

Dynamic Drainage of Froth with Wood Fibers

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Understanding froth drainage with fibers (or simply called fiber drainage in froth) is important for improving fiber yield in the flotation deinking operation. In this study, the data of water and fiber mass in foams collected at different froth heights were used to reconstruct the time-dependent and spatially resolved froth density and fiber volumetric concentration in the froth. The results revealed that fiber drainage is caused by water carryover. However, fiber drainage stopped in the upper part of the froth and at longer draining time because of decreased draining water flux, as indicated by a critical water flux of about $5 \text{ mg s}^{-1} \text{ cm}^{-2}$. As a result, some fibers were permanently trapped in the froth and were unrecoverable. These findings indicate that only the upper part of a froth should be removed in flotation deinking operations to reduce fiber yield loss.

Introduction

Froths or foams are formed by dispersing gas in the form of bubbles stabilized by surfactant into a liquid and have many applications in industry. Therefore, the study of froth or foam not only has scientific importance in chemistry and physics but also has practical significance, especially in petrochemical processing, food processing, biorefining processes, mineral flotation, flotation deinking for paper recycling, and fire and explosion suppression. The drainage of a froth is a natural process caused by the density difference between the gas and the liquid in the froth that causes the gas to rise and the liquid to flow down under gravity. The drainage process is critical to the study of froth because it is responsible for the formation and evolution of a froth and has practical significance for separation in industrial processes.

The theory of drainage dynamics of an aqueous froth has been well formulated. Early works on the subject include Mysels et al.¹ and Bikerman et al.² Current theory on froth drainage dynamics is based on the assumption that all liquids are drained through the Plateau boarder (PB) channels in the form of Poiseuille flow. The one-dimensional unsteady continuity equation has been applied to describe the liquid flow in PB channels based on this assumption in the form of the liquid fraction or cross-sectional area of the PB channels.^{3–7} Narsimhan⁸ and Bhakta and Ruckenstein^{9,10} modeled the froth height decay caused by drainage using the unsteady material balance of gas in the froth. By coupling with the one-dimensional unsteady continuity equation of liquid, they were able to solve the moving boundary problem of froth decay. The forced drainage of froth has also been studied and described by Weaire et al.^{5,11} These studies on aqueous froth lay the foundation for the study of drainage of industrial foams that are nonaqueous in nature and containing solid materials such as colloidal or noncolloidal particles and fiber suspensions.

The study of froth drainage with solid materials (or simply called solid drainage) in solid-containing froths is important because these froths are common occur-

rences in most industrial applications and are often related to the performance of operating processes, such as fiber yield in flotation deinking to recover secondary fibers and mineral grade selectivity in the mining industry. However, most studies on the subject are related to mineral recovery,^{12–15} which is different from recovery of fibers in deinking in terms of the recovery process and properties of the material (geometry, density, flexibility, etc.). In froth flotation deinking of wastepaper, the ink particles detached from fibers during the pulping process can stick to air bubbles because of their hydrophobic property. Subsequently, inks are separated from fibers through the rejection of froth. Unfortunately, the rejected froth also contains wood fibers that are hydrophilic in nature and are entrained into the froth during flotation.^{16,17} Robertson et al.¹⁸ and Zhu et al.¹⁹ hypothesized that water flow in froth also carries fiber along. These researchers found that either water or surfactant spray applied to the top of their froth created increased liquid flow in the froth and resulted in decreased fiber rejection loss in their flotation deinking experiments. They explained that the reduction may be caused partially by the washing effect (fiber carryover by liquid flow) of the liquid spray. However, they did not present any direct evidence of fiber carryover within their froth. One objective of this study is to answer the fundamental question of whether fiber carryover by liquid flow occurs in froth. A positive answer to this question has practical significance because it can help develop technologies to improve fiber yield in flotation deinking, which will conserve natural resources and reduce the environmental impact of wood pulping processes. This study will also verify whether the aqueous froth drainage theory presented in the literature can be applied to solid-containing froth. We will then try to develop a simple model to predict the fiber carryover by liquid flow in froth.

Theoretical Model

The free drainage problem of froth has been solved^{3–6} analytically and numerically.⁷ However, the drainage of solid materials within froth has not been dealt with. As a first step to understanding fiber drainage in a froth, the present model focuses on the initial value problem

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of the fiber draining process, that is, the temporal evolution of the spatially averaged fiber mass or consistency in froth. This simplified spatially averaged model can answer the questions of under what conditions and how fast fibers drain in froth. These answers have significant importance in improving fiber yield in paper recycling using froth flotation deinking. The simplified model can be verified easily by experiments as will be demonstrated in this study and lays the foundation for the development of spatially resolved fiber drainage models in the future.

