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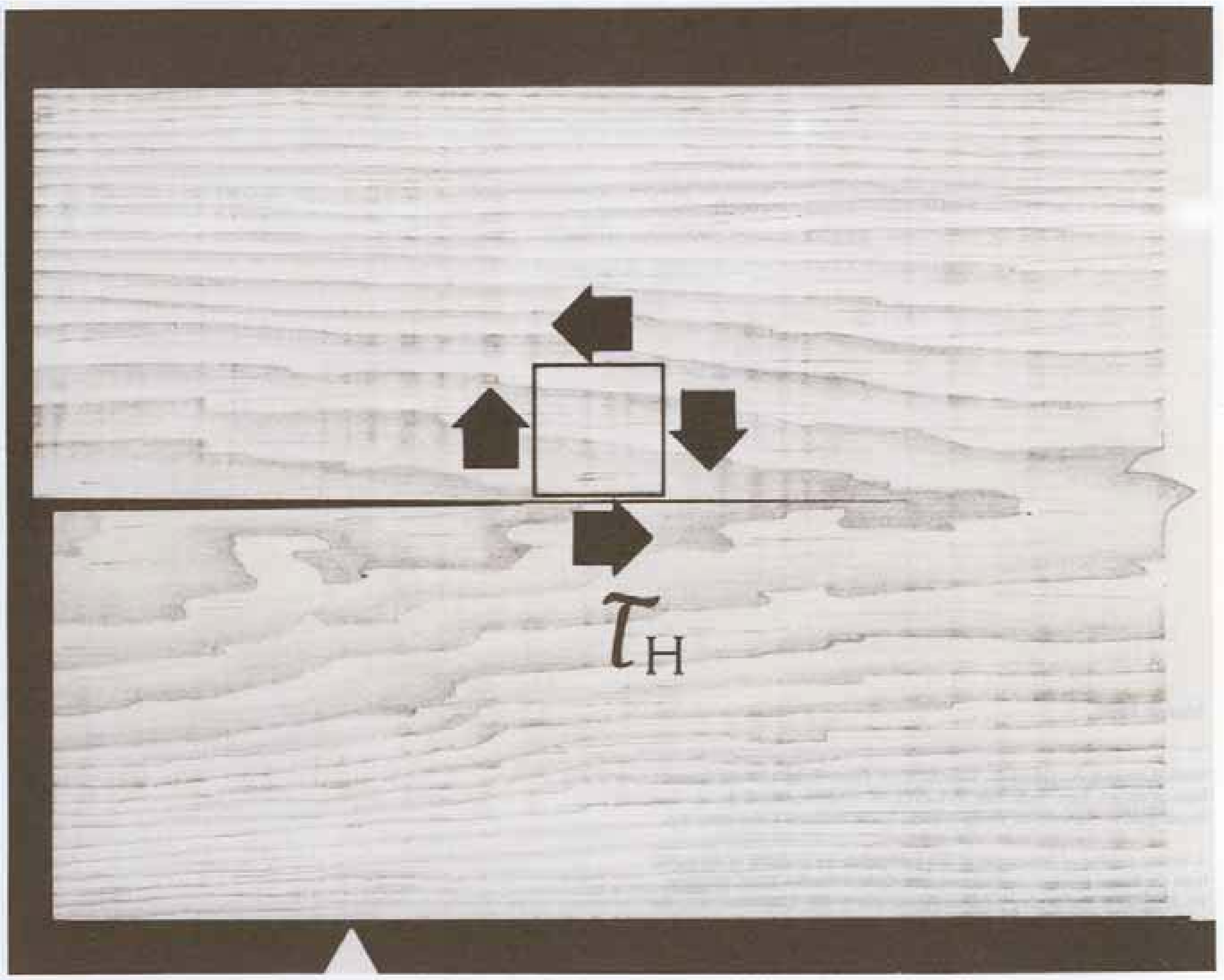
Shear Design of Wood Beams

State of the Art

Lawrence A. Soltis
Terry D. Gerhardt



Abstract

Current shear design technology in the United States for lumber or glued-laminated beams is confusing. This report summarizes shear stress and strength research including both analytical and experimental approaches. Both checked and unchecked beams are included. The analytical work has been experimentally verified for only limited load conditions and span-to-depth ratios. Future research is required to better define the effects of beam size, load configuration, checks, and combined stresses on shear design.

