Two Dimensional Finite Element Heat Transfer Models for Softwood

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ABSTRACT

The anisotropy of wood creates a complex problem for solving heat and mass transfer problems that require analyses be based on fundamental material properties of the wood structure. Most heat transfer models use average thermal properties across either the radial or tangential directions and have not differentiated the effects of cellular alignment, earlywood/latewood differentiation, ring orientation, and moisture content. Two 2-Dimensional finite element models have been developed that take these parameters into consideration. The first model is used to determine the effective thermal conductivities of softwood cellular structure as a function of cell alignment, cell porosity or density, and moisture content. The second model uses the results from the first model to help explain the transient heat transfer effects of ring orientation for any board "cut" from any location in a log, earlywood/latewood ratio, earlywood and latewood densities, and growth rate. This paper, briefly discusses the two models and their development. Initial results are presented showing the effects of density and moisture content on the effective thermal conductivity values for softwood cell structure. Comparisons are made with empirical equations for thermal conductivity of wood in the literature. The second finte element board model is introduced to show the effects of ring orientation at 0% moisture content for several boards "cut" from several locations in a log. These new models are useful for enhancing our understanding of fundamental heat transfer effects in various wood boards.

Keywords: finite element modeling, thermal conductivity, transient heat transfer, cellular structure, growth ring, moisture content

The effects of anisotropy in heat and mass transfer for wood boards continues with the description of where boards are cut from a log (Fig. 3), the tree species, whether fast or suppressed growth conditions, and density. Heat and mass transfer in wood on a wood board scale has been studied extensively which has resulted in numerous empirical and theoretical models. Development of these models has been reviewed by Kamke & Vanek 1992 and 1994. Most heat transfer models for wood are 2D and ignore the longitudinal heat transfer effects because of its relatively long heat transfer path compared to the transverse heat transfer in the board. Most models also do not differentiate thermal properties with respect to the ideal axial symmetry, radial, or tangential directions, but assume averaged thermal heat transfer characteristics (Forest Products Laboratory, 1999 and MacLean, 1941) based only on an average species-density and moisture relationship. However, in some boards, the radial and tangential orientation of the rings can curve such that the radial directions could act both horizontally and vertically in the same board that could have a significant effects on the thermal properties. Earlywood and latewood differentiation of the thermal characteristics have only been studied in limited efforts in the modeling field. Significant thermal differences can occur based on ring orientation, ring density, and radial/tangential cell alignment.

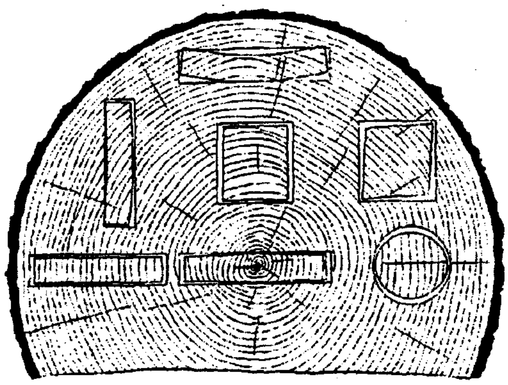

Figure 3 - Orientation of rings in boards cut from various locations in a log can be significantly different,

In addition to the main structural, species, and growing conditions of wood, moisture has significant effects on heat and mass transfer. To measure this effect is not easy. Early in the 1940's, MacLean (1941) pointed out that the conductivity of wood with certain moisture content as determined under steady-state conditions does not represent the true conductivity of the wood under the original moisture distribution conditions because, in the process of conducting the experiment causes moisture redistribution resulting in slight errors in the measurement. Therefore, the true value of thermal conductivity of wood can only be obtained by theoretical modeling due to the limitation of physical tests. Significant research has been done to measure the thermal conductivities of different species and some regression models from those test data have been drawn (MacLean 1941, Stamm 1960, Wangaard 1943 and Hendricks 1962). However, the difference in thermal conductivities between radial and tangential directions and between earlywood and latewood has not been theoretically studied and fully understood.

There is a need to develop heat transfer models which considers all these parameters in order to more accurately determine the heat transfer effects in softwood, especially for transient heating and cooling conditions.

