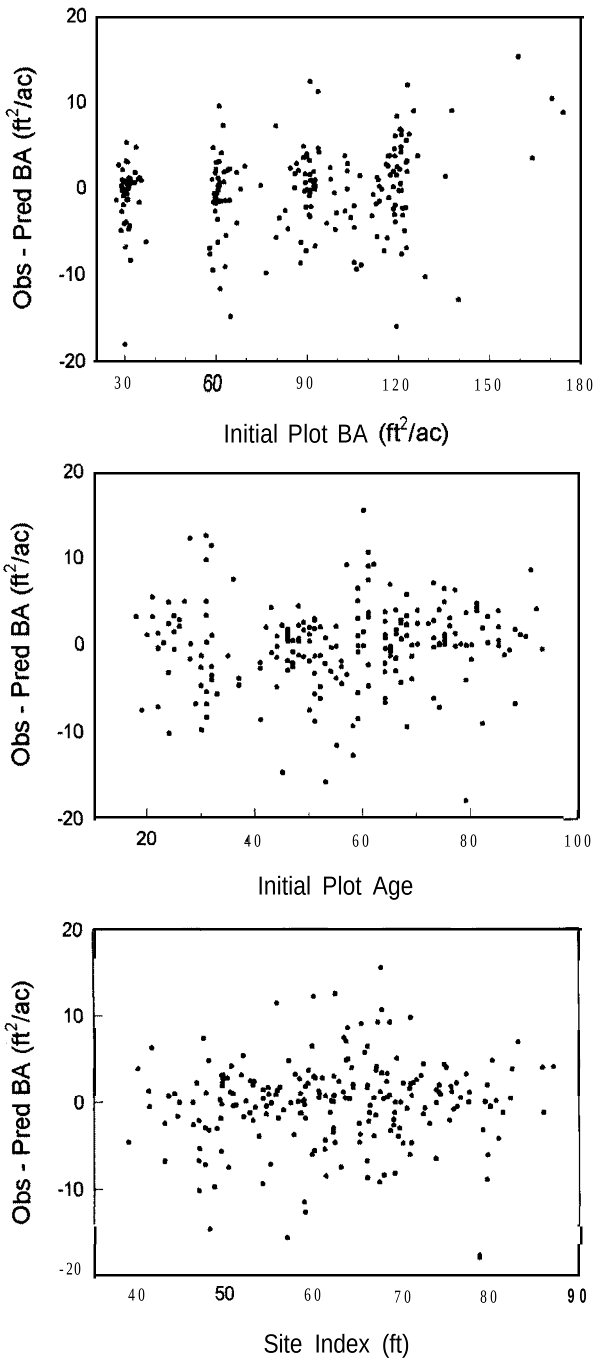

Figure 1. Residual values (observed minus predicted from the Shortleaf Pine Stand Simulator) of final stand basal area (ft²/ac) versus initial plot age, stand basal area, and plot site index; based on 217 plots in natural, even-aged shortleaf pine stands in western Arkansas and southeastern Oklahoma.

