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Predicting Volumes and Numbers of Logs by Grade from Hardwood Cruise Data

Daniel A. Yaussy
Robert L. Brisbin
Mary J. Humphreys

Abstract

The equations presented allow the estimation of quality and quantity of commercial logs produced from a hardwood stand based on cruise data. The gross International ¼-inch board-foot volume and number of logs by log grade for trees of 11 species can be predicted based on dbh, merchantable height, and tree grade. A two-stage estimation procedure was developed utilizing regression techniques and linear discriminant analysis. The volumes by log grade are estimated with multivariate multiple linear regression equations. These volumes, the tree dimensions, and grade are then used in discriminant functions to estimate the number of logs by log grade. This information, when packaged in appropriate computer software, will provide the mill manager with the means to estimate the quantity and quality of logs that would be added to a mill yard inventory from a timber sale. The software also will help a timber buyer or landowner set purchase price based on stumpage prices or delivered log prices and conversion costs.

The Authors

DANIEL A. YAUSSY, research forester, received a B.S. degree in natural resources (forestry) from the Ohio State University in 1976, an M.S. degree in forest biometrics from Virginia Polytechnic Institute and State University in 1978, and an M.S. degree in statistics from Ohio State in 1984. He joined the USDA Forest Service in 1979 and is with the Northeastern Forest Experiment Station project on management and utilization alternatives for nonindustrial private forests at Delaware, Ohio.

ROBERT L. BRISBIN, research forest products technologist, received a B.S. degree in forestry from Iowa State University in 1961 and an M.S. degree in wood technology from the Pennsylvania State University in 1969. He joined the Northeastern Station in 1961, and is located at Delaware, Ohio, on the staff of the Station's forest inventory, analysis, and economics project.

MARY J. HUMPHREYS, research forest technician, is working toward a B.S. degree from Michigan Technological University. She joined the USDA Forest Service in 1987 and is currently assigned to the project on management and utilization alternatives for nonindustrial private forests at Delaware, Ohio.

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Introduction

The equations in this paper were developed to be used in the computer program STUMP (Brisbin et al. 1988) for estimating the quality and quantity of commercial logs that can be produced from individual hardwood trees. The gross International ¼-inch board-foot volume and number of logs by log grade for each tree are predicted based on diameter at breast height (dbh or D), merchantable height to an 8-inch top (H), and USDA Forest Service tree grade (Hanks 1976). When used in conjunction with a taper equation, a bucking simulator can be developed to estimate scaling diameter, log length, and log grade for each log cut from a tree. This bucking routine can be integrated into a growth and yield simulator. This information, when packaged in appropriate computer software, will provide a mill manager with the means to estimate the quantity and quality of logs that would be added to a mill yard inventory from a timber sale. This same software will help a timber buyer set stumpage prices based on conversion costs and delivered log prices.

Blinn et al. (1983) developed a method for estimating the multiproduct value of a hardwood stand. The method is field intensive and does not lend itself to computer simulations. Another method of partitioning volume into round-product classes was developed by Yaussy and Sonderman (1984), but the classes were too broad to accommodate estimation of numbers of logs by log grade. These methods require that an estimate of total height be obtained in some manner. Reed et al. (1987) distributed gross board-foot volume of a tree between the log grades of sugar maple. Reed's method could be substituted as the volume estimator for this study. For practical application of the results of this study, the independent variables used in these equations were limited to those commonly collected in a field cruise.

Data

The data set used in this study is part of an extensive data base collected in the 1960's and early 1970's and includes the major Appalachian, central, and northern hardwood species. The standing trees were measured and graded using Forest Service tree grades. They were then felled, bucked, and diagrammed, and the resulting logs were measured and graded using Forest Service log grades (Rast et al. 1973). The logs were bucked in lengths from 8 to 16 feet to maximize log grade and minimize scalable defects such as sweep, crook, and rot. The study reported here includes the 11 species listed in Table 1 of the Appendix. The data set contained 2,800 trees. One-quarter of the trees from each species were set aside in a validation data set.

Methods

The object of this research was to provide a means of predicting the volume and the number of logs by log grade that a standing tree would produce based on cruise data. Analysis indicated that the only way to obtain a satisfactory prediction of the number of logs by log grade could be obtained only the use of volume by log grade. Since neither of these variables is measured in a normal cruise, a two-stage estimation procedure was appropriate. An equation was developed to predict volumes by log grade. These volumes were then used in a second set of equations to estimate the number of logs by log grade.

Multivariate linear regression analysis was used to develop the equations for predicting gross International ¼-inch board-foot volumes by log grade. This form of analysis is preferable to Ordinary Least Squares (OLS) methods since there is a relationship between the log grade volumes within the tree, i.e., they must sum to the total factory grade sawlog volume. The usual variables representing base diameter (D), basal area (D^2), surface area (DH), and cylindrical volume (D^2H) were used in conjunction with the dummy variables representing tree grade (G_2, G_3, defined in Table 2) in univariate stepwise regressions to find the five most descriptive variables for each log grade in each species. These variables were then combined in a multivariate linear equation across log grades for each species (Table 3). Variables were deleted by means of backward elimination if the associated coefficients were not statistically different from zero at the .05 level of significance based on the Wilk's Lambda criteria. Coefficients are found in Table 4.

