

# Additional results for perpendicular distance sampling

Michael S. Williams, Harry T. Valentine, Jeffrey H. Gove, and Mark J. Ducey

**Abstract:** Over the last decade a number of new methods have been proposed to sample coarse woody debris. Of the new methods, both field trials and computer simulations suggest that perpendicular distance sampling is often the most efficient method for estimating the volume and surface area of coarse woody debris. As with any new sampling technique, further research and field testing are required to address some of the practical problems associated with the implementation of perpendicular distance sampling. This paper provides further results associated with the sampling of curved and multitemmed logs and field techniques for both slope correction and the measurement of elevated logs.

**Résumé:** Au cours de la dernière décennie, plusieurs méthodes nouvelles ont été proposées pour échantillonner les débris ligneux grossiers. Parmi ces nouvelles méthodes, tant les essais sur le terrain que les simulations par ordinateur indiquent que l'échantillonnage à distance perpendiculaire est souvent la méthode la plus efficace pour estimer le volume et la superficie des débris ligneux grossiers. Comme c'est le cas avec n'importe quelle nouvelle méthode d'échantillonnage, d'autres travaux de recherche et essais sur le terrain sont nécessaires pour trouver une solution à certains des problèmes pratiques associés à l'implantation de l'échantillonnage à distance perpendiculaire. Cet article apporte d'autres résultats concernant l'échantillonnage des billes courbes ou comportant plusieurs tiges ainsi que les méthodes permettant de corriger pour la pente et de mesurer des billes surélevées sur le terrain.

[Traduit par la Rédaction]

## Introduction

In the past decade there has been a substantial amount of interest in efficient methods for collecting information on coarse woody debris (CWD) in forest ecosystems (Ståhl et al. 2001). CWD is of current interest because it serves as habitat for many plant and animal species, a carbon sink for the study of global climate change, and a fuel source for forest fires.

Estimation techniques for CWD can be divided into two categories. The first category comprises techniques for estimating an attribute for a single log. For example, one might wish to determine the surface area of an individual log to evaluate the area of microhabitat it provides for some organism. The second category comprises sampling techniques for estimating the total or mean of an attribute over an area. For example, a survey could be performed to either assess the total volume of logs that meet the definition of 1000-h fuels (Rothermel 1972) or to estimate the total carbon budget for eastern Canada. None of the existing sampling techniques for estimating CWD volume perform this type of analysis directly, so additional information is collected to group sampled logs into specific classes (e.g., size, habitat,

fuel class, decay status), and models are used to convert the volume estimate to total biomass or carbon (Waddell 2002).

Williams and Gove (2003) proposed a new method, referred to as perpendicular distance sampling (PDS), for assessing CWD volume. Theoretical and simulation results show that when estimating CWD volume, the PDS estimator is design unbiased and generally has the smallest variance of all the current sampling strategies. Field testing of PDS in the northeastern United States has shown that PDS is substantially more efficient than line intersect sampling, with PDS field crews achieving the precision of line intersect sampling in one-half to one-sixth the time (M.J. Ducey, S. Roberge, M.S. Williams, and J.H. Gove, unpublished data). Williams et al. (2005) extend PDS to the assessment of surface area for CWD with similar results and suggest using fixed-area sampling to estimate the number of logs.

As with any new forest sampling technique, additional details must be worked out to account for the problems encountered in the field. Williams and Gove (2003) discuss some of these issues, but four important issues were not carefully addressed:

- (1) Field techniques for estimating the cross-sectional area of a log.
- (2) Field techniques for curved logs.
- (3) Sampling and estimation methods for multitemmed logs.
- (4) Field techniques for inclined logs and slope correction.

The purpose of this note is to propose solutions to these issues.

## Perpendicular distance sampling

The sampling and estimation of the volume and surface area of CWD with PDS can be described in a number of different

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**M.S. Williams.**<sup>1</sup> Mathematical Statistician, Rocky Mountain Research Station, USDA Forest Service, 2150 A Centre Avenue, Suite 361, Fort Collins, CO 80526, USA.

