

Assessing surface area of coarse woody debris with line intersect and perpendicular distance sampling

Michael S. Williams, Mark J. Ducey, and Jeffrey H. Gove

Abstract: Coarse woody debris (CWD) plays an important role in many terrestrial and aquatic ecosystem processes. In recent years, a number of new methods have been proposed to sample CWD. Of these methods, perpendicular distance sampling (PDS) is one of the most efficient methods for estimating CWD volume in terms of both estimator variance and field effort. This study extends the results for PDS to the estimation of the surface area of CWD. The PDS estimator is also compared to two line intersect sampling (LIS) estimators, where one of the LIS estimators requires the measurement of surface area on each log and the other estimates surface area using a single measurement of log circumference at the point of intersection between the log and the line. The first estimator approximates the true surface area by assuming either a conic or parabolic stem form and requires measurements of the end diameters of each log, which is more time consuming than a single measurement. The performance of the three estimators was compared using a computer simulation. The results of the simulation indicate that, given the same number of pieces of CWD sampled at each point, equal variances can be achieved with PDS using sample sizes that range from about 10% to in excess of 100% the size of a comparable LIS estimator. When the LIS estimators were compared, the estimator that required the measurement of surface area was only about 3%–6% more efficient than the alternative estimator, but the bias associated with assuming a conic or parabolic stem form ranged from roughly 5% to 15%. We conclude that PDS will generally outperform either of the LIS estimators. Another important conclusion is that the LIS estimator based on a measured surface area is likely to have a higher mean squared error than an LIS estimator that employs a single measurement of circumference. Thus, LIS sampling strategies that require the least amount of field work will often have the smallest mean square error.

Résumé : Les débris ligneux grossiers (DLG) jouent un rôle important dans plusieurs processus écosystémiques terrestres et aquatiques. Plusieurs nouvelles méthodes ont récemment été proposées pour échantillonner les DLG. Parmi ces méthodes, l'échantillonnage à distance perpendiculaire (EDP) est l'une des méthodes les plus efficaces pour estimer le volume des DLG, tant du point de vue de la variance de l'estimateur que de l'effort requis sur le terrain. Cet article élargit la portée des résultats de l'EDP pour estimer la superficie des DLG. L'estimateur de l'EDP est aussi comparé à deux estimateurs d'échantillonnage par ligne d'intersection (ELI), où un des estimateurs de l'ELI requiert la mesure de la superficie de chaque bille et l'autre évalue la superficie à partir d'une seule mesure de la circonférence de la bille au point d'intersection de la ligne avec la bille. Le premier estimateur fournit une approximation de la vraie superficie en assumant que la forme de la tige est soit conique, soit parabolique, et nécessite la mesure du diamètre aux extrémités de chaque bille, ce qui prend plus de temps qu'une seule mesure. La performance des trois estimateurs a été comparée à l'aide d'une simulation par ordinateur. Si le même nombre de tronçons de DLG est échantillonné à chaque point, les résultats de la simulation indiquent que l'EDP permet d'obtenir des variances égales avec une taille d'échantillons qui varie de 10 % à plus de 100 % de la taille d'un estimateur comparable de l'ELI. Lorsqu'on compare les estimateurs de l'ELI, l'estimateur qui requiert la mesure de la superficie est seulement 3 % à 6 % plus efficace que l'autre estimateur mais le biais associé au fait d'assumer que la tige a une forme conique ou parabolique varie en gros de 5 % à 15 %. Les auteurs concluent que l'EDP donnera généralement de meilleurs résultats que l'un ou l'autre des estimateurs de l'ELI. Une autre conclusion importante est le fait que l'estimateur de l'ELI qui requiert la mesure de la superficie risque d'avoir un écart-type plus grand qu'un estimateur de l'ELI qui utilise une seule mesure de circonférence. Par conséquent, les stratégies d'inventaire par ELI qui exigent le moins de travail sur le terrain auront souvent le plus faible écart-type.

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Introduction

The past decade has seen substantial interest in the role of woody debris (CWD) in virtually every ecosystem process (Spies et al. 1988). Given the broad range of interest in CWD, many different attributes must be estimated for each application. For example, CWD volume must be estimated to assess carbon budgets. Another important attribute for CWD is surface area. In fact, many different types of surface area are of interest. Examples of studies related to different types of surface area are

- (1) Rubino and McCarthy (2003) identified the aboveground surface area of CWD as a key habitat component for macrofungal and myxomycete communities.
- (2) Pyron et al. (1999) studied the role of CWD surface area in relation to freshwater shrimp habitat.
- (3) Maschhoff and Dooley (2001) discussed properties of manufactured CWD. They note that a high degree of surface roughness and a high surface area to volume ratio are key design parameters.
- (4) Ralph et al. (1994), Beechie and Sibley (1997), and Bailie and Davies (2002) studied the relationship between channel morphology and CWD.
- (4) O'Hanlon-Manners and Kotanen (2003) studied the importance of nurse logs in protecting seeds from pathogenic soil fungi. While there is no standard definition of what constitutes a nurse log, only the upper portion of a piece of CWD can realistically serve as a nurse log (Fig. 1).

Estimation techniques for CWD fall into two categories, depending on the application. The first category comprises techniques for estimating an attribute, such as surface area or volume, for individual logs. For example, we might wish to determine the surface area of an individual log to evaluate the area of microhabitat it provides for some organism. The second category comprises sampling techniques for estimating the total or mean of an attribute over an area. For example, a survey could be performed to assess the total amount of carbon stored in CWD in a single stand or over the entire western hemisphere.

Numerous techniques have been proposed for estimating the surface area of a standing bole or a piece of CWD. Many studies assume that a stem is adequately modeled by a geometric solid, with the form of this solid being based on either a circular cone (e.g., Swank and Schreuder 1974; Rubino and McCarthy 2003) or a cylindrical or paraboloidal stem shape (Lescaffette 1951; Husch et al. 1982, p. 88). The concern with these methods is the difficulty in acquiring the necessary measurements and the potential for these estimates to be biased when the stems differ significantly from the assumed shape. An alternative approach to the direct measurement of each stem is to estimate surface area using various Monte Carlo integration techniques. Examples of this approach include the use of randomized branch sampling, importance sampling (Gregoire et al. 1993, 1995; Gregoire and Valentine 1996; Lynch 2002), and critical height sampling (Lynch 1986).

Of the various sampling techniques for CWD, line intersect sampling (LIS) is certainly the most flexible in its ability to assess many attributes associated with CWD. One of its primary

advantages is that many attributes can be estimated without actually measuring the specific attribute (e.g., the volume of CWD can be estimated with only a measurement of cross-sectional area on individual pieces). One drawback of LIS is that a substantial field effort is usually required to achieve estimates with high precision. For example, Pickford and Hazard (1978) recommend total line lengths of at least 1 km. Surveys with lines of this length are often impractical, particularly in aquatic surveys.

Estimating the surface area of a log or tree bole poses some unique challenges not found in the estimation of other attributes such as the estimation of volume, biomass, length, and number of logs. The first consideration is that the surface of a bole is often covered by bark, which can be heavily furrowed. These concavities in the surface are very difficult to measure, so this component of the surface area is usually ignored in most forestry and ecological applications, as will be the case in this study.