The next step involved developing equations to predict the number of bucked logs by log grade that an individual tree would produce. OLS regression analysis was not appropriate for predictions of this type since the dependent variable is not continuous. A prediction of 2.38 grade 2 logs would not be acceptable. Linear discriminant analysis develops equations which can be used to estimate the probability that an observation fits into a certain category. In this instance, the categories were the numbers of logs of each log grade that a tree will produce (i.e., 0,1,2,3,4). This meant that there would be an equation for each possible category in each of three log grades, in each of 11 species, 198 equations. With this in mind, it was decided to limit the number of variables in each equation to five. The linear discriminant function used was:

$$f_{in} = a_{in0}+a_{in1}{}^*G_2+a_{in2}{}^*G_3+a_{in3}{}^*V_1/D^2$$
$$+a_{in4}{}^*V_2/D^2+a_{in5}{}^*V_3/D^2.$$

The coefficients for the linear discriminant functions are listed in Table 5. Many species in the development data set had trees which produced four or more logs per log grade, others did not. That is why some species in Table 5 have fewer coefficients than others. For example, the paper birch sample had no trees that produced more than one grade 1 log; therefore, it only has coefficients for two equations.

Validation

The multivariate equations were applied to the trees in the validation data set. The results (Table 6) indicate that total merchantable volume by species is predicted within 5 percent of the actual. With the exception of log grade 3 chestnut and northern red oaks, all log grade volumes are within 20 percent. Actual log grade volumes of the validation trees were used as input to the discriminant functions for testing. The total predicted numbers of logs is within 10 percent for all species except one, and is acceptable for each log grade within species except for grade 3 northern red oak. It is interesting that the multivariate equations overestimated the grade 3 volume for this species by such a wide margin, yet the discriminant functions underestimated the number of logs by the same percentage. This might be due to the low number of grade 3 northern red oak logs in the study (Table 2).

For the predicted values that deviated from the actual, the deviations are usually negative. This is due to the nature of the system being modeled. The probability of a tree producing no logs or one log of a certain grade is much higher than the probability of producing two, three, or four logs of that grade. And this model, like most models, tends toward the mean and avoids the extremes. This yields a slight negative bias even when the model is used with the development data set.

Applying the Models

Log grade volumes are estimated for each tree based on equation set (1). The discriminant equations are then applied to the values of dbh, merchantable height, tree grade, and the predicted volumes from equation set (1). The largest resulting values indicate the numbers of logs by log grade the tree is most likely to produce.

One of the drawbacks of additive models, like the multivariate equations, is that a volume is always estimated even if it is negative or close to zero. These small volumes produce no logs when entered in the discriminant functions. Log grade volumes of this type may be handled in two ways:

1. The volume is distributed evenly to the volumes of the remaining log grades and the value is set to zero. In this manner the total merchantable sawlog volume for the tree is maintained.
2. The volume is added to the volume of the next tree of the same species. This will more closely estimate the log grade volume and number of logs of the stand.

An example of the use of the equations is given in Table 7.

Conclusion

The objective of this study was to produce equations to predict volumes and numbers of logs by log grade from information that is commonly tallied in a timber cruise. The resulting models work satisfactorily. More accurate models probably could be developed if more descriptive, less obtainable variables were considered.

The next phase of this research is to apply a taper function to the volumes and number of logs to predict log length and small-end diameter for each log in a tree.

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Appendix

Table 1.—Number of trees and logs by species and tree grade for the development and validation data sets (numbers in parentheses are percentages)

		Development				Validation			
		No. of	No. of logs			No. of	No. of logs		
Species and code	Tree grade	trees	Grade 1	Grade 2	Grade 3	trees	Grade 1	Grade 2	Grade 3
Basswood (BAS)	1	44 (41)	46 (98)	51 (46)	61 (30)	17 (47)	17 (77)	23 (55)	20 (32)
	2	30 (28)	0 (0)	43 (38)	53 (26)	14 (39)	5 (23)	18 (43)	29 (46)
	3	34 (31)	1 (2)	18 (16)	92 (45)	5 (14)	0 (0)	1 (2)	14 (22)
	Total	108	47	112	206	36	22	42	63
Black cherry (BLC)	1	63 (47)	68 (92)	62 (43)	78 (40)	24 (53)	25 (89)	30 (45)	33 (59)
	2	42 (31)	3 (4)	62 (43)	57 (29)	14 (31)	3 (11)	27 (41)	12 (21)
	3	30 (22)	3 (4)	21 (14)	60 (31)	7 (16)	0 (0)	9 (14)	11 (20)
	Total	135	74	145	195	45	28	66	56
Black oak (BLO)	1	79 (47)	91 (96)	84 (52)	74 (39)	29 (52)	38 (100)	24 (53)	27 (38)
	2	55 (33)	2 (2)	70 (43)	67 (35)	16 (29)	0 (0)	18 (40)	22 (31)
	3	33 (20)	2 (2)	9 (6)	50 (26)	11 (20)	0 (0)	3 (7)	22 (31)
	Total	167	95	163	191	56	38	45	71
Chestnut oak (CHO)	1	90 (37)	90 (87)	84 (34)	84 (30)	33 (41)	35 (83)	39 (44)	18 (26)
	2	92 (38)	13 (13)	116 (48)	99 (36)	37 (46)	7 (17)	45 (51)	35 (51)
	3	61 (25)	0 (0)	44 (18)	93 (34)	11 (14)	0 (0)	5 (6)	15 (22)
	Total	243	103	244	276	81	42	89	68