Keywords: Beams, design, shear strength, shear stress, shear tests, timber, lumber, glued laminated.

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Shear Design of Wood Beams

State of the Art

Lawrence A. Soltis, Supervisory Research Engineer
Forest Products Laboratory, Madison, WI

Terry D. Gerhardt, Senior Research Associate
Sonoco Products Co., Madison, WI

Introduction

Current shear design technology in the United States for lumber or glued laminated beams is not well founded, may be too conservative in some cases, and is frequently ignored by engineering designers. Shear design requirements (American Institute of Timber Construction (AITC) 1986, National Forest Products Association (NFPA) 1986) are confusing and little understood. The objective of this report is to review the state of the art in shear design of rectangular wood beams.

Shear stresses and strengths must first be defined. Local shear stresses act at a point in the beam and are dependent on the beam geometry and load configuration. Local shear failure occurs when the shear stresses parallel to the grain, i.e. shear stresses acting in the longitudinal-tangential (LT) or longitudinal-radial (LR) planes, exceed the shear strength of the beam.

The shear strength of a wood member is difficult to quantify. It depends on test method, member size, load configuration, checks, splits or knots, grain slope, moisture content, and temperature. A pure local shear failure is difficult to attain since shear often interacts with bending and perpendicular-to-grain stresses. Additionally, the wood member may fail by cracks propagating from end splits or side checks.

For unchecked members, both shear stress and strength are dependent on member size and load configuration. This complicates a generalized design procedure for all sizes and configurations. For checked members, different failure modes further complicate a generalized procedure.

Shear Stress

Shear stresses (τ) are local phenomena acting at a point in the beam. The resultant shear force (V) is defined as the integral of τ over a beam cross section:

$$V = \int \tau \, dA \tag{1}$$

The distribution of V along the length of the beam is easily calculated from applied loads and reactions at each support. The shear stress distribution at a cross section away from a support is approximated by elementary beam theory and is dependent on V at that cross section. The shear stress distribution at a cross section near a support, however, cannot be determined from elementary beam theory (Cowan 1962). These stresses can be computed using analytical or numerical techniques (Liu and Cheng 1979).

Cowan (1962) experimentally found that the maximum shear stress near a support is about 33 percent greater than the maximum shear stress predicted by elementary beam theory. The distribution of stress, however, is considerably different at a support than the parabolic distribution of elementary beam theory for rectangular cross sections.

Near supports, bending and perpendicular-to-grain stresses also occur in addition to shear stresses (Liu and Cheng 1979). Shear forces, therefore, can be associated with nonshear-type failures. For example, Bickel (1983) documented bending failures in floor trusses caused by shear reactions acting on top chord bearing extensions. In that case, the bending failure was located quite close to the support. Such locations are usually associated with shear failures. Several orthotropic failure criteria have been investigated for combined stresses in wood members (Cowin 1979, Goodman and Bodig 1971, Keenan 1974, Liu 1984a, Norris 1950, Tsai and Wu 1971). These failure criteria require strength properties (discussed later) that are difficult to measure.

Shear Strength

The evolution of allowable shear strength values has been traced from 1906 (Ethington et al. 1979). The early research summarized in that report recognized that flexural tests of lumber resulted in lower shear strengths than found from tests of small, clear, straight-grained specimens. That study traced the evolution of adjustment factors based on experience and compromise.

Current shear strengths of small, clear specimens are given in American Society for Testing and Materials (ASTM) Designation D 2555. Adjustments for strength ratio and splits and checks to visually graded lumber are given in ASTM Designation D 245. Standard methods of establishing glued-laminated timber values are given in ASTM Designation D 3737.