INTRODUCTION

Wood is an anisotropic, porous material with complicated cellular and macro scale structure features and material properties. The structurally induced anisotropic effects on heat and mass transfer have significant implications for drying lumber, heating logs in veneer mills, or hot pressing wood composites. Anisotropy of wood is due to wood fiber's radial, tangential, and longitudinal orientation (Fig. 1) and the structural differences between the development of earlywood and latewood bands for each annual ring in softwood (Fig. 2). Earlywood cells are formed in the fast growing spring season and are low-density wood cells with large cavities and thin walls (Fig. 2. left). Latewood cells are formed later in the year and are high-density cells, characterized by smaller cavities and thick walls (Fig. 2, right). Softwood cells tend to align in straight radial rows because they originate from the same cambial mother cells, but cells are not necessarily aligned in tangential rows (Fig. 2). For the tangential direction the alignment can vary from 0% up to a 50% offset, which is defined as the maximum misalignment by Hart (1964). Longitudinal differences also occur but are not within the scope of this paper. While growing conditions and tree species impart numerous variations between earlywood and latewood cells, general assumptions can be made and modeled. For heat transfer modeling, earlywood and latewood cells are made of essentially the same material within the wall substance. Cell porosity -- percentage of openings in a wood cell -- may vary from 90% to 70% in earlywood and from 30% to 10% in latewood (Gu, 2001).

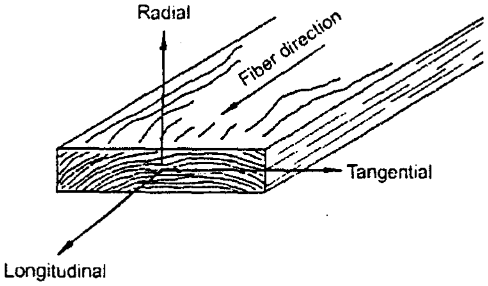

Figure 1 - Three principal axes of wood with respect to fiber direction and growth rings.

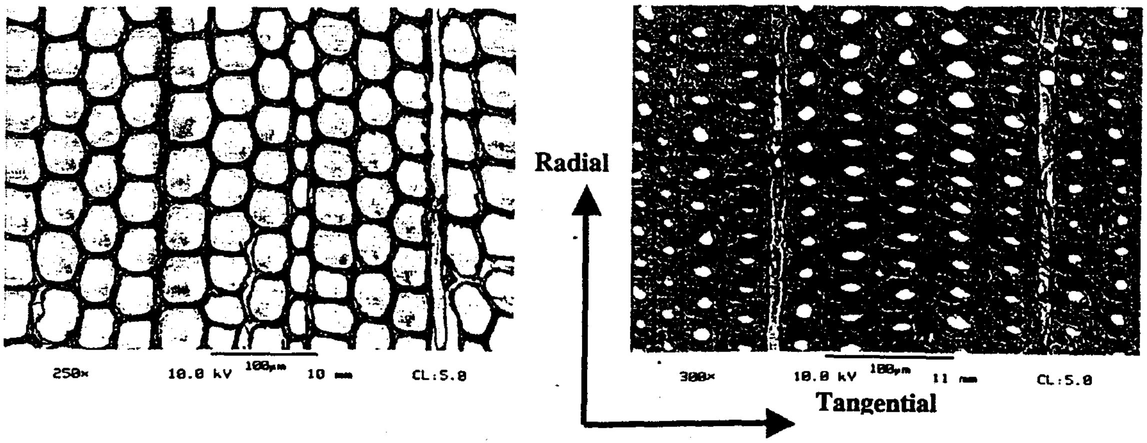

Figure 2 - Microscope images of softwood structure in earlywood (left) and latewood (right) regions.

FINITE ELEMENT MODELS

Cellular Model

A cellular finite element model was developed using ANSYS finite element software (ANSYS, 2004). Finite element type PLANE35, a 2-D 6-node triangular thermal solid element, was used to conduct theoretical heat transfer analyses. The mathematical solution for this element's conduction heat transfer is based on the first law of thermodynamics -- energy conservation law, Eq. 1.