Continued

Table 1.—Continued

Species and code	Tree grade	Development: No. of trees	Development: No. of logs Grade 1	Development: No. of logs Grade 2	Development: No. of logs Grade 3	Validation: No. of trees	Validation: No. of logs Grade 1	Validation: No. of logs Grade 2	Validation: No. of logs Grade 3
Northern red oak (NRO)	1	104 (68)	161 (91)	145 (69)	49 (51)	31 (60)	46 (87)	50 (59)	19 (45)
	2	36 (23)	15 (9)	51 (24)	30 (31)	16 (31)	7 (13)	30 (35)	14 (33)
	3	14 (9)	0 (0)	13 (6)	18 (19)	5 (10)	0 (0)	5 (6)	9 (21)
	Total	154	176	209	97	52	53	85	42
Paper birch (PAB)	1	22 (20)	22 (81)	13 (12)	24 (14)	11 (29)	11 (92)	4 (13)	15 (28)
	2	54 (48)	5 (19)	71 (67)	79 (46)	16 (42)	1 (8)	21 (66)	19 (36)
	3	36 (32)	0 (0)	22 (21)	70 (40)	11 (29)	0 (0)	7 (22)	19 (36)
	Total	112	27	106	173	38	12	32	53
Red maple (REM)	1	70 (31)	81 (92)	77 (35)	86 (23)	27 (36)	30 (83)	39 (51)	25 (24)
	2	79 (35)	5 (6)	112 (51)	113 (30)	26 (35)	5 (14)	35 (46)	33 (31)
	3	75 (33)	2 (2)	31 (14)	172 (46)	21 (28)	1 (3)	2 (3)	48 (45)
	Total	224	88	220	371	74	36	76	106
Sugar maple (SUM)	1	84 (37)	102 (87)	101 (38)	60 (22)	30 (40)	37 (93)	36 (41)	17 (22)
	2	83 (37)	11 (9)	127 (48)	89 (33)	20 (27)	3 (8)	35 (40)	15 (19)
	3	58 (26)	4 (3)	36 (14)	122 (45)	25 (33)	0 (0)	17 (19)	45 (58)
	Total	225	117	264	271	75	40	88	77

Continued

Table 1.—Continued

Species and code	Tree grade	Development: No. of trees	Development: No. of logs Grade 1	Development: No. of logs Grade 2	Development: No. of logs Grade 3	Validation: No. of trees	Validation: No. of logs Grade 1	Validation: No. of logs Grade 2	Validation: No. of logs Grade 3
White oak (WHO)	1	100 (36)	99 (88)	125 (46)	94 (27)	37 (40)	37 (80)	42 (55)	35 (30)
	2	119 (43)	13 (12)	135 (50)	148 (43)	35 (38)	9 (20)	32 (42)	42 (36)
	3	57 (21)	0 (0)	9 (3)	106 (30)	20 (22)	0 (0)	2 (3)	39 (34)
	Total	276	112	269	348	92	46	76	116
Yellow birch (YEB)	1	81 (38)	88 (90)	83 (40)	67 (25)	18 (25)	18 (90)	18 (23)	15 (17)
	2	71 (33)	9 (9)	100 (48)	84 (32)	32 (45)	1 (5)	51 (65)	36 (40)
	3	60 (28)	1 (1)	27 (13)	115 (43)	21 (30)	1 (5)	9 (12)	38 (43)
	Total	212	98	210	266	71	20	78	89
Yellow-poplar (YEP)	1	98 (40)	110 (89)	126 (47)	123 (29)	22 (28)	28 (82)	26 (31)	28 (18)
	2	75 (31)	10 (8)	108 (41)	125 (29)	31 (39)	5 (15)	46 (55)	61 (39)
	3	71 (29)	4 (3)	32 (12)	183 (42)	27 (34)	1 (3)	12 (14)	68 (43)
	Total	244	124	266	431	80	34	84	157

Table 2.—List of variables and definitions

D	=	dbh, tree diameter, outside bark, at 4.5 feet
H	=	merchantable height, height to an 8-inch diameter, inside bark, or to the point that the main stem breaks up
V_i	=	gross International ¼-inch board-foot volume for log grade i, i=1,2,3
G_k	=	dummy variable indicating tree grade, k=2,3:

Tree grade	G_2	G_3
1	0	0
2	1	0
3	0	1

b_{ij}	=	coefficients determined by regression, j=0,...,11
f_{in}	=	linear discriminant function value for log grade i and number of logs n, n=0,...,4
a_{inm}	=	coefficients determined by discriminant analysis, m=0,...,5
L_i	=	number of grade i logs in a tree, L_i = the "n" associated with $\max_n(f_{in})$

Table 3.—Model forms and variables used for predicting volumes by log grade

Equation set (1) model form:

$V_i = b_{i0} + b_{i1}X_{i1} + b_{i2}X_{i2} + \ldots + b_{i11}X_{i11} \qquad i = 1,2,3$