An ideal shear strength specimen would have a critical failure plane acted on exclusively by uniform shear stress. Such a state of stress is difficult to achieve. Experimental stress analysis of the standard test specimen ASTM D 143 has revealed a nonuniform shear stress distribution with a stress concentration near the corner (Coker and Coleman 1935, Radcliffe and Suddarth 1955). Keenan (1974) used wood tubes loaded in torsion to obtain a more uniform state of shear stress. It is difficult to fabricate wood specimens with proper grain orientation and without the development of residual stresses, which limits this approach. A butterfly-shaped specimen has been used to measure shear strength (Arcan et al. 1978) and shear fracture properties (Banks-Sills and Arcan 1983) of composite materials. Experimental results (Arcan et al. 1978) indicate a relatively uniform shear stress distribution in the failure plane. Liu (1984b) recently measured the shear strength of Douglas-fir using this specimen.

The butterfly-shaped specimen appears promising for measuring clear wood shear strength. However, setting allowable shear strength values for beams from these data present certain difficulties. To overcome these difficulties, various researchers have considered the effects on shear strength of the following:

1. Member size (Keenan 1974, Longworth 1977, Madsen and Nielsen 1978).

2. Moisture content and temperature (Gerhards 1982).

3. Grain slope variation (Liu and Floeter 1984).

4. End splits and checks (Barrett and Foschi 1977, Murphy 1979, 1980).

5. Knots (Cramer and Goodman 1983).

The shear strengths of timber beams have long been recognized as being lower than the shear strengths as measured in shear block tests. Wilson and Cottingham (1952) tested glued-laminated wood beams (both horizontally and vertically laminated) under two-point loading. The beams were 5- by 12-inch Douglas-fir and had a length-to-depth ratio, l/d, of 13.5. Longworth (1977) tested a number of sizes of Douglas-fir glued-laminated beams under two point loading. The beams had l/d ratios of approximately 5. Keenan et al. (1985) tested small sizes of several species of glued-laminated beams under a concentrated midspan load with l/d ratios of 1.5 to 5. Results of average shear strengths from these three references are included in figure 1.

Foschi and Barrett developed both a theory (1976) and a design procedure (1977) to predict the shear strengths of glued-laminated beams. The theory is based on finite element simulations using quadratic isoparametric quadrilateral elements (Zienkiewicz 1976) with nonlinear constitutive laws for compression parallel and perpendicular to the grain. From the numerical stress analyses, Foschi and Barrett (1976) concluded that:

1. The high shear stress near the support corners decays rapidly and contributes only to bearing-type failures.

2. The maximum shear stress in regions removed from loads and supports is conservatively estimated by the elementary beam equation.

Foschi and Barrett used the computed shear stresses in a two-parameter Weibull model to develop a strength theory that incorporates the effects of support, loading condition, and size. The Weibull parameters were calculated from data collected by Longworth (1977) for 5 beam configurations with 30 replications of each. The parameters were calculated from a least squares procedure using only the mean values from each group. Foschi and Barrett (1976) then compared model predictions to shear failure loads for Griplam nail connections (Foschi and Longworth 1975), torque tubes (Keenan 1974, Madsen 1972), and beams (Keenan 1974).

In a subsequent paper, Foschi and Barrett (1977) developed a design procedure by simplifying the analysis and incorporating the effects of support width. The final result was a set of tables for designing rectangular and tapered glued-laminated beams under various loading conditions for shear strength. This design methodology is now prescribed in the Canadian Standards Association design code (1984).