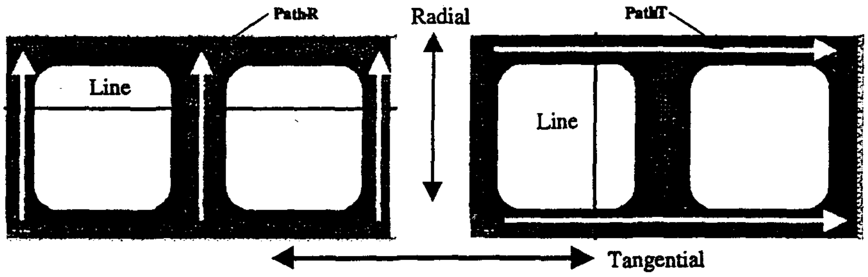

Figure 4 - Model of wood cells with 50% porosity in the fully aligned case (0% offset) of cellular structure with no free water in the lumen.

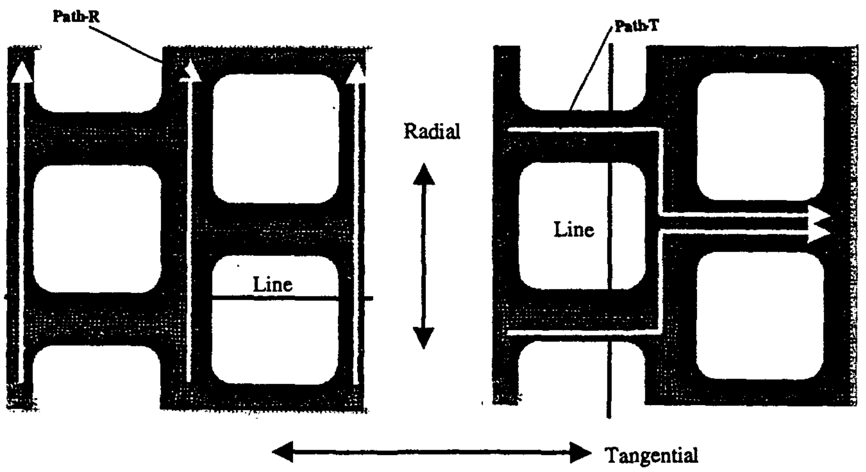

Figure 5 - Model of wood cells with 50% porosity in the fully misaligned case (50% offset) of cellular structure with no free water in the lumen.

For this paper, four moisture conditions with relatively easier-to-describe moisture relationships were used and analyzed:

1.) 0% MC in the cell wall and dry air in the lumen;
2.) fiber saturation point (FSP);
3.) fully saturated cell wall and 50/50 volume ratio of water/water vapor in the lumen;
4.) fully saturated cell wall and 100% free water in the lumen.

$$\rho C_p \frac{\partial T}{\partial t} = k_{eff,X} \frac{\partial^2 T}{\partial x^2} + k_{eff,Y} \frac{\partial^2 T}{\partial y^2} \tag{1}$$

where r is the density of material, C_p is the heat capacity, and $k_{eff,X}$, $k_{eff,Y}$ are the effective thermal conductivities in x and y (radial and tangential) directions. From the cellular structure of softwood observed under the microscope (Fig. 2), the model was developed to simulate the structural cell porosity and cell alignment/misalignment. The cell porosity is the fractional void volume of a wood cell. Cell porosity is assumed in the model to range from 10% to 90%. In softwood, cell porosities range from 70% to 90% for earlywood and 10% to 30% for latewood (Gu, 2001). Softwood cells tend to align in straight radial rows (vertical in Fig. 2). Whereas, softwood cells are much less aligned in the tangential direction (horizontal in Fig. 2) and this alignment or misalignment between cells varies from 0% to 50% maximum. A fully aligned cellular structure model is shown in Fig. 4. Conversely, a fully misaligned or 50% offset between the two rows of cells is shown in Fig. 5.

The FSP is assumed to be at 30% MC (Siau 1995), which yields a cell wall substance volume of 70.7% and bound water volume of 29.3%. The model for the 50/50 volume ratio for water and water vapor in the lumen assumes that 50% of the outside lumen volume contains free water due to the surface energy tension at the lumen surface. The free water was assumed to spread "evenly" around the inside lumen surface leaving a circular 50% open volume, Fig. 6. It is assumed that the open volume is water vapor only and no air.