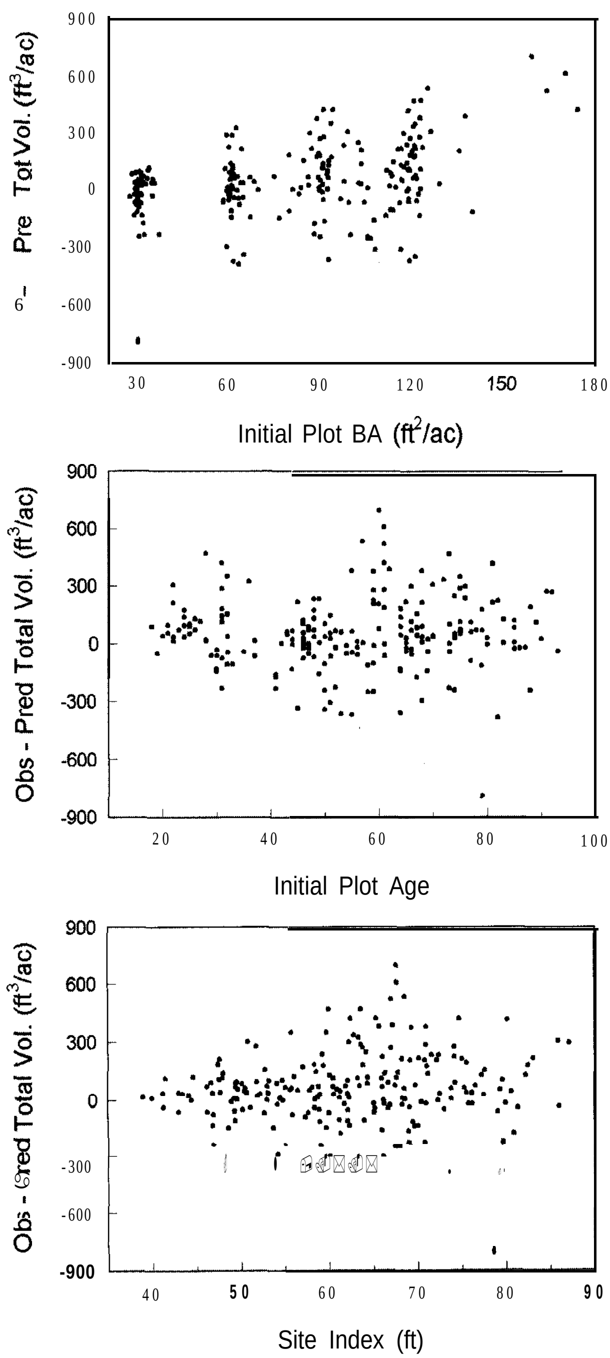

Figure 2. Residual values (observed minus predicted from the Shortleaf Pine Stand Simulator) of final total volume inside bark (ft³/ac) versus initial plot age, stand basal area, and plot site index; based on 217 plots in natural, even-aged shot-deaf pine stands in western Arkansas and southeastern Oklahoma.

used for evaluation was also used to fit parameters; a truly independent evaluation data set was unavailable.

The simulator's performance was also evaluated by growing hypothetical stands and comparing their mean annual increments (MAI) of basal area (ft²/ac) and total volume (ft³/ac). In one case, 20-yr-old stands with site index (base age of 50 yr) of 60 ft, and initial basal areas of 30, 60, 90, and 120 ft²/ac were grown for 80 yr. Figure 3 indicates that the basal area MAIs of all four stands converge by age 100. Convergence occurs earlier in the total volume MAIs (Figure 4).

Interestingly, the volume MA1 curve for the 90 ft²/ac stand crosses that of the 120 ft²/ac stand at about age 55, probably because the 120 ft²/ac stand experiences greater mortality and the average tree in the 90 ft²/ac stand contains more volume. However, the difference between the two MA1 curves is very small after they cross.

In the second case, 20-yr-old stands with initial basal area of 7 1 ft²/ac and site indices of 50, 60, 70, and 80 ft were grown for 80 yr. The total volume MA1 curves for the second case are shown in Figure 5.

Table 12. Summary of residual analysis for the Shottleaf Pine Stand Simulator, based on 217 plots in natural, even-aged stands in western Arkansas and southeastern Oklahoma.

Attribute	Difference (observed–predicted) Average	Stand. Dev.	Minimum	Maximum
Trees/ac	1.3	10.8	-97.0	36.1
Quadratic mean dbh (in.)	-0.02	0.23	-0.90	0.69
Basal area (ft^2/ac)	-0.1	4.8	-17.9	15.5
Volume (ft' ib/ac)				
Total	54.8	191.2	-792.3	697.5
To 4 in. top dob	51.7	186.1	-774.6	667.5
To 7 in. top dib	42.6	177.3	-745.2	657.1
Volume (bd ft/ac)				
Doyle	139.1	685.0	–3,459.1	2,007.7
Scribner	211.6	954.9	–4,388.2	2,982.4
International-l/4	406.0	1128.0	–4,875.5	3,859.6
Green weight (tons/ac)				
Total	1.7	6.7	–27.6	24.4
To 4 in. top dob	1.8	6.5	-27.0	23.3
To 7 in. top dib	1.5	6.1	-25.9	22.7

