VALIDATION OF VOLUME AND TAPER EQUATIONS FOR LOBLOLLY, SHORTLEAF AND SLASH PINE[1]

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Abstract-Inside-bark diameter measurements at 6.6-ft intervals of 137 loblolly, 52 shortleaf, and 64 slash pines were used to calculate the actual volume and taper of each species for comparison with volumes and tapers predicted from published equations. The loblolly pine were cut in Texas (TX) and Louisiana (LA) while the shortleaf was sampled only in TX. The slash pine were sampled in Mississippi (MS). The models had been developed using trees that had the same diameter and height distributions as the validation data; however, the validation trees were generally older. For loblolly and slash pine, the volume equations over-predicted the average volume by 1.5 and 3.7 ft^3/tree, respectively, or about 5 to 10 percent of tree volume. The taper equation for loblolly was adequate, but the slash pine taper equation did not fit the data well. There is no appropriate volume or taper model for shortleaf pine. However, the loblolly equations predicted both volume and taper of the shortleaf pine data as well as or better than they did for the loblolly data. The loblolly and slash pine equations can be used with confidence-about half of the errors appear to be random, indicating that the best equations will have an error of about 1.5 ft^3/tree regardless of tree size. Most prediction errors occurred In larger trees, indicating that some improvement could be made in taper-volume models by adding age as a variable. Species seems to be less important, and future modeling efforts should evaluate combined species equations.

INTRODUCTION

Valid volume and taper models are required for estimating tree volumes and stem shapes from simple measurements such as total tree height and diameter at breast height (d.b.h.). The models are used to estimate stand volume and site productivity, as well as to develop growth and yield models. There are numerous volume and taper models available for loblolly pine (e.g., Amateis and Burkhart 1987, Baldwin and Feduccia 1991, Lenhart and others 1987) and slash pine (e.g., Lohrey 1985, Thomas and Parresol 1991). The usual procedure to develop a model is to collect data over a suitably wide range of tree sizes, and then fit appropriate regression equations to the data set. Although validation with independent data is desirable, sufficient resources are rarely available to repeat the data collection process.

In the process of establishing long-term soil productivity (LTSP) plots on the national forests, we collected stem analysis data to determine the height growth patterns of existing stands over time. The upper-stem diameters collected as part of stem analysis are also suitable for validation of current volume and taper equations. One of the strengths of these data is that an intensive soil survey was conducted on each site, and the plots within each site were located within a stand on uniform soils. Our objective was to use these data to validate some existing volume and taper models.

METHODS

Sample trees were selected on four LTSP sites on the Kisatchie National Forest in Louisiana (LA), three on the Davey Crockett National Forest in Texas (TX), and three on the DeSoto National Forest in Mississippi (MS). Each site consisted of nine l-acre plots within stands that were to be harvested. A tenth plot was established on two sites in LA and one site in MS. After establishing the plots, we measured the d.b.h. of all pines with a diameter tape. Three sample trees on each plot were selected from the low, mid, and highest one-third of the diameter distribution. The sample trees were felled and inside bark diameter was recorded every 8.8 ft, beginning at the base. We calculated the volume of each tree using Smalian's formula and recorded total height and species. Tree age was determined by counting the number of rings at the base.

For the validation procedure, tree volume and taper for each tree were estimated using equations for loblolly and slash pine that were developed in LA. The volume equation tested for thinned loblolly pine (Baldwin and Feduccia 1987) was:

$$\ln(V)=-8.885331+2.040995\ln(D)+1.150022\ln(H) \quad (1)$$

where
V = estimated total volume inside bark (ft^3),
D = d.b.h. (in.), and
H = total height (ft).

For planted slash pine, we tested the volume equation of Lohrey (1985):

$$\ln(V)=-6.93385+2.07195\ln(D)+1.13645\ln(H) \quad (2)$$

We tested prediction accuracy of both volume models using the Fit Index (Schlaegel 1982), expressed as:

$$FI=1-\frac{\sum(V_i-\hat{V}_i)^2}{\sum(V_i-\bar{V})^2}, \quad (3)$$

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[1] Paper presented at the Tenth Biennial Southern Silvicultural Research Conference, Shreveport, LA, February 16-18, 1999.