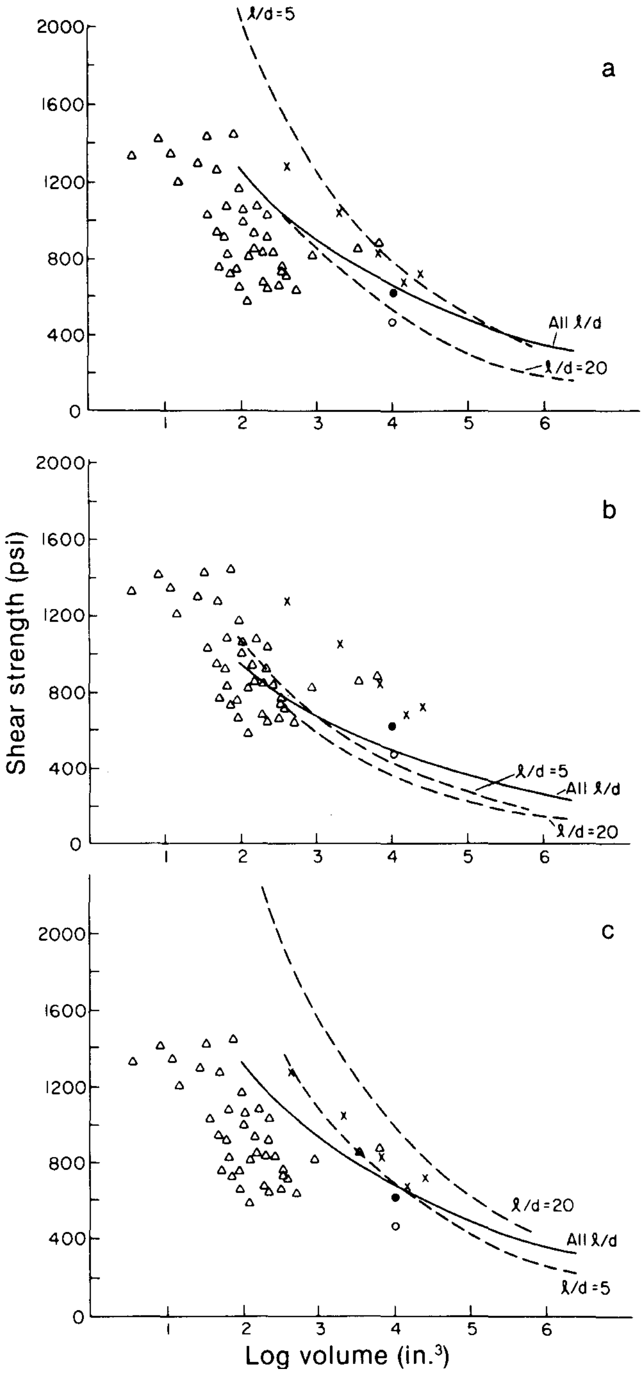

Figure I – Predicted fifth percentile shear strengths (Liu theory --- , Foschi and Barrett theory–)for various beam sizes and length-to-depth (l/d) ratios for the following loading configurations: (a) uniform load, (b) concentrated midspan load, and (c) concentrated load at 0.1 l from the support. Experimental results for mean shear strength found by single or two-point concentrated loads are also included. x = Longworth (two-point load); o = Wilson and Cottingham (horizontally laminated); • = Wilson and Cottingham (vertically laminated); r = Keenan et a1. (MLR875533)

Liu (1980) also developed a shear design methodology for glued-laminated beams. It was similar to that just described, in the sense that a stress analysis was substituted into a two-parameter Weibull model. Liu, however, used elementary beam theory to compute shear stresses. As mentioned previously, Foschi and Barrett (1976) concluded that this was a conservative approximation. By using these rather simple expressions for shear stress, Liu derived closed form expressions for design. The expressions are based on shear resultant force distributions in the beam. The stresses can be easily determined for any loading conditions. Liu also calculated the two Weibull parameters using Longworth's data (1977). He used a method described by Mitchell (1967) and Johnson (1964), which included the effects of the distributional characteristics and the suspended data (caused by beams that failed in bending). Because of differences in assumed shear stresses and in analysis of Longworth's data, the Weibull parameters computed by Liu (1980) and Foschi and Barrett (1976) are somewhat different. Liu (1981) also extended his approach to model tapered beams.

In recent work, Zakic (1983, 1984) developed a theory for predicting the shear strengths of wood beams based on the plastic bending theory. This approach is applicable only to extremely clear wood beams. Thus, it would not be appropriate for a design approach and will not be reviewed in this report.