An additional concern, which makes the estimation of surface area unique, is that it is not physically possible to take the necessary measurements. The easiest way to illustrate this concern is to contrast the calculation of surface area with the calculation of the volume of a tree bole. To illustrate, assume a straight and circular tree bole of length H , with its diameter adequately explained by the taper function $d(h)$. The volume of the bole is calculated by integrating the cross-sectional area ($\pi r^2 = \pi d(h)^2/4$) along all points $0 \leq h \leq H$. Thus,

$$[1] \quad V = \int_0^H \pi \frac{d(h)^2}{4} dh$$

where V denotes the total volume of the bole. Following this same reasoning, it would seem logical that the surface area of the bole could be derived by integrating the circumference of the bole, given by $2\pi d(h)/2$, along the length. Thus, intuitively, the formula for surface area would be given by

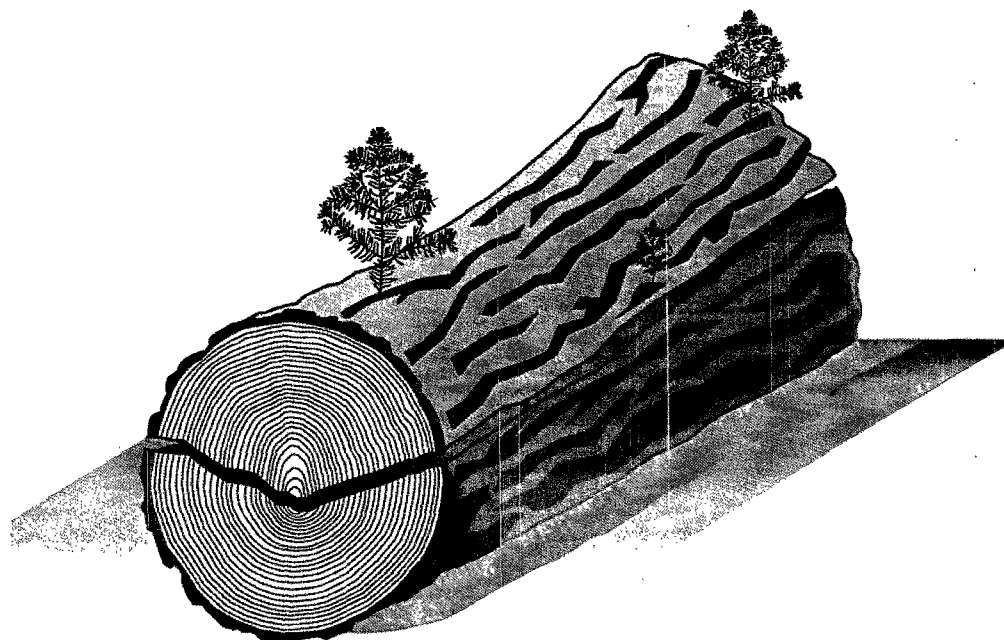
$$[2] \quad S = \int_0^H 2\pi \frac{d(h)}{2} dh$$

where S denotes the surface area. However, this is incorrect because rather than integrating the circumference along the length of the bole, the appropriate formula integrates the arc length of $d(h)$ (Swokowski 1979, p. 643). Thus, the correct formulation of the integral is

$$[3] \quad S = \int_0^H \pi d(h) \sqrt{1 + \frac{d'(h)^2}{4}} dh$$

where $d'(h)$ is the rate of change in the taper of the log with respect to length (or height for standing trees). Equations 2 and 3 differ by the term $\sqrt{1 + d'(h)^2/4}$, which will be referred to as the "differential". This term is greater than or equal to one at all points along the bole. Thus, ignoring this term causes an estimator of surface area to be biased downward (i.e., an underestimate of the true surface area). This bias will exist in almost every estimation technique because it would be nearly impossible to measure the arc length in the field with any degree of accuracy. This problem has been discussed by Lynch (1986, 2002), who approximated the error associated with ignoring $\sqrt{1 + d'(h)^2/4}$ using a cone frustum and a rate of taper for the diameter of 0.39 cm (1 in.) every 2.4 m (8 ft). Given this rate of

Fig. 1. Example of a nurse log. Note that only the upper portion of the log is such that it could “catch” a seed. The amount of suitable surface area will be at least partially determined by the number and depth of the furrows, which is determined by the roughness of the bark.



taper, the differential term is $\sqrt{1 + d'(h)^2/4} = 1.0000136$, so the bias incurred is most likely small in applications where the differential term contains the rate of change in diameter with respect to height (i.e., the rate of taper for a log is “slow”). However, the bias can be substantial in any application that involves the rate of change in height with respect to diameter ($h'(d)$) because a small change in diameter can occur over a great length. An application where the bias would be substantial is the estimation of surface area of standing trees using critical height sampling and employing the method of cylindrical shells (Lynch 1986).

Williams and Gove (2003) proposed a new method for assessing CWD volume, referred to as perpendicular distance sampling (PDS). Theoretical and simulation results show that the PDS estimator of volume is design-unbiased and, given an equal sampling effort, it is likely to have the smallest variance of all the current sampling strategies that satisfy the condition of being design-unbiased. Field testing of PDS in the northeastern and central United States suggest that the sampling effort for estimating CWD volume can be anywhere from two to six times less than competing methods (M.J. Ducey, unpublished data).

Williams and Gove (2003) allude to the possibility of estimating CWD surface area with PDS, but fail to provide sufficient detail. Thus, the goals of this study are

- (1) Develop estimators of CWD surface area using PDS.
- (2) Quantify the magnitude of the bias associated with ignoring the term $\sqrt{1 + d'(h)^2/4}$ in an estimator of surface area.
- (3) Compare the performance of PDS to sampling strategies based on line intersect sampling.

Data description

The differential term given in eq. 3 poses a number of unique problems. One problem is that data sets that describe the true surface area for pieces of CWD cannot realistically be collected because measuring the instantaneous change in taper ($d'(h)$) at all points of a log would be impossible. Another problem is that the form and taper of a piece of CWD is likely to be much less consistent than that of a standing tree. This would be due to breakage of the stem and the change in form due to decay and the resultant loss of structure. For these reasons, it is assumed that the potential bias associated with the differential term is probably much larger for CWD than for standing trees. To address these problems, a modified version of the methodology used in Williams and Gove (2003) was used to create an artificial data set that contained pieces of CWD with a known rate of taper at all points on the piece. This was accomplished by starting with a taper equation for standing trees and modifying it to produce a new log taper equation that differed for every log. The size and form of each log was loosely based on real data.

The original data comprised $N = 183$ ponderosa pine (*Pinus ponderosa* Dougl. ex Laws.) trees. For each tree, diameter outside bark d and length h measurements were taken every 1.22 m (4 ft) on felled trees collected at various timber sale sites on the Black Hills National Forest, USA. A total of 2010 diameter and height measurements were taken, with most trees having between 3 and 10 sections. Diameter at breast height D (i.e., 1.37 m (4.5 ft) above the ground) and total height H from a 0.305-m (1-ft) stump to the tip were also recorded for each tree.