Early measurements of temporal evolution of the spatially averaged liquid fraction in aqueous froth were documented by Bikerman et al.² and were found to follow an inverse relation with time as proposed by an early study.²⁰ This inverse relationship can be derived from the solution of the free drainage equation of froth when the variation of pressure along the PB channel caused by capillary action is neglected and has been found to agree with experimental data.⁵ For a froth of height (L) with zero mass flux at the top of the froth (free draining), a control volume analysis of the material balance can be applied to a PB channel of cross-sectional area (A) to elaborate the basics of the model for predicting temporal evolution of liquid drainage in a PB channel:

$$\frac{d(\rho AL)}{dt} = -\rho uA \quad (1a)$$

$$A(t = 0) = A_0 \quad (1b)$$

where u is the draining velocity and ρ is the density of the draining liquid. One can obtain the same classical free drainage model²⁰ to predict temporal evolution of spatially averaged liquid fraction in froth by using the Stokes equation for liquid flow velocity (ignoring the pressure variation along the PB channel caused by capillary action³⁻⁶), that is, $u = \rho gA/\eta$:

$$\frac{A}{A_0} = \frac{1}{1 + \left(\frac{\rho g}{\eta} \frac{A_0}{L}\right)t} = \frac{1}{1 + kt} \quad (2)$$

where η is the viscosity of the liquid.

Because fiber mass in froth is a conserved scalar, control volume analysis of the material balance can be applied by using the continuum concept for fibers to obtain the following equation and initial condition:

$$\frac{d(\rho_f FL)}{dt} = -\rho_f u_f F \quad (3a)$$

$$F(t = 0) = F_0 \quad (3b)$$

where F is the equivalent cross-sectional area filled with fiber mass that is simply the liquid cross-sectional area of the PB channels filled with liquid times the fiber consistency (defined as the mass ratio of fiber over liquid) (χ) in froth and the density ratio between the liquid and fiber, that is, $F = A\chi\rho/\rho_f$; u_f is the average bulk draining velocity of the fibers. One can imagine that fibers are compacted to a real continuum with a density equal to the density of the fiber itself and draining in a channel with cross-sectional area of F at velocity u_f .

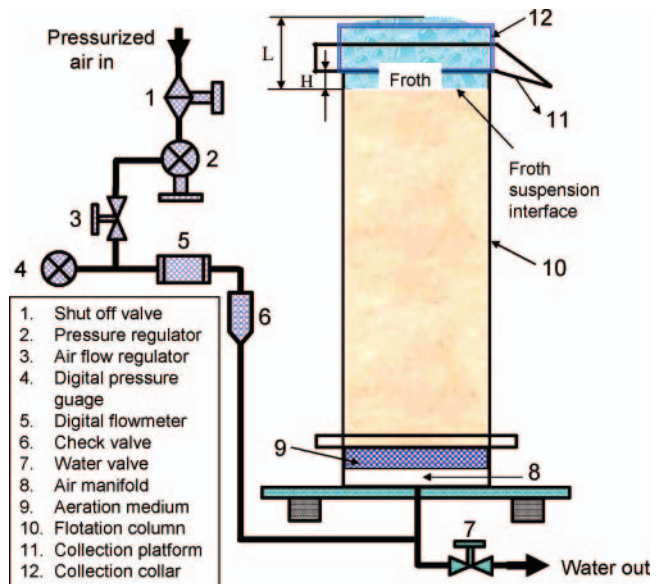


Figure 1. Diagram of the column flotation cell.

Following the Stokes law for liquid velocity in PB channels, we assume that the imaginary fiber flow velocity is also proportional to the equivalent cross-sectional area (F) but to the power of a constant, that is

$$u_f = bF^\alpha \quad (4)$$

Equation 4 is an empirical correlation. Then eq 3a can be rewritten as

$$\frac{dF}{dt} = -(b/L)F^{1+\alpha} \quad (5)$$

Solving eq 5 along with eq 3b, we have

$$\frac{F}{F_0} = \frac{1}{(1 + b\alpha F_0^\alpha/Lt)^{1/\alpha}} = \frac{1}{(1 + k_f t)^{1/\alpha}} \quad (6a)$$

Therefore, the normalized fiber consistency in froth can be expressed as

$$\frac{\chi}{\chi_0} = \frac{1 + kt}{(1 + k_f t)^{1/\alpha}} \quad (6b)$$

Equations 6a and 6b are the fiber free drainage model that describes the temporal evolution of the normalized spatially averaged fiber mass fraction remaining and normalized spatially averaged fiber consistency in a froth.