Species	X_{i1}	X_{i2}	X_{i3}	X_{i4}	X_{i5}	X_{i6}	X_{i7}	X_{i8}	X_{i9}	X_{i10}	X_{i11}
BAS	D	H	$D*H$	D^2G_2	G_3	$D*G_3$	D^2*G_3				
BLC	D^2	D^2*H	D^2*G_2	D^2*H*G_2	D^2*G_3	$D*H*G_3$					
BLO	D	H	D^2*H	D^2*G_2	D^2*H*G_3						
CHO	D	D^2	H	$D*H$	D^2*H	$D*G_2$	D^2*G_2	$D*H*G_2$	D^2*H*G_2	$D*G_3$	D^2*G_3
NRO	$D*H$	D^2*H	$D*G_2$	D^2*H*G_2	$D*G_3$	D^2*G_3	$H*G_3$	D^2*H*G_3			
PAB	D^2	H	D^2*G_2	$H*G_2$	$D*H*G_2$	D^2*H*G_2	D^2*G_3	$D*H*G_3$	D^2*H*G_3		
REM	D	D^2	D^2*H	$D*G_2$	D^2*G_2	D^2*H*G_2	$D*G_3$	D^2*G_3	D^2*H*G_3		
SUM	D	H	D^2*H	$D*G_2$	D^2*H*G_2	D^2*G_3	D^2*H*G_3				
WHO	D^2	$D*H$	D^2*H	D^2*G_2	D^2*G_3	D^2*H*G_3					
YEB	D	H	$D*H$	D^2*H	$D*G_2$	$H*G_2$	$D*H*G_2$	D^2*H*G_2	$D*G_3$	D^2*G_3	
YEP	$D*H$	D^2*H	$D*G_2$	D^2*G_2	$D*H*G_2$	D^2*H*G_2	D^2*G_3	D^2*H*G_3			

Table 4.—Multivariate regression coefficients to predict volumes by log grade

i	b_{j0}	b_{j1}	b_{j2}	b_{j3}	b_{j4}	b_{j5}	b_{j6}	b_{j7}	b_{j8}	b_{j9}	b_{j10}	b_{j11}
						BAS						
1	124.4676	-1.56267	-116.795	7.428632	-0.41316	-.96.3311	17.87017	-1.118				
2	89.09193	-6.2618	-45.0312	4.814644	0.207677	792.8332	-111.084	3.587002				
3	-500.424	32.86177	145.2025	-8.09048	0.138984	-316.591	47.72614	-1.22285				
						BLC						
1	-40.0789	0.505623	0.022593	-0.42124	0.023633	-0.72066	3.041069					
2	-25.5242	0.199181	0.044957	0.152841	0.073134	0.312790	-1.91674					
3	40.92406	-0.23153	0.12770	0.000909	0.014509	0.107651	0.813458					
						BLO						
1	-28.5531	8.349625	-41.2948	0.131248	-0.39456	-0.15321						
2	-14.9287	3.456186	-34.9253	0.131475	0.245983	-0.02877						
3	24.74719	-4.09842	40.43329	0.039805	0.097044	0.095157						
						CHO						
1	-1030.04	119.5710	-2.93541	503.5838	-59.2919	1.658894	-25.0837	0.902412	10.68945	-0.50995	-0.31793	-0.32453
2	-355.341	11.95751	0.271448	4.420796	5.984719	-0.19765	54.70315	-2.44229	-19.2414	0.927707	13.72388	-0.56886
3	1526.879	-137.292	2.862889	-500.197	47.80283	-0.97528	-42.1716	2.165500	14.07657	-0.73187	-22.2291	1.285556
						NRO						
1	122.3440	-10.2673	0.549595	-1.07319	-0.11966	-7.85511	0.042438	81.16464	-0.24130			
2	-25.4511	5.259903	-0.08475	-1.23754	0.096733	60.56266	-2.88872	-271.742	0.585807			
3	56.47212	-0.34953	0.009915	-3.03426	0.086864	-86.7301	4.877621	303.4664	-0.88190			
						PAB						
1	-16.5397	0.485952	-6.53406	-0.35776	178.6421	-23.0975	0.747528	-0.41788	1.991921	-0.09785		
2	83.24077	0.001010	-15.6132	-0.24188	-204.682	23.27547	-0.43230	-0.32030	-7.48109	0.646159		
3	-80.6021	0.053629	60.26620	0.386348	-120.077	14.25352	-0.58485	0.317018	0.068102	-0.07493		
						REM						
1	-22.0244	-7.97003	0.975629	-0.01742	9.197141	-0.89884	0.001650	8.517887	-0.81466	-0.01730		
2	167.5897	-18.5903	0.283858	0.173881	-4.93892	0.992506	-0.15668	-14.2928	1.395580	-0.21456		
3	-237.854	35.95310	-1.20282	0.073460	-0.13171	-0.26224	0.124409	8.686255	-0.71907	0.172893		
						SUM						
1	82.83013	-0.03570	-44.1681	0.201989	-2.05552	-0.11932	-0.27442	-0.05718				
2	40.18536	3.837562	-41.2307	0.096812	-3.59342	0.18487	-0.54040	0.209947				
3	-238.336	9.391353	78.66990	-0.07997	3.420974	-0.03584	0.835311	-0.17571				
						WHO						
1	82.42034	0.114695	-4.90782	0.257628	-0.30051	-0.30031	-0.03631					
2	-55.2884	0.115733	0.790178	0.088963	0.112273	0.409885	-0.23695					
3	-85.6513	0.120743	4.268837	-0.09586	0.158384	0.195562	0.089931					
						YEB						
1	-39.5768	7.565891	18.43794	-7.16096	0.350975	-5.01494	-58.9563	11.48668	-0.44884	7.149255	-0.71674	
2	-224.931	13.58281	-21.9086	8.410861	-0.29970	5.568782	169.6781	-23.8634	0.723178	2.365217	-0.18458	
3	109.5665	-8.02168	113.6866	-11.0371	0.367923	1.142733	-228.611	22.79333	-0.54453	-9.70899	0.820814	
						YEP						
1	124.1392	-6.04278	0.346507	-19.8908	0.721813	7.488989	-0.41292	-0.34244	-0.03172			
2	32.01724	-1.95196	0.185181	-17.8547	1.036300	6.049651	-0.28623	-0.41000	0.078513			
3	0.409898	2.144933	-0.02501	7.890494	-0.45646	-4.78722	0.296917	0.211873	0.077016			