For this study, the diameter and rate of change in taper were needed at every point on the stem as well as the true surface area.

Individual taper functions could not be fit to each tree because many of the smaller trees had only three to five measurements. To address this problem, a single taper function was first fitted to the data. To create a unique taper equation for each of the 183 logs, an individual taper function for each log was created using the following method. From the original taper function, a set of 30 (d, h) values that represented the profile of an "average" tree was derived by using the mean D and H values from the data set. The next step was to create a new unique taper equation for each log. This was done by perturbing each d value with a random multiplicative error, where the errors were $\epsilon \sim \text{Uniform}(0.6, 1.4)$. A new taper equation was fitted to each log. The model used was

$$d(h)\epsilon = D \left(\hat{\beta}_1 H' + \hat{\beta}_2 H'^2 + \hat{\beta}_3 H'^3 + \hat{\beta}_4 H'^4 + \hat{\beta}_5 H'^5 + \hat{\beta}_6 H'^6 \right) + \epsilon'$$

where $H' = 1 - h/H$ and $\epsilon' \sim N(0, \sigma^2 \epsilon)$. The taper functions for each log in the data sets are displayed in Fig. 2a, where the taper of logs is far greater than would be observed on standing trees. The large amount of taper was used to show that the differential term can be ignored with little impact on bias.

CWD is defined as logs whose minimum diameter is greater than some lower limit. There is no standard definition for this lower limit, but it is often near 7.5 cm (≈ 3 in.), corresponding to the boundary between 100- and 1000-h fuels in fire behavior models (Rothermel 1972). The trees in the original data set differed from CWD because each tree tapered to a point. To create a log taper equation that more closely mimics CWD, the length of each tree was truncated by a random percentage. This percentage was derived by setting $H_{\text{trunc}} = \epsilon H$, where $\epsilon \sim \text{Beta}(3, 2)$. The final form of each log is displayed in Fig. 2b. Each taper function was used to calculate the true and approximate surface area of each tree using eqs. 2 and 3.

One of the goals of the study is to illustrate the effects of using a simple approximation to the true surface area. To meet this goal, the surface area was approximated by both a truncated right circular cone and a paraboloid, as suggested by Rubino and McCarthy (2003) and Husch et al. (1982, p. 88), respectively. When the form of the log follows a cone, the radius is described by $d(h)/2 = \beta_c h$, with the surface area given by

$$S_{\text{cone}} = \int_0^H 2\pi\beta_c h \sqrt{1 + \beta_c^2} dh$$

where $\beta_c = D_{\text{base}}/2H$ and D_{base} is the diameter of the log at the base. When the form of the log follows a paraboloid, the radius is described by $[d(h)/2]^2 = \beta_p h$, where $\beta_p = D_{\text{base}}^2/4H$ for each tree. Thus,

$$S_{\text{para}} = \int_0^H 2\pi\sqrt{\beta_p h} \sqrt{1 + \frac{\beta_p^2}{4h}} dh$$

Summary statistics for the populations are given in Table 1. For this data set, the surface area approximation based on the parabolic stem form is much closer to the true surface area than the approximation based on the assumed cone.

Once the set of logs was constructed, the next step was to arrange them as pieces of downed CWD within a forest stand

Table 1. Summary statistics for the ponderosa pine data set.

Attribute	Min.	Mean	Max.
D	0.18	0.28	0.56
D_{base}	0.19	0.32	0.68
H	1.2	10.7	22.0
S	1.0	8.2	27.6
S_{cone}	1.1	7.1	23.5
S_{para}	1.2	8.7	30.0

Note: All units are in metres. See text for variable definitions.

from which a sample could be drawn. A series of populations was created, with each population being a square with total area 1.44 ha. Each log was given a random orientation ϕ between 0 and 2π and then randomly located in accordance with a Uniform distribution. A buffer strip was then added along the boundary of the population to avoid the design bias associated with boundary overlap (Ducey et al. 2004). This creates some large-scale clustering in the distribution of logs at the center of the population. Whether this distribution of locations accurately depicts real log populations is unknown. Additional testing suggests that the difference in the performance of the estimators decreases as the population becomes more clustered. To illustrate, if all logs are placed into a single cluster (i.e., a slash pile), then the estimates derived at each sample point are either zero (i.e., the sample point missed the slash pile) or very close to the population total (i.e., the point falls in the slash pile and the majority of logs will be tallied).

One trait of downed CWD that could influence the performance of the estimators is the orientation of the logs. In steep terrain and areas with high winds, the orientation of the logs is unlikely to be random. To address this issue, the distribution of the ϕ angle was varied. The first data set, referred to as RAND, was created using a completely random orientation, with the distribution of ϕ being $\phi_{\text{uni}} \sim \text{Uniform}(0, \pi/2)$. The second data set was generated so that the orientation of the logs would have a consistent north-south pattern in their orientation. The angle of orientation for the north-south data set was calculated using $\phi = (1 - b) * \phi_{\text{uni}} + b * \phi_{9\pi/20}$, where $\phi_{9\pi/20} \sim \text{Uniform}(9\pi/20, \pi/2)$ and $b \sim \text{Bernoulli}(0.6)$. Using this distribution, roughly 80% of the logs are oriented within $8\pi/10$ radians (36°) of the y-axis, which is considered the direction of north-south orientation. This data set will be referred to as NorS. Figures 3a and 3b shows the distribution of the size and orientation of logs in the NorS and RAND data sets, respectively.

Adapting PDS to estimate surface area

While the mathematical expression for calculating surface area is somewhat complicated, the adaptation of PDS to estimating this attribute is straightforward. Historically, many sampling methods in forestry and natural resources were viewed as an extension of finite population sampling theory. However, over the last 10–15 years areal sampling strategies such as fixed-area and variable-radius plot (VRP) sampling, critical height, LIS, and PDS have been increasingly viewed as applications of infi-

Fig. 2. Part (a) illustrates the difference in taper for each log in the data set, where each log is scaled by relative diameter (d/DBH) and relative height diameter (h/H). Part (b) shows the actual length, diameter, and taper for each log. The logs were created by truncating the top of each log in (a).