Both Foschi and Barrett's and Liu's theories agree with Longworth's data for two-point loading and l/d ratios of 5. We applied the two theories to other common types of loading —uniform load, concentrated load at midspan, and concentrated load near a support - for a range of sizes and for other l/d ratios. The results (fig. 1) are fifth percentile predictions from theory compared to average values of data. The data were obtained from either centerpoint or two-point loading conditions. Several items are of interest:

1. Liu's theory gives predicted results independent of l/d ratio, whereas Foschi and Barrett depend on l/d. This is because Liu neglected the differences that exist between the ordinary beam theory and the finite element analysis at the supports.

2. There is no constant reliability for either theory for the different types of loading. Different types of loading result in different predicted shear strengths for a given wood member.

3. Current allowable shear values (AITC 1986, NFPA 1986) range from 65 to 200 pounds per square inch for various species of solid-sawn and glued-laminated beams. No size effect is included. Both theories and data indicate this to be conservative for nonchecked smaller sizes.

Two-Beam Theory

In the United States, current design methods to check beam shear strength (AITC 1986, NFPA 1986) are based on both elementary beam theory and the Two-Beam Theory (Newlin et al. 1934). A designer usually applies the elementary beam theory equation, which is known to be overly conservative in many situations. This is, however, the only. recommended procedure for unchecked beams in the Timber Construction Manual (AITC 1986). But for checked and unchecked beams that fail to qualify under the elementary beam theory equation, the National Design Specification for Wood Construction (NFPA 1986) allows the application of the Two-Beam Theory.

Newlin et al. (1934) proposed a shear strength model for wood beams containing checks located at beam middepth as depicted in figure 2. The model is based on a combined analytical-experimental approach. The checks were of constant depth along the length of the beam and existed on both lateral faces. The following assumptions were made in the analysis:

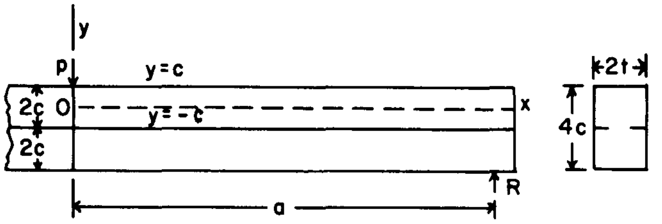

Figure 2–Wood beam uniformly checked on both lateral faces. (ML875534)

1. Only the portion of the beam $x > 0$ and $y > -c$ was considered. The effect of the lower beam section ($y < -c$) was assumed to be a constant shear stress (J) acting on the plane $y = -c$. The effect of the region $x < 0$ was assumed to be a support at $x = 0$.

2. The effect of the support at $x = a$ and $y = -c$ on the analyzed portion was assumed to be a force one-half the magnitude of the reaction force (P/2).

3. The stress perpendicular to the grain (σ_y) was assumed to be zero throughout the beam.

4. The beam was assumed to have isotropic properties.

Based on these assumptions, Newlin et al. (1934) solved the problem of a cantilever beam with concentrated end load (P/2) and uniform distributed shear load (J) on $y = -c$ as shown in figure 3. The authors derived a plane stress, elasticity solution for this idealized problem. The boundary conditions assumed were

$$u = \frac{\partial u}{\partial y} = 0 \text{ at } x = y = 0 \quad (2)$$

where u is the displacement component in the x direction. Newlin et al. (1934) derived expressions for τ, σ_x, and u (given in their equations (9), (10), and (21), respectively) that satisfy all governing differential equations and boundary conditions for the elasticity problem. Gerhardt and Liu (1983) derived beam equations for arbitrary shear and normal loading that reduce to the same expressions for the stresses.

From the elasticity solution, Newlin et al. (1934) derived an expression that is the basis of the Two-Beam Theory:

$$R = B + A/a^2 \quad (3)$$

where

$$A = Eu^*bh^2/6 \quad (4)$$

and

$$B = 2Jbh/3 \quad (5)$$

E is Young's modulus, u* is u evaluated at x = a and y = - c, b is beam width, and h is beam height. They argue from equation (3) that the reaction R has two parts. The part represented by B is associated with middepth shear stress as predicted by elementary beam theory. The second term is independent of shear stress and increases as the point of loading approaches the support. It is called the two-beam portion of the reaction since it is caused by the beam sections y > - c and y < - c acting independently.