Effective thermal conductivities were determined by simple conduction problems across the cellular models for cell porosities from 10% to 90% at increments of 10%. A temperature difference of 80°K across the two opposing boundaries was used with the other two boundaries set as adiabatic boundaries. The material properties used for input variables for the cell wall substance, air in the lumen, water vapor in the lumen, and water in the lumen, are listed in Table 1. The saturated cell wall thermal properties were calculated by the rule of mixtures. The total heat flux (q''_x) across a random line (shown in Figs. 4 and 5) was summed to determine the effective thermal conductivity (k_{eff}) using the definition (Incropera & DeWitt, 1981) in Eq. 6.

$$q''_x = -k_{eff}\frac{dT}{dx} \tag{6}$$

Where, q''_x = heat flux [W/m^2], k_{eff} = effective thermal conductivity [W/m-K], dT = temperature change [°K], dx = linear distance across the cells (m).

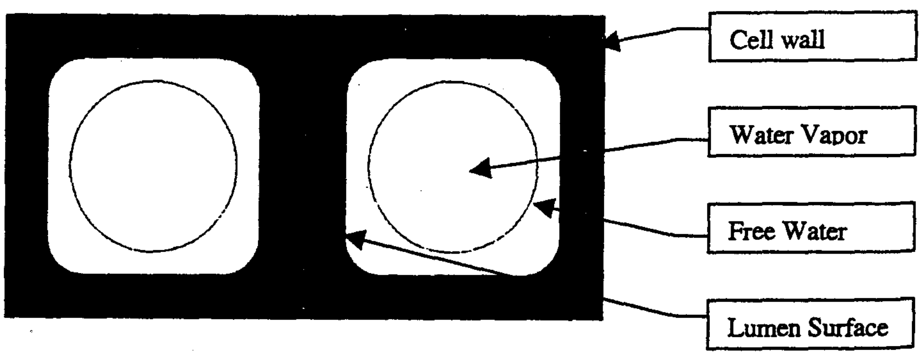

Figure 6 –Model of wood cells with 50% porosity in the fully aligned case (0% offset) of cellular structure with 50% free water and 50% water vapor in the lumen.

Table 1 - Physical and Thermal Properties

	Material Properties in the cellular model		
	Thermal Conductivity [W/m.K]	Density [Kg/m3]	Specific Heat [J/Kg·K]
Cell wall substance (0%MC) [1]	0.410	1540	1260
Air in the lumen (0%MC) [2]	0.026	1.161	1007
Bound water in cell wall [3]	0.680	1115	4658
Saturated cell wall (FSP) [4]	0.491	1412	2279
Water vapor in cell lumen [5]	0.018	0.734	2278
Free water in cell lumen [2]	0.610	1003	4176

1. Siau 1995.

2. Incropera 1981

3. Density (Siau 1995). Thermal conductivity and specific heat obtained based on water property values and assumption of their linear relationship with free water density.

4. Property of saturated cell wall was obtained by rule of mixture

5. Ierardi 1999

Board Model

When a board is cut from a log (Fig. 3) it can have significantly different number and orientation of rings depending on the location from where it was cut. As shown in the Fig. 3, wood shrinkage can be quite different due to the ring orientation and earlywood/latewood difference, and in the same way, the heat transfer properties can be different due to these reasons, too (Hunt and Gu, 2004). A 2-dimensional finite element softwood board model was developed to simulate a log of any size from which any size board could be "cut" from any location and analyzed in transient heat transfer conditions. Input parameters for the model include: 1.) earlywood and latewood densities; 2.) earlywood/latewood ratio (E/L); 3.) earlywood + latewood ring width; 4.) assigned thermal conductivities for earlywood and latewood based on density; and 5.) board size and location within the log.

The board model is briefly presented here to demonstrate the significant heat transfer effects due to the earlywood and latewood ring characteristics and varying ring orientations. These characteristics need to be considered rather than assuming softwood has homogeneous heat transfer properties. In this paper, transient heat transfer effects due only to changing ring orientation are presented, while keeping all other parameters constant. The earlywood density is assumed to be 370 Kg/m^3 and the latewood density was 1263 Kg/m^3 based on assumed 76% and 18% porosity of earlywood and latewood, respectively, and the MC was 0%. The board model uses an average softwood ring width value of 7.1 1 mm and E/L ratio of 5/2. The finite element coordinate system was cylindrical and concurrent with the log pith. The effective thermal conductivities, K_{eff}, for earlywood and latewood softwood at room temperature were determined by curve fitting the results from the finite element cellular model at 0% MC conditions described by Hunt and Gu (2004) and briefly described above.