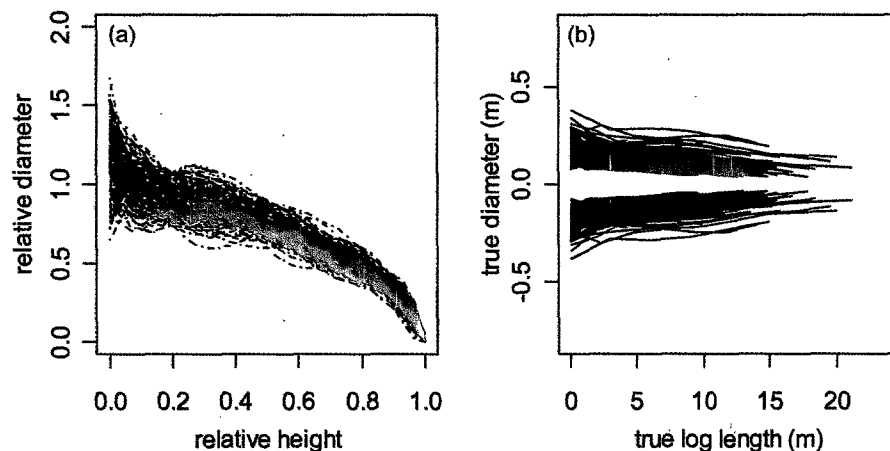
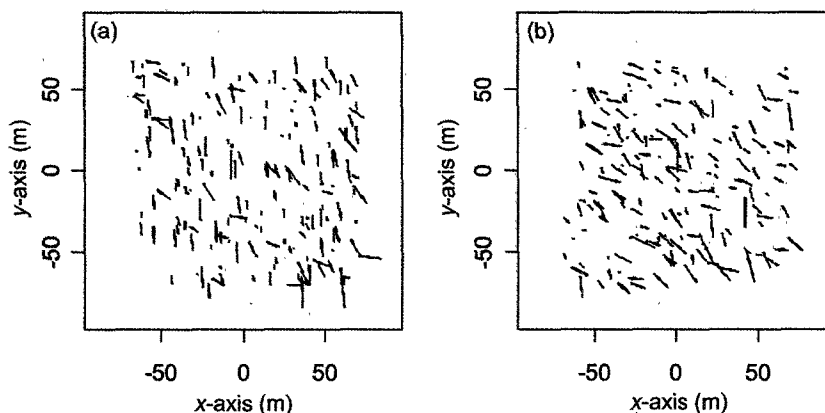


Fig. 3. Log orientations for the (a) NorS (north-south) and (b) RAND (random orientation) data sets. Note that the boundaries of the stand have been extended to create the buffer used to correct for sampling along the edge of the stand.



nite population sampling or Monte Carlo integration techniques (e.g., Eriksson 1995; Valentine et al. 2001; Williams 2001). The infinite population approach will be used in the following example to demonstrate how PDS could be used to estimate surface area.

Assume that a straight log is completely covered by bark and lies on a flat plane, denoted by A , with known area $|A|$ (Fig. 4). Let S be the surface area of the bark. Next, assume that the bark can be split along the central axis of the log, "peeled" away from the log, and laid flat on A . Thus, the area of the piece of bark that covers a portion of A is equal to S (Fig. 4). If sampling locations (points) are distributed uniformly over A , then the probability that a point falls on the peeled section of bark is $\pi = \frac{S}{|A|}$, which is the inclusion probability for the log based on its surface area. Thus, the peeled bark forms a log-centered plot (Husch et al. 1982, pp. 224–227) or "inclusion zone" about the central axis of the log, whose area is equal to the surface area of the log.

This example can be extended to N logs, each with surface area S_i ($i = 1, 2, \dots, N$), and inclusion probability $\pi_i =$

$S_i/|A|$. The estimator considered here is

$$\hat{Z} = \sum_{i=1}^{n(x,y)} \frac{S_i}{\pi_i} = \sum_{i=1}^{n(x,y)} S_i \frac{|A|}{S_i} = |A|n(x,y)$$

where S_i is the surface area for tree i , π_i is its probability of inclusion, and $n(x, y)$ is the number of pieces of CWD tallied at the point with coordinates (x, y) . The estimator $\hat{Z}$ is design-unbiased (Appendix A) when the possibility of pieces of bark extending beyond the boundary of A is ignored. Note that no measurements are required on the log because the inclusion probability is a function of the true surface area.

Some of the obvious problems with this example are that the bark does not lay flat on the ground and few if any logs would be selected in any sample because only a small portion of A would be covered by the peeled pieces of bark (i.e., the probability of the point falling on one of the pieces of peeled bark is too small to be of practical value). The solution to the first problem is to realize that a simple distance rule can be applied to determine

whether a point would fall in the area that would be covered by the peeled bark. Specifically, the algorithm for determining when a point falls within the inclusion zone is

- (1) Measure the perpendicular distance, denoted by $|d_{\perp}|$, from the sample location (x, y) to the central axis of the log.
- (2) Select the log if $|d_{\perp}| \leq c_{\text{circ}}/2$, where c_{circ} is the circumference of the log at the point of measurement.

The solution to the problem of too few logs being selected is to realize that the size of the inclusion zone can always be increased by a multiplicative factor, as in VRP sampling. This is illustrated in Fig. 4, where a series of inclusion zones are drawn about a log. The boundaries above and below the log are generated by multiplying the circumference of the log by a multiplicative factor K_{PDS} at every point along the log, where K_{PDS} is a constant similar to the basal area factor in VRP sampling (see Husch et al 1982, p. 228). For example, if $K_{\text{PDS}} = 2$ then the distance from the central axis of the log to the boundary of inclusion zone is two times the circumference of the log. Note that for the peeled bark example given previously, the log is selected when the perpendicular distance is $c_{\text{circ}}/2$, whereas in this case the limiting distance is $K_{\text{PDS}}c_{\text{circ}}$. This formulation of the problem is used to maintain consistency in the derivation of PDS for the estimation of both surface area and volume. Also note that the same units of measure are used for the circumference and distance from the log. If the circumference is

measured in centimetres, the limiting distance can be expressed in metres by dividing K_{PDS} by 100.

When PDS is used in the estimation of CWD volume, each inclusion zone has area $2K_{\text{PDS}}V$, where V is the true volume of the log and $2K_{\text{PDS}}$ is a constant multiplier (Williams and Gove 2003). Intuitively, when adapting PDS to the assessment of surface area, the area of each inclusion zone would be $2K_{\text{PDS}}S$. Once again, the differential term poses a problem. The source of the problem is illustrated in Fig. 4, where it should be noted that the slope of the line (i.e., $d'(h)$) defining the inclusion zone increases with increasing K_{PDS} values. This is a source of bias because the differential in the surface area integral is such that the inclusion zone for each log has area

$$[4] \quad a_{\text{PDS}} = 4\pi K_{\text{PDS}} \int_0^H d(h) \sqrt{1 + K_{\text{PDS}}^2 d'(h)^2} dh \neq 2K_{\text{PDS}}S$$

because

$$[5] \quad 2K_{\text{PDS}}S = 4\pi K_{\text{PDS}} \int_0^H d(h) \sqrt{1 + d'(h)^2} dh$$

Thus, the area of the inclusion zone is such that a design bias will exist for all $K_{\text{PDS}} \neq 1$. The difference between these two areas, and the potential bias associated with PDS, expressed as a percentage, is given by

$$[6] \quad B_{\text{PDS}} = 100 \frac{a_{\text{PDS}} - 2K_{\text{PDS}}S}{2K_{\text{PDS}}S} = \frac{4\pi K_{\text{PDS}} \int_0^H d(h) \sqrt{1 + K_{\text{PDS}}^2 d'(h)^2} dh - 2K_{\text{PDS}}S}{2K_{\text{PDS}}S}$$

Also note that K_{PDS} is not the same when estimating surface area and volume because K_{PDS} is expressed in inverse metres when estimating volume, and it is unitless when estimating surface area.