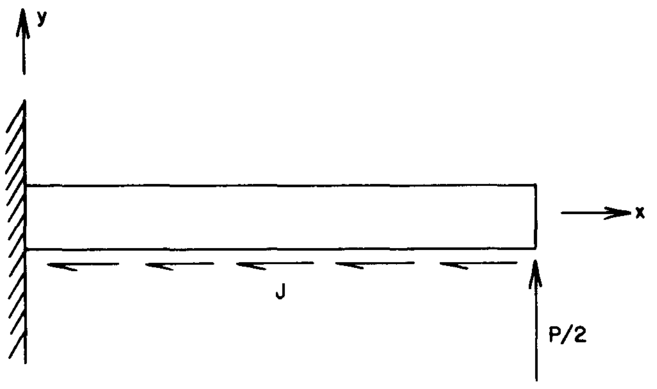

Figure 3 –Cantilever beam with concentrated end load and uniform shear load. (ML875537)

Newlin et al. (1934) used equation (3) from the elasticity solution to predict beam failure. Since J and u* at failure cannot be measured in a simple manner, the authors determined A and B empirically from strength tests. They fabricated checked beams by gluing clear pieces together. The beams had dimensions h = 4.5 inches, b = 2.5 inches, and spans (l) of 28, 42, and 63 inches. The beams contained checks 0.875 to 1.125 inches deep (70 to 90 pct of beam width) and were loaded at positions (a) varying from 7 to 31.5 inches. The two-beam term in equation (3) enabled the model to predict the minimum failure load at a location several beam heights from the support, as is indicated by experiments. Elementary beam theory predicts the minimum failure load at a location just adjacent to the support.

The strength data were analyzed in a rather peculiar fashion. For example, in table 1 of Newlin et al. (1934), failure reaction forces from all three spans were averaged together for a given distance (a). Thus, any dependency on the ratio (a/l) was eliminated. (In a sense, the analysis was consistent with scaling by (a/h) since h was not varied in the experiment. Note that the "two-beam" portion of equation (3) can be easily reformulated in terms of (a/h).) The authors compared the mean reaction from each distance a to the mean reaction from the a = 7 inches group. By assuming that J and u* at failure were independent of load and position, the authors calculated A and B. In regards to the pairing, the authors state:

> "It is true that if the test results of this series are combined in pairs in other ways than that just used, some considerable deviations from the values recorded will appear, but none of such magnitude as to cast doubt on the conclusion that Equation (3) describes approximately the behavior of a checked beam."

Table 1 –Coefficients of $P(l-a)/l$

a/h	Equation(7)	Equation(10)
0	0	1/2
1	1/3	6/7
2	2/3	21/22
3	9/11	46/41
4	8/9	81/82

As indicated in the Newlin et al. (1934) tables 1, 2, and 3, the coefficients A and B depend on both beam dimensions and check depth. Determination of A and B for a given group, therefore, does not result in a usable design equation. For discussion purposes, define Q as the ratio of two-beam reaction to total reaction:

$$Q = (A/a^2)/R \quad (6)$$

Newlin et al. derived a design equation by making several additional assumptions based on the strength results:

1. For a beam of given size and a, Q is independent of check depth.

2. For beams of varying dimensions, Q is constant for a fixed a/h.

3. Q is approximately equal to 2/11 when a/h = 3.

From these assumptions, equation (3), and the equilibrium equation $R = P(1-a/l)$ the design equation for concentrated load is easily derived:

$$B = \frac{P(\ell - a)(a/h)^2}{\ell[2 + (a/h)^2]} \quad (7)$$

By integrating along the length of the beam, the authors derived an expression for the uniform load (W) case:

$$B = W\ell/2 - Wh\sqrt{2}\tan^{-1}[\ell/(h\sqrt{2})] + Wh^2 2.3\log_{10}[(\ell^2 + 2h^2)/2h^2]/\ell \qquad (8)$$

The authors also approximated equation (8) by

$$B = 9W(\ell/2 - h)/10 \qquad (9)$$

Equations (7) and (9) appear in the U.S. design code (NFPA 1986) with slight modifications. These equations are applied in design by relating B to allowable shear strength through equation (5).