**H.T. Valentine and J.H. Gove.** Research Forester, USDA Forest Service, Northeastern Research Station, P.O. Box 640, Durham, NH 03824, USA.

**M.J. Ducey.** University of New Hampshire, Department of Forestry, Durham, NH 03824, USA.

<sup>1</sup> Corresponding author (e-mail: [mswilliams@fs.fed.us](mailto:mswilliams@fs.fed.us)).

ways. A summary is provided here for the estimation of volume. The premise of PDS is that a log-centered inclusion zone can be created about each piece of CWD such that the area of the inclusion zone is equal to the actual volume of the log times a constant multiplier ( $2K$ ). The constant  $K$  can be viewed as a "volume factor", and it can be chosen so that each tallied piece of CWD represents a convenient quantity. For example,  $K$  can be chosen so that each tallied log represents a volume of  $80 \text{ m}^3 \cdot \text{ha}^{-1}$ .

The advantage of PDS is that pieces of CWD are selected with probability proportional to their actual volume and that measurements are required only for "borderline" logs.

We consider a tract of land  $\mathcal{A}$  with horizontal area  $|\mathcal{A}|$ . Of interest is the population of  $N$  pieces of fallen CWD within the boundaries of the tract. The attribute of interest ( $Z$ ) is the aggregate volume of these pieces of CWD. For convenience, we shall call an individual piece of CWD a "log".

PDS protocols define an inclusion zone for each log. The land area of each inclusion zone is proportional to the volume of the log. Figure 1 illustrates a series of inclusion zones for different values of  $K$ . Each log has a central axis of length  $H$ . The width of the inclusion zone at any point  $h$  ( $0 \leq h \leq H$ ) along the central axis is  $2Kg(h)$ , where  $g(h)$  is the cross-sectional area perpendicular to the central axis. In practice, when a sample point falls near a log, one sights on the log along a line perpendicular to the log axis, and having measured (or, in many cases, estimated)  $g(h)$ , one compares the distance to the log to a table or chart of  $Kg(h)$ . If the actual distance is less than  $Kg(h)$ , the log is tallied. Note that in many cases ocular estimation of  $g(h)$  and the distance is adequate to determine accurately whether the log should be included from a sample point; measurement is required when the log is "borderline" and its inclusion is doubtful. The land area covered by the inclusion zone is

$$2K \int_0^H g(h) dh = 2KV$$

This inclusion area is proportional to the volume of the log ( $V$ ) because

$$[1] \quad V = \int_0^H g(h) dh$$

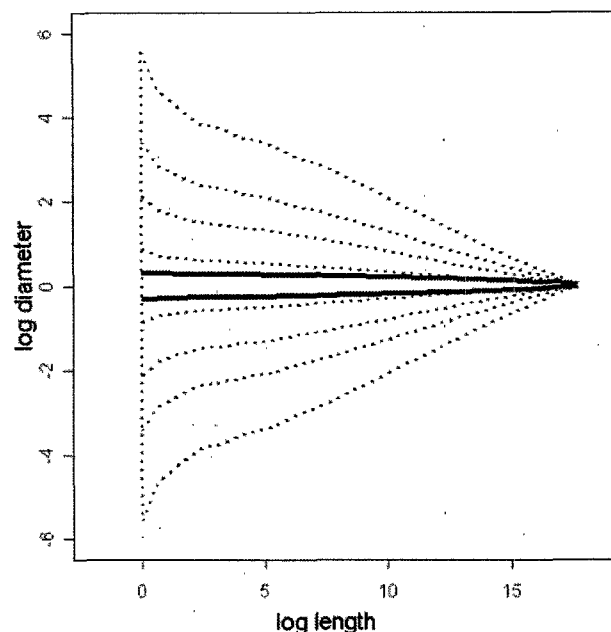
is the true volume of the log.