The estimator

$$[7] \quad \hat{Z}_{\text{PDS}}(x, y) = \sum_{i=1}^{n(x,y)} S_i |A| / a_{\text{PDS}i} \approx \sum_{i=1}^{n(x,y)} S_i \frac{|A|}{2K_{\text{PDS}}S_i} = \frac{|A|n(x, y)}{2K_{\text{PDS}}}$$

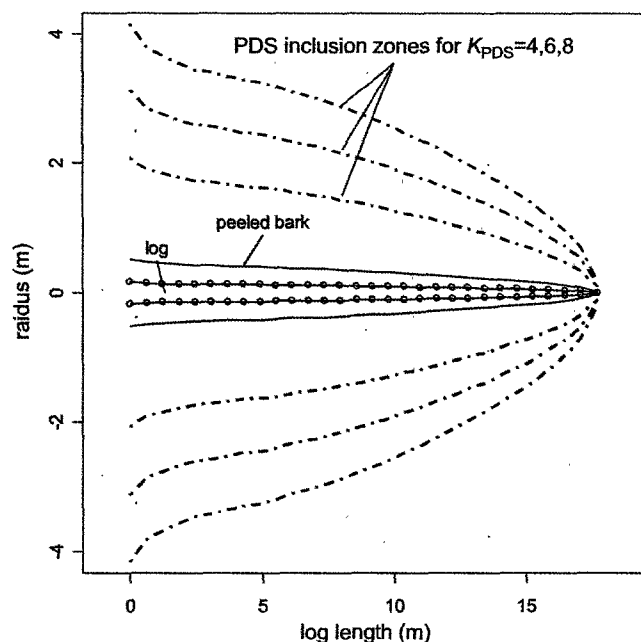
is proposed for estimating CWD surface area. Note the actual surface area is never measured and that just as in VRP sampling, logs where $|d_{\perp}|$ is obviously less than $K_{\text{PDS}}c_{\text{circ}}$ can be selected into the sample with no actual measurements. Thus, the advantage for PDS is that only the small proportion of borderline logs needs to be measured to determine their circumference. Borderline logs can then be carefully measured to accurately determine whether their circumference is large enough to warrant inclusion in the sample. If the circumference is measured with a linear tape, concave areas can be accounted for by pressing the

tape into these areas to achieve a nearly exact measurement of the circumference. While this seems like a complex set of measurements, a series of these measurements would be required on every sampled log to estimate the true surface area using a technique such as importance sampling.

Line intersect sampling to assess surface area

Many options exist when implementing LIS. One option for LIS is determining whether logs are selected when the line only partially intersects the log. In this study, the central axis of the log is described by a line, and the log is selected into the sample whenever the line crosses this axis. The log was viewed as a line because it simplified the implementation of the simulation study (see de Vries 1986, pp. 242–250 for examples of viewing the log as a needle). Additional considerations include whether inference about the population is made using either the design- or model-based approach. However, with the exception of Kaiser (1983), Williams and Gove (2003), and the collected works associated with Gregoire (Gregoire 1998; Valentine et al. 2001; Gregoire and Valentine 2003), authors have generally failed to

Fig. 4. A log is centered along the x -axis. The solid lines depict the inclusion zone that would be formed by the peeled bark, and the broken lines show the boundaries of three different perpendicular distance sampling (PDS) inclusion zones. Note that the slope of the lines defining the boundaries of the log-centered plots increases with increasing K_{PDS} values, particularly near the tip of the log.



explicitly state whether the design- or model-based approach to inference was employed. One of the interesting historical aspects of LIS is that it appears to be one of the few applications in forest inventory where the model-based approach to inference is used almost exclusively (Canfield 1941; Warren and Olsen 1964; de Vries 1986; Bell et al. 1999; Stahl et al. 2001; Waddell 2002). For this reason, the version of LIS used in this study employs the model-based approach to inference and closely mimics the work of de Vries (1986).

At least three different estimators have been derived for the estimation of CWD surface area under the model-based approach (Brown 1971; de Vries 1986). Two different estimators will be studied here. For this study, a randomly chosen point (x, y) within A is visited and a line of length L and fixed orientation is centered about the point. All logs whose central axis is crossed by the line are selected into the sample. The first estimator is

$$[8] \quad \hat{Z}_{LIS1} = \sum_{i=1}^{n(x,y)} \frac{S_i}{\pi_i^{(m)}} = \sum_{i=1}^{n(x,y)} \frac{|A|\pi S_i}{2LH_i}$$

where S_i is the measured surface area for tree i , $\pi_i^{(m)}$ is its probability of inclusion under the assumed model, $n(x, y)$ is the number of pieces of CWD tallied at the point with coordinates (x, y) , and the π value in the numerator is the usual $\pi \approx 3.1416$.

De Vries (1986) provides a detailed derivation for the probability of a needle intersecting a randomly placed line of length L . In order to derive the probability that the line intersects a log, the following assumptions are made

- (1) The length of the line L is "long" in comparison to the length of the needles.
- (2) The pieces of CWD are randomly oriented.
- (3) The pieces of CWD are randomly located within A .
- (4) A is sufficiently large so that edge effect can be ignored.

The notation $\pi^{(m)}$ will be used to denote the probability of intersection of log i and the line, where the superscript (m) denotes the fact that inference is with respect to the model.

Unbiasedness of $\hat{Z}_{LIS1}$ requires that the surface area of each log be measured without error, which as noted by de Vries (1986, p. 258), "...generally will be prohibitive from an economical point of view". The second estimator replaces the measurement of S with a measurement of the circumference at the point of intersection. Following de Vries' (1986, pp. 258–260) approach to volume estimation, the estimator is

$$[9] \quad \hat{Z}_{LIS2} = \sum_{i=1}^{n(x,y)} \frac{|A|\pi^2 d_i(x', y')}{2L}$$

where $d_i(x', y')$ is the diameter of the log at the point (x', y') where the log and line intersect. Note that as with PDS, this estimator also ignores the differential term, which will result in some bias.

Magnitude of the bias associated with ignoring the differential

The differential term causes two possible sources of bias. The first is the bias associated with measuring the log, and the other is the bias in the area of the PDS inclusion zone, whose magnitude is given in eq. 6.

Figure 5 illustrates the size of the differential term at every point on the bole for each piece of CWD in the data set. While this maximum value would indicate the possibility of a substantial bias, the integral "averages" the differential across the entire length of the log. To assess the magnitude of the bias associated with ignoring the differential term, the surface area was calculated using eqs. 2 and 3. The largest bias was 0.004 m^2 , which is inconsequential in comparison to the average surface area of 8.2 m^2 .