Thermal properties of wood, including effective thermal conductivity and heat capacity, change with temperature (Kuhlmann 1962, Skarr 1972, Hendricks 1962). The relationship between the thermal properties and temperature was documented in Siau's book (1995) and are programmed into the board model. A rectangular wood board 44.5 by 88.9 mm (1.75 by 3.5 inches) was generated at several locations. The convective heat transfer coefficients were 12.93 W/m^2·K for the side boundaries and 9.15 W/m^2·K for the top and bottom boundaries (Incropera & DeWitt, 1981). Details of the calculations can be found in Gu's dissertation (Gu, 2001). A temperature of 100°C was applied to all outside boundaries with the board initial temperature set at 20°C. A series of transient heat transfer analyses were simulated for the boards. Temperature rise at the core of each board was determined and plotted versus time.

RESULTS AND DISCUSSION

Cellular Model

The effective thermal conductivities as a function of increasing porosity for both alignment cases and the four moisture conditions are plotted in Fig. 7. For each MC line, 0% porosity (an impossible case, but shown for theoretical proposes) is on the right and 90% porosity is on the left with increments of 10% data points between these two extremes. For all cases, the model predicted less than 1% difference between radial and tangential thermal conductivity values. Therefore, the average effective thermal conductivities from model predictions are plotted in Fig. 7. As density decreases there is a significant decrease, as expected, in thermal conductivity. The thermal conductivity increases with moisture. As water increases in the lumen as shown with the 50/50 water/vapor and fully water filled lumen there is an increase in thermal conductivity. At some eventual point (somewhere between 50% to 100% water in the lumen) the thermal conductivity will begin to decrease with increasing density. At fully saturated conditions the thermal conductivity through the water dominants the thermal conductivity effect through the cell wall. Thus the higher the porosity (lower density), the more the water in wood, the higher the effective thermal conductivity of the wood. Theoretically, the maximum thermal conductivity approaches that of water (0.61 W/m.K) as porosity approaches 100%.

In Fig. 7, two empirical equations developed by MacLean (1941) are plotted; one for MC under 40% (0% MC and fiber saturated conditions) and one for above 40% MC. At 0% moisture content, the finite element model and the empirical model agree fairly well. The finite element model is a non-linear function whereas the empirical fit uses a linear equation. As MC increases to the fiber saturation point, the two models begin to show significant differences. As related earlier by MacLean (1941), the process of conducting the experiment causes moisture redistribution resulting in slight interference in the measurement. As MC increases, the effect of moisture redistribution would become more pronounced. The lines from MacLean's equation plotted for 50/50 water/vapor and fully saturated conditions are significantly different than the finite element model. MacLean's data was primarily focused around the fiber saturation point and below with limited tests on a narrow set of species in the green or fully saturated state. It's possible that the moisture redistribution effect caused significant errors in the results. Our plots of MacLean's data also extrapolate beyond his measured data which yields significantly erroneous plots. The bump in MacLean's 50/50 water/vapor line is due to the change in equations used for over and below 40% MC. Another factor to consider is the samples that MacLean used to measure the conductivity. The samples were not uniform density material, but were an average measurement across both earlywood and latewood portions over several rings and for various ring orientations. All of these significantly influence the empirical data and the resulting curve. Whereas, the finite element model assumes no redistribution of the MC and is based on the fundamental properties of homogeneous softwood material over the full range of porosity.

The finite element model can accommodate a more geometrical description of the cell, including the interior radius of the lumen as part of the heat transfer effects, not possible at the cellular level with other models. The authors believe the cellular finite element model can be used to determine effective thermal conductivities through uniform density earlywood and latewood bands representing a more realistic characterization of the heat flux path than those obtained from either the averaged wood thermal conductivity experiments. Effective finite element thermal conductivity values from the cellular model were then applied to the board finite element model to determine the effects of earlywood and latewood and ring orientation.