Table 5.—Linear discriminant function coefficients for predicting the number of logs by log grade

i	n	a_{in^0}	a_{in^1}	a_{in^2}	a_{in^3}	a_{in^4}	a_{in^5}
				BAS			
	0	-23.9025	29.04879	29.23947	17.37410	24.63011	22.38560
	1	-49.7957	9.611913	9.897940	213.7596	13.24815	15.58454
1	2	-153.658	14.77046	14.68596	390.9944	5.712198	10.98221
	3	—	—	—	—	—	—
	4	—	—	—	—	—	—
	0	-19.0834	16.18397	18.67562	46.24672	9.097859	23.60029
	1	-23.0732	16.97077	17.09449	48.98229	61.45964	15.52180
2	2	-40.4423	17.03788	17.49262	52.78757	98.03809	12.52378
	3	-69.3095	15.11483	20.74589	55.66289	138.8155	9.981720
	4	-73.9881	10.98870	16.33567	44.38489	148.6951	6.368033
	0	-19.3260	17.22115	19.42133	50.94276	21.14459	10.09766
	1	-18.3642	16.58939	16.93973	46.63900	24.06213	19.96918
3	2	-19.2071	17.21500	16.64529	43.51611	20.71456	28.73801
	3	-21.8346	14.84871	15.72571	41.29689	19.20619	35.98256
	4	-36.4457	15.14349	17.11509	42.54693	20.53033	50.50779
				BLC			
	0	-22.5481	22.19483	23.05132	34.75790	28.78846	32.42247
	1	-49.4566	11.08843	13.71652	209.6073	25.73694	26.02659
1	2	-127.953	19.36905	21.11360	352.6378	16.21002	20.37389
	3	—	—	—	—	—	—
	4	—	—	—	—	—	—
	0	-22.8420	20.42635	22.95071	62.38852	9.994734	32.39095
	1	-23.1738	13.84200	17.28160	56.03624	60.47682	27.59525
2	2	-39.4128	9.930851	18.13481	55.08602	103.8535	19.42101
	3	-56.3846	15.76417	25.84078	71.68060	116.3020	17.05780
	4	—	—	—	—	—	—
	0	-21.2797	15.92640	18.09682	55.57552	33.22392	29.97801
	1	-21.5397	15.68623	17.09623	57.10345	31.49688	49.35207
3	2	-28.1725	14.18401	15.23348	57.54535	33.62051	72.71268
	3	-41.0953	14.89772	16.18072	62.76297	37.91150	93.31171
	4	—	—	—	—	—	—
				BLO			
	0	-15.9154	19.28829	21.99984	-8.34473	16.32115	16.34590
	1	-23.1756	-0.17521	1.875626	94.70103	10.32862	8.692768
1	2	-56.4027	-1.36803	0.212283	159.2314	5.018671	-1.05359
	3	-108.190	-1.19847	-0.76108	224.2160	-0.66486	-7.64187
	4	—	—	—	—	—	—
	0	-11.6549	9.011345	13.58125	24.84197	6.457632	16.98172
	1	-18.5561	9.639161	12.23819	30.93405	60.87163	9.927046
2	2	-39.0530	7.038277	12.73075	33.48478	105.9056	3.372948
	3	-79.7210	4.571225	15.51461	38.88869	158.9338	2.442764
	4	—	—	—	—	—	—

Continued

Table 5.—Continued

i	n	a_{in^0}	a_{in^1}	a_{in^2}	a_{in^3}	a_{in^4}	a_{in^5}
	0	-11.9714	9.402555	13.54840	28.21173	16.90298	7.998236
	1	-14.7220	10.08981	9.189846	24.03876	14.04622	47.24703
3	2	-28.8003	9.091627	5.761827	24.30541	16.53631	84.35400
	3	-48.2771	9.519774	4.062538	25.42952	18.51981	112.4821
	4	—	—	—	—	—	—
				CHO			
	0	-15.2882	13.04472	17.00297	17.97675	26.31331	23.00735
	1	-26.6371	9.547395	11.15577	116.9472	17.08381	11.87230
1	2	-91.2358	15.48256	17.22099	232.0805	12.52570	1.459532
	3	—	—	—	—	—	—
	4	—	—	—	—	—	—
	0	-13.4197	9.348615	12.90514	31.39438	12.35994	23.11646
	1	-20.3143	9.351390	13.00943	37.36661	73.66879	12.47214
2	2	-38.5448	10.22643	15.47643	44.38555	112.0860	6.182095
	3	-74.6479	6.630709	15.85712	48.39762	165.7705	3.338323
	4	—	—	—	—	—	—
	0	-12.5756	9.603141	12.32869	32.33747	22.97032	18.77531
	1	-16.9860	9.597020	11.68221	31.53606	25.44669	52.91449
3	2	-27.5531	9.840385	11.43332	31.16356	26.84734	82.39610
	3	-49.1926	10.24557	10.88015	33.83450	30.60698	117.2000
	4	—	—	—	—	—	—
				NRO			
	0	-23.0945	16.61669	24.83825	11.03181	33.27270	36.93330
	1	-25.4662	6.658784	8.469391	93.75824	21.07350	23.90157
1	2	-62.7533	7.252860	8.149256	160.9441	16.10322	20.80553
	3	-117.172	6.799620	7.411188	223.7037	14.83475	20.79236
	4	-148.976	6.393699	6.070625	252.0507	8.382677	15.49281
	0	-14.0535	7.174951	10.84611	24.30704	17.73957	29.02894
	1	-15.8991	6.542946	9.558031	25.89077	45.11660	22.35797
2	2	-24.3557	5.211690	9.313187	27.58869	65.61830	18.52543
	3	-35.3584	4.918304	11.53423	30.43460	80.66497	18.75486
	4	-53.4519	8.848654	11.12551	30.90317	98.71328	13.76342
	0	-13.4704	7.080354	11.05702	25.50874	25.33196	17.62662
	1	-15.4163	6.541392	7.236086	22.25135	24.80644	51.82179
3	2	-28.9663	5.609066	2.677372	21.05326	26.49242	93.36548
	3	-56.5002	5.267545	11.39122	22.43139	25.77277	128.2080
	4	—	—	—	—	—	—
				PAB			
	0	-23.4349	26.06787	29.37293	66.64879	28.48493	28.99417
	1	-90.0192	32.28707	35.12669	439.9145	19.06164	6.080971
1	2	—	—	—	—	—	—
	3	—	—	—	—	—	—
	4	—	—	—	—	—	—