In a field application of PDS, the steps for determining when a point falls within the inclusion zone are

- (1) Select a sample point  $j$  ( $j = 1, 2, \dots, m$ ) at random within  $\mathcal{A}$ .
- (2) Measure the perpendicular distance, denoted by  $d_\perp$ , from the sample point to the central axis of a log.
- (3) Select the log if  $d_\perp \leq Kg$ , where  $g$  is the cross-sectional area of the log at the point of measurement.

Field trials in the northeastern United States indicate that CWD volume can be assessed in slightly less than 1 min per point using a logger's tape, calipers, and a table of limiting distances based on log diameter (M.J. Ducey, S. Roberge, M.S. Williams, and J.H. Gove, unpublished data). One factor that greatly reduces the amount of time required to implement PDS

Fig. 1. An overhead view illustrating a log and its inclusion zones based on perpendicular distance sampling. The outline of the log is depicted by the solid lines. The four different inclusion zones are based on  $K = 10, 25, 40, 65$  (dotted lines).



is that experienced field crews can visually assess the diameter and perpendicular distance to candidate logs. Thus, only a small number of "borderline" logs require a measurement of diameter and distance.

When the buffering approach is used to correct for boundary overlap (Ducey et al. 2004), a Horvitz-Thompson estimator is design unbiased for CWD volume, viz.,

$$[2] \quad \begin{aligned} \hat{Z}_j &= \sum_{i=1}^{n_j} \frac{V_i}{\pi_i} \\ &= \sum_{i=1}^{n_j} \frac{|\mathcal{A}| V_i}{2K V_i} \\ &= \frac{|\mathcal{A}|}{2K} n_j \end{aligned}$$

where  $\pi_i$  is the inclusion probability of log  $i$ ,  $|\mathcal{A}|$  is the area of the stand being assessed,  $V_i$  is the volume of log  $i$ , and  $n_j$  is the number of logs tallied at the  $j$ th sample point. Williams and Gove (2003) and Ducey et al. (2004) discuss boundary correction techniques for PDS.

In practice,  $m$  points are randomly located within  $\mathcal{A}$ , so the estimator of total volume is

$$\hat{Z} = \frac{1}{m} \sum_{j=1}^m \hat{Z}_j$$

where  $\hat{Z}_j$  is the estimator for sample point  $j = 1, \dots, m$ . Let  $\hat{Z}(x, y)$  be an estimate obtained at any sample point with coor-

ordinates  $(x, y)$  in  $\mathcal{A}$ . The variance of the estimator is

$$[3] \quad \text{Var}[\hat{Z}] = \frac{1}{m|\mathcal{A}|} \iint_{\mathcal{A}} (\hat{Z}(x, y) - Z)^2 dx dy$$

When the number of points is fixed, the design-unbiased estimator of the variance is

$$[4] \quad v[\hat{Z}] = \frac{1}{m(m-1)} \sum_{j=1}^m (\hat{Z}_j - \hat{Z})^2$$

## Correction techniques

Williams and Gove (2003) and Williams et al. (2005) originally developed PDS assuming circular, straight, single-stemmed logs. While these assumptions are often used in many other applications (e.g., taper and volume equations, variable-radius plot sampling), they are unlikely to be true in practice. The first section provides techniques for noncircular logs. This is followed by a correction technique to address the problem of curved logs. Field techniques for multistemmed logs are not as straightforward because the method of correction depends on how CWD is defined, so two different techniques are proposed.