Summary and Conclusions

A Shortleaf Pine Stand Simulator (SLPSS) for even-aged natural shortleaf pine stands has been developed from research plots established and remeasured on the Ozark and Ouachita National Forests of western Arkansas and eastern Oklahoma. This distance-independent individual-tree simulator uses equations that predict basal area growth and survival for individual shortleaf pine trees. A height prediction and projection system is used to predict total height for trees of given dbh's in stands for which the average total height of dominants and codominants is known. A site index equation (Graney and Burkhart 1973) predicts average total heights of dominants and codominants in stands for which site index and age are known. Taper equations (Farrar and Murphy 1987) estimate total and merchantable volumes in cubic and board feet. Green weights to specified merchantable top limits are also predicted.

Stand tables or inventory data are required as initial conditions to begin stand simulation. Predicted stand conditions are given in terms of per acre values by dbh class. SLPSS can conduct low thinnings, free thinnings, or remove specified numbers of trees by dbh class. These options make the simulator a useful tool for those interested in managing even-aged natural shortleaf pine stands.

Because the stochastic mortality function causes stand predictions to vary, the authors recommend that users average several projections for a particular stand. Information concerning installation and use of SLPSS is given in a user's manual developed by Huebschmann et al. (1998), and can be obtained from this article's senior author.

Literature Cited

BOLTON, R.K., AND R.S. MELDAHL. 1990. User's guide to a multipurpose forest projection system for southern forests. Bull 604. Alabama Agric. Exp. Sta. 15 p.

BRINKMAN, K.A. 1967. Stand volume equations for shortleaf pine in Missouri. USDA For. Serv. Res. Note NC-24.4 p.

BRINKMAN, K.A., N.F. ROGERS, AND S.F. GINGRICH. 1965. Shortleaf pine in Missouri: Stand density affects yield. USDA For. Serv. Res. Pap. cs-14. 14 p.

CURTIS, R.O. 1967. Height-diameter and height-diameter-age equations for second growth Douglas-fir. For. Sci. 13:365–375.

DYER, M.E., AND H.E. BURKHART. 1987. Compatible crown ratio and crown height models. Can. J. For. Res. 17:572–574.

FARRAR, R.M., JR., AND P.A. MURPHY. 1987. Taper functions for predicting product volumes in natural shortleaf pines. USDA For. Serv. Res. Pap. SO-234.9 p.

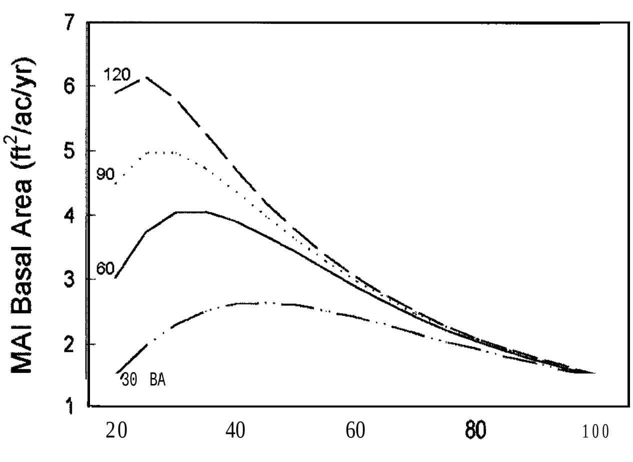

Figure 3. Mean annual increments of basal area in hypothetical natural, even-aged shortleaf pine stands with initial stand basal area stocking levels of 30, 60, 90, and 120 ft^2/ac.