Experimental Section

A column flotation cell as shown schematically in Figure 1 was made in-house using a Plexiglas (AtoHaas Americas, Inc., Philadelphia, PA) tube with 15.9 cm i.d. The length of the flotation cell was 108 cm. The aeration medium was a circular Plexiglas plate of 6.4 mm thickness with 16 drilled holes, 0.3 mm in diameter, uniformly distributed to produce a froth equivalent to that seen in flotation deinking operations. To ease froth collection, a stainless steel collar with a platform was mounted at the top of the flotation column. Half of the surface of the platform was leveled with the top end of the flotation cell, and the other half was an inclined

channel enabling liquid or froth to easily flow out of the column for collection. In-house pressurized air was used for flotation. An air flow regulator and a flow meter (model FMA-A2317, OMEGA, Stamford, CT) were used to regulate the air flow rate into the column. A pressure of about 6.2 kPa was set during flotation. A check valve was also installed in the air flow line to prevent water back-flow to the air flow meter.

A mechanical wood pulping process was employed to produce thermomechanical pulp (TMP) fibers primarily from lodgepole pine (47%), Douglas fir (42%), and some western hemlock (8%) in a commercial pilot plant disk refiner (Andritz, Springfield, OH). The Canadian standard freeness of the pulp sample was 130 mL. Freeness is a measure of water drainage of a pulp mat and the degree of refining of wood fibers. It was measured in this study using TAPPI standard method T 227-om-94.²¹ The fiber lengths of the TMP pulp fibers were analyzed by Kajaani FS-100 fiber analyzer (Kajaani Automation, Norcross, GA).

Flotation experiments were conducted using Triton X-100 (TX-100), a nonionic octyl phenyl ethoxylate frother ($C_8Ph(EO)_{10}$; Ph = phenyl), analytical grade, (J. T. Baker, Phillipsburg, NJ), as the surfactant to stabilize froth.

A fiber consistency of 0.5% was used for all the flotation experiments. The air flow rate was set at 20 standard liters per minute (slpm). The height of the suspension level was varied to obtain spatial distribution of water and fiber in froth. A cylindrical collar made of the same Plexiglas tube as that of the flotation column was used to collect froth. The collar was placed on top of the flotation cell on the platform aligned with the flotation column as shown in Figure 1 during experiments. The bottom of the collar was attached with a rubber ring using silicon glue to prevent water leakage during froth collection or flotation. Froth rose with aeration and the application of surfactant and gradually filled the collar. The flotation air was shut off after 10 s. A circular aluminum disk was inserted into the junction between the collection platform (or top of the flotation column) and the bottom of the collar to collect the froth. A stopwatch was used to record the aeration time and the time of froth collection. Froth was collected immediately after shutting off the flotation air flow in the first experiment. The collected froth along with the collar container was weighed. Before starting another flotation experiment, forced air was used to completely destroy the froth produced in the previous experiment. Furthermore, the suspension was thoroughly stirred to make sure that the fibers were well mixed. The above-described experiment was then repeated; however, froth collection was not performed immediately after shutting off the flotation air but was delayed. The delay time was varied in each subsequent experiment to allow the liquid and fibers to drain within the froth. The froth draining time is defined as the time lapse from the time when the flotation air was shut off to the time of froth collection. Therefore, the history of the dynamic drainage of liquid and fibers in the froth can be reproduced through a series of froth collection experiments with different froth draining times. The entire series of the experiments described above was repeated to obtain duplicate data at a given froth draining time. However, the froth collection experiments in the second series were conducted backward; that is, froth collection at the

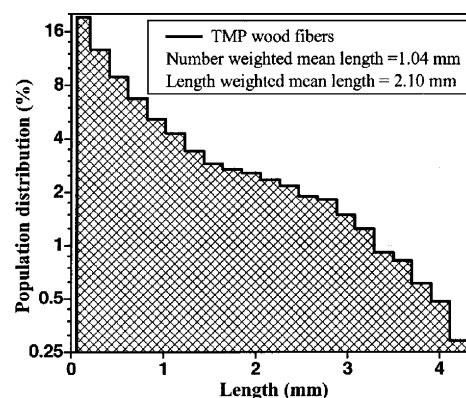


Figure 2. Fiber length distribution of the wood fibers measured by a time of transit technique.