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where

FI = fit index,

V_i = measured volume of tree,

$I, \hat{V}_i$ = predicted volume of tree I, and

$\bar{V}$ = average volume of all trees.

The FI ranges from 0 to 1 with 1 indicating a perfect fit. To estimate inside-bark diameter (d)as inches at a specified height (h) as feet, we used the taper equation for thinned loblolly pine (Baldwin and Feduccia 1991):

$$d = D(1.07996 + 0.28760 \ln(1 - 0.9776(\frac{h}{H})^{\frac{1}{3}})) \,, \qquad (4)$$

where

D = d.b.h. outside bark, expressed as inches; and

H = total height, expressed as feet.

The prediction equation for inside-bark diameters of thinned slash pine taper (Thomas and Parresol 1991) is:

$$\frac{d^2}{D^2} = -0.741(\frac{h}{H} - 1) + 0.0345 \sin(1.5\pi\frac{h}{H}) + 0.00391 \, cotan(\frac{\pi}{2}\frac{h}{H}). \qquad (5)$$

A Fit Index for testing the taper equations was calculated using weighting based on the square of the diameter of each disk sample. The index is:

$$FI = 1 - \sum_{i=1}^{T}\sum_{j=1}^{S}\frac{d_{ij}^2}{\sum_{j=1}^{S} d_{ij}^2}(d_{ij} - \hat{d}_{ij})^2 / \sum_{i=1}^{T}\sum_{j=1}^{S}\frac{d_{ij}^2}{\sum_{j=1}^{S} d_{ij}^2}(d_{ij} - \bar{d})^2 \qquad (6)$$

where

= the measured inside bark diameter and

4 = the predicted diameter (ib) of the jth disk of the ith sample tree,

S = the number of disk samples for the ith tree, and

T = the total number of trees sampled.

Weighting was based on the square of the diameter, so the Fit Index is an indicator of accuracy in predicting volume rather than diameter.

The 138 loblolly pines sampled for validation on the LTSP plots in LA and TX ranged in age from 21 to 87 years, averaging 48. The average height was 75 ft and ranged from 36 to 105 ft. The d.b.h averaged 12.5 in. and ranged from 4 to 22.5 in. Distribution of the heights and diameters of loblolly pines sampled for validation agreed with the data set used to develop the models (fig 1 a), but the validation data set contained older trees. The age of the original data set averaged 30 years and the maximum age was 55 years (Baldwin and Fectuccia 1991).

The age range of the 84 slash pines sampled for validation on LTSP plots in MS was 18 to 62 years, and the average age was 53 years; although the age distribution was concentrated near the mean. Only two trees were under 40 years old. The average height was 84 ft and ranged from 30 to 104 ft. The d.b.h averaged 13.0 in. and ranged from 3.5 to 17.9 in. As with the loblolly pine, height and diameter distribution of the slash pines sampled for validation agreed with the data set used to develop the models (fig 1 b), although the validation data set contained older trees. The age of trees in the original data set averaged 31 years and the maximum age was 48 years (Lohrey 1985).