We carefully analyzed the assumptions, theory, and data presented by Newlin et al. (1934). In our opinion, several difficulties make the proposed method inapplicable for predicting shear strengths of either glued-laminated or lumber beams.

Two-Beam Theory predicts incorrect shear stress distributions for unchecked beams. In figure 4, predicted stresses are reproduced for a beam of dimensions h = 4.5 inches, b = 2.5 inches, and ℓ = 42 inches with a 1-inch check depth. Note the shear stress at the neutral axis, J, is independent of position: a condition imposed by the theory. The "two-beam" portion of the reaction causes a redistribution of shear stresses near the support as shown. This redistribution is, therefore, the fundamental premise of the theory. Other researchers have analyzed stresses in glued-laminated or clear wood beams. Experimental (Cowan 1962), analytical (Gerhardt and Liu 1983, Liu and Cheng 1979), and numerical (Foschi and Barrett 1976, Keenan 1974) techniques all indicate shear stress distributions near supports quite different than the parabolic redistribution predicted in figure 4. Furthermore, Norris (1962b) loaded unchecked beams (h = 5.6 in, b = 1.6 in, ℓ = 86 in) to failure with the distance a varied from 5 to 30 inches. Norris found the failure reactions calculated by Two-Beam Theory to be two to three times greater than those measured. Thus, Two-Beam Theory cannot predict shear failure in glued-laminated or clear wood beams. Note that the intent of Newlin et al. (1934) was to model deeply checked beams.

The validity of equation (7) is restricted to the deeply checked, fabricated beams tested by Newlin et al. (1934). Norris (1962a) considered the problem defined by Newlin et al. and depicted in figure 2. Using a well-established analytic procedure, Norris derived the equation

$$B = \frac{P(\ell - a)[0.2 + (a/h)^2]}{\ell[0.4 + (a/h)^2]} \qquad (10)$$

The only assumption made was that E/G = 16, where G is the shear modulus. Although equations (7) and (10) have a similar form, they predict quite different coefficients of $P(\ell - a)/\ell$ as indicated in table 1. From this comparison, Norris concluded that ". . . equation (7) is only an empirical equation that fits the results of the tests reported."