The magnitude of the bias associated with the area of the PDS inclusion zone was also assessed. This assessment quantified the mean bias of all 183 pieces of CWD for K_{PDS} values ranging from $K_{PDS} = 1, 2, \dots, 12$, where the diameter, length, and perpendicular distance are all expressed in metres. These results are given in Fig. 6. While the bias increases quadratically with increasing K_{PDS} values, the bias only exceeds 1% when $K_{PDS} = 12$. To put this in perspective, if a circular piece of CWD had a diameter of 30 cm (≈ 12 in.), then it would be sampled any time the point fell within $|d_{\perp}| = 5.6 \text{ m}$ (18.5 ft) of the log. Thus, K_{PDS} values in this range are probably about as large as would be used in most field applications.

Thus, we conclude that the bias associated with the missing differential is of little consequence in comparison with the potential biases associated with the various assumptions for areal sampling (e.g., Schreuder et al. 1993, p. 119) and the measurement of tree boles (Matérn 1956; 1990).

Fig. 5. Each line in the figure is the magnitude of the differential term $(\sqrt{1 + d'(h)^2})$ for each log in the data set. The magnitude tends to be the largest near the tip of each log.

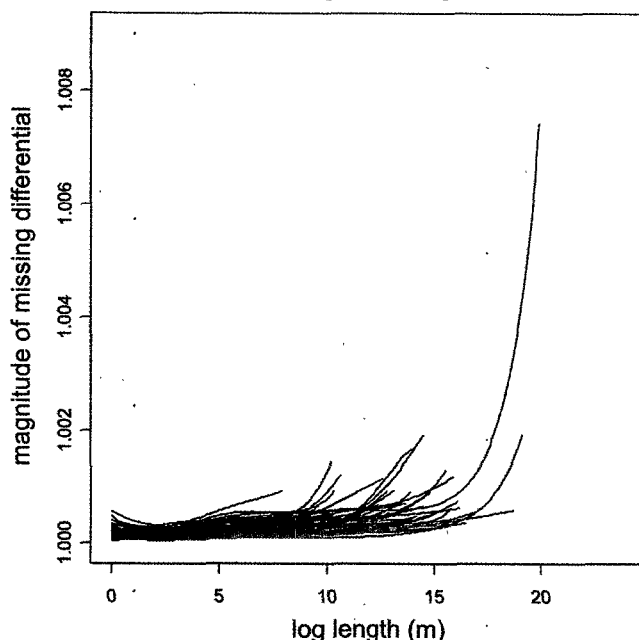
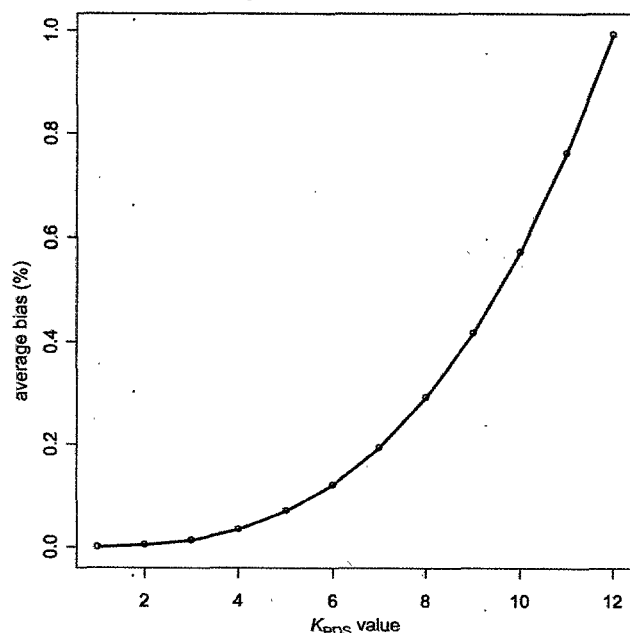


Fig. 6. Average percent bias, calculated from eq. 6, as a function of K_{PDS} . In this graph, the K_{PDS} value is calculated assuming the circumference of each log is measured in metres.



Simulation study

As mentioned by Sarndal et al. (1992, p. 234), "We judge estimators by their design-based qualities, such as design expectation and design variance, under repeated sampling with a given sampling design from the fixed finite population". So

regardless of an estimator's properties with respect to a model, the true test is the performance of the estimator with respect to the design. The simulation study assessed the design bias and variance of the three estimators. The bias and variance for each estimator was derived from a sample of locations from a very fine grid covering A . The grid spacing used was 0.2 m (7.8 in.).

The method of edge correction implemented in this study was to set the area of the population A to be such that the inclusion zone for every log fell completely within A (Ducey et al. 2004 and references therein). This method is known to increase the variance of the estimators over the alternative edge-correction methods, such as the reflection method proposed for LIS (Gregoire and Monkevich 1994) or the walk-through methods proposed by Ducey et al. (2004).

In the field, pieces of CWD would be selected using very different measurement techniques, and the amount of time required to draw a sample will vary depending on factors such as size of the logs, terrain, height and density of the understory, and the instrumentation used. Thus, a direct comparison is not possible. Because these factors cannot be realistically mimicked in a simulation study, the method of "standardizing" the three techniques was to equalize the expected number of pieces of CWD sampled at each point. This was done by setting the expected number of logs to be tallied at each point ($E[n(x, y)]$) to be 2.

An important assumption for this study is that all measurements of length, diameter, and cross-sectional area and surface area are made without error. Thus, given the assumption that no measurement error exists in any variable, the estimator $\hat{Z}_{PDS}$ will still exhibit a small design bias. Even though the two LIS estimators are model-unbiased and all of the assumptions are satisfied for the RAND data set, some design bias is expected in $\hat{Z}_{LIS1}$ and $\hat{Z}_{LIS2}$ (see Gregoire 1998, example 2 and references therein). We expect that the two LIS estimators will be more susceptible to problems with measurement errors because measurements are required on every log.

The standard error of the estimators is given by $\sqrt{V[\hat{Z}_*]}$, where $*$ denotes the sampling strategy (LIS1, LIS2, PDS) used. To facilitate the comparison between the two LIS estimators and PDS, the relative efficiency

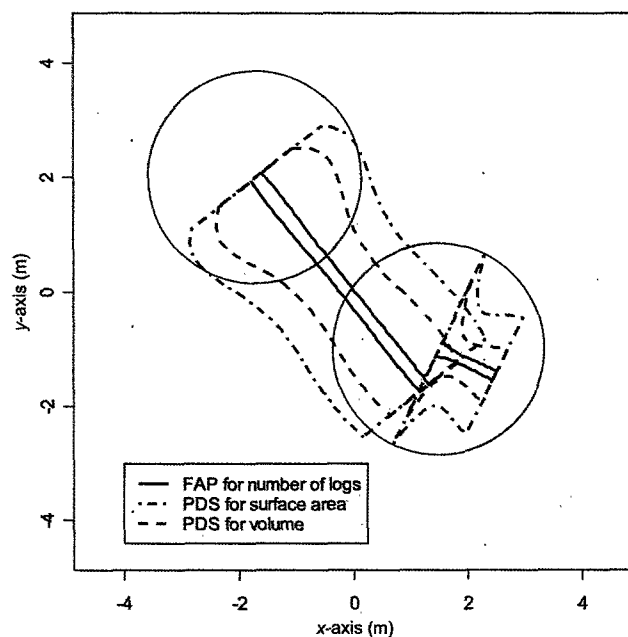
$$RE(PDS) = \sqrt{\frac{V[\hat{Z}_{LIS*}]}{V[\hat{Z}_{PDS}]}}$$

was used to express how many times larger the standard error of the other methods was when compared to PDS. Of additional interest is the difference in the efficiency between $\hat{Z}_{LIS1}$ and $\hat{Z}_{LIS2}$, which will indicate how much of a reduction in the variance can be achieved by measuring the surface area ($\hat{Z}_{LIS1}$) over a sampling strategy where only the circumference is measured at the point of intersection between the log and the line. This metric is given by

$$RE(LIS) = \sqrt{\frac{V[\hat{Z}_{LIS1}]}{V[\hat{Z}_{LIS2}]}}$$

The advantage of using RE to compare estimators is that the number of points required to achieve equal variance with each