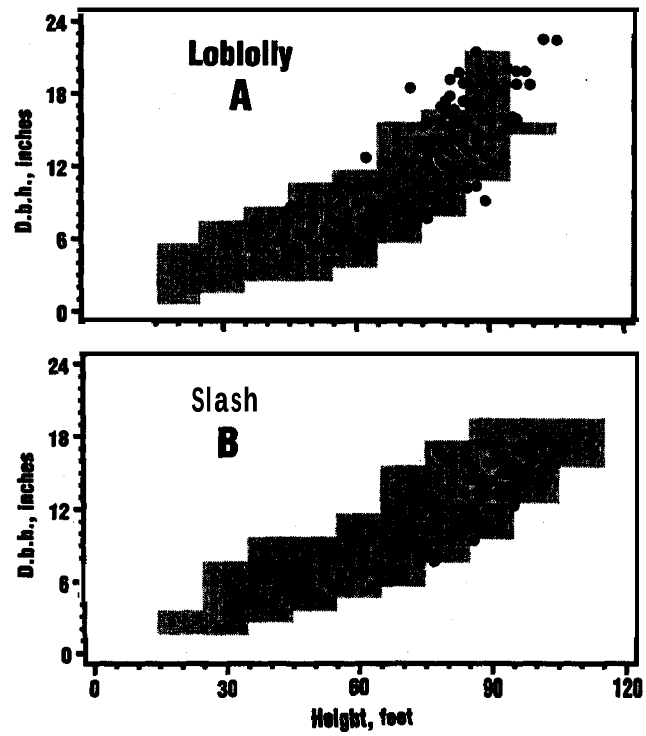

Figure l-Diameter and height distribution of the data sets used to develop volume and taper equations (shaded areas) and data used to validate the equations for loblolly (A) and slash(B) pine.

The age range of the 52 shortleaf pine sampled in TX was 25 to 90 years. Heights ranged from 39 to 103 ft, and d.b.h. ranged from 4.7 to 19.9 in. No shortleaf model was available for validation, so the shortleaf data were compared to predictions of the loblolly models for volume and taper.

RESULTS AND DISCUSSION

Volume Prediction

The Baldwin and Feduccia (1987) equation predicted inside-bark volume of loblolly pine adequately (fig. 2a) with a Fit Index of 0.976. The mean measured volume of the 138 trees was 33.3 ft^3/tree while the mean predicted volume was 31.8 ft^3/tree. The equation under-predicted actual volume, with the greatest discrepancies in the larger trees.

No volume equation was available for shortleaf pine, so we used the loblolly equation to predict the volume of 52 shortleaf pine that were sampled. Like loblolly pine, the prediction of inside-bark volume of shortleaf pine was acceptable (fig. 2b) with a Fit Index of 0.971. Differences between the predicted and measured volumes of shortleaf pine were evenly distributed with tree size, unlike the loblolly pine. Overall, the shortleafs predicted volume was 30.7 ft^3/acre—the same as its measured volume.