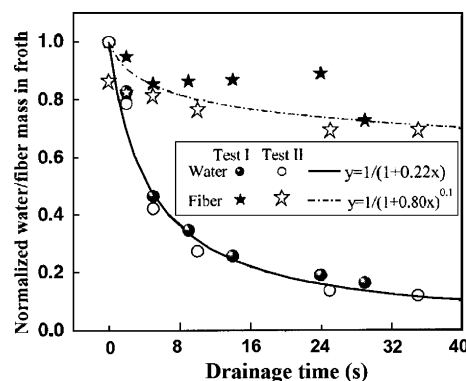


Figure 3. Repeatability tests of the temporal evolution of water and fiber content in a froth along with validation of model predictions.

longest draining time was conducted first and the zero draining time was conducted last.

The gravimetric method was used to determine fiber rejection in froth flotation. The wet mass of the collected froth was first determined by a balance. The wet reject was then transferred into an aluminum foil boat (weighing about 6–7 g). Fiber-containing froth collected at the same delay time was combined into one boat. Then the boats were put in an oven to remove water and moisture. The oven temperature was held at 105 °C. The boat remained in the oven overnight and then was removed and weighed for the determination of dry fiber in froth.

Result and Discussion

Experimental Repeatability. The repeatability of the experiments was demonstrated by conducting two sets of replicate measurements of water and fiber in froth to reproduce the histories of two draining events under the same experimental conditions. At an aeration rate of 20 slpm, the water level in the column was pushed up to about 16 mm from the top of the flotation column because of gas holdup. The two sets of experiments were conducted several days apart using TMP wood fibers with the same surfactant (TX-100) concentration of 70 mg/L in fiber suspension. After 10 s of aeration, a froth height of 124 mm was produced. The fiber length distribution of the wood fiber was measured by a Kajaani fiber analyzer (FS-100) (Figure 2). The number-weighted and length-weighted mean fiber lengths were 1.04 and 2.10 mm, respectively. Figure 3 shows the measured time-dependent water and fiber content in froth. Each data point was averaged across

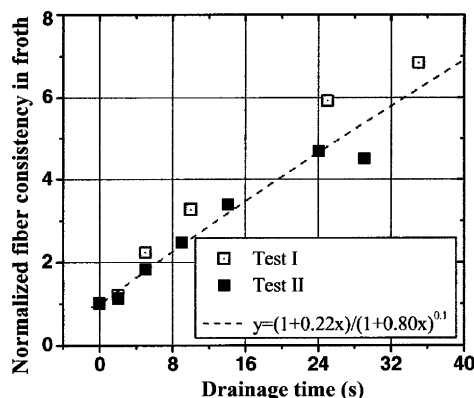


Figure 4. Validation of model-predicted fiber consistency in froth.

two individual runs as described in the Experimental Section. The data were then normalized by their corresponding values at zero drainage time, respectively. Good experimental repeatability was obtained as indicated by the data (Figure 3), especially for water drainage. The standard deviations of the drainage data of an individual run were approximately 5% and 10% for water and fiber, respectively. The calculated fiber consistencies in froth based on the measured water and fiber data normalized by the fiber consistency in the suspension of 0.5% also agree well (Figure 4). Results shown in Figures 3 and 4 validate the present approach of using a set of experiments conducted under the same experimental conditions to reproduce the histories of the fiber and water draining process. The measured fiber mass data shown in Figure 3 clearly answer the fundamental question of whether fiber drains in froth under the experimental conditions we used.

Spatially Averaged Drainage Model. The simple model in eqs 2 and 6a was used to conduct regression analysis of the measured water and fiber data to obtain the time constants for the water and fiber drainage. As shown in Figures 3 and 4, the temporal evolution of spatially averaged water and fiber mass and fiber consistency in froth can be predicted with eqs 2 and 6b, respectively. The time constants for water and fiber drainage were $k = 0.22$ and $k_f = 0.80$, respectively. The power index for the imaginary fiber flow velocity was $\alpha = 10$, that is, $u_f = bF^{10}$.¹⁰ The results in Figures 3 and 4 also indicate that the assumed power law empirical correlation for fiber flow velocity can be used to correlate fiber drainage in froth.