Continued

Table 5.—Continued

i	n	a_{in^0}	a_{in^1}	a_{in^2}	a_{in^3}	a_{in^4}	a_{in^5}
	0	-27.4134	28.84188	34.69084	94.09483	30.17883	30.45710
	1	-32.7225	32.28813	37.20074	97.58112	75.48165	20.16537
2	2	-55.3268	38.34194	45.67319	116.3879	110.5138	17.45698
	3	-73.4775	41.34100	49.83071	122.3341	134.5598	9.309786
	4	—	—	—	—	—	—
	0	-24.0953	25.45822	30.47835	82.32999	28.75793	16.32866
	1	-24.3399	25.73898	27.70859	78.95217	27.29664	39.74486
3	2	-30.8661	25.26648	26.66292	77.38389	29.32994	60.83406
	3	-37.1439	26.16656	25.09376	73.19734	25.39991	73.26560
	4	—	—	—	—	—	—
				REM			
	0	-26.4061	26.35559	31.43903	10.91842	28.74853	33.89331
	1	-62.4073	9.358912	10.92691	274.4579	14.96292	17.85137
1	2	-157.652	10.36062	11.48646	450.4057	10.21550	10.60369
	3	—	—	—	—	—	—
	4	—	—	—	—	—	—
	0	-21.8294	16.89662	21.79864	52.46328	14.31698	33.55805
	1	-23.7319	16.02399	20.84694	56.61049	51.57732	24.08137
2	2	-37.8591	13.65067	22.15043	61.93538	83.17152	19.79676
	3	-53.3398	11.45874	23.06293	66.45825	104.2850	18.80464
	4	—	—	—	—	—	—
	0	-23.0443	18.88870	23.90431	59.48075	25.30625	18.82301
	1	-20.8599	16.96302	20.99925	52.83543	24.68597	35.68753
3	2	-23.7116	15.07098	19.06962	49.76605	24.97658	50.32214
	3	-36.1654	14.08447	19.84916	50.80167	25.79768	70.60117
	4	-32.6209	15.55027	19.65262	54.11300	26.95622	59.25131
				SUM			
	0	-20.7318	16.12579	18.67638	34.67152	33.30147	31.41859
	1	-43.8758	9.515562	12.39526	192.0065	28.31902	24.18096
1	2	-123.829	11.86022	15.11249	336.3506	30.76344	26.94164
	3	-144.672	11.90146	15.88579	361.7504	33.67433	28.35919
	4	—	—	—	—	—	—
	0	-19.0979	12.07538	16.40248	43.71495	20.86768	28.96448
	1	-19.1366	10.48098	13.87071	44.05449	50.46624	22.51217
2	2	-30.8173	9.767247	16.16728	47.93929	76.04320	19.39242
	3	-48.1050	7.942084	16.87634	48.05779	100.9165	16.30143
	4	—	—	—	—	—	—
	0	-17.5169	11.66168	14.62629	45.00846	30.17266	29.53050
	1	-19.7453	11.85508	12.40125	45.08740	30.88589	50.70147
3	2	-29.4020	13.15799	12.56379	48.87861	33.76373	72.36822
	3	-44.7700	13.09899	11.65129	54.90436	36.33276	97.38596
	4	-48.7705	12.93442	13.16876	52.42079	34.54720	98.86170