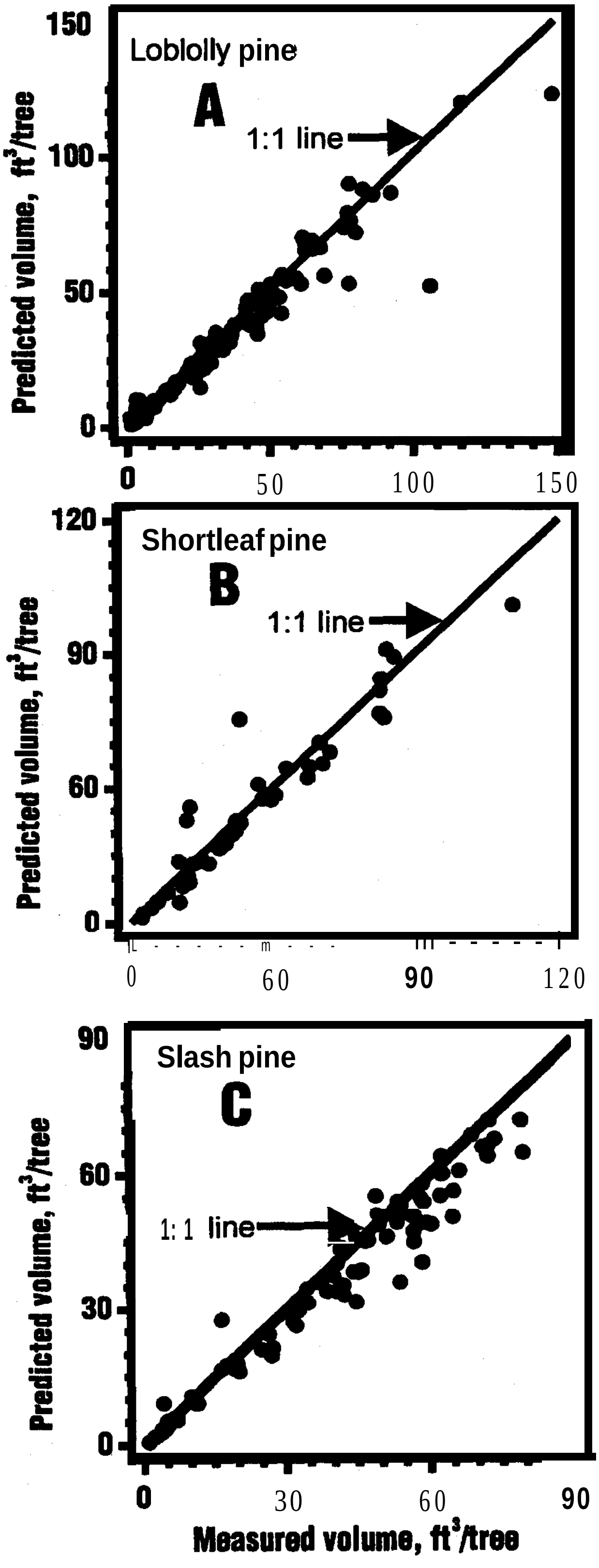

Figure 2-Total volume (ib) predicted by volume equations compared to measured volume for loblolly (A) shortleaf (B) and slash (C) pine. A loblolly pine equation was used to predict shortleaf pine volume.

The equation we used to predict slash pine volume (Lohrey 1985) was suitable (fig. 2c) and had the highest Fit Index (0.982). As with loblolly pine, the greatest under-predictions occurred with the largest trees, so the mean predicted volume was 36.5 ft^3/acre when the measured volume was 40.2 ft^3/acre.

Taper Prediction

The taper equation used to predict upper-stem diameters of loblolly pine (Baldwin and Feduccia 1991) fit the validation data well (Fit Index = 0.952). The largest deviations between predicted and measured inside-bark diameters occurred in the upper 80 percent of the stem, where the model predicted diameters too large (fig. 3a). Because the Fit Index was weighted by the square of the diameter, the lack of fit in the upper stem would have a smaller effect than it would in the lower part. Of the 138 trees in the data set, two did not follow the same pattern in the diameter-to-height plot. Field notes do not indicate any problems, but disease or physical damage may have been responsible.

A taper equation was not available for shortleaf pine, so we used the loblolly equation to predict the sample's inside-bark diameters. The loblolly equation fit the shortleaf stem taper with a Fit Index of 0.936. As with loblolly, the greatest error was in the upper part of the stem, where the equation predicted diameters that were too large (fig. 3b).

Of the three species' taper equations, the one used for slash pine was least acceptable, having a Fit Index of only 0.884. Although the fit was adequate for the top and bottom of the trees, the equation under-predicted diameters in the middle portions of the slash pine boles (fig. 3c).

Model Improvement

In general, the species-specific volume and taper equations performed reasonably well, thus validating those equations. However, our analysis indicates that stem volume and taper modeling might be improved in two ways. First, the greatest deviations from the models occurred in larger trees. Although a tree's form may be affected by size, it is more likely that form changes with age and that the largest trees are older. Because the validation data set included sample trees that were older than those used to develop the models, age should be considered as an additional variable. Thomas and others (1995) found age to be an important variable in longleaf pine which ranged to 55 years. In our most extreme case, slash pine in the validation data set was 20 years older than the slash pine used to develop the model. Slash pine volume and taper models had lower Fit Indices than the loblolly pine, where ages of trees in model and validation data sets were more closely matched. We would need to balance the cost of including age in prediction equations with the value of improved predictions.