Spatially Resolved Water and Fiber Drainage. Assuming that the froth structure, height, fiber, and water content, etc. remained exactly the same (the froths were identical) when the variations in suspension level, H (Figure 1), were orders of magnitude smaller than the flotation column height, then different heights of the “identical” froth were collected when the suspension level changed because the collection location (the collection platform) was fixed. The froth collected at a smaller H contains the identical froth collected at a larger H . The measured water and fiber contents in froth from experiments conducted under the same fiber suspension of 0.5%, surfactant concentrations of 70 mg/L (TX-100), aeration flow rate of 20 slpm, but different suspension levels of $H_1 = 13$, $H_2 = 16$, and $H_3 = 22$ mm from the top of the flotation column can therefore be used to reconstruct the spatially resolved froth structure. The froth with a total height of L (Figure 1) can be divided into three sections from the froth

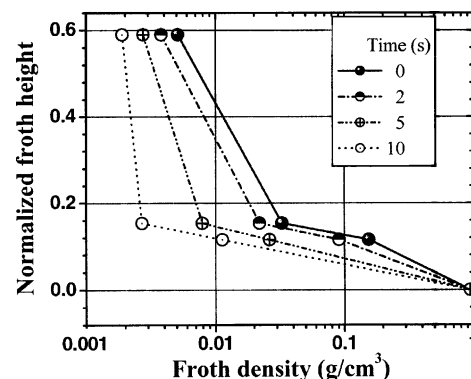


Figure 5. Spatially and temporally resolved froth density.

suspension interface with heights of $H_2 - H_1$, $H_3 - H_2$, and $L - H_3$ ($L = 124$ mm), respectively. The amount of water and fiber in the first two sections can be obtained from the difference of the measured water and fiber data at two consecutive suspension levels. The amounts of measured fiber and water at suspension level $H_3 = 22$ mm are the fiber and water contents in the top section of the froth of height $L - H_3$. The fitted values from eqs 2 and 6a (Figure 3) were used to calculate the differences to eliminate possible difficulties in using actual data caused by measurement uncertainties. The froth density or fiber volumetric concentration in each section can therefore be calculated by dividing the water and fiber mass in each section by the volume of the section. A constant froth volume of a section was used without accounting for the froth volume variation with time caused by froth decay; that is, the same froth volume was used for various delay times.

Figure 5 shows the time-dependent and spatially resolved normalized density of the froth. The density of the froth at the suspension–froth interface of 0.94 g/cm³ (froth height = 0) was assumed to be the density of the bulk suspension (with gas holdup) and was used to normalize the data. The results show that the froth density decreases rapidly along the froth vertical direction. Even at the moment immediately after shutting off of the aeration flow (because of the air bubble residence time of about 3 s, there were still some bubbles flowing upward to the suspension–froth interface), froth density was only about 10% of the density at about 15% of the froth height. Froth density dropped by another order of magnitude at the half-height of the froth. As drainage progressed, froth density decreased, and the density curve shifted to the left of the axis (smaller values). This temporal evolution of the froth density curve qualitatively agrees with the analytical and numerical results^{5,7} and experimental data⁶ in the literature. However, the froth density was uniform at time = 0 in the literature, which is not the case in this study as clearly shown in Figure 5. Also, froth decay was not significant within a time frame of 10 s after the flotation air was shut off. Video indicates that froth height had decreased about 15% after 10 s of draining.

The normalized fiber volumetric concentration (by the bulk volumetric fiber concentration in the suspension of 5 g/m³; the fiber concentration at the suspension–froth interface was assumed to be equal to the bulk concentration, i.e., 5 g/m³) within the froth showed similar behavior to that of froth density (Figure 6). The main feature in Figure 6 is that the deviation among the fiber volumetric concentration curves is small after a draining time of 2 s, indicating that fiber drainage

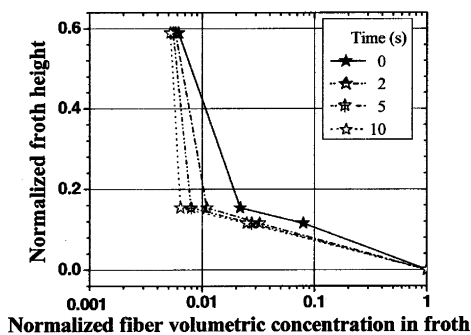


Figure 6. Spatially and temporally resolved fiber volumetric concentration in froth.