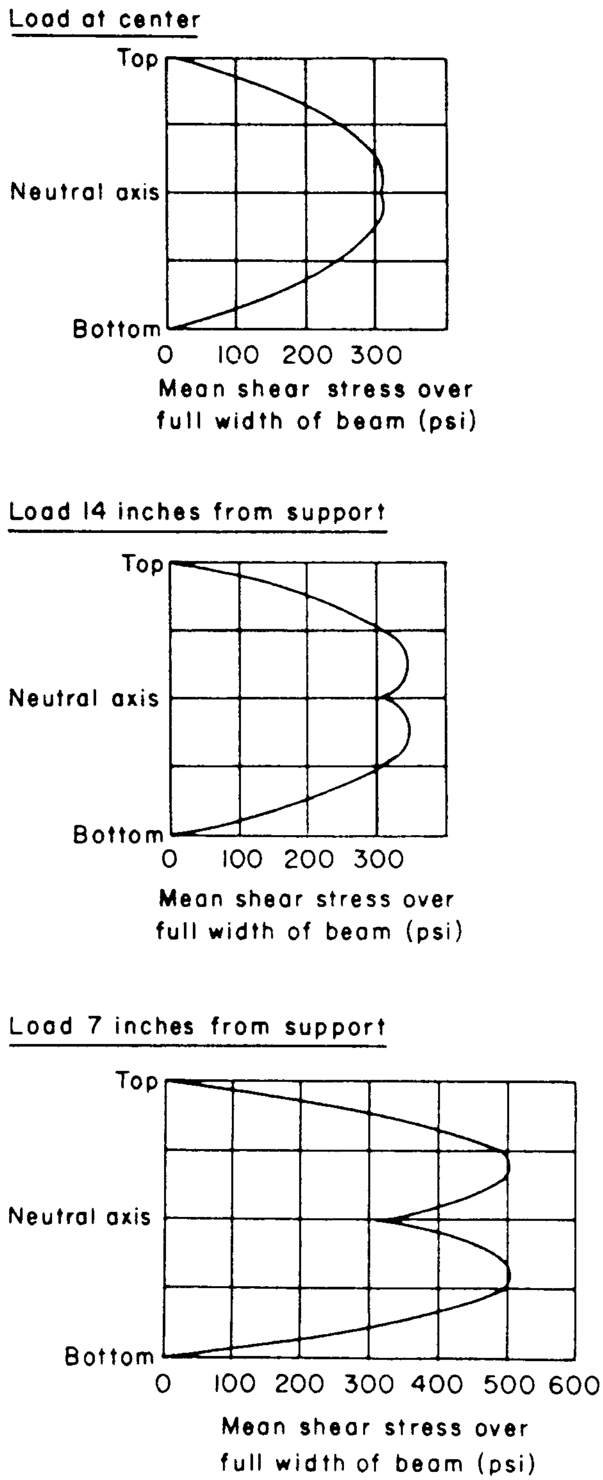

Figure 4 – Theoretical variation of horizontal shear stress with distance from neutral axis of checked beams for various positions of a single concentrated load (beams, 2.5 by 4.5 by 45 in; span, 42 in; depth of checks, 1 in). (ML875538)

Effect of Check Depth on Strength

The design equations derived by Newlin et al. (1934) are independent of check depth. Both single and two-beam failure reactions computed by the authors show a strong dependence on check depth. This dependence was eliminated from the design equation in the derivation of equation (7) as previously described. Murphy (1980) used a fracture mechanics approach to predict the relation between check depth and strength. This approach, which assumes a singularity at the crack tip, modeled the data of Newlin et al. quite well. As illustrated in figure 11 of Murphy's 1980 report, strength is extremely dependent on check depth when the depth is greater than 30 percent of beam width. In light of contemporary fracture mechanics theory, a strength model that is independent of crack depth is untenable.

The shear strength of lumber is governed by the presence of defects such as end splits and side checks. Barrett and Foschi (1977) noted that the theory developed for glued-laminated beams does not predict the shear strength of dimension lumber. Recent literature has employed linear-elastic fracture mechanics theory to model such defects (Barrett 1981). Although this methodology can predict strength, it requires knowledge of defect size and location.

Barrett and Foschi (1977) considered wood beams containing end splits at beam center depth. The authors presented expressions for the stress intensity factor in regression equations involving crack length and beam span-to-depth ratio. Murphy (1979) considered the same problem and greatly simplified the expression for stress intensity factor by relating it to nominal resultant bending moment and shear force. In this manner, the analysis can be applied to checked beams under any loading conditions.

Murphy (1980) also considered beams containing side checks identical to those tested by Newlin et al. (1934). Using fracture mechanics theory, Murphy predicted strength results that agreed with those measured by Newlin et al.

Summary

Current shear design technology in the United States for lumber or glued-laminated beams is confusing. In the United States, a designer usually applies elementary beam theory on a tacitly assumed unchecked beam. But for both checked and unchecked beams that fail to qualify under the elementary beam theory equation, the designer may apply the Two-Beam Theory. Glued- laminated beams, however, are designed on the basis of an unchecked beam. Theories exist for both checked and unchecked beams, but they have only been verified for limited load conditions and l/d ratios.

Shear strength is difficult to measure because a pure shear stress condition is difficult to attain. Shear strength is dependent on beam size, but this relationship has not been adequately verified for inclusion in current design.

Future research is required to define the effects of beam size, load configuration, checks, and combined stresses on shear design. Existing theories should be verified over a broader range of variables.

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