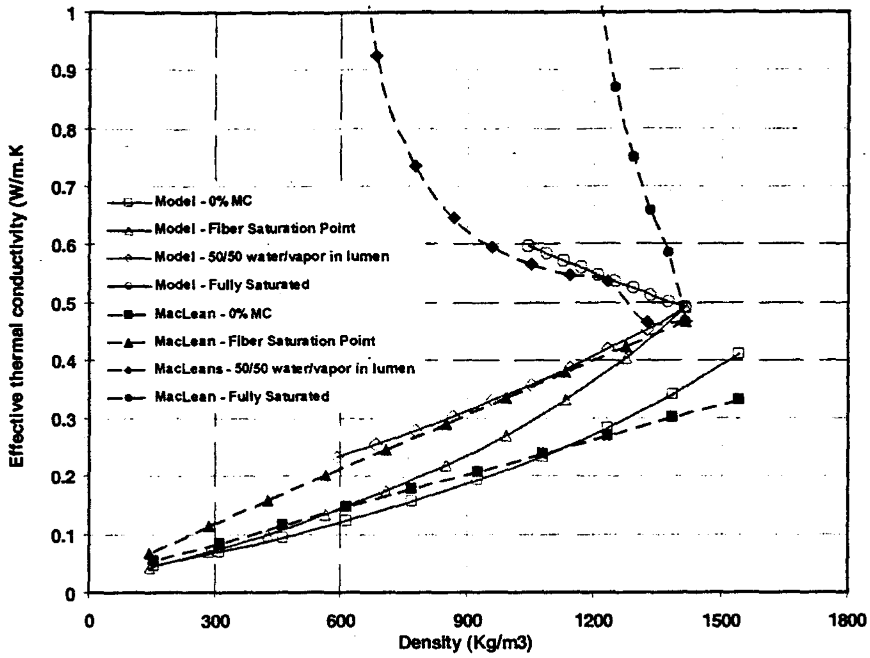

Figure 7 - Thermal conductivity values from the finite element model compared with MacLean's (1941)data.

Board Model

The board model is presented here to briefly introduce the finite element model as a useful tool for analysis and to demonstrate the transient heat transfer effects of ring orientation and earlywood-latewood regions in softwood. A more complete description of the board model is provided by Gu and Hunt (2004).

Several boards, 44.45 by 88.9 mm (1.75 by 3.5 inches), were "cut" from several locations within a log (Fig. 8) and were analyzed. Thermal conductivity values are assigned to the earlywood and latewood regions based on theoretical density estimates from the cellular finite element model. The mid-point temperature was plotted for each of the boards. Fig. 8 shows there are significant effects of due to the ring orientation while keeping all other parameters constant. The board "cut" from the center of the log (board C) had the slowest core transient temperature rise. This is due to the concentric bands of less dense earlywood that slow the heat transfer into the core. Whereas the board "cut" from the perimeter with verticle ring orientation, (board B4), has the fastest core transient temperature rise. This is due to the orientation of the denser latewood bands that have higher thermal conductivity than the earlywood. Fig. 9 shows the higher heat flux vectors in latewood (thinner rings) than in the earlywood regions. Heat flux vectors show the relative magnitude and direction with significant heat flux following up into the board through the latewood rings. Thermal energy is shown being transferred through the latewood rings, then across into the earlywood ring from one latewood ring to the next and continuing toward the center of the board (see the magnified plot in Fig. 9). Implications of this higher heat flux in the latewood bands can result in thermal expansion and higher stresses at the earlywood and latewood boundaries. Differential heating along the ring orientation of wood is clearly shown. Therefore different heating rates are anticipated for boards with different ring orientation and ring densities. These effects as well as E/L ratio, ring count, and density effects are discussed in more detail by Gu and Hunt (2004).

CONCLUSIONS

The two new finite element heat transfer models provide the ability to study heat transfer effect in wood boards from the moisture and actual structural characteristics of the wood including cellular structure, earlywood and latewood densities (porosity), ring orientation, growth rate, and earlywood/latewood ratio. The first cellular model results in a theoretical estimation of the effective thermal conductivity of wood in a full range of porosity (density) and full range of moisture content. The second softwood board model results in a better understanding of the heat transfer process in any board, of any size, "cut" from any location in a log, of any species, from any growth condition. Such a fundamental approach to studying heat transfer issues in wood has numerous practical applications which include: optimizing drying schedules for different cut boards; determining heat treatment times to kill insects; and determining heat curing times for solid wood and wood composites.