The second way equations may be improved is by combining species data sets during the modeling process, and statistically testing whether separate taper or volume equations are required for each species. In our study, the volume and taper equations for loblolly pine fit the validation data for shortleaf pine, as well as for the target species. In building models, stand origin- including planted vs direct seeded (Lohrey 1985), natural vs planted (Amateis and Burkhart 1987) and old-field vs cut-over (Lenhart and others

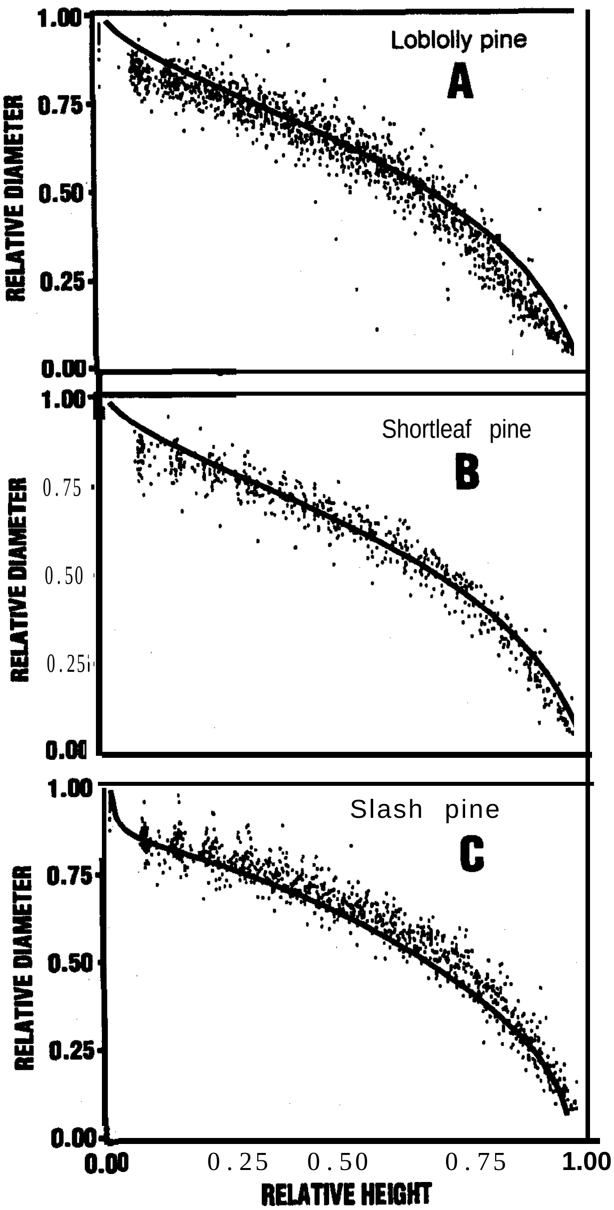

Figure 3-Plots of relative height (height of observation/total height) vs relative diameter (diameter of observation/ diameter at base) for loblolly (A) shortleaf (B) and slash (C) pines, with the line representing the equation used to predict taper for each species.

1987)—has been found to affect volume and taper prediction models. Silvicultural treatments, including thinning (Baldwin and Feduccia 1991, Thomas and **Parresol 1991)** **and** fertilization (Shoulders and others **1989),** also have been found to significantly affect the taper of pines. The effect of species on tree form is usually assumed to be important, but in this study species had little effect. The Fit Index was always greater than 0.97 when loblolly or slash pine volume equations were used to predict the volume of any of the three species. The fit index of the equation for loblolly taper was 0.954 when used to predict slash taper, **compared** to 0.952 when used to predict loblolly taper. The slash pine taper equation was slightly better in predicting loblolly (0.898) than slash (0.884). Because of this apparent lack of species effect, we suggest testing groupings of species as models are developed. Such evaluation should examine a wide range of genetic material from each species.

SUMMARY AND CONCLUSIONS

Volume predicted by models agreed reasonably well with measured volume throughout the range of the data. The tested taper equation for loblolly pine was adequate for the lower bole, but predicted less taper than was found **in crown** portions of the bole. The taper equation for slash pine underestimated diameter, especially in the mid-bole area. In both loblolly and slash pine, measured trees were older than those used to develop the model. Hence, tree age appears to affect taper. Shottleaf pine volume and taper were predicted well using equations developed for loblolly pine--indicating that species may have a small influence on volume. Future modeling efforts should combine similar species and use age as a variable in prediction equations.

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