Continued

Table 5.—Continued

i	n	a_{in^0}	a_{in^1}	a_{in^2}	a_{in^3}	a_{in^4}	a_{in^5}
				WHO			
	0	-13.9463	15.68075	21.11756	4.079934	18.33989	11.57775
	1	-35.8546	9.169029	9.541529	172.4429	4.326000	2.796366
1	2	-110.141	12.56293	14.12656	307.8128	3.543687	-4.27862
	3	—	—	—	—	—	—
	4	—	—	—	—	—	—
	0	-11.6153	11.57852	16.02093	30.44647	2.933688	11.90309
	1	-16.6179	10.55119	13.50678	33.76878	53.81944	5.778838
2	2	-38.7096	7.585372	12.93143	39.46238	101.7026	3.001601
	3	-58.5846	8.542273	15.11850	45.32825	126.3576	0.180557
	4	-104.519	4.006737	11.87170	40.36565	178.1794	-2.96547
	0	-12.2429	12.40428	17.31752	32.96593	14.89575	2.072930
	1	-11.4624	11.11991	13.48480	29.00572	12.96263	21.28108
3	2	-19.4357	11.39273	13.88476	29.43159	13.61550	40.51661
	3	-31.1054	9.339869	11.75160	26.75495	14.32855	60.26592
	4	-38.6369	5.033271	7.235757	20.60108	15.21827	71.53659
				YEB			
	0	-19.9894	15.06560	22.61053	34.31837	39.11721	30.17830
	1	-59.0659	9.373844	13.42175	282.9954	32.23796	23.04481
1	2	-176.465	12.68512	16.89758	502.5163	34.92327	22.13156
	3	-269.895	15.48035	18.71390	623.8901	31.13832	18.24740
	4	—	—	—	—	—	—
	0	-18.2458	11.77936	19.09565	49.66864	29.08326	29.38369
	1	-22.9397	9.186274	16.85377	52.98685	82.02176	24.59036
2	2	-42.6691	8.112024	19.18828	58.92343	128.6083	21.20388
	3	-59.6309	7.092160	20.00756	64.13038	154.6574	23.52150
	4	—	—	—	—	—	—
	0	-16.8052	10.51737	17.28062	48.07580	35.54930	24.06342
	1	-18.6357	11.54614	16.05029	48.76921	35.59520	44.40267
3	2	-27.1038	10.91787	14.79576	49.58979	38.02054	72.01447
	3	-32.1643	11.08783	15.60289	48.91072	37.24817	78.89615
	4	-42.5177	11.26117	14.59744	50.69773	39.73598	95.40305
				YEP			
	0	-15.4393	16.69808	18.00787	14.23959	13.88934	14.02178
	1	-39.8433	8.860768	8.757671	165.2932	9.961304	8.956738
1	2	-121.012	15.77201	14.73622	299.9025	6.603205	4.720561
	3	—	—	—	—	—	—
	4	—	—	—	—	—	—
	0	-13.0543	10.21522	12.13002	25.77630	1.225529	15.41413
	1	-18.6616	10.39665	11.17330	33.27572	50.88939	8.361107
2	2	-35.4771	9.635215	12.23943	37.50107	85.80538	3.174894
	3	-67.1102	9.542296	13.85012	45.53780	123.2446	-0.20739
	4	-106.834	4.464091	13.49012	39.91286	163.2957	-6.42525

Continued

Table 5.—Continued

i	n	a_{in^0}	a_{in^1}	a_{in^2}	a_{in^3}	a_{in^4}	a_{in^5}
	0	-12.7651	10.28436	11.99476	28.13005	14.24970	15.43093
	1	-15.1381	10.91596	10.91514	30.20579	15.49488	31.76439
3	2	-22.0202	11.18303	10.65391	31.43890	17.60187	47.73145
	3	-34.9036	11.30793	10.14854	33.85821	19.97653	66.08001
	4	-51.1457	12.06043	10.77125	37.72719	22.42329	82.13253

Table 6.—Comparison of actual values and those predicted by the models using the validation data set

Tree	Value	BAS	BLC	BLO	CHO	NRO	PAB	REM	SUM	WHO	YEB	YEP	Total
						GROSS BOARD-FOOT VOLUME (Int. ¼ inch)							
	Actual	3300	4995	7935	9980	13360	1465	5545	7980	9945	3785	6590	74880
1	Predicted	2923	5043	7292	9638	14645	1498	5742	7645	8828	4024	6479	73757
	% Difference	-11.4	1.0	-8.1	-3.4	9.6	2.3	3.6	-4.2	-11.2	6.3	-1.7	-1.5
	Actual	3750	6340	6590	11925	11345	2615	8005	10535	10565	8190	12800	92660
2	Predicted	3978	5179	7838	10440	10522	2601	7548	10459	12478	7183	12970	91196
	% Difference	6.1	-18.3	18.9	-12.5	-7.3	-0.5	-5.7	-0.7	18.1	-12.3	1.3	-1.6
	Actual	3645	3120	6845	5175	4270	3100	6080	6650	12170	5525	15690	72270
3	Predicted	3798	3618	6383	7298	5182	2723	5976	7834	11313	5734	14852	74711
	% Difference	4.2	16.0	-6.7	41.0	21.4	-12.2	-1.7	17.8	-7.0	3.8	-5.3	3.4
	Actual	10695	14455	21370	27080	28975	7180	19630	25165	32680	17500	35080	239810
Total	Predicted	10699	13840	21513	27376	30349	6822	19266	25938	32619	16941	34301	239664
	% Difference	0.0	-4.3	0.7	1.1	4.7	-5.0	-1.9	3.1	-0.2	-3.2	-2.2	-0.1
						NUMBER OF LOGS							
	Actual	22	28	38	42	53	12	36	40	46	20	34	371
1	Predicted	22	28	34	41	51	12	36	40	47	21	34	366
	% Difference	0.0	0.0	-10.5	-2.4	-3.8	0.0	0.0	0.0	2.2	5.0	0.0	-1.3
	Actual	42	66	45	89	85	32	76	88	76	78	84	761
2	Predicted	38	58	45	92	72	31	75	86	74	75	83	729
	% Difference	-9.5	-12.1	0.0	3.4	-15.3	-3.1	-1.3	-2.3	-2.6	-3.8	-1.2	-4.2
	Actual	63	56	71	68	42	53	106	77	116	89	157	898
3	Predicted	63	52	64	68	33	53	105	68	115	78	158	857
	% Difference	0.0	-7.1	-9.9	0.0	-21.4	0.0	-0.9	-11.7	-0.9	-12.4	0.6	-4.6
	Actual	127	150	154	199	180	97	218	205	238	187	275	2030
Total	Predicted	123	138	143	201	156	96	216	194	236	174	275	1952
	% Difference	-3.1	-8.0	-7.1	1.0	-13.3	-1.0	-0.9	-5.4	-0.8	-7.0	0.0	-3.8