Fig. 7. The inclusion zones for an inventory estimating the number of logs, surface area, and volume, where a fixed-area plot is centered about the large end of each log and PDS inclusions zones for surface area and volume are centered about the long axis of each log.



of the techniques, denoted by m_{EV} , is

$$m_{EV} = m_{RE}$$

To illustrate the bias associated with estimating the surface area of each piece of CWD using a simple approximation, the simulation was also run where the S_i for each log was approximated using the truncated cone and paraboloid. The bias was expressed as a percentage using

$$B_* = 100 \frac{\hat{Z}_{LIS1}}{\sum_{i=1}^N S_i^*}$$

where $*$ = cone, para. The bias of the cone and parabolic approximations are not relevant for either the LIS2 or PDS estimators because S_i does not appear in the estimators.

Results

Results for the RAND data set

The results for the simulation study are given in Table 2. As expected, the PDS estimator consistently has the smallest variance. Using RE, results show that roughly 10% and 12% more lines would need to be sampled in order for $\hat{Z}_{LIS1}$ and $\hat{Z}_{LIS2}$, respectively, to achieve the same efficiency as that of $\hat{Z}_{PDS}$. Also of interest is how similar the two LIS methods are in terms of variance, with $\hat{Z}_{LIS2}$ needing less than a 3% larger sample to achieve a variance equal to that of $\hat{Z}_{LIS1}$. Therefore, the utility of $\hat{Z}_{LIS1}$ is questionable given the substantial savings in field effort realized by $\hat{Z}_{LIS2}$.

Table 2. Simulation results for estimating the surface area of CWD, where LIS denotes line intercept sampling and PDS denotes perpendicular distance sampling.

Data set	LIS1	LIS2	PDS
RAND			
$B(\text{cone})$	121.2	na	na
$B(\text{para})$	99.2	na	na
$B(\text{design})$	105.3	106.0	99.6
RE(PDS)	1.10	1.12	na
RE(LIS)	1.03	na	na
NorS			
$B(\text{cone})$	159.2	na	na
$B(\text{para})$	130.6	na	na
$B(\text{design})$	137.9	140.1	99.6
RE(PDS)	2.06	2.18	na
RE(LIS)	1.06	na	na

Note: Two types of bias (B) are given. The first is the design bias of the estimators. The second is the bias in $\hat{Z}_{LIS1}$ due to approximating the surface area using a cone (S_{cone}) or paraboloid (S_{para}). RAND, random orientation; NorS, north-south orientation; RE, relative efficiency; na, not applicable.

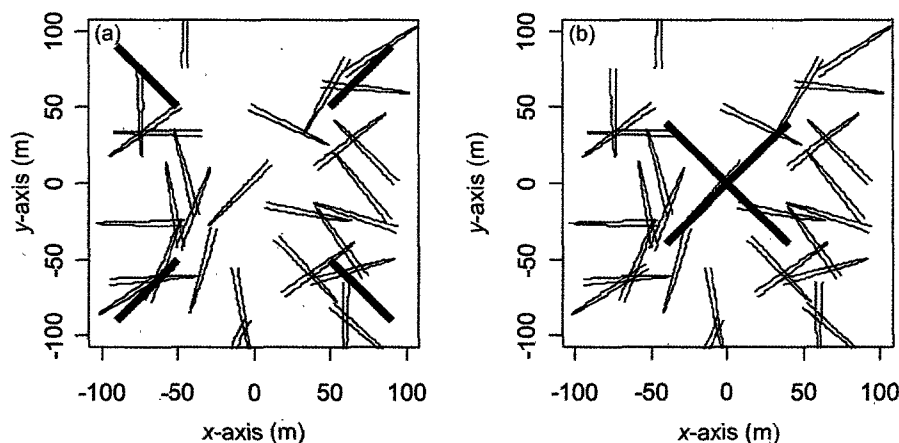
The design bias, expressed as a percentage for $\hat{Z}_{LIS1}$ and $\hat{Z}_{LIS2}$, was 105.3% and 106.0%, respectively. Thus, it is reasonable to expect biases in excess of 5% in either of these estimators even when all of the assumptions for these estimators are satisfied. As expected, the bias in $\hat{Z}_{PDS}$ is less than 1%. The bias of $\hat{Z}_{LIS1}$, when the surface area is calculated using either a cone or parabola, ranged from 99.2% to 121.3%. Thus, in general it seems reasonable to conclude that approximating the surface area induces a bias in an estimator of CWD surface area by as much as 15%.

Results for the NorS data set

The results for the NorS data set are given in the lower portion of Table 2. As expected, the PDS estimator consistently had the smallest variance, with the increase in efficiency being much larger than for the RAND data. Using RE shows that more than twice as many lines would need to be sampled for $\hat{Z}_{LIS1}$ and $\hat{Z}_{LIS2}$ to achieve the same efficiency as that of $\hat{Z}_{PDS}$. Once again it can be seen that the gain in efficiency of roughly 6% for $\hat{Z}_{LIS1}$, in comparison with $\hat{Z}_{LIS2}$, is trivial in comparison to additional fieldwork necessary to assess the surface area of each sampled piece of CWD.

The LIS estimators are known to be biased when the orientation of the logs is not random (Bell et al. 1996), with the estimators overestimating the true surface by roughly 40%. The use of the cone approximation further increased the bias to roughly 60%, and the parabolic approximation reduced the bias to around 31%, though a bias of 30% would probably be considered unacceptable in any application. Whether the parabolic approximation would consistently be superior to the cone approximation cannot be determined.

Fig. 8. An example contrasting a line intersect sample (LIS) of CWD using either (a) four lines systematically arranged over the area or (b) sampling with a single multisegmented LIS plot.



Discussion

There are a number of issues related to the assessment of CWD surface area using either PDS or LIS that were not covered in the simulation study. A number of issues related to a field implementation of PDS are also discussed. These are broken down into separate subsections.

Limitations and extensions of PDS

While the simplicity and performance of PDS is encouraging, these results do not suggest that PDS should be used in every application. For example, in field situations where the understory contains many logs that are completely obscured, LIS may be a better alternative. However, personal experience indicates that even in deep grass that obscures small logs, PDS is easy to implement because a sweep of the area with your foot around the point is usually sufficient to “feel” small logs, and very large logs are relatively easy to locate.