Two common assumptions are that logs either lay flat on the ground or that the area of interest is a flat plane. If either of these assumptions fail to hold, the vertical cross-section of the stem is most likely not circular. The basis for the proposed corrections is to note that while the volume of a straight, circular log oriented along an “ $x$ -axis” is

$$[5] \quad V = \int_0^H \frac{\pi}{4} d(x)^2 dx$$

the volume for any solid can be determined by

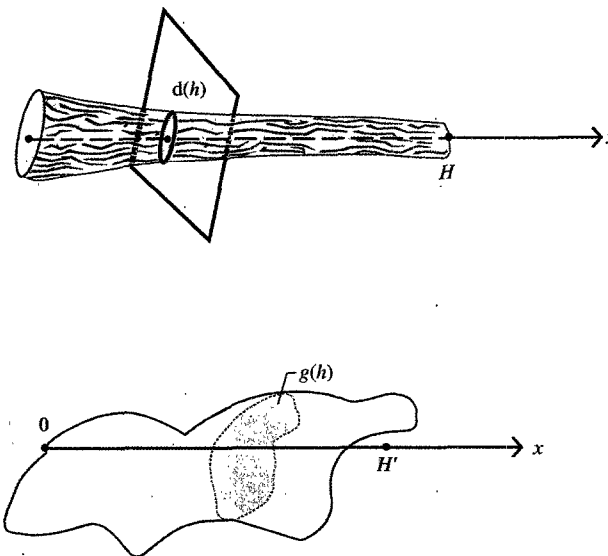
$$[6] \quad V = \int_0^{H'} g(x) dx$$

where  $g(x)$  is the cross-sectional area at any point and  $H'$  is the total length of an object projected onto the  $x$ -axis (Swokowski 1979, p. 295). This is illustrated in Fig. 2.

## Noncircular logs

The practical problem with many of the correction techniques is that they require a measurement of cross-sectional area at any angle across the log. This requires the field crew to estimate the area of a long, elliptical cross-section in some cases. It can be very difficult to accurately determine the true cross-sectional area when the measurement is made perpendicular to the central axis of the log, much less an inclined log. One option to simplify the estimation of the cross-section would be to assume the cross-sectional area is adequately estimated by assuming that the log can be modeled by a frustum of a cone or other solid. However, this assumption would be suspect in light of the results of Matern and others (1956, 1990 and reference therein), who studied the suitability of assuming that the cross-section of a bole could be modeled by an ellipse. As Matern states, “...the convex closure of a stem differs from circular form in what can only be described as “irregular”.”

**Fig. 2.** The volume of a log is often calculated assuming the straight cylindrical log (top). However, the volume of any solid can be calculated by integrating the cross-sectional area along the length of the object (bottom).



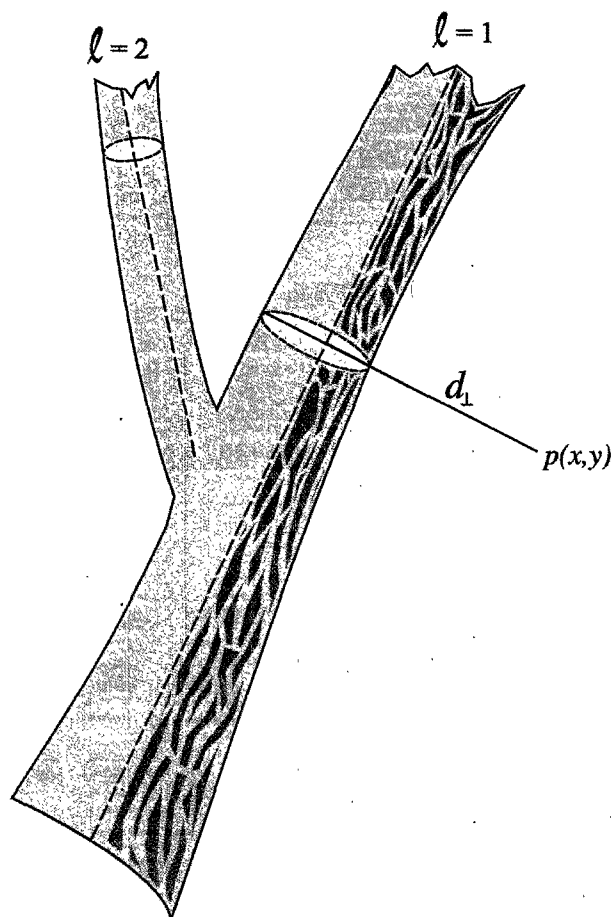
Matern (1956, 1990) suggests that the mean of multiple diameter measurements can be used to accurately assess the convex closure of the cross-section. Thus, a reasonable solution would be to take multiple caliper measurements of the log and average the measurements. Another option is to measure the diameter with a tape, which is the expectation of diameter measured with a caliper. These measurements can be difficult, time consuming, and prone to measurement error. However, it should be noted that all of the sampling techniques for CWD would require many more of these same measurements for every log, rather than just for borderline logs.