Table 7.—Example of the use of the models

Species:	Black cherry (BLC)
Dbh (D):	20 inches
Merchantable height (h):	3 logs
Tree grade:	2 (G_2=1, G_3=0)

Multivariate equations:

V_1 = –40.0789+.505623*D^2+.022593*D^2*H–.42124*D^2*G_2

+.023633*D^2*H*G_2–.72066*D^2*G_3+3.041069*D*H*G_3

= 31 board feet

V_2 = 210

V_3 = 62

Discriminant functions:

f_{10} = –22.5491+22.1948*G_2+23.0513*G_3+34.7579*V_1/D^2

+28.7885*V_2/D^2+32.4225*V_3/D^2

= 22.5

f_{11} = –4.6

f_{12} = –69.6

f_{13} = no calculations

f_{14} = no calculations

L_1 = 0 grade 1 logs ($f_{10} = \max_n(f_{1n})$)

f_{20} = –22.8420+20.4264*G_2+22.9507*G_3+62.3885*V_1/D^2

+9.99473*V_2/D^2+32.3910*V_3/D^2

= 12.7

f_{21} = 31.0

f_{22} = 32.3

f_{23} = 28.6

f_{24} = no calculations

L_2 = 2 grade 2 logs

Continued

Table 7.—Continued

f_{30} = $-21.2797+15.9264*G_2+18.0968*G_3+55.5755*V_1/D^2$

$+33.2239*V_2/D^2+29.9780*V_3/D^2$

= 21.0

f_{31} = 22.8

f_{32} = 19.4

f_{33} = 13.0

f_{34} = no calculations

L_3 = 1 grade 3 log

After adjusting the grade 1 log volume to reflect the estimate of no grade 1 logs:

	Grade 1	Grade 2	Grade 3
Volume	0	225	78
Number of logs	0	2	1

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Table 7.--Example of the use of the models

Species:	Black cherry (BLC)
Dbh (D):	20 inches
Merchantable height (h):	3 logs
Tree grade:	2 $(G_2=1,\ G_3=0)$

Multivariate equations:

$$V_1 = -40.0789+.505623*D^2+.022593*D^2*H-.42124*D^2*G_2$$
$$+.023633*D^2*H*G_2-.72066*D^2*G_3+3.041069*D*H*G_3$$
$$= 49 \text{ board feet}$$

$$V_2 = 257$$

$$V_3 = 119$$

Discriminant functions:

$$f_{10} = -22.5491+22.1948*G_2+23.0513*G_3+34.7579*V_1/D^2$$
$$+28.7885*V_2/D^2+32.4225*V_3/D^2$$
$$= 32.1$$

$$f_{11} = 11.7$$

$$f_{12} = -48.8$$

f_{13} = no calculations

f_{14} = no calculations

L_1 = 0 grade 1 logs ($f_{10} = \max_n(f_{1n})$)

Continued

Table 7. Continued

$$f_{20} = -22.8420+20.4264*G_2+22.9507*G_3+62.3885*V_1/D^2 +9.99473*V_2/D^2+32.3910*V_3/D^2$$

$$= 21.3$$

f_{21} = 44.6

f_{22} = 49.8

f_{23} = 48.0

f_{24} = no calculations

L_2 = 2 grade 2 logs

$$f_{30} = -21.2797+15.9264*G_2+18.0968*G_3+55.5755*V_1/D^2 +33.2239*V_2/D^2+29.9780*V_3/D^2$$

$$= 31.8$$

f_{31} = 36.1

f_{32} = 36.4

f_{33} = 33.7

f_{34} = no calculations

L_3 = 2 grade 3 logs

After adjusting the grade 1 log volume to reflect the estimate of no grade 1 logs:

	Grade 1	Grade 2	Grade 3
Volume	0	282	144
Number of logs	0	2	2

Yaussy, Daniel A.; Brisbin, Robert L.; Humphreys, Mary J. 1988. **Predicting volumes and numbers of logs by grade from hardwood cruise data.** Res. Pap. NE-613. Broomall, PA: U.S. Department of Agriculture, Forest Service, Northeastern Forest Experiment Station. 15 p.

The equations presented allow the estimation of quality and quantity of logs produced from a hardwood stand based on cruise data. When packaged in appropriate computer software, the information will provide the mill manager with the means to estimate the value of logs that would be added to a mill yard inventory from a timber sale.

ODC 832.15—525.1:853

Keywords: Multivariate regression; discriminant functions

Headquarters of the Northeastern Forest Experiment Station are in Broomall, Pa. Field laboratories are maintained at:

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- Durham, New Hampshire, in cooperation with the University of New Hampshire.
- Hamden, Connecticut, in cooperation with Yale University.
- Morgantown, West Virginia, in cooperation with West Virginia University, Morgantown.
- Orono, Maine, in cooperation with the University of Maine, Orono.
- Parsons, West Virginia.
- Princeton, West Virginia.
- Syracuse, New York, in cooperation with the State University of New York College of Environmental Sciences and Forestry at Syracuse University, Syracuse.
- University Park, Pennsylvania, in cooperation with the Pennsylvania State University.
- Warren, Pennsylvania.