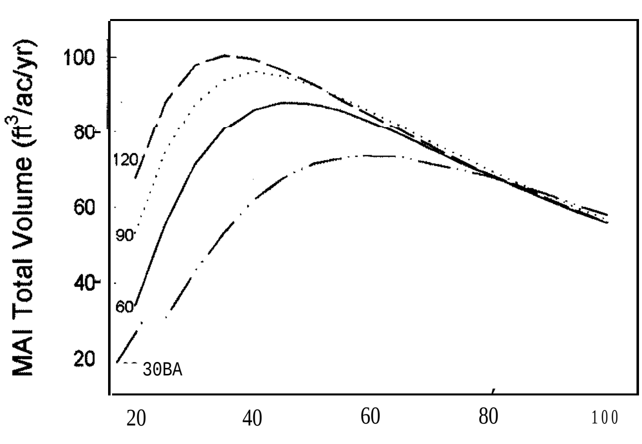

Figure4. Mean annual incrementsof total volume in hypothetical natural, even-aged shortleaf pine stands with initial stand basal area stocking levels of 30, 60, 90, and 120 ft^2/ac.

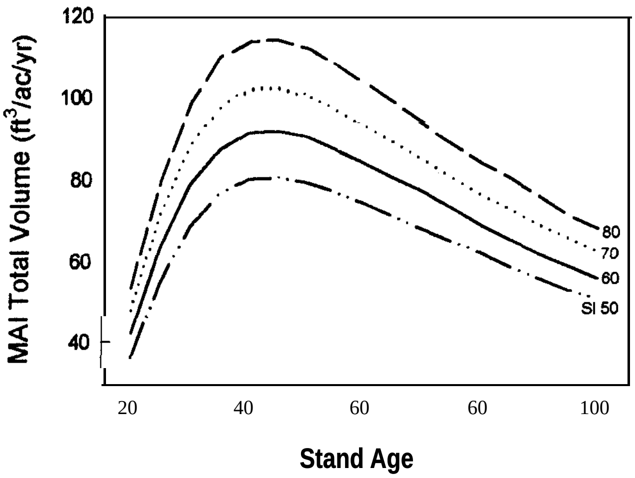

Figure5. Mean annual increments of total volume in hypothetical natural, even-aged shortleaf pine stands with the same initial stand basal area, but site index (base age = 50 yr) values of 50, 60, 70, and 80 ft.

GRANEY, D.L., AND H.E. BURKHART. 1973. Polymorphic site index curves for shortleaf pine in the Ouachita mountains. USDA For. Serv. Res. Pap. SO-85. 12 p.

HAMILTON, D.A. 1976. Modeling the probability of individual tree mortality. USDA For. Serv. Res. Pap. INT-185. 22 p.

HAMILTON, D.A. 1974. Event probabilities estimated by regression. USDA For. Serv. Res. Pap. INT-152. 18 p.

HITCH, K.L. 1994. A distance-independent individual tree basal area growth model for natural, even-aged stands of shortleaf pine in eastern Oklahoma and western Arkansas. M.S. thesis, Oklahoma State Univ., Stillwater, OK. 78 p.

HUEBSCHMANN, M.M., T.B. LYNCH, AND P.A. MURPHY. 1998. Shortleaf pine stand simulator: an even-aged natural shortleaf pine growth and yield model user's manual. Res. Rep. P-967, Okla. State Univ. Agric. Exp. Sta. 25 p.

LENHART, J.D., AND J.L. CLUTTER. 197 1. Cubic foot yield tables for old field loblolly pine plantations in the Georgia Piedmont. Ga. For. Res. Counc., Rep. 22 Series 3. 12 p.

LYNCH, T.B., AND P.A. MURPHY. 1995. A compatible height prediction and projection system for individual trees in natural, even-aged shortleaf pine stands. For. Sci. 41(1):194–209.

LYNCH, T.B., M.M. HUEBSCHMANN, AND P.A. MURPHY. A survival model for individual shortleaf pine trees in even-aged natural stands. P. 00–00 *in* Proc. Conf. on Integrated Tools for Natural Resource Inventories in the 2 I st Century. (In press.)

LYNCH, T.B., P.A. MURPHY, AND E.R. LAWSON. 1991. Stand volume equations for managed natural even-aged shortleaf pine in eastern Oklahoma and western Arkansas. Res. Rep. P-921. Okla. State Univ. Agric. Exp. Sta. 12 p.