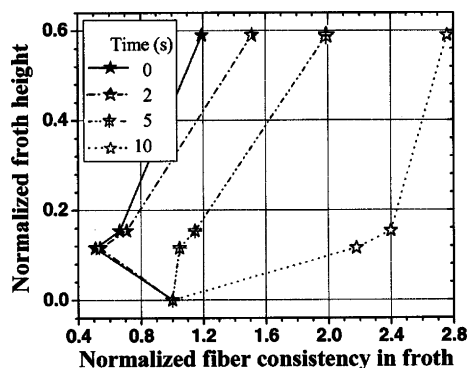


Figure 7. Spatially and temporally resolved fiber consistency in froth.

may have been close to completion, which is different from that of froth density (mainly from water) as shown in Figure 5. Furthermore, the normalized fiber volumetric concentration reached an asymptotic value of 0.005 at the top of the froth, indicating that fiber draining stopped at the top of the froth where the water volumetric fraction was low; that is, less than 1% according to Figure 5. Therefore, some fibers were permanently trapped in froth.

From the data shown in Figures 5 and 6, we calculated the fiber consistency in froth. Figure 7 shows the normalized fiber consistency (by the consistency of the suspension of 0.5%) in froth. The data clearly show that normalized fiber consistency in froth increases as draining time increases because of water drainage. At a longer draining time (>5 s), fiber consistency also increases along the froth height because of water drainage. The increased fiber consistency in froth and in time indicates that water drainage is much faster than fiber drainage. However, within a short draining time, there was a reduction in fiber consistency in the bottom part of the froth (within about 30% of the froth from the suspension-froth interface). This can be explained by two factors: the fiber consistency in the froth is lower than that in the suspension, and fiber draining is fast enough to be able to maintain a low fiber consistency in froth for a period of time at the beginning of the draining process in the bottom part of the froth where liquid fraction is high (about 10% according to Figure 5). Figure 7 indicates that this time period is only 5 s when the fiber consistency in froth is beginning to be greater than that in the initial suspension of 0.5%.

Water and Fiber Draining Flow Flux. We also calculated the water and fiber flow rates per unit area of the froth cross section (or fluxes) during drainage tests. The water flow flux spatial-distribution curve

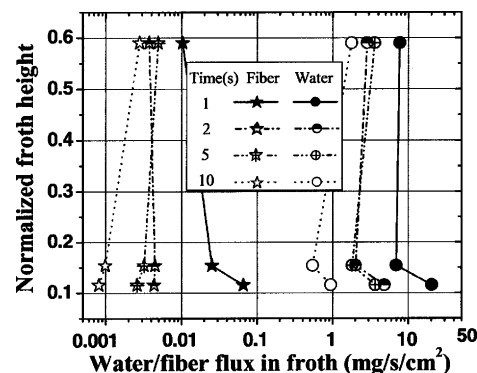


Figure 8. Time-dependent water and fiber flow rates in froth.

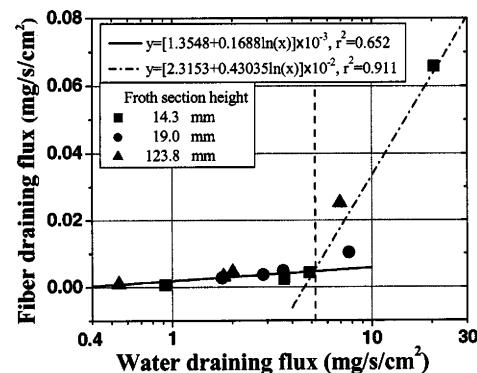


Figure 9. Correlation between water flux and fiber flux in froth at different draining time and different froth heights.

shifts to the left of the axis (small values) as draining progresses because of the decreased amount of water available in the froth (Figure 8). All the water flow flux curves have similar behavior; that is, the flux decreased rapidly in the bottom part of the froth and then maintained a constant value. The slight increase in the water flow flux in the upper part of the froth could well be caused by the decay of froth; that is, breakup of the air bubbles increased the water flow downward. This is especially true at longer draining times of 10 s as clearly shown in Figure 8. The data also show that a maximum water flux of $20 \text{ mg s}^{-1} \text{ cm}^{-2}$ was obtained.