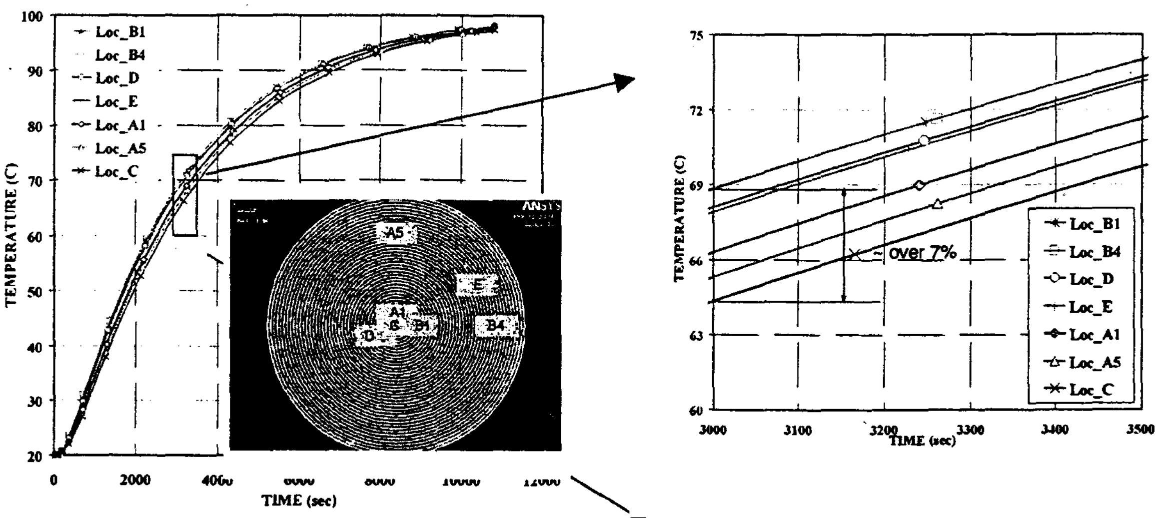

Figure 8. Transient temperature at the center point of each board cut from different locations on a log (small picture in the chart showing the location of the boards).

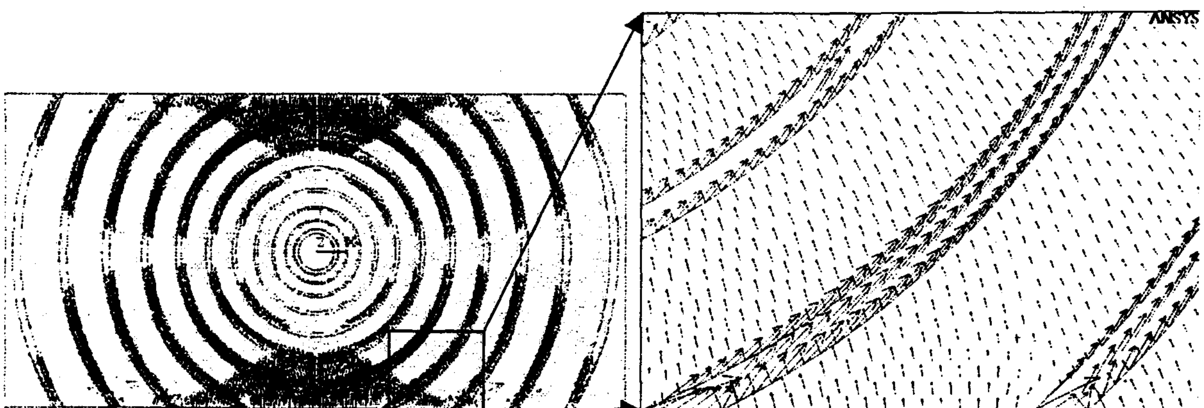

Figure 9. Vector plot of heat fluzx for the board cutting from the center of a log. (left: entire plot for the board; right: magnified plot showing high and low heat flux regions in earlywood and latewood).

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