## Curved logs

PDS was originally developed assuming straight logs, which is unlikely to be true in practice. The proposed solution is based on the notion of defining the central axis of each curved log to be a needle, which is also the solution proposed by de Vries (1986, p. 249) for line intersect sampling. One way to view the correction technique is that it transforms a curved log into a shorter and “fatter” straight log whose volume is equal to the volume of the curved log. This needle must be uniquely defined for each log in order for the geometrically defined inclusion probability to hold.

While there are a number of possible solutions, the one proposed is identical to the one given by de Vries (1986, p. 249), where the needle connects the ends of each log. The most accurate way to implement this method is to stretch a tape between range poles erected at each end of the needle (i.e., use the pith at the large end, and the most distant terminal apex, to define the needle). The taut tape between the range poles defines the straight, central axis. Then the perpendicular distance ( $d_{\perp}$ ) is measured to the taut tape, rather than the midpoint of the log, with the cross-sectional area being measured perpendicular to the tape. Logs that curve away from the sample point ( $p(x, y)$ )

**Fig. 3.** A multistemmed log treated as two pieces of CWD. In this case, each stem is sampled independently and the measurements of cross-sectional area are taken perpendicular to the needles defining each central axis.



and meet the condition of  $d_{\perp} \leq Kg$  do not require the definition of the needle because the log is obviously “in” the sample when the cross-sectional area is “moved” to the needle. Borderline logs that curve toward the sample point will require a measurement from the sample point to the needle to ensure that  $d_{\perp} \leq Kg$ .

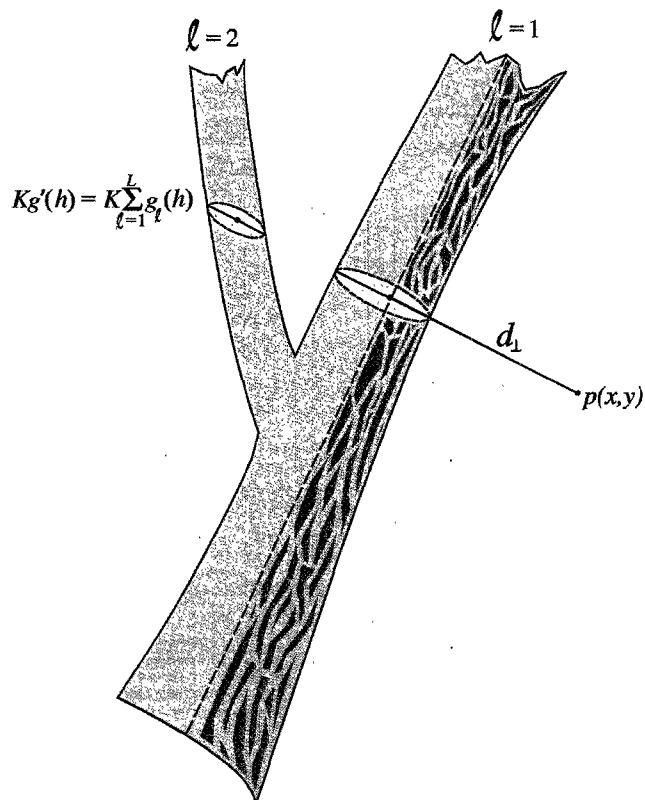
The only concern with this approach is that it may require two people to implement, and if a log has an extreme degree of curvature (i.e. J-shaped), there can be some volume that extends past the end on the needle. If this happens, the estimator is no longer unbiased because the area of the inclusion zone is no longer proportional to the volume.