The other drawback of PDS is that it cannot be used to assess the number of pieces of CWD, because this would require the measurement of volume or surface area to calculate the inclusion probability. A solution to this problem is to assess the number of pieces of CWD using a circular fixed-area plot, with the easiest solution being to include logs whenever the large end of the log falls within the plot. Thus, the volume, surface area, and number of pieces of CWD can be estimated while only taking measurements to determine whether any of the “borderline” logs should be included in the sample. An example is given in Fig. 7, where the inclusion zones for two logs is illustrated.

Further results for LIS

One of the most interesting results is that the efficiency of $\hat{Z}_{LIS2}$ is nearly equal to that of $\hat{Z}_{LIS1}$. In view of these results, the utility of $\hat{Z}_{LIS1}$ is questionable given both the substantial savings in field effort realized by $\hat{Z}_{LIS2}$ and the potential for a reduction in bias associated with approximating the surface area.

A common solution to the problem of orientation bias has been to use multiple lines oriented in an L-, X-, Y-, or fan-shaped arrangement. This approach has been shown to reduce

the bias of the LIS estimators studied here, but at a cost that is rarely mentioned. This solution presents two main problems.

To illustrate the first problem, consider Fig. 8a, where a sample of four lines with different orientations are systematically placed within a small forest stand. In Fig. 8b, the four lines have been moved to form a single X-shaped plot. The point of the two figures is that a multisegmented LIS plot is nothing more than an open-cluster plot (de Vries 1986, p. 167), whose individual plots are placed sufficiently close as to be touching. Placing the individual subplots (lines) this closely is known to reduce the efficiency of a sampling strategy based on an open-cluster plot because it reduces, rather than increases, the within-cluster variability. Thus, a reduction in bias with this approach will almost certainly increase the variance. A more appealing solution to the problem of orientation bias is to install a single long transect at each point where the orientation is random. The other alternative is to rely on the design-based approach to inference and employ the LIS estimators presented by Kaiser (1983). These sampling strategies will sometimes require the measurement of the angle of orientation between the log and the line, but their property of being design-unbiased eliminates any concern regarding orientation bias.

The second problem relates to the frequent use of LIS in multiresource inventories that employ fixed-area plot sampling for standing trees, where the center points for each sampling method are colocated at the same point. A concern raised by field personnel is that it is very difficult to take all the necessary measurements on the plot without inflicting at least some inadvertent damage to the CWD due to trampling (A. Moreno, USDA Forest Service, personal communication, 2004). Multisegmented LIS plots increase this concern because the majority of the sampling effort is focused in the area where the most trampling is likely to occur (i.e., the center of the plot). An LIS sampling method that employs a single line does not focus as much of the sampling effort near the center of the plot. While it is true that this concern can be reduced by performing the CWD sampling before the sampling of standing trees, this solution does not work for an inventory where the plot is revisited to assess resource change, because at least part of the change could be attributed to mechanical damage, rather than

actual decay of the CWD. A more appealing solution would be to extend the line beyond a fixed-area plot boundary so that a test for significant differences between the on-plot and off-plot estimates of CWD could be constructed, but this renders the LIS cluster less compact.

Implementation guidelines and instrumentation for PDS

When logs are assumed to be circular, inclusion of a log under PDS is determined by a distance rule based on the diameter of the log (i.e., include the log when $d_L \leq K_{PDS}\pi d(x, y)$). Note that the same type of distance rule is applied when VRP sampling is used to select standing trees. Thus, an angle gauge or prism, rotated by 90° , can be used to quickly assess whether candidate logs should be included in the sample or whether additional measurements are required for borderline logs. The concept of using an angle gauge in a CWD inventory is not new. For example, Bebbier and Thomas (2003) proposed diameter relascope sampling as an angle gauge based method for the assessment of CWD and found that laser dendrometers and prisms work well in situations where logs are not completely obscured by the understory.

For VRP sampling, the angle chosen for a prism or angle gauge is selected so that each tree in the sample represents a convenient and constant amount of basal area per unit area (i.e., square metres per hectare or square feet per acre), where this constant is the basal area factor F . For example, if the metric basal area factor is $F = 4$, then each tree counted at a point represents $4 \text{ m}^2/\text{ha}$ of basal area. The basal area factor, F , can be related to the K_{PDS} value through the equations

$$K_{PDS} = \sqrt{\frac{2500}{\pi^2 F}}$$

and

$$K_{PDS} = \sqrt{\frac{10\,890}{\pi^2 F}}$$

for metric and English basal area factors, respectively.

The K_{PDS} value can also be chosen to yield a convenient surface area factor, denoted SAF. For example, if each tallied log is to represent $10 \text{ m}^2/\text{ha}$, then the appropriate K_{PDS} can be determined by solving

$$\text{SAF} = 10 = \frac{10\,000}{2K_{PDS}}$$

to give $K_{PDS} = 500$. This value of K_{PDS} assumes that the circumference of the log is measured in metres. A conversion from metres to centimetres can be accomplished by dividing K_{PDS} by 100 to yield $K_{PDS} = 5$.

Conclusions

The differential term adds much complexity to the estimation of CWD surface area. In fact, with the exception of the special case of PDS with the $K_{PDS} = 1$, there are no sampling strategies that can realistically claim to be unbiased for estimating surface area. However, in the case of estimating CWD surface area, the bias associated with ignoring the differential term is trivial compared to other sources of bias (i.e., measurement error).

The efficiency and simplicity of PDS makes it an attractive method for assessing many different surface area attributes. However, the results of this study raise some concerns with its application. One concern is that studies that relate CWD to other ecological processes often estimate surface area based on conic or paraboloid approximations. The results of these studies are then used to determine management guidelines. However, if $\hat{Z}_{PDS}$ or $\hat{Z}_{LIS2}$ are employed in a survey that monitors compliance with these guidelines, it is quite possible that bias in the approximations could erroneously indicate whether the management guidelines are achieved. For this reason, it is important to match the data collection methods for both special studies and any subsequent monitoring surveys so that biases in either approach are roughly equivalent. Thus, if PDS is to be employed in a monitoring survey, we suggest that an estimation technique with similar bias properties, such as importance sampling, be employed in any study whose outcomes might be used to set management guidelines.

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Appendix A.

A number of different approaches can be used to prove that the surface area PDS estimator, with $K_{PDS} = 1$, is design-unbiased (Eriksson 1995). Williams and Gove (2003) use an infinite population approach, and a finite population sampling approach will be given here.

The expectation of $\hat{Z}_{PDS}$ is derived by defining the indicator variable

$$\delta_i = \begin{cases} 1, & \text{if } i \text{ in the sample} \\ 0, & \text{otherwise.} \end{cases}$$

Then

$$\begin{aligned} E[\hat{Z}_{PDS}] &= \sum_{i=1}^{n(x,y)} \frac{S_i}{\pi_i} \\ [A.1] \quad &= \sum_{i=1}^N \frac{S_i \delta_i}{\pi_i} \\ &= \sum_{i=1}^N S_i \end{aligned}$$

because $E[\delta_i] = \pi_i$.