The fiber flow flux distribution curve showed similar left-shift behavior compared with that of water caused by the reduction of water flow as draining progressed. This phenomenon can be explained by the fact that fiber draining is mainly caused by water flow carryover and not by the fiber gravity itself. When the water flow is significantly decreased, the fluid dynamic force becomes too small to overcome the fiber friction force to move the fiber downward, and draining of fiber will stop. At the beginning of the draining process (draining time of 1 s) fiber flow flux decreased continuously and rapidly along the froth height. The fiber flow flux became uniform in froth as draining progressed (draining time of 2 and 5 s). Froth decay became significant at longer draining time (10 s), and fiber flow flux increased along the froth height. The maximum fiber draining flow flux was found to be less than $0.1 \text{ mg s}^{-1} \text{ cm}^{-2}$.

To further elaborate upon the carryover of fiber by the water flow in froth, we plotted the correlation between the water and fiber flux data (Figure 9) presented in Figure 8. It was found that the fiber flux data correlate to the water flux data well at large water flux of above $5 \text{ mg s}^{-1} \text{ cm}^{-2}$, clearly validating the fiber draining mechanism; that is, fiber flow caused by water

flow carryover. However, there is a critical water flux below which fiber flow flux is significantly decreased with a low correlation coefficient (more than an order of magnitude smaller) between the water flux and fiber flux data, indicating that the stated fiber draining mechanism may not be valid in dry froth (water fraction <0.1). A minimal water flux is required for the water flow to drive the fiber to flow downward. The critical water flux was found to be about $5.1 \text{ mg s}^{-1} \text{ cm}^{-2}$ for the froth and fiber suspension system in this study. When the water flux is below the critical value, the small value of the measured fiber flux shown in Figure 9 is probably caused by the slow decay of the froth, and fiber draining by carryover has probably stopped because of decreased fluid dynamic force.

Implications of Deinking Pulp System. The virgin mechanical pulp suspension used in this study is certainly different from industrial deinking pulp systems. Industry deinking pulp systems always contain ink (probably less than 0.1% of the fiber mass) and ash (for example, fillers of calcium carbonate). The ash content varies depending on paper grade. Most newsprint recycling operations in the United States purposely mix 5–20% magazine to increase the recycled pulp brightness and bonding strength (magazine papers are made of bleached chemical pulps and have a better strength than mechanical pulps used to make newsprint papers), which gives an ash content of 2–10% by mass of the total solid materials or 0.014–0.07% of the suspension assuming suspension consistency of 0.7%. Furthermore, the differences in aeration process, scale, and deinking chemical formulation also make industry froth different from that in this study. For example, froths in industry operations have larger bubbles than those seen in this study (on the order of 1–3 cm based on video recorded during experiments). However, froth drainage is mainly dictated by liquid flow in PB channels that is primarily driven by gravity and, to a lesser extent, capillary effect (ignored in deriving eq 2). The rheology of the suspension and the liquid is not significantly affected by some variations in deinking chemistry and the presence of low amount of ink and ash particles ($<100 \mu\text{m}$ for ink and $<2 \mu\text{m}$ for ash). Therefore, we believe that the results obtained in this study are still relevant to paper recycling practice.

Conclusions

This study revealed the important fact that fiber draining does occur in froth that is similar to those of laboratory flotation deinking operations. Furthermore, fiber downward flow is caused by the carryover of the draining water flow in froth. The temporal evolution of the spatially averaged water and fiber drainage can be predicted by a simple model with inverse of time functionality. The measured froth density in froth agrees qualitatively with those of analytical and numerical results found in the literature. The fiber volumetric concentration in froth decreases rapidly upward along the froth and reaches an asymptotic value, indicating that fiber gradually stops draining because of decreased fluid dynamic forces and water fraction at low water flow rate in the upper part of the froth. Further analysis of the water and fiber flux data obtained at different draining times and from different froth sections indicated that there is a critical water flux value of $5.1 \text{ mg s}^{-1} \text{ cm}^{-2}$, below which fiber flux is small and probably is not caused by water carryover but

rather by froth decay, indicating that some fibers may be permanently trapped in the upper part of the froth and cannot be recovered. Fiber consistency in froth increased because of water draining. These findings are significant to flotation deinking to achieve high fiber yield. For example, by selectively removing the upper part of a froth at an appropriate froth rejection rate, one could achieve an acceptable deinking level and fiber yield. Use of a liquid spray^{18,19,22} could promote fiber drainage in froth, improving fiber yield without affecting ink removal.

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