#### Multistemmed logs

To describe the solution for multistemmed logs, let the  $l = 1, 2, \dots, L$  index the individual stems contained in a multistemmed log.

Williams and Gove (2003) provide a brief discussion of the problem of sampling multistemmed logs. The solution proposed was the following:

**Fig. 4.** A multistemmed log can be treated as a single piece of CWD. In this case, the measurements of cross-sectional area are summed for each stem and cross-sectional area is taken perpendicular to the needle defining the central axis of the main stem.



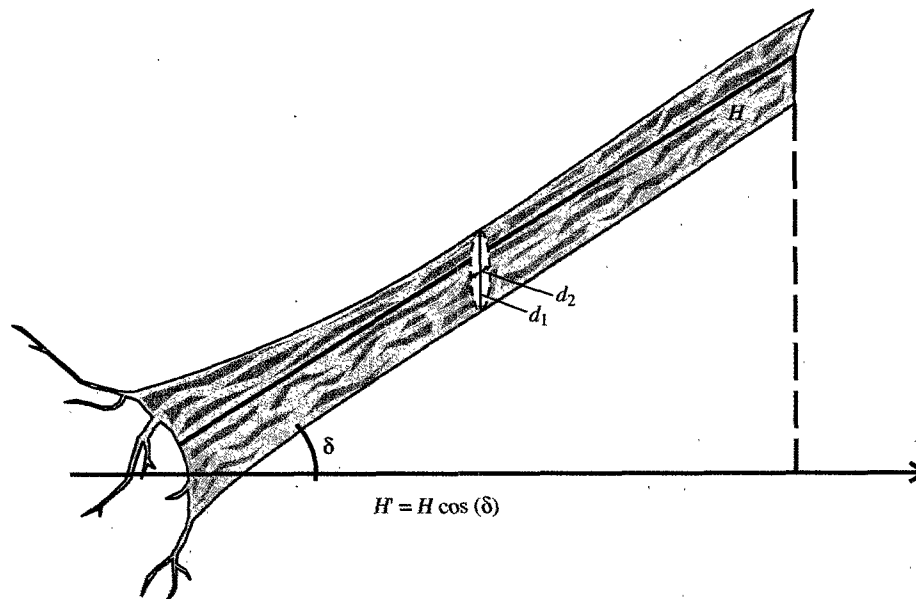
The solution for PDS is straightforward and views a multistemmed log as a collection of internodes or segments. Some concerns about this approach are determining consistent rules for proportioning volume to each segment at the location where the fork occurs and the difficulty of determining cross-sectional area at the fork. However, it should be noted that none of the methods for sampling CWD are particularly adept at handling this situation.

This is depicted in Fig. 3, where the branches of a two-stemmed log are viewed as two separate logs. Note that for the point  $p(x, y)$ , the smaller stem is not tallied in this example.

While dividing a log into segments is one possible solution, another is to treat a multistemmed log as what it really is: a single multistemmed log. In this case, we measure and sum the cross-sectional areas of all the stems perpendicular to the central axis. The central axis is defined exactly as in the case of a curved log. In this situation the cross-sectional area for each stem is summed using

$$g'(h) = \sum_{l=1}^L g(h)_l$$

**Fig. 5.** The inclination adjustment changes the orientation of the measurement of the cross-sectional area to be aligned with the vertical axis.



where  $g(h)_l$  is the cross-sectional area of stem  $l$  at the point  $h$ ,  $0 \leq h \leq H'$  on the central axis and  $H'$  is the horizontal length of the central axis. The log is selected if  $d_{\perp} \leq K g'(h)$ . This is depicted in Fig. 4. Note that the orientation of the measurement of the cross-sectional area on each stem differs between the two approaches, and it is always perpendicular to the central axis of the log.