MINER, CL.. N.R. WALTERS. AND M.L. BELLI. 1989. A technical guide to TWIGS for the North Central U.S. USDA For. Serv. Gen. Tech, Rep. NC-125. 105 p.

MURPHY, P.A. 1982. Sawtimber growth and yield for natural even-aged stands of shortleaf pine in the West Gulf. USDA For. Serv. Res. Pap. SO-181. 13 p.

MURPHY, P.A. 1986. Growth and yield of shortleaf pine. P. 159-I 77 *in* Proc. Symp. on the Shortleaf Pine Ecosystem. Arkansas Coop. Ext. Serv., Univ. of Arkansas, Monticello, AR.

MURPHY. P.A. 1988a. Establishment and progress report: Growth and yield of thinned natural shortleaf pine stands in the Ozark and Ouachita National Forests. South. For. Exp. Sta., For. Sci. Lab., Monticello AR. 52 p.

MURPHY, P.A. 1988b. Study plan and progress report: Growth relationships for even-aged stands of shortleaf pine. South. For. Exp. Sta.. For. Sci. Lab., Monticello AR. 22 p.

MURPHY, P.A.. AND R.C. BELTZ. 198 1. Growth and yield of shortleaf pine in the West Gulf region. USDA For. Serv. Res. Pap. SO- 169. 15 p.

MURPHY, P.A.. AND R.M. FARRAR, JR. 1985. Growth and yield of uneven-aged shortleaf pine in the Interior Highlands. USDA For. Serv. Res. Pap. SO-218. 11 p.

MURPHY, P.A.. AND M.G. SHELTON. 1996. An individual-tree basal areagrowth model for loblolly pine stands. Can. J. For. Res. 26:327–331.

MURPHY, P.A.. E.R. LAWSON, AND T.B. LYNCH. 1992. Basal area and volume development of natural even-aged shortleaf pine stands in the Ouachita mountains. South. J. Appl. For. 16(1):30–34.

RICHARDS, F.J. 1959. A flexible growth function for empirical use. J. Exp. Bot. 10:290–300.

ROGERS, R. 1983. Guides for thinning shortleaf pine. P. 2 17-225 in Proc. of Second Bienn. South. Silv. Res. Conf. USDA For. Serv. Gen. Tech. Rep. SE-24.

ROGERS, R., AND I.L. SANDER. 1985. Intermediate thinnings in a Missouri shortleaf pine stand: A 30-year history. P. 2 16–2 19 *in* Proc. Third Bienn. South. Silv. Res. Conf. USDA For. Serv. Gen. Tech. Rep. SO-54.

SANDER, I.L.. AND R. ROGERS. 1979. Growth and yield of shortleaf pine in Missouri: 21-year results from a thinning study. P. 14-27 *in* Proc. Symp. Management of Pines in the Interior South. USDA For. Serv. Tech. Publ. SA-TP2.

SAUCIER, J.R.. D.R. PHILLIPS, AND J.G. WILLIAMS, JR. 198 1. Section II shortleaf pine yield tables. P. 25-37 *in* Green weight, volume. board-foot and cord tables for the major southern pine species. Georgia For. Res. Pap. 19. Res. Div., Georgia For. Comm.

SCHUMACHER, F.X., AND T.S. COILE. 1960. Growth and yield of natural stands of the southern pines. T.S. Coile, Inc., Durham. NC. 1 15 p.

SHIFLEY. S.R., AND G.J. BRAND. 1084. Chapman-Richards growth function constrained for maximum tree size. For. Sci. 30(4): 1066–1070.

SMALLEY, G.W., AND R.L. BAILEY. 1974. Yield tables and stand structure for shortleaf pine plantations in Tennessee. Alabama. and Georgia highlands. USDA For. Serv. Res. Pap. SO-97. 57 p.

USDA FOR. SERV. 1929 (rev. 1976). Volume, yield and stand tables for second-growth southern pines. USDA Misc. Publ. 50. 202 p.

WITTWER, R.F.. T.B. LYNCH, AND M.M HUEBSCHMANN. 1996. Thinning improvesprowthofcroptrees innatural shortleafpine stands. South. J. Appl. For. 29(4):182–1 87.