The choice between these two approaches depends on the definition of CWD. However, there is no generally accepted definition of CWD (Helms 1998). The first option is appropriate if each stem of a multistemmed log represents a piece of CWD. The second option is appropriate in situations where a multistemmed log only represents a single piece of CWD.

#### Corrections for slope and inclined logs

One of the common assumptions used when deriving results for CWD sampling techniques is that logs lie flat on the ground. This assumption is rarely tenable given that it is common to find logs on steep slopes or situations where one end of a log will be elevated by an obstacle such as another tree or boulder. Note that there are two different components that need to be addressed. The first is the perpendicular distance from the sample point to the log ( $d_{\perp}$ ). The adjustment of this distance will be referred to as the "slope correction", whose angle will be denoted by  $\theta$ . The second is an adjustment to account for a situation when the elevations of the two ends of a log differ, which will be referred to as the "inclination correction" and whose angle will be denoted by  $\delta$ .

When the logs to be sampled lie on a flat plane, both  $\theta = 0$  and  $\delta = 0$ . These two corrections are necessary so that when the inclusion zone of the log is projected onto a flat plane, the resulting area of the inclusion zone is  $2KV$ , regardless of the slope of the ground or inclination of the log.

Slope correction for PDS is similar to the methods used for

other areal sampling techniques. If the sample location is located above or below the log with an angle  $\theta$  and the distance along the slope is  $d_{\text{slope}}$ , then the slope-corrected perpendicular distance is given by  $d_{\perp} = d_{\text{slope}} \cos(\theta)$ . Thus, logs whose slope distance appears to exceed  $Kg$  may actually be within the inclusion zone after the slope correction. When the slope is expressed as a percentage (i.e.,  $\theta_{\%}$ ), the correction distance is  $d_{\perp} = \sqrt{1 + (\theta_{\%}/100)^2} d_{\text{slope}}$ . General guidelines for when to include slope correction to avoid a significant bias in the estimator range from about 10% (Shiver and Borders 1996, p. 91) to 15% (Avery and Burkhart 2002, p. 240). The magnitude of the bias is determined by the degree to which  $\cos(\theta)$  differs from 1. However, it seems unlikely that reasonable bounds on the bias could be constructed without making additional assumptions, such as the spatial distribution of the size and number of logs being independent of changes in the slope across  $A$ .

The solution to the inclination correction problem for PDS is quite simple and is accomplished by measuring the vertical cross-sectional area. The vertical cross-sectional area can be approximated by calipering the bole vertically ( $d_1$ ) and "on the slant" ( $d_2$ ) and calculating  $\pi d_1 d_2 / 4$ , which is the vertical cross-sectional area, assuming circularity along the slanted axis. Here, the vertical measurement is not necessarily perpendicular to the slanted log axis, but is perpendicular to the ground surface. The measurement "on the slant" is taken horizontally and pierces the slanted axis of the log along the perpendicular line-of-sight from the sample point. In other words, the cross-sectional area of the log is being measured in the plane defined by the line of sight perpendicular to the log axis and by the geometric vertical axis. The motivation for this solution is to note that the volume of any object is given by eq. [6]. Assume that the long axis of the log falls along the  $x$ -axis in the usual Cartesian coordinate system, but that it is inclined at an angle  $\delta$ . The length of the vertical projection of the log is  $H' = H \cos(\delta)$ . If the measurement of

the cross-sectional area is taken in the plane defined by the y- and z-axes, the resulting inclusion area projected onto the flat plane will have area  $2KV$  (Fig. 5).

## Conclusion

Adapting PDS to account for sampling real populations is fairly straightforward. One way of looking at the correction techniques is that in every case the attributes of the log and its inclusion zone are always projected onto one or more of the axes of the Cartesian coordinate system. While some of these correction techniques can be time consuming, it should be noted that any sampling technique for CWD requires many more of the same measurements for every log, rather than just for borderline logs. The elimination of measurements on the majority of logs is what makes PDS appealing and